$b-ind$ and $b-Ind$

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Abstract

We study some functions by using $b-open\ \ set$ which are small and large inductive dimension (ind and Ind) and called by $b-ind$ and $b-Ind$. Also we study some relations between them .

Introduction

The concept of $b-open$ set in topological space was introduced in [2]. We recall the definition of ind and Ind in [7]. In this paper we study similar definitions using $b-open$ sets which are called $b-ind$ and $b-Ind$ and we study some relations between them .

Section one : On $b-open$ sets

1.1.Definition [2]

Let X be a topological space and $A \subseteq X$. A is called $b-open$ set in X if $A \subseteq \overset{\circ}{\bar{A}} \cup \bar{\overset{\circ}{A}}$. The complement of $b-open$ set is called $b-closed$ set ,that is , A is $b-closed$ set if $\bar{\overset{\circ}{A}} \cap \overset{\circ}{\bar{A}} \subseteq A$

It is clear that every open set is $b-open$ and every closed set is $b-closed$,but the converse may be not true in general .As the following example .

1.2.Example

Let $X = \{a,b,c\}$, $\mathrm{T} = \{\{a\}, X, \phi\}$. The $b-open$ sets are :- $\{a\}, \{a,b\}, \{a,c\}, \phi, X$. Then $\{a,b\}$ is $b-open$ set but not open.

1.3.Proposition [4]

Let X be topological space then G is an open set in X iff $\overline{G \cap \bar{A}} = \overline{G \cap A}$ for each $A \subset X$.

1.4.Remark [6]

The intersection of an $b-open$ set and an open set is $b-open$.

1.5.Example

The intersection of two $b-open$ sets may be not $b-open$ in general . Example let $X=\{a,b,c\}$, $\mathrm{T}=\{\{a\},\{b\},\{a,b\},X,\phi\}$. Then each of $\{a,c\},\{b,c\}$ is an $b-open$ set , where as $\{a,c\}\cap\{b,c\}=\{c\}$ is not $b-open$.

1.6.Proposition [6]

Let $\{A_\lambda\}_{\lambda\in\wedge}$ be a collection of $b-open$ set in a topological space X , then $\cup_{\lambda\in\wedge} A_\lambda$ is $b-open$.

1.7.Definition [3]

Let X be a topological space and $A\subseteq X$. A is called semi-open ($s-open$) set in X if $A\subseteq \overline{\overset{\circ}{A}}$. The complement of $s-open$ set is called semi-closed ($s-closed$) that is, A is $s-closed$ set if $\overset{\circ}{\overline{A}}\subseteq A$. The intersection of all $s-closed$ subsets of X containing A is called semi-closur ($s-closur$) of A and the union of all $s-open$ subsets of X contained in A is called semi- interior ($s-\text{int}\,erior$) of A , and are denoted by $\overline{A}^s$, $A^{\circ s}$ respectively.

1.8.Definition [5]

Let X be a topological space and $A\subseteq X$. A is called pre-open set in X if $A\subseteq \overset{\circ}{\overline{A}}$. The complement of $pre-open$ set is called $pre-closed$ that is A is $pre-closed$ set if $\overline{\overset{\circ}{A}}\subseteq A$. The intersection of all $pre-closed$ subsets of X containing A is called $pre-closur$ of A and the union of all $pre-open$ subsets of

X contained in A is called $pre-interior \ \ of \ \ A$, and denoted by $\overline{A}^{p}$, $A^{\circ p}$ respectively.

1.9.Proposition [1]

Let X be a topological space and $A \subseteq B \subseteq X$,then :

(i) $\overline{A}^{p} \subseteq \overline{B}^{p}$

(ii) $A^{\circ p} \subseteq B^{\circ p}$

1.10.Proposition [2]

For any subset A of a space X the following statements are equivalent:-

(i) A is $b-open$ set

(ii) $A \subseteq A^{\circ p} \cup A^{\circ s}$

(iii) $A \subseteq (A^{\circ p})^{-p}$

1.11.Proposition

For any subset A of a space X , the following statements are equivalent :-

(i) A is $b-open$ set

(ii) $\overline{A}^{p} = \overline{A^{\circ p}}^{p}$

(iii) There exists an $pre-open$ set G in X such that $G \subseteq A \subseteq \overline{G}^{p}$.

Proof :

$(i) \to (ii)$ since $A \subseteq \overline{A^{\circ p}}^{p}$ by proposition 1.10 , then $\overline{A}^{p} \subseteq \overline{A^{\circ p}}^{p}$ and since $\overline{A^{\circ p}}^{p} \subseteq \overline{A}^{p}$, then $\overline{A}^{p} = \overline{A^{\circ p}}^{p}$.

$(ii) \to (iii)$ let $G = A^{\circ p}$ .Since $\overline{A}^{p} = \overline{A^{\circ p}}^{p}$ and $A^{\circ p} \subseteq A \subseteq \overline{A^{\circ P}}^{p}$. Then $G \subseteq A \subseteq \overline{G}^{p}$.

$(iii) \to (i)$ Suppose that there exists $pre-open$ set G such that $G \subseteq A \subseteq \overline{G}^{p}$. Then $G = G^{\circ p}$ and since $A \subseteq \overline{G}^{p} = \overline{G^{\circ p}}^{p} \subseteq \overline{A^{\circ p}}^{p}$, then A is $b-open$ set by proposition 1.10 .

1.12.Proposition

For any subset A of a space X, the following statements are equivalent :-

(i) A is $b-open$ set

(ii) There exists an open set G in X such that $G \subseteq A \subseteq \overline{G} \cup \overline{\mathring{A}}$.

Proof :

(i)→(ii) suppose that A is $b-open$ set . let $G = \mathring{A}$, then $\mathring{A} = G \subseteq A \subseteq \overline{A^{\circ}} \cup \overline{\mathring{A}} = \overline{G} \cup \overline{\mathring{A}}$. Hence $G \subseteq A \subseteq \overline{G} \cup \overline{\mathring{A}}$.

(ii)→(i) Suppose that there exists an open set G in X such that $G \subseteq A \subseteq \overline{G} \cup \overline{\mathring{A}}$.Then $A \subseteq \overline{G} \cup \overline{\mathring{A}} = \overline{\mathring{G}} \cup \overline{\mathring{A}} \subseteq \overline{\mathring{A}} \cup \overline{\mathring{A}}$ by definition 1.1 .Hence $A \subseteq \overline{\mathring{A}} \cup \overline{\mathring{A}}$, therefore A is $b-open$ set .

1.13.Proposition

Let X be a topological space . let Y be an open subset of X and A is $b-open$ set in Y. Then there exists a $b-open$ set B in X such that $A = B \cap Y$.

Proof:

Let A is $b-open$ set in Y, then by proposition 1.12 there exists an open set U in Y such that $U \subseteq A \subseteq \overline{U}^{Y} \cup \overline{\overset{\circ Y}{A}}^{Y}$,then there exists an open set W in X such that $W \cap Y = U$. Let $B = A \cup W$, then $B \cap Y = (A \cup W) \cap Y = (A \cap Y) \cup (W \cap Y) = A \cup U = A$ to prove $W \subset B \subset \overline{W} \cap \overline{\overset{\circ}{B}}$, since $W \subset A \cup W = B$, then $W \subset B$, since $B = AUW \subset \overline{W} \cup A$

$\subset \overline{W} \cup \overline{U}^{Y} \cup \overline{\overset{\circ Y}{A}}^{Y} \subseteq \overline{W} \cup (\overline{U} \cap Y) \cup \overline{\overset{\circ Y}{A}}^{Y} \subset \overline{W} \cup (\overline{W} \cap Y) \cup \overline{\overset{\circ Y}{A}}^{Y} \subset \overline{W} \cup \overline{A}^{Y\,\circ Y} =$
$\overline{W} \cup (\overline{A} \cap Y)^{\circ Y} = \overline{W} \cup (\overset{\circ Y}{\overline{A}} \cap \overset{\circ Y}{Y}) = \overline{W} \cup (\overset{\circ Y}{\overline{A}} \cap Y) = \overline{W} \cup (\overset{\circ Y}{\overline{A}} \cap \overset{\circ}{Y})$

(since $Y = \overset{\circ}{Y}$)$= \overline{W} \cup \overline{\overset{\circ}{A}} \subset \overline{W} \cup \overline{\overset{\circ}{B}}$ (since $A \subset A \cup W = B$).Therefore

$W \subseteq B \subseteq \overline{W} \cup \overline{\overset{\circ}{B}}$.

1.14.Proposition

Let X be a topological space and $Y \subseteq X$. If G is a $b-open$ set in X and Y is an open set in X, then $G \cap Y$ is $b-open$ set in Y.

Proof :

Since G is a $b-open$ set in X, then $G \subseteq \overline{\overset{\circ}{G}} \cup \overline{\overset{\circ}{G}}$. But

$\overline{(G\cap Y)}^{Y\circ Y} \cup \overline{(G\cap Y)^{\circ Y}}^{Y} \supseteq \overline{(G\cap Y)}^{Y\,\circ} \cup \overline{(G\cap Y)^{\circ}}^{Y} = (\overline{G\cap Y}^{\circ} \cap Y^{\circ}) \cup (\overline{G^{\circ} \cap Y^{\circ}} \cap Y) =$

$(\overline{(\overline{G}\cap Y)}^{\circ} \cap Y^{\circ}) \cup (\overline{(\overline{G^{\circ}} \cap Y^{\circ})} \cap Y)$ by proposition 1.3 $\supseteq ((\overline{G} \cap Y)^{\circ} \cap Y^{\circ}) \cup$ $(\overline{G^{\circ}} \cap Y^{\circ} \cap Y) = (\overset{\circ}{\overline{G}} \cap Y^{\circ}) \cup (\overline{G^{\circ}} \cap Y^{\circ})$

$=(\overset{\circ}{\overline{G}}\cup \overline{G^{\circ}})\cap Y^{\circ}=(\overset{\circ}{\overline{G}}\cup \overline{G^{\circ}})\cap Y \supseteq G\cap Y$ .Then $G\cap Y$ is a $b-open$ in Y

1.15.Proposition

Let X be a topological space .Let Y be an open subset of X and A is $b-open$ set in Y **.**Then A is $b-open$ in X **.**

Proof :

Let A is $b-open$ set in Y, then by proposition 1.12 there exist an open set G in Y such that $G\subseteq A\subseteq \overline{G}^{Y}\cup \overline{A}^{Y\ \circ Y}$,then there exist an open set W in X such that $W\cap Y=G$,

$A\subset \overline{A}^{Y\ \circ Y}\cup \overline{A^{\circ Y}}^{Y}=(\overline{A}^{\circ Y}\cap Y^{\circ Y})\bigcup(\overline{A^{\circ Y}}\cap Y)\subseteq$

$\overline{A}^{\circ}\bigcup(\overline{\overline{A^{\circ Y}}\cap Y})=\overline{A}^{\circ}\bigcup(\overline{A^{\circ Y}\cap Y^{\circ}})=\overline{A}^{\circ}\cup\overline{A^{\circ}}$. Then $A\subseteq\overline{A}^{\circ}\cup\overline{A^{\circ}}$. Hence A is $b-open$ in X .

1.16.Proposition

For any subset A of a space X, the following statements are equivalent :

(i) A is $b-closed$ set in X .

(ii)There exist a closed set C in X such that $C^{\circ}\bigcap\overline{A^{\circ}}\subseteq A\subseteq C$.

Proof :

(i)→(ii) Suppose that A is $b-closed$ set , then A^{c} is $b-open$ set . By proposition 1.12 there exists an open set G such that

$G\subseteq A^{c}\subseteq\overline{G}\bigcup\overset{\circ}{\overline{A^{c}}}$, $(\overline{G}\bigcup\overset{\circ}{\overline{A^{c}}})^{c}\subseteq A\subseteq G^{c}$

$(\overline{G} \cup \overset{\circ}{\overline{A^c}})^c = \overline{G}^c \cap \overline{A^c}^{\circ c}$

$= G^{c^\circ} \cap \overline{A^{c^{c^{\circ c^c}}}}^{c^c} = G^{c^\circ} \cap \overline{A^\circ}$

Then $\overline{G} \cup \overline{A} \subseteq A \subseteq G$,let $C = G^c$. Then $C^\circ \cap \overline{A^\circ} \subseteq A \subseteq C$.

(ii)→(i) Suppose that there exists a closed set C such that $C^\circ \cap \overline{A}^\circ \subseteq A \subseteq C$, then $C^c \subseteq A^c \subseteq (C^\circ \cup \overline{A^\circ})^c$,

$(C^\circ \cup \overline{A^\circ})^c = C^{\circ^c} \cup \overline{A^\circ}^c = \overline{C^c} \cup \overline{A^c}^{c^{c^{\circ c^c}}}$, let $C^c = G$, $G \subseteq A^c \subseteq \overline{G} \cup \overset{\circ}{\overline{A^c}}$, then by proposition 1.12 A^c is $b-open$ set . Therefore A is $b-closed$ set .

1.17.Proposition [8]

A space X is $regular$ space iff for every $x \in X$ and each open set U in X such that $x \in U$ there exists an $open$ set W such that $x \in W \subseteq \overline{W} \subseteq U$.

1.18.Proposition[8]

A space X is normal space iff for every closed set $C \subseteq X$ and each open set U in X such that $C \subseteq U$ there exists an open set W such that $C \subseteq W \subseteq \overline{W} \subseteq U$.

Section TWO

On ind and $b-ind$

2.1.Definition [7]

Let X be a topological space then $ind\,X=-1$ iff $X=\phi$, and if n is a positive integer or 0 then it is said that $ind\,X\le n$ iff the following satisfied :

For each $x\in X$ and for each open set G containing x , there exists an open set U such that $x\in U\subseteq G$ and $ind\,b(\mathrm{U})\le \mathrm{n}-1$. If there exists no integer n for which $ind\,X\le \mathrm{n}$ then we put $ind\,X=\infty$

This suggests the following :-

2.2.Definition

Let X be a topological space. we say that $b-ind\,X=-1$ iff $X=\phi$, and if n is a positive integer or 0 then we say that $b-ind\,X\le \mathrm{n}$ iff the following satisfied:

For each $x\in X$ and for each open set G containing x, there exists an $b-open$ set U such that $x\in U\subseteq G$ and $b-ind\,b(\mathrm{U})\le \mathrm{n}-1$.If there exists no integer n for which $b-ind\,X\le \mathrm{n}$ then we put $b-ind\,X=\infty$.

To find $b-ind\,b(\mathrm{U})\le \mathrm{n}-1$ we have to know the topology on $b(\mathrm{U})$, then we have to get $b-ind$ of the boundary of open set in the topology of $b(\mathrm{U})$.

2.3.Example

The following example of a space X with $ind\,\mathrm{X}=\mathrm{b}\text{-}ind\,X=0$. Let $X=\{a,b,c\}$, $T=\{\{c\},\{a,b\},\phi,X\}$ the $b-open$ sets are $\{c\},\{a,b\},\phi,X,\{a\},$ $\{b\},\{b,c\},\{a,c\}$. Since $\{a,b\}$ is an open and closed then $b\{a,b\}=\phi$,$indb\{a,b\}=-1$ hence $indX\leq 0$ and since $X\neq\phi$ then $indX=0$. And by theorem 2.6 then $b-indX=0$.

2.4.Example

Example 1.5 gives $ind\,\mathrm{X}=\mathrm{b}\text{-}ind\,X=1$.Since $a\in\{a\}\subseteq X$ such that $b\{a\}=\overline{\{a\}}-\{a\}^{\circ}=\{a,c\}-\{a\}=\{c\}$ .The topology on $\{c\}$ is indiscrete then $ind\,b\{a\}=-1$ *if* $b\{a\}=\phi$ *or* $indb\{a\}=0$ *if* $b\{a\}\neq\phi$ since X is not regular then $ind\,\mathrm{X}\neq 0$ and since $ind\,b\{a\}=0$ then $ind\,\mathrm{X}=1$.Since e $\{a\}$ is $b-open$ then $\mathrm{b}\text{-}ind\,X=1$

2.5.Proposition

Let X be a topological space .If $indX$ is exist then $b-ind\,X\leq indX$

Proof:

By induction on n. It is clear $n=-1$. Suppose that it is true for $n-1$. Now suppose that $ind\,X\leq n$, to prove $b-ind\,X\leq n$, let $x\in X$ and G is an open set in X such that $x\in G$ since $ind\,X\leq n$, then there exists an open set V in X such that $x\in U\subseteq G$ and $ind\,b(U)\leq n-1$ and since each open set is $b-open$ then U is an

$b-open$ set such that $x \in U \subseteq G$ and $b-ind\,b(U) \leq n-1$. Hence $b-ind\,X \leq n$.

2.6.Theorem

Let X be a topological space , then $ind\,X=0$ iff $b-ind\,\mathrm{X}=0$.

Proof :

By **2.5** if $ind\,X=0$ then $b-ind\,\mathrm{X}\leq 0$ and since $X \neq \phi$, then $b-ind\,\mathrm{X}=0$. Now let $b-ind\,\mathrm{X}=0$, let $x \in X$ and G an open set in X such that $x \in G$. Since $b-ind\,\mathrm{X}=0$ then there exists an $b-open$ set U such that $x \in U \subseteq G$ and $b-ind\,b(\mathrm{U}) \leq -1$. Then $b(\mathrm{U})=\phi$, and thus $ind\,b(\mathrm{U})=-1$, so that $indX \leq 0$, and since $X \neq \phi$,then $indX=0$.

It is known that if X is a topological space with $ind\,\mathrm{X}=0$ then X is a regular space (see [7])

Then we have the following :

2.7.Corollary

Let X be a topological space , if $b-ind\,X=0$ then X is a regular space .

Proof:

Let $x \in X$ and G an $open$ set such that $x \in G$. Since $b-ind\,X=0$ then there exists an $b-open$ set V such that $x \in V \subseteq G$ and $b-ind\,b(V)=-1$ then $b(V)=\phi$ hence V is an open and closed set .Therefore $x \in V \subseteq \overline{V} \subseteq G$ by proposition 1.17 Then X is a regular space .

It is known that a space X satisfies $ind\,X=0$ iff it is not empty and has a base for its topology which consists of open and closed sets. (see[7])

Similarity, we have :

2.8.Corollary

A space X satisfies $b-ind\,X=0$ iff it is not empty and has a base for its topology which consists of open and closed sets.

2.9.Theorem

If A is an open subspace of a space X, then $b\text{-}ind\,A \le b-ind\,X$.

Proof:

By induction on n. It is clear $n=-1$. Suppose that it is true for $n-1$. Now suppose that $b\text{-}ind\,X \le n$, to prove $b-ind\,A \le n$, let $x \in A$ and G is an open subset in A such that $x \in G$, since G is an open in A, then there exists U open set in X such that $U \cap A = G$. Since $x \in U$ and $b-ind\,X \le n$, then there exist an $b-open$ set W in X such that $x \in W \subseteq U$ and $b-ind\,b(W) \le n-1$, then $V = W \cap A$ is $b-open$ in A by proposition 1.14.

$$x \in V = W \cap A \subset U \bigcap A = G$$

$$\begin{aligned}
b_A(V) \subseteq b(V) \cap A &= (V - \overset{\circ}{V}) \cap A \subset (\overline{W} - \overset{\circ}{V}) \cap A) \\
&= (W \cap \overset{\circ c}{V}) \cap A \\
&= \left[\overline{W} \cap (\overset{\circ c}{W} \cup \overset{\circ c}{A}) \right] \cap A \\
&= \left[(\overline{W} \cap \overset{\circ c}{W}) \cup (W \cap A^c) \right] \cap A \subseteq \left[b(W) \cup A^c \right] \bigcap A \\
&= \left(b(W) \cap A \right) \bigcup (A^c \cap A) \\
&= b(W) \cap A \subseteq b(W)
\end{aligned}$$

Since $b-ind\,b(w) \le n-1$, then $b-ind\,b_A(v) \le n-1$, therefore $b-ind\,A \le n$.

Section Three : On Ind and $b-Ind$

3.1.Definition [7]

Let X be a topological space . It is said that $Ind\,X=-1$ iff $X=\phi$ and if n is a positive integer or 0 then we say that $Ind\,X\le n$ iff the following is satisfied :

For each closed set C in X and each open set G, $C\subseteq G$, there exists an open set U such that $C\subseteq U\subseteq G$ and $Ind\,b(\mathrm{U})\le n-1$. If there exists no integer n for which $Ind\,X\le \mathrm{n}$ then we put $Ind\,X=\infty$

This suggest the following :

3.2.Definition

Let X be a topological space . we say that $b-Ind\,X=-1$ iff $X=\phi$, and if n is a positive integer or 0 then we say that $b-Ind\,X\le n$ iff the following is satisfied :

For each closed set C in X and each open set G, such that $C\subseteq G$, there exists an $b-open$ set U such that $C\subseteq U\subseteq G$ and $b-Ind\,b(\mathrm{U})\le n-1$. If there exists no integer n for which $b-Ind\,X\le \mathrm{n}$ then we put $b-Ind\,X=\infty$ .To find $b-Ind\,b(\mathrm{U})\le \mathrm{n}-1$ we have to know the topology on $b(\mathrm{U})$, then we have to get $b-Ind$ of the boundary of open set in the topology of $b(\mathrm{U})$.

3.3.Example

The following example of a space X with $b-Ind\,X = Ind\,X = 0$. Let $X = \{a,b,c,d\}$, $T = \{\{d\},\{b,c\},\{b,c,d\},\phi,X\}$. The $b-open$ sets are $\{b\},\{c\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{d\},\{b,c\},\{b,c,d\},\{a,d\},\{a,b,c\},\phi,X$ .Since $\{a,b.c\} \subseteq X \subseteq X$ $,b(X) = \phi$ *then* $Indb(X) = -1$ *hence* $IndX = 0$ and since X is an $b-open$ then $b-Ind\,X = 0$.

3.4.Example

The following example of a space X with $Ind\,X = b-Ind\,X = 1$ .Let $X = \{a,b,c,d,e\}$, $T = \{\{a\},\{c,d\},\{a,c,d\},\{a,b,d,e\},\{d\},\{a,d\},X,\phi\}$.

The $b-open$ sets are:-
$\{a\},\{c,d\},\{a,c,d\},\{a,b,d,e\},\{d\},\{a,d\},\{a,b,d\},\{a,d,e\},$
$\{a,b,c,d\},\{a,c,d,e\}\{b,d,e\},\{b,c,d,e\},\{a,e\},\{b,d\},\{d,e\},\{c,d,e\},\{b,c,d\},\{a,b,e\},\{a,b\},\phi,X$
..Since $\{c\} \subset \{c,d\} \subset X$ such that $b\{c,d\} = \overline{\{c,d\}} - \{c,d\}^{\circ} = \{b,c,d,e\} - \{a,c,d\} = \{b,e\}$ .The topology on $\{b,e\}$ is an indiscrete then $Ind\,b\{c,d\} = -1$ *if* $b\{c,d\} = \phi$ *or* $Indb\{c,d\} = 0$ *if* $b\{c,d\} \neq \phi$ since X is not normal then $Ind\,\mathrm{X} \neq 0$ and since $Ind\,b\{c,d\} = 0$ then $Ind\,\mathrm{X} = 1$ .Since $\{c,d\}$ is $b-open$ then $\mathrm{b}\text{-}ind\,X = 1$

3.5.Proposition[7]

Let X be a topological space , if $Ind\,X=0$ then X is normal space.

3.6.Corollary

Let X be a topological space , if $b-Ind\,X=0$, then X is normal space .

Proof:

Let C be a closed set in X and U an open set such that $C\subseteq U$ .Since $b-Ind\,X=0$,then there exist an $b-open$ set W such that $b-Ind\,b(W)=-1$,hence W is an open and closed set .Therefore $C\subseteq W\subseteq \overline{W}\subseteq U$ by proposition 1.18 .Then X is a normal.

3.7.Proposition:

Let X be a topological space .If $IndX$ is exist then $b-Ind\,X\leq Ind\,X$.

Proof:

By induction on n. It is clear $n=-1$. Suppose that it is true for $n-1$. Now suppose that $Ind\,X\leq n$, to prove $b-Ind\,X\leq n$, let C be a closed set in X and G is an open set in X such that $C\in G$ since $Ind\,X\leq n$, then there exists an open set V in X such that $C\in U\subseteq G$ and $Ind\,b(U)\leq n-1$ and since each open set is $b-open$ then U is an $b-open$ set such that $C\in U\subseteq G$ and $b-Ind\,b(U)\leq n-1$. Hence $b-Ind\,X\leq n$.

The following analogous to theorem 2.6 and its proof is similar and hence is omitted .

3.8.Theorem

Let X be a topological space . Then $Ind\,X = 0$ iff $b - Ind\,X = 0$.

It is known and easy to see that if X is T_1-space, then $ind\,X \le Ind\,X$ (see[7]).

Similarity we have :

3.9.Proposition

Let X be a topological space . If X is $T_1 - space$ then $b - ind\,X \le b - Ind\,X$.

Proof:

Let $x \in X$ and G be an open set such that $x \in G$,since X is $T_1 - space$ then $\{x\} \subseteq G$ and since $b - Ind\,X \le n$ then there exist an $b - open$ set V such that $\{x\} \subseteq V \subseteq G$ and $Indb(V) \le n-1$ .Hence $indb(V) \le n-1$,then $ind X \le n$.

It is known and easy to see that if X is a regular topological space ,then $ind\,X \le Ind\,X$. (see[7])

Similarity , we have

3.10.Theorem

Let X be a topological space . If X is regular space then $b - ind\,X \le b - Ind\,X$.

Proof :

By induction on n. If $n=-1$ then $b-Ind\,X=-1$ and $X=\phi$, so that $b-Ind\,X=-1$. Suppose that the statement is true for $n-1$. Now , let $b-Ind\,X\leq n$. Let $x\in X$ and G be an open set such that $x\in G$. Since X is regular space then there exists an open set V such that $x\in V\subseteq\overline{V}\subseteq G$ by proposition 1.17. Also since $b-Ind\,X\leq n$ and $\overline{V}$ is closed , $\overline{V}\subseteq G$, then there exists an $b-open$ set U such that $\overline{V}\subseteq U\subseteq G$ and $b-Ind\,b(\mathrm{U})\leq n-1$, then $b-ind\,b(\mathrm{U})\leq n-1$[by indication] and $b-ind\,X\leq n$.

3.11.Proposition[7]

If A is a closed subset of a space X, then $Ind\,A\leq Ind\,X$

We have the following :

3.12.Theorem

If A is an open and closed subspace of a space X , then $b-Ind\,A\leq b-Ind\,X$.

Proof :

By induction on n. It is clear if $n=-1$, suppose that it is true for $n-1$. Now suppose that $b-Ind\,X\leq n$, to prove $b-Ind\,A\leq n$, let C is a closed subset of A and G is an open subset in A such that $C\subseteq G$,since $C$is closed in A and A is closed in$X$, then C is closed in X. Since G is an open in A , then there exists U open set in X such that $U \cap A = G$. Since $C \subset U$ and $b-Ind\, X \leq n$, then there exists an $b-open$ set W in X such that $C \subseteq W \subseteq U$ and $b-Ind\, b(\mathrm{W}) \leq n-1$, since A is an open set then $V = W \cap A$ is $b-open$ set in A by proposition 1.14 ,

$$C \subset V = W \cap A \subset U \cap A = G$$

$$b_A(V) \subseteq b(V) \cap A = (\overline{V} - V^{\circ}) \cap A \subset (\overline{W} - V^{\circ}) \cap A = (\overline{W} \cap \overset{c}{\overset{\circ}{V}}) \cap A$$

$$= \left[\overline{W} \cap (\overset{c}{W^{\circ}} \cup \overset{c}{A^{\circ}})\right] \cap A$$

$$= \left[(\overline{W} \cap \overset{c}{W^{\circ}}) \cup (\overline{W} \cap A^{c})\right] \cap A$$

$$\subseteq \left[b(W) \cup A^{c}\right] \cap A$$

$$= (b(W) \cap A) \cup (A^{c} \cap A) = b(W) \cap A \subseteq b(W)$$

Since $b-Ind\, b(\mathrm{W}) \leq n-1$, then $b-Ind\, \mathrm{b_A}(\mathrm{V}) \leq n-1$, therefore $b-Ind\, A \leq n$.

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