Strongly Regular Proper Mappings

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Abstract

The main goal of this work is to create a special type of proper mappings namely, strongly regular proper mappings and we introduce the definition of a new type of compact and coercive mappings and give some properties and some equivalent statements of these concepts , as well as explain the relationship among them .

Introduction

One of the very important concepts in topology is the concept of mapping . There are several types of mappings , in this work we study an important class of mappings , namely , strongly regular proper mappings .

Proper mapping was introduced by Bourbaki in [1] .

Let A be a subset of topological space X . We denote to the closure and interior of A by $\overline{A}$ and A° respectively .

James Dugundji in [2] defined the regular open set as a subset A of a space X , such that $A = \overline{A}^{\circ}$. Stephen Willard in [8] defined the regular open set similarly with Dugundji's definition .

This work consists of three sections .

Section one includes the fundamental concepts in general topology , and the proves of some related results which are needed in the next section .

Section two contains the definitions of strongly regular compact mapping and strongly regular coercive mapping . Also the relationship among these concepts is introduced and some of its related results are proved .

Section three introduces the definition of strongly regular proper mapping and some of its related are proved .

1- Basic concepts

Definition 1.1 , [2] : A subset B of a space X is called **regular open (r- open)** set if $B = \overset{\circ}{\overline{B}}$. The complement of a regular open set is defined to be a **regular closed (r-closed)** set .

Proposition 1.2 , [2] : A subset B of a space X is r- closed if and only if $B = \overline{\overset{\circ}{B}}$.

Its clearly that every r- open set is an open set and every r- closed set is closed set , but the converse is not true in general as the following example shows :

Example 1.3 : Let X = {a, b, c, d} be a set and T = {Ø, X, {a}, {a, b}, {a, c, d}} be a topology on X . Notice that {a, b} is an open set in X , but its not r- open set , and {b} is a closed set in X , but its not r- closed set .

Corollary 1.4 : A subset B of a space X is clopen (open and closed) if and only if B is r- clopen (r- open and r- closed) .

Proposition 1.5 : Let $A \subseteq Y \subseteq X$. Then :

(i) If A is an r- open set in Y and Y is an r- open set in X , then A is an r- open set in X .

(ii) If A is an r- closed set in Y and Y is an r- closed set in X, then A is an r- closed set in X .

Remark 1.6 : If A is an r- closed set in X and B is a clopen set in X , then A∩B is r- closed in B .

Definition 1.7 : Let A be a subset of a space X . A point $x \in A$ is called **r- interior** point of A if there exists an r- open set U in X such that $x \in U \subseteq A$.

The set of all r- interior points of A is called **r- interior** set of A and its denoted by $A^{\circ r}$.

Proposition 1.8 : Let (X , T) be a space and $A \subseteq X$. Then :

(i) $A^{\circ r} \subseteq A^{\circ}$.

(ii) $(A^{\circ r})^{\circ} = (A^{\circ})^{\circ r}$.

(iii) A is r- open if and only if $A^{\circ r} = A$.

Definition 1.9 : Let A be a subset of a space X . A point x in X is said to be **r- limit** point of A if for each r- open set U contains x implies that U∩A \ {x} ≠ Ø .

The set of all r- limit points of A is called **r- derived** set of A and its denoted by $A^{\prime r}$.

Definition 1.10 : Let X be a space and $B \subseteq X$. The intersection of all r- closed sets containing B is called the **r- closure** of B and denotes by $\overline{A}^{r}$.

Proposition 1.11 : Let X be a space and A , B $\subseteq$ X . Then :

(i) $\overline{A}^{r}$ is an r- closed set .

(ii) A $\subseteq \overline{A}^{r}$.

(iii) A is r- closed if and only if $\overline{A}^{r} = A$.

(iv) x $\in \overline{A}^{r}$ if and only if A$\cap$U $\neq \emptyset$, for any r- open set U containing x .

Proposition 1.12: Let X and Y be two spaces , and A $\subseteq$ X , B $\subseteq$ Y . Then :

(i) A , B are r- open subsets of X and Y respectively if and only if A$\times$B is r- open subset in X$\times$Y .

(ii) A , B are r- closed subsets of X and Y respectively if and only if A$\times$B is r- closed subset in X$\times$Y .

(iii) A , B are clopen subsets of X and Y respectively if and only if A$\times$B is clopen subset in X$\times$Y .

(iv) A , B are r- clopen subsets of X and Y respectively if and only if A$\times$B is r- clopen subset in X$\times$Y .

Definition 1.13 , [3] : Let X be a space and B be any subset of X . **A neighborhood of B** is any subset of X which containing an open set containing B .

The neighborhoods of a subset {x} , consisting of a single point are also called **neighborhood of a point x** .

The collection of all neighborhoods of the subset B is denoted by **N(B)** . In particular the collection of all neighborhoods of x is denoted by **N(x)** .

Proposition 1.14 , [1] : Let X be a set . If to each element x of X , there corresponds a collection β(x) of subsets of X , satisfying the properties :

(i) Every subset of X which contains a set belongs to **β(x)** , itself belongs to **β(x)** .

(ii) Every finite intersection of sets of **β(x)** belongs to **β(x)** .

(iii) The element x is in every set of **β(x)** .

(iv) If V belongs to **β(x)** , then there is a set W belonging to **β(x)** such that for each y $\in$ W , V belongs to β(y) .

Then there is a unique topological structure on X such that , for each x $\in$X , **β(x)** is the collection of neighborhoods of x in this topology .

Definition 1.15 : Let X be a space and $B \subseteq X$. An **r- neighborhood of B** is any subset of X which contains an r- open set containing B . The r- neighborhoods of a subset {x} consisting of a single point are also called **r- neighborhoods** of the point x .

Let us denote the collection of all r- neighborhoods of the subset B of X by **Nr(B) .** In particular , we denote the collection of all r- neighborhoods of x by **Nr(x)** .

Definition 1.16 , [1] : Let $f : X \to Y$ be a mapping of spaces .Then :

(i) f is called continuous mapping if $f^{-1}(A)$ is an open set in X for every open set A in Y .

(ii) f is called open mapping if $f(A)$ is an open set in Y for every open set A in X .

(iii) f is called closed mapping if $f(A)$ is a closed set in Y for every closed set A in X .

Definition 1.17 : A mapping $f : X \to Y$ is called r- irresolute if $f^{-1}(A)$ is an r- open set in X for every r- open set A in Y .

Definition 1.18 : Let X and Y be spaces and $f : X \to Y$ be a mapping . Then :

(i) f is called **a strongly r- open (st- r- open)** mapping if the image of each r- open subset of X is an r- open set in Y .

(ii) f is called **a strongly r- closed (st- r- closed)** mapping if the image of each r- closed subset of X is an r- closed set in Y .

Definition 1.19 : Let X and Y be spaces . Then the mapping $f : X \to Y$ is called **st- r- homeomorphism** if

(i) f is bijective .

(ii) f is continuous .

(iii) f is st- r- open (or st- r- closed) .

Proposition 1.20 : A mapping $f : X \to Y$ is st- r- closed if and only if $\overline{f(A)}^{r} \subseteq f(\overline{A}^{r})$, $\forall\, A \subseteq X$.

Proof : $\to$) Let $f : X \to Y$ be a st- r- closed mapping and $A \subseteq X$. Since $\overline{A}^{r}$ is an r- closed set in X , then $f(\overline{A}^{r})$ is an r- closed subset of Y , and since $A \subseteq \overline{A}^{r}$, then $f(A) \subseteq f(\overline{A}^{r})$. Thus $\overline{f(A)}^{r} \subseteq \overline{f(\overline{A}^{r})}^{r} = f(\overline{A}^{r})$, hence $\overline{f(A)}^{r} \subseteq f(\overline{A}^{r})$.

$\leftarrow$) Let $\overline{f(A)}^{r} \subseteq f(\overline{A}^{r})$, for all $A \subseteq X$. Let F be an r- closed subset of X , i.e , $F = \overline{F}^{r}$, thus by hypothesis $\overline{f(F)}^{r} \subseteq f(\overline{F}^{r}) = f(F)$. But $f(F) \subseteq \overline{f(F)}^{r}$, then $f(F) = \overline{f(F)}^{r}$. Hence $f(F)$ is an r- closed set in Y , thus $f : X \to Y$ is a st- r- closed mapping .

Proposition 1.21 : Let X and Y be spaces . If f : X → Y is a st- r- closed , continuous mapping . Then for each clopen subset T of Y , $f_T : f^{-1}(T) \rightarrow T$ is a st- r- closed mapping .

Proof : Let T be a clopen subset of Y . Since f is continuous , then $f^{-1}(T)$ is a clopen set in X . Let F be an r- closed set in $f^{-1}(T)$, by Corollary (1.4) , and Proposition (1.5) , F is r- closed in X . Since f is a st- r- closed mapping , then f(F) is r- closed in Y , hence by Remark (1.6) , T∩f(F) is r- closed in T . But $f_T(F) = T \cap f(F)$, then $f_T(F)$ is an r- closed set in T . Therefore f_T is a st- r- closed mapping .

Proposition 1.22: Let X , Y and Z be spaces and f : X → Y , g : Y → Z be mappings . Then :

(i) If f and g are st- r- closed , then go$f$:X → Z is st- r- closed mapping .

(ii) If go$f$ is a st- r- closed mapping and f is onto , r- irresolute , then g is st- r- closed .

(iii) If go$f$ is a st- r- closed mapping and g is one to one , r- irresolute , then f is st- r- closed .

Proof :

(i) Let F be an r- closed subset of X , then f(F) is an r- closed set in Y and then g(f(F)) = (go$f$)(F) is an r- closed set in Z . Hence (go$f$) is a st- r- closed mapping .

(ii) Let F be an r- closed subset of Y , since f is r- irresolute , then $f^{-1}(F)$ is r- closed in X . Since go$f$ is a st- r- closed mapping , then (go$f$)($f^{-1}(F)$) is an r- closed set in Z . But f is onto , then (go$f$)($f^{-1}(F)$) = g(F) , thus g(F) is an r- closed set in Z . Hence g is st- r- closed .

(iii) Let F be an r- closed subset of X , then (go$f$)(F) is an r- closed set in Z . Since g is one to one , r- irresolute , then $g^{-1}((gof)(F) = f(F)$ is an r- closed set in Y . Hence f is a st- r- closed mapping .

Proposition 1.23 : Let X be a space . If A is an r- closed subset of X , then the inclusion mapping i_A : A → X is st- r- closed .

Proof : Let F be an r- closed set in A , since A is r- closed in X , then by Proposition (1.5), F is r- closed in X . But $i_A(F) = F$, then $i_A(F)$ is an r- closed set in X . Hence the inclusion mapping i_A : A → X is st- r- closed .

Proposition 1.24 : Let X and Y be spaces , f : X → Y be a st- r- closed mapping . If F is an r- closed subset of X , then the restriction mapping $f_{|F}$: F → Y is st- r- closed .

Proof : Since F is an r- closed set in X , then by Proposition (1.23) , the inclusion mapping i_A : F → X is st- r- closed . Since f is st- r- closed mapping , then by Proposition (1.22) , $f o i_A$: F → Y is a st- r- closed mapping . But $f o i_A = f_{|F}$, then the restriction mapping $f_{|F}$: F → Y is st- r- closed .

Proposition 1.25 : A bijective mapping $f : X \to Y$ is st- r- closed if and only if is st- r- open .

Proof : $\to$) Let $f : X \to Y$ be a bijective , st- r- closed mapping and U be an r- open subset of X , thus U^c is r- closed . Since f is st- r- closed then $f(U^c)$ is r- closed in Y , thus $(f(U^c))^c$ is r- open . Since f is bijective mapping , then $(f(U^c))^c = f(U)$, hence $f(U)$ is r- open in Y , therefore f is a st- r- open mapping .

$\leftarrow$) Let $f : X \to Y$ be a bijective , st- r- open mapping and F be an r- closed subset of X , thus F^c is r- open . Since f is st- r- open then $f(F^c)$ is r- open in Y , thus $(f(F^c))^c$ is r- closed . Since f is a bijective mapping , then $(f(F^c))^c = f(F)$, hence $f(F)$ is r- closed in Y . So f is st- r- closed mapping .

Theorem 1.26 , [8] : Let X be a space and A be a subset of X , $x \in X$ .Then $x \in \overline{A}$ if and only if there is a net in A which converges to x .

Lemma 1.27 , [5] : If (χ_d) is a net in a space X and for each $d_o \in D$, $A_{do} = \{\chi_d \mid d \geq d_o\}$, then $x \in X$ is a cluster point of (χ_d) if and only if $x \in \overline{A_d}$, for all $d \in D$.

Definition 1.28 : Let $(\chi_d)_{d\in D}$ be a net in a space X , $x \in X$. Then $(\chi_d)_{d\in D}$ **r- converges** to x [written $\chi_d \xrightarrow{r} x$], if $(\chi_d)_{d\in D}$ is eventually in every r- nbd of x . The point x is called **an r- limit point** of $(\chi_d)_{d\in D}$.

Definition 1. 29 : Let $(\chi_d)_{d\in D}$ be a net in a space X , $x \in X$ .Then $(\chi_d)_{d\in D}$ is said to have x as **an r- cluster point** [written $\chi_d \overset{r}{\propto} x$] if $(\chi_d)_{d\in D}$ is frequently in every r- nbd of x .

Proposition 1.30 : Let (X , T) be a space and $A \subseteq X$, $x \in X$ .Then $x \in \overline{A}^r$ if and only if there exists a net $(\chi_d)_{d\in D}$ in A and $\chi_d \overset{r}{\propto} x$.

Proof : $\to$) Let $x \in \overline{A}^r$, then $U \cap A \neq \emptyset$, for every r- open set U , $x \in U$. Notice that $(Nr(x) , \subseteq)$ is a directed set , such that for all $U_1, U_2 \in Nr(x)$, $U_1 \geq U_2$ if and only if $U_1 \subseteq U_2$. Since for all $U \in Nr(x)$, $U \cap A \neq \emptyset$, then we can define a net $\chi : Nr(x) \to X$ as follows : $\chi(U) = \chi_U \in U \cap A$, $U \in Nr(x)$. To prove that $\chi_U \overset{r}{\propto} x$. Let $B \in Nr(x)$, thus $B \cap U \in Nr(x)$. Since $B \cap U \subseteq U$, then $B \cap U \geq U$, $\chi(B \cap U) = \chi_{B\cap U} \in B \cap U \subseteq B$. Hence $\chi_U \overset{r}{\propto} x$.

←) Let $(\chi_d)_{d\in D}$ be a net in A , such that $\chi_d \overset{r}{\propto} x$, and let U be an r- open set , $x \in U$. Since $\chi_d \overset{r}{\propto} x$, then $(\chi_d)_{d\in D}$ is frequently in U . Thus $U\cap A \neq \emptyset$, for all r- open set U , $x \in U$. Hence $x \in \overline{A}^{r}$.

<u>Proposition 1.31 :</u> Let X be a space and $(\chi_d)_{d\in D}$ be a net in X , for each $d_o \in D$, such that $A_{do} = \{\chi_d \mid d \geq d_o\}$, then a point x of X is r- cluster point of $(\chi_d)_{d\in D}$ if and only if $x \in \overline{A_{do}}^{r}$, for all $d_o \in D$.

<u>Proof :</u> →) Let x be an r- cluster point of $(\chi_d)_{d\in D}$ and let N be an r- open set contain x , then $(\chi_d)_{d\in D}$ is frequently in N , thus $A_{do}\cap N \neq \emptyset$, $\forall\, d_o \in D$, then by Proposition (1.11) , $x \in \overline{A_{do}}^{r}$.

←) Let $x \in \overline{A_{do}}^{r}$, $\forall\, d_o \in D$, and suppose that x is not r- cluster point of $(\chi_d)_{d\in D}$, then there exists r- nbd N of x , such that $A_{do}\cap N = \emptyset$, $\forall\, d_o \in D$, $\chi_d \notin D$, $d \geq d_o$ $d \geq d_o$, then $x \notin \overline{A_{do}}^{r}$. This is contradiction . Hence x is r- cluster point of $(\chi_d)_{d\in D}$.

2- Certain types of strongly regular proper mappings

<u>Definition 2.1 , [6] :</u> A space X is called **Hausdorff (T_2)** if for any two distinct points x , y of X there exists disjoint open subsets U and V of X such that $x \in U$, $y \in V$.

<u>Proposition 2.2 :</u> Let (X,T) is a T_2- space , then the set {x} is an r- closed in X , for all $x \in X$.

<u>Proof :</u> To prove that $\{x\} = \overline{\{x\}}^{r}$, let $y \in X$, such that $x \neq y$. Since X is a T_2- space, then , X is an r- T_2 , so there is an r- open set U in X , such that $y \in U$, $x \notin U \rightarrow \{x\} \subseteq U^{c}$. But U^{c} is an r- closed set , then $\overline{\{x\}}^{r} \subseteq U^{c}$, therefore $y \notin \overline{\{x\}}^{r}$, for all $y \in X$ and $y \neq x$. Then $\{x\} = \overline{\{x\}}^{r}$, (i.e) , $\overline{\{x\}}^{r}$ is an r- closed set in X .

<u>Definition 2.3 , [7] :</u> A space X is called **compact** if every open cover of X has a finite subcover .

<u>Theorem 2.4 , [7] :</u>

(i) A closed subset of compact space is compact .

(ii) In any space , the intersection of a compact set with a closed set is compact .

(iii) Every compact subset of T_2- space is closed .

Theorem 2.5 , [6] : A space X is compact if and only if every net in X has a cluster point in X .

Definition 2.6 : A space X is called **r- compact** if every r- open cover of X has a finite subcover .

Proposition 2.7 : Every compact space is r- compact space .

The converse of Proposition (2.7) , is not true in general as the following example shows :

Example 2.8 : Let $T = \{A \subseteq R \mid Z \subseteq A\} \bigcup \{\emptyset\}$, be a topology on R . Notice that the topological space (R,T) is r- compact , but its not compact .

Theorem 2.9 : A space X is an r- compact if and only if every net in X has r- cluster point in X .

Theorem 2.10 :

(i) An r- closed subset of compact space is r- compact .

(ii) Every r- compact subset of T_2- space is r- closed .

(iii) In any space , the intersection of an r- compact set with an r- closed set is r- compact .

(iv) In a T_2- space , the intersection of two r- compact sets is r- compact .

Proposition 2.11 : Let X be a space and Y be an r- open subspace of X , $K \subseteq Y$. Then K is an r- compact set in Y if and only if K is an r- compact set in X .

Proof : $\rightarrow$) Let K be an r- compact set in Y . To prove that K is an r- compact set in X . Let $\{U_\lambda\}_{\lambda\in\Lambda}$ be an r- open cover in X of K , let $V_\lambda = U_\lambda \cap Y$, $\forall\lambda\in\Lambda$. Then V_λ is r- open in X , $\forall\lambda\in\Lambda$. But $V_\lambda \subseteq Y$, thus V_λ is r- open in Y, $\forall\lambda\in\Lambda$. Since $K \subseteq \bigcup_{\lambda\in\Lambda} V_\lambda$, then $\{V_\lambda\}_{\lambda\in\Lambda}$ is an r- open cover in Y of K, and by hypothesis this cover has finite subcover $\{ V_{\lambda_1}, V_{\lambda_2}, \ldots, V_{\lambda_n} \}$ of K , thus the cover $\{U_\lambda\}_{\lambda\in\Lambda}$ has a finite subcover of K . Hence K is an r- compact set in X .

$\leftarrow$) Let K be an r- compact set in X . To prove that K is an r- compact set in Y. Let $\{U_\lambda\}_{\lambda\in\Lambda}$ be an r- open cover in Y of K . Since Y is an r- open subspace of X , then by Proposition (1.5) , $\{U_\lambda\}_{\lambda\in\Lambda}$ is an r- open cover in X of K . Then by hypothesis there exists $\{\lambda_1 , \lambda_2 , \ldots , \lambda_m\}$, such that $K \subseteq \bigcup_{\lambda=1}^{m} U_\lambda$, thus the cover $\{U_\lambda\}_{\lambda\in\Lambda}$ has a finite subcover of K . Hence K is an r- compact set in Y .

Definition 2.12 : Let X be a space and $W \subseteq X$. We say that W is **compactly r- closed set** if $W \cap K$ is r- compact , for every r- compact set K in X .

Proposition 2.13 : Every r- closed subset of a space X is compactly r- closed .

The converse of Proposition (2.13), is not true in general as the following example shows :

Example 2.14 : Let X = {a, b, c} be a space and T = {X, Ø, {a, b}} be a topology on X . Notice that the set A = {a, b} is compactly r- closed , but its not r- closed set .

Theorem 2.15 : Let X be a T_2 - space .A subset A of X is compactly r- closed if and only if A is r- closed .

Remark 2.16 : Let X be a compact , T_2 - space and $A \subseteq X$. Then :

(i) A is closed if and only if A is r- closed .

(ii) A is compact if and only if A is r- compact .

Definition 2.17 [6]: Let X and Y be spaces . We say that the mapping $f : X \to Y$ is **a compact mapping** if the inverse image of each compact set in Y , is an compact set in X .

Definition 2.18 : Let X and Y be spaces . We say that the mapping $f : X \to Y$ is **a st- r- compact mapping** if the inverse image of each r- compact set in Y , is an r- compact set in X .

Examples 2.19 :

(i) The identity mapping is st- r- compact .

(ii)Any mapping from a finite topological space into any topological space is st- r- compact .

Proposition 2.20 : Let X and Y be spaces , and $f : X \to Y$ be a st- r- compact , r- irresolute , mapping . If T is an r- clopen subset of Y , then $f_T : f^{-1}(T) \to T$ is a st- r- compact mapping .

Proof : Let K be an r- compact subset of T . Since T is an r- open set in Y, then by Proposition (2.11) , K is an r- compact set in Y . Since f is a st- r- compact mapping , then $f^{-1}(K)$ is r-compact in X .

Now , since T is an r- closed set in Y , and f is an r- irresolute mapping , then $f^{-1}(T)$ is an r- closed set in X , thus by Proposition (2.10) , $f^{-1}(T) \cap f^{-1}(K)$ is an r- compact set . But $f_T^{-1}(K) = f^{-1}(T) \cap f^{-1}(K)$, then $f_T^{-1}(K)$ is an r- compact set in $f^{-1}(T)$. Therefore is a st- r- compact mapping .

Proposition 2.21 : Let X , Y and Z be spaces , and $f : X \to Y$, $g : Y \to Z$ be mappings . Then :

(i) If f and g are st- r- compact mapping , then $g \circ f : X \to Z$ is a st-r- compact mapping .

(ii) If $g \circ f$ is a st- r- compact mapping and f is r- irresolute , onto , then g is st- r- compact .

(iii) If $g o f$ is a st- r- compact mapping and g is r- irresolute , one to one , then f is st- r- compact .

Proof :

(i) Let K be an r- compact set in Z . Then $g^{-1}(K)$ is an r- compact set in Y , and then $f^{-1}(g^{-1}(K)) = (g o f)^{-1}(K)$ is an r- compact set in X . Hence $g o f : X \to Z$ is a st- r- compact mapping .

(ii) Let K be an r- compact set in Z . Then $(g o f)^{-1}(K)$ is an r- compact set in X , and then $f((g o f)^{-1})(K)$ is r- compact in Y . Now , since f is onto , then $f((g o f)^{-1})(K) = g^{-1}(K)$, hence $g^{-1}(K)$ is an r- compact set in Y . Therefore g is a st- r- compact mapping .

(iii) Let K be an r- compact set in Y . Since g is an r- irresolute , then g(K) is an r- compact set in Z , thus $(g o f)^{-1}(g(K))$ is an r- compact set in X . Since g is one to one , then $(g o f)^{-1}(g(K)) = f^{-1}(K)$, hence $f^{-1}(K)$ is an r- compact set in X . Thus f is a st- r- compact mapping .

Proposition 2.22 : For any r- closed subset F of a space X , the inclusion mapping $i_F : F \to X$ is a st- r-compact mapping .

Proof : Let K be an r- compact set in X , then by Proposition (2.10) , $F \cap K$ is an r- compact set in F . But $i_F^{-1}(K) = F \cap K$, then $i_F^{-1}(K)$ is an r- compact set in F . Therefore the inclusion mapping $i_F : F \to X$ is st- r- compact .

Proposition 2.23 : Let X and Y be spaces , and $f : X \to Y$ be a st- r- compact mapping . If F is an r- closed subset of X , then $f_{|F} : F \to X$ is a st- r- compact mapping .

Proof : Since F is an r- closed subset of X , then by Proposition (2.22) , the inclusion $i_F : F \to X$ is a st- r- compact mapping . But $f_{|F} \equiv f o i_F$, then by Proposition (2.21) , $f_{|F}$ is a st- r- compact mapping .

Definition 2.24 , [4] : Let X and Y be spaces , A mapping $f : X \to Y$ is called **a coercive** if for every compact set $J \subseteq Y$, there exists a compact set $K \subseteq X$ such that $f(X \setminus K) \subseteq Y \setminus J$.

Definition 2.25 : Let X and Y be spaces , the mapping $f : X \to Y$ is called **a st- r- coercive** if for every r- compact set $J \subseteq Y$, there exists an r- compact set $K \subseteq X$, such that $f(X \setminus K) \subseteq Y \setminus J$.

Examples 2.26 :

(i) The identity mapping on any space is st- r- coercive .

(ii) If $f : (X,T) \to (Y,\tau)$ is a mapping , such that X is r- compact space , then f is st- r- coercive .

Proposition 2.27 : Every st- r- compact mapping is a st- r- coercive mapping .

Proof : Let J be an r- compact set in Y . Since f is a st- r- compact mapping , then $f^{-1}(J)$ is an r- compact set in X . But $f(X \setminus f^{-1}(J)) \subseteq Y \setminus J$. Hence $f : X \to Y$ is a st- r- coercive mapping .

Proposition 2.28 : Let X and Y be spaces , such that Y is a T_2 – space , and $f : X \to Y$ is an r- irresolute mapping . Then f is a st- r- coercive if and only if f is a st- r- compact .

Proof : $\rightarrow$) Let J be an r- compact set in Y . To prove that $f^{-1}(J)$ is an r- compact set in X . Since Y is a T_2 – space and f is an r- irresolute mapping , then $f^{-1}(J)$ is an r- closed set in X . Since f is a st- r- coercive mapping , then there exists an r- compact set K in X , such that $f(X \setminus K) \subseteq Y \setminus J$. Then $f(K^c) \subseteq J^c$, therefore $f^{-1}(J) \subseteq K$. Thus by Proposition (2.10) , $f^{-1}(J)$ is an r- compact set in X . Hence $f : X \to Y$ is a st- r- compact mapping .

$\leftarrow$) By Proposition (2.25) .

Proposition 2.29 : Let X , Y and Z be spaces . If $f : X \to Y$, $g : Y \to Z$ are st- r- coercive mapping , then $gof : X \to Z$ is a st- r- coercive mapping .

Proof : Let J be an r- compact set in Z . Since $g : Y \to Z$ is a st- r- coercive mapping , then there exists an r- compact set K in Y, such that $g(Y \setminus K) \subseteq Z \setminus J$.

Since $f : X \to Y$ is a st- r- coercive mapping , then there exists an r- compact set H in X , such that $f(X \setminus H) \subseteq Y \setminus K \Rightarrow g(f(X \setminus H) \subseteq g(Y \setminus K) \subseteq Z \setminus J \Rightarrow (gof)(X \setminus H) \subseteq Z \setminus J$.

Hence gof is a st- r- coercive mapping .

Proposition 2.30 : Let X and Y be spaces , and $f : X \to Y$ be a st- r- coercive mapping . If F is an r- closed subset of X , then the restriction mapping $f_{|F} : F \to Y$ is a st- r- coercive mapping .

Proof : Since F is an r- closed subset of X , then by Proposition (2.22) , and Proposition (2.27) , the inclusion mapping $i_F : F \to X$ is a st- r-coercive mapping . But $f_{|F} \equiv f o i_F$, then by Proposition (2.21) , $f_{|F}$ is a st- r- coercive mapping .

3- Strongly Regular Proper Mapping :

Definition 3.1 , [1] : Let X and Y be spaces , and $f : X \to Y$ be a mapping . We say that f is **a proper mapping** if :

(i) f is continuous .

(ii) $f \times I_Z : X \times Z \to Y \times Z$ is closed , for every space Z .

Definition 3.2 : Let X and Y be spaces , and $f : X \to Y$ be a mapping . We say that f is **a strongly regular proper (st-r- proper) mapping** if :

(i) f is continuous .

(ii) $f\times I_Z : X\times Z \to Y\times Z$ is st- r- closed , for every space Z .

Example 3.3 : Let X = {a, b} , Y = {x, y} be sets and T = {Ø, X, {a}, {b}} , τ = {Ø, Y, {x}, {y}} be topologies on X and Y respectively . The mapping $f : X \to Y$ which is defined by : $f(a) = f(b) = x$ is st- r- proper .

Remarks 3.4 :

(i) Every st- r- proper mapping is st- r- closed .

(ii) Every st- r- homeomorphism is st- r- proper .

The converse of Remark (3.4.i) , is not true in general as the following example shows :

Example 3.5 : Let $f : (R , U) \to (R , U)$ be the mapping which is defined by $f(x) = 0$, for every $x \in R$. Notice that f is a st- r- closed mapping but f is not st- r- proper mapping , since for the usual space (R , U) the mapping $f\times I_Z : R\times R \to R\times R$, such that $(f\times I_R)(x,y) = (0,y)$, for every $(x,y) \in R$ is not st- r- closed mapping .

The converse of Remarks (3.4.ii) , is not true in general as the following example shows :

Example 3.6 : Let X = {a, b, c} , Y = {x, y} be sets and T = {Ø, X, {a}, {a, b}} , τ = {Ø, Y, {x}} be topologies on X and Y respectively . Let $f : X \to Y$ be a mapping which is defined by : $f(a) = f(b) = x$, $f(c) = y$. Notice that f is a st- r- proper mapping , but f is not one to one mapping , therefore f is not st- r- homeomorphism .

Proposition 3.7 : Let X and Y be spaces , and $f : X \to Y$ be a st- r- proper mapping . If T is a clopen subset of Y , then $f_T : f^{-1}(T) \to T$ is a st- r- proper mapping .

Proof : Since $f : X \to Y$ is a continuous mapping , then f_T is a continuous mapping . To prove that $f_T\times I_Z : f^{-1}(T)\times Z \to T\times Z$ is a st- r- closed mapping , for every space Z . Notice that $f_T \times I_Z \equiv (f\times I_Z)_{T\times Z}$, where $f\times I_Z$ is a st- r- closed mapping , since T is a clopen subset of Y, then by Proposition (1.12) , T×Z is a clopen subset of Y×Z , thus by Proposition (1.21) , $(f\times I_Z)_{T\times Z} \equiv (f_T\times I_Z)$ is a st- r- closed mapping , hence $f_T : f^{-1}(T) \to T$ is a st- r- proper mapping .

Proposition 3.8 : Let X and Y be spaces , and $f : X \to Y$ be a st- r- proper mapping . If Y is a T_2- space , then $f_{\{y\}} : f^{-1}(\{y\}) \to \{y\}$ is a st- r- proper mapping , for all $y \in Y$.

Proof : Since $f : X \to Y$ is a continuous mapping , then $f_{\{y\}}$ is a continuous mapping . To prove that $f_{\{y\}}\times I_Z : f^{-1}(\{y\})\times Z \to \{y\}\times Z$ is a st- r- closed mapping , for every space Z . Let $F \subseteq f^{-1}(\{y\})\times Z$, then : $\overline{(f_{\{y\}}\times I_Z)(F)}^{r} \subseteq \overline{(\{y\}\times Z)\cap(f\times I_Z)(F)}^{r} \subseteq \overline{\{y\}\times Z}^{r} \cap \overline{(f\times I_Z)(F)}^{r}$.

Since Y is a T_2- space , then by Proposition (2.2) , {y} is an r- closed set , for all $y \in Y$, so {y}×Z is an r- closed in Y×Z , then $\overline{\{y\}\times Z}^{r} = \{y\}\times Z$. Since $f\times I_Z : X\times Z \to Y\times Z$ is a st- r-closed mapping and $F \subseteq f^{-1}(\{y\})\times Z \subseteq X\times Z$, then by Proposition (1.20) , $\overline{(f\times I_Z)(F)}^{r} \subseteq (f\times I_Z)(\overline{F}^{r})$. Thus $\overline{(f_{\{y\}}\times I_Z)(F)}^{r} \subseteq \{y\}\times Z \cap (f\times I_Z)(\overline{F}^{r})$.

Since $(f_{\{y\}}\times I_Z)(\overline{F}^{r}) = (f\times I_Z)_{\{y\}\times Z}(\overline{F}^{r}) = (\{y\}\times Z)\cap(f\times I_Z)(\overline{F}^{r})$, then $\overline{(f_{\{y\}}\times I_Z)(F)}^{r} \subseteq (f_{\{y\}}\times I_Z)(\overline{F}^{r})$, therefore by Proposition (1.20) , $f_{\{y\}}\times I_Z$ is a st- r- closed mapping . Hence $f_{\{y\}} : f^{-1}(\{y\}) \to \{y\}$ is a st- r- proper mapping .

Theorem 3.9 : Let $f : X \to P = \{w\}$ be a mapping on a space X . If f is a st- r- proper mapping , then X is an r- compact space , where w is any point which does not belong to X .

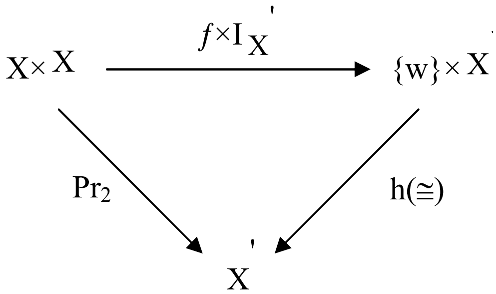
Proof : To prove that X is an r- compact space . Let $(\chi_d)_{d\in D}$ be a net in X , and let $X' = X\bigcup\{w\}$. Consider : $\beta(x) = \{U \subseteq X' : x \in U\}$, $x \in X$.

$\beta(w) = \{M\bigcup\{w\} \mid M \subseteq X$ and (χ_d) is eventually in $M\}$.

Clearly that for each $x \in X$, the family $\beta(x)$ satisfies the conditions of Proposition (1.14) , and therefore we can define a topology on X' by : $T' = \{U \subseteq X' \mid \forall\, x \in U \Rightarrow U \in \beta(x)\}$, such that the family $\bigcup\{\beta(x)\}_{x\in X'}$ is the neighborhood system of the space (X' , T') .

Now , suppose $w \in \overline{X}^{r}$ with respect to T' . Let $U \in T'^{r}$, $w \in U$, then U is an r- open set , and then U is an α- open set , then there exists an open set $V \in T'$ such that $V \subseteq U \subseteq \overline{V}^{\circ} \subseteq \overline{V}$, hence $w \in \overline{V}$, thus for all open set $U_1 \in T'$, such that $w \in U_1$, $U_1\cap V \neq \emptyset$. Since the set $U_1 = X\bigcup\{w\} \in T'$ and $w \in U_1$, then $U_1\cap V \neq \emptyset \to (X\bigcup\{w\})\cap V \neq \emptyset \to (X\cap V) \bigcup (\{w\}\cap V) \neq \emptyset$.

Claim $X\cap V \neq \emptyset$, if $X\cap V = \emptyset$, then $\{w\}\cap V \neq \emptyset \to w \in V \in T' \to V \in \beta(w)$, thus $V = M_2\bigcup\{w\}$, where $M_2 \subseteq X$ and a net (χ_d) is eventually in M_2 . But $X\cap V = \emptyset$, then $X\cap(M_2\bigcup\{w\}) = \emptyset$, hence $X\cap M_2 = \emptyset$, and this is a contradiction .Thus $X\cap V \neq \emptyset \to X\cap U \neq \emptyset$ $(V \subseteq U)$, thus $w \in \overline{X}^{r}$. Now , let Δ be the diagonal set of $X\times X$ in T , and let $F = \overline{\Delta}$, consider the commutative diagram :



Where $h : \{w\} \times X' \to X'$ is the homeomorphism and $Pr_2 : X \times X' \to X'$ is the projection map . Since $f : X \to \{w\}$ is a st- r- proper mapping , then $f \times I_Z : X \times X' \to \{w\} \times X'$ is a st- r- closed mapping .

Claim $X \subseteq Pr_2(F)$, if $x \in X \to (x,x) \in \Delta \subseteq \overline{\Delta}^r = F \to x = Pr_2(x,x) \in Pr_2(F) \to X \subseteq Pr_2(F)$. Since $w \in \overline{X}^r \subseteq \overline{pr_2(F)}^r = Pr_2(F)$, then $w \in Pr_2(F)$. Therefore there exists a point $x \in X$, such that $(x,w) \in F = \overline{\Delta}^r$. Let U be an r- open set in X contains x and V be any subset of X, such that a net (χ_d) is eventually in V .Thus $V \cup \{w\} \in \beta(w)$, $w \in V \cup \{w\}$. Thus by Proposition (1.12) , $U \times (V \cup \{w\})$ is an r- open set in $X \times X'$ containing (x,w) , since $(x,w) \in \overline{\Delta}^r$, then $U \times (V \cup \{w\}) \cap \Delta \neq \emptyset \to U \cap V \neq \emptyset$. So for all r- open set U in X containing x and for all subset V of X , such that a net (χ_d) is eventually in V , $U \cap V \neq \emptyset$.

Since (χ_d) is eventually in $A_{do} = V \subseteq X$, then $A_{do} \cap U \neq \emptyset$, for all $d_o \in D$ and all r- open set contains x . Thus $x \in \overline{A_{do}}^r$, $\forall d_o \in D$, therefore Proposition (1.31) , $\chi_d \overset{r}{\propto} x$. Hence by Proposition (2.9) , X is an r- compact space .

Theorem 3.10 : Let X and Y be spaces , and $f : X \to Y$ be a continuous mapping . If Y is a T_2- space , then the following statements are equivalent :

(i) f is a st- r- proper mapping .

(ii) f is a st- r- closed mapping and $f^{-1}(\{y\})$ is r- compact for each $y \in Y$.

(iii) If $(\chi_d)_{d \in D}$ is a net in X and $y \in Y$ is an r- cluster point of $f(\chi_d)$, then there is an r- cluster point $x \in X$ of $(\chi_d)_{d \in D}$, such that $f(x) = y$.

Proof :

(i→ii). Let $f : X \to Y$ be a st- r- proper mapping , then $f \times I_Z : X\times Z \to Y\times Z$ is a st- r- closed for every space Z . Let Z = {t} , then $X\times Z = X\times\{t\} \cong X$ and $Y\times Z = Y\times\{t\} \cong Y$, and we can replace $f\times I_Z$ by f , thus f is a st- r- closed mapping . Now , let $y \in Y$, then by Proposition (3.8) , the mapping $f_{\{y\}} : f^{-1}(\{y\}) \to \{y\}$ is a st- r- proper . Thus by Theorem (3.9) , $f^{-1}(\{y\})$ is an r- compact set .

(ii→iii). Let $(\chi_d)_{d \in D}$ be a net in X and $y\in Y$ be an r- cluster point of a net $f(\chi_d)$ in Y . Assume that $f^{-1}(y) \neq \emptyset$, if $f^{-1}(y) = \emptyset$, then $y \notin f(x) \to y \in (f(X))^c$, since X is an r- closed set in X and f is a st- r- closed mapping , then $f(X)$ is an r- closed set in Y . Thus $(f(X))^c$ is an r- open set in Y . Therefore $(f(\chi_d))$ is frequently in $(f(X))^c$.

But $f(\chi_d) \in f(X)$, $\forall\, d \in D$, then $f(X) \cap (f(X))^c \neq \emptyset$, and this is a contradiction . Thus $f^{-1}(y) \neq \emptyset$, therefore $\exists\, x \in X$, such that $f(x) = y$.

Now , suppose that the statement (iii) , is not true , that means , for all $x \in f^{-1}(y)$ there exists an r- open set U_X in X contains x , such that (χ_d) is not frequently in U_X . Notice that $f^{-1}(y) = \bigcup_{x\in f^{-1}(y)} \{x\}$. Therefore the family $\{U_X \mid x \in f^{-1}(y)\}$ is an r-open cover for $f^{-1}(y)$. But $f^{-1}(y)$ is an r- compact set , thus there exists $x_1, x_2, \ldots, x_n \in f^{-1}(y)$, such that $f^{-1}(y) \subseteq Ux_1 \bigcup Ux_2 \ldots \bigcup Ux_n$, then $f^{-1}(y) \bigcap [\bigcup_{i=1}^{n} U_{xi}]^c = \emptyset \to f^{-1}(y) \bigcap [\bigcap_{i=1}^{n} U_{xi}^c] = \emptyset$. But $(x_i)_{i\in\Lambda}$ is not frequently in $\bigcup_{i=1}^{n} U_{xi}$, but $\bigcup_{i=1}^{n} U_{xi}$ is an r- open set in X , then $\bigcap_{i=1}^{n} U_{xi}^c$ is an r- closed set in X . Thus $f(\bigcap_{i=1}^{n} U_{xi}^c)$ is an r- closed set in Y .

Claim $y \notin f(\bigcap_{i=1}^{n} U_{xi}^c)$, if $y \in f(\bigcap_{i=1}^{n} U_{xi}^c)$, then there exists $x \in \bigcap_{i=1}^{n} U_{xi}^c$, such that $f(x) = y$, thus $x \notin \bigcup_{i=1}^{n} U_{xi}$, but $x \in f^{-1}(y)$, therefore $f^{-1}(y)$ is not a subset of $\bigcup_{i=1}^{n} U_{xi}$, and this is a contradiction . Hence there is an r- open set A in Y , such that $y \in A$ and $A \bigcap f(\bigcap_{i=1}^{n} U_{xi}^c) = \emptyset \to f^{-1}(A) \bigcap f^{-1}(f(\bigcap_{i=1}^{n} U_{xi}^c)) = \emptyset \to f^{-1}(A) \bigcap [\bigcap_{i=1}^{n} U_{xi}^c] = \emptyset \to$

$f^{-1}(A) \subseteq \bigcup_{i=1}^{n} U_{xi}$. But $(f(\chi_d))$ is frequently in A , then (χ_d) is frequently in $f^{-1}(A)$, and then (χ_d) is frequently in $\bigcup_{i=1}^{n} U_{xi}$ .This is contradiction , and this is complete the proof .

(iii → i). Let Z be any space . To prove that $f : X \to Y$ is a st- r- proper mapping , i.e , to prove that $f\times I_Z : X\times Z \to Y\times Z$ is a st- r- closed mapping . Let F be an r- closed set in X×Z . To prove that $(f\times I_Z)(F)$ is an r- closed set in Y×Z . Let $(y,z) \in \overline{(f\times I_Z)(F)}^{r}$, then by Proposition (1.30) , there exists a net $\{(y_d , z_d)\}_{d\in D}$ in $(f\times I_Z)(F)$ such that $(y_d , z_d) \overset{r}{\propto} (y,z)$, where $(y_d , z_d) = ((f\times I_Z)(x_d , y_d))$, and $\{(x_d , y_d)\}_{d\in D}$ is a net in F . Thus $(f(x_d) , I_Z(z_d)) \overset{r}{\propto} (y,z)$, so $f(x_d) \overset{r}{\propto} y$ and $z_d \overset{r}{\propto} z$. Then $\exists\, x \in X$, such that $x_d \overset{r}{\propto} x$ and $f(x) = y \to (x_d , z_d) \overset{r}{\propto} (x,z)$ and $\{(x_d , z_d)\}_{d\in D}$ is a net in F , thus $(x,y) \in \overline{F}^{r}$. Since $F = \overline{F}^{r}$, then $(x,y)\in F \to (y,z) = ((f\times I_Z)(x,y)) \to (y,z) \in (f\times I_Z)(F)$, and then $\overline{(f\times I_Z)(F)}^{r} = (f\times I_Z)(F)$, thus $(f\times I_Z)(F)$ is an r- closed set in Y×Z . Hence $f\times I_Z : X\times Z \to Y\times Z$ is a st- r- closed mapping . Therefore $f : X \to Y$ is a st- r- proper mapping .

Proposition 3.11 : If X is an r- compact space , then the mapping $f : X \to P = \{w\}$ on a space X is st- r- proper , where w is any point which does not belongs to X .

Proof : Let X be an r- compact space . Since P is a single point , then f is a continuous mapping . To prove that $f : X \to P = \{w\}$ is a st- r- proper mapping :

(i) Since $f^{-1}(P) = X$, then $f^{-1}(P)$ is an r- compact set .

(ii) Let F is an r- closed subset of X , then either : $f(F) = \emptyset$ or $f(F) = \{w\}$. Then f is st- r- closed mapping , hence by Theorem (3.10) , f is a st- r- proper mapping .

Proposition 3.12 : Let X , Y and Z be spaces . If $f : X \to Y$ and $g : Y \to Z$ are st-r- proper maps , then $gof : X \to Z$ is a st- r- proper mapping .

Proof : Since f and g are st- r- proper maps , then $f\times I_W$ and $g\times I_W$ are st-r- closed , for every space W , then by Proposition (1.22) , $(g\times I_W)o(f\times I_W)$ is st- r- closed mapping . But $(g\times I_W)o(f\times I_W) = (gof)\times I_W$, then $(gof)\times I_W$ is st- r- closed , and since gof is continuous . Hence gof is an st- r- proper mapping .

Proposition 3.13 : Let X , Y and Z be spaces , and $f : X \to Y$, $g : Y \to Z$ be continuous maps , such that $gof : X \to Z$ is a st- r- proper mapping . If g is one to one , r- irresolute , then f is a st- r- proper mapping .

Proof : Let W be any space . To prove that $f\times I_W : X\times W \rightarrow Y\times W$ is a st- r- closed mapping . Since $gof : X \rightarrow Z$ is a st- r- proper , then $(gof)\times I_W : X\times W \rightarrow Z\times W$ is a st- r- closed mapping , so we can write $(gof)\times I_W = (g\times I_W)o(f\times I_W)$. Since $g\times I_W$ is one to one , r- irresolute mapping , then by Proposition (1.22) , $f\times I_W$ is a st- r- closed . Hence $f : X \rightarrow Y$ is a st- r- proper mapping .

Proposition 3.14 : Let $f_1 : X_1 \rightarrow Y_1$ and $f_2 : X_2 \rightarrow Y_2$ be maps . Then $f_1\times f_2 : X_1\times X_2 \rightarrow Y_1\times Y_2$ is a st- r- proper mapping if and only if f_1 and f_2 are st- r- proper .

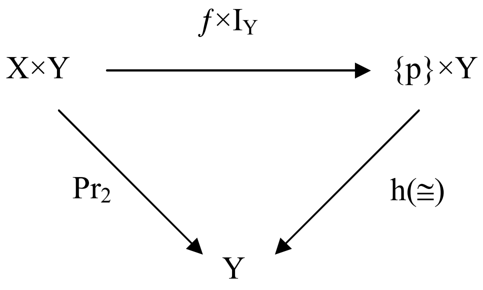
Proof : $\rightarrow$) To prove that f_2 is a st- r- proper . Since $f_1\times f_2$ is continuous , then both f_1 and f_2 are continuous . To prove that $f_2\times I_Z : X_2\times Z \rightarrow Y_2\times Z$ is st- r- closed , for every space Z . Let F be an r- closed subset of $X_2\times Z$, since X_1 is an r- closed set in X_1 , then by Proposition (1.12) , $X_1\times F$ is an r- closed set in $X_1\times X_2\times Z$. Since $f_1\times f_2$ is st- r- proper , then $(f_1\times f_2\times I_Z)(X_1\times F)$ is an r- closed set in $Y_1\times Y_2\times Z$ .But $(f_1\times f_2\times I_Z)(X_1\times F) = f_1(X_1)\times(f_2\times I_Z)(F)$, thus $f_1(X_1)\times(f_2\times I_Z)(F)$ is an r- closed set in $Y_1\times Y_2\times Z$, then by Proposition (1.12) , $(f_2\times I_Z)(F)$ is an r- closed set in $Y_2\times Z$, then $f_2\times I_Z : X_2\times Z \rightarrow Y_2\times Z$ is a st- r- closed mapping . Therefore $f_2 : X_2 \rightarrow Y_2$ is a st- r- proper mapping .

Similarly , we can prove that $f_1 : X_1 \rightarrow Y_1$ is a st- r- proper mapping .

$\leftarrow$) To prove that $f_1\times f_2 : X_1\times X_2 \rightarrow Y_1\times Y_2$ is a st- r- proper . Since f_1 and f_2 are continuous , then $f_1\times f_2$ is continuous mapping . Let Z be any space . Notice that : $f_1\times f_2\times I_Z = (Iy_1\times f_2\times I_Z)o(f_1\times Ix_2\times I_Z)$, since f_1 and f_2 are st- r- proper maps , then $(Iy_1\times f_2\times I_Z)$ and $(f_1\times Ix_2\times I_Z) = f_1\times Ix_{2\times Z}$ are st- r- closed maps . Therefore by Proposition (1.22) , the mapping $f_1\times f_2\times I_Z$ is a st- r- closed . Hence $f_1\times f_2$ is a st- r- proper mapping .

Proposition 3.15 : Let X be an r- compact space , and Y any space , then the projection $Pr_2 : X\times Y \rightarrow Y$ is a st- r- proper mapping .

Proof : Consider the commutative diagram :



Where $h : \{p\}\times Y \rightarrow Y$ is the homeomorphism of $\{p\}\times Y$ onto Y and $Pr_2 : X\times Y \rightarrow Y$ is the projection of X×Y into Y . Since X an is r- compact space , then by Proposition (3.11) , $f : X \rightarrow \{p\}$ is a st- r- proper and $I_y : Y \rightarrow Y$ is a st- r- proper , then $f\times I_y$ is a st- r- proper .

Therefore $ho(f\times I_y)$ is a st- r- proper mapping . But $Pr_2 = ho(f\times I_y)$, then Pr_2 is a st- r- proper mapping .

Proposition 3.16 : Let X and Y be spaces , and $f : X \to Y$ be a st- r- proper mapping . If F is a clopen subset of X , then the restriction map $f_{|F} : F \to Y$ is a st- r- proper mapping .

Proof : To prove that $f_{|F}\times I_Z : F\times Z \to Y\times Z$ is a st- r- closed mapping for every space Z . Since F is an clopen subset of X , then by Proposition (1.12) , $F\times Z$ is a clopen subset of $X\times Z$. Since $f\times I_Z$ is a st- r- closed mapping , then by Proposition (1.21) , $(f\times I_Z)_{F\times Z}$ is a st- r- closed mapping . But $f_{|F}\times I_Z = (f\times I_Z)_{F\times Z}$, thus $f_{|F}\times I_Z$ is a st- r- closed mapping . Since $f_{|F}$ is continuous , hence $f_{|F} : F \to Y$ is a st- r- proper mapping .

Proposition 3.17 : Let X and Y be spaces . If $f : X \to Y$ is a st- r- proper mapping , then f is a st- r- compact mapping .

Proof : Let A be an r- compact subset of Y . To prove that $f^{-1}(A)$ is an r- compact set in X , let $(\chi_d)_{d\in D}$ be a net in $f^{-1}(A)$, then $f(\chi_d)$ is a net in A . Since A is an r- compact set in Y , then by Proposition (2.9) , there exists $y \in A$, such that y is an r- cluster point of $f(\chi_d)$. Since f is st- r- proper , then by Theorem (3.10) , there exists $x \in X$, such that x is an r- cluster point of (χ_d) , and $f(x) = y$. Thus every net in $f^{-1}(A)$ has r- cluster point in itself , then by Proposition (2.9) , $f^{-1}(A)$ is an r- compact set in X .Therefore $f : X \to Y$ is a st- r- compact mapping .

The converse of Proposition (3.17) , is not true in general as the following example shows :

Example 3.18 : Let X = {a, b, c, d} , Y = {x, y, z} be sets and T = {Ø, X, {a, b}, {c, d}} , τ = {Ø, Y, {z}} be topologies on X and Y respectively . Let $f : X \to Y$ be a mapping which is defined by : $f(a) = f(b) = f(c) = y$, $f(d) = z$.

Notice that f is a st- r- compact mapping , but f is not st- r- proper mapping . Since {c,d} is an r- closed set in X , but $f(\{c,d\}) = \{y,z\}$ which is not r- closed set in Y , then f is not st- r- closed mapping .

Theorem 3.19 : Let X and Y be spaces , such that Y is a T_2 - space , and $f : X \to Y$ is a continuous , r- irresolute mapping . Then f is a st-r- proper mapping if and only if f is a st- r- compact mapping .

Proof : →) By Proposition (3.17) .

←) To prove that f is a st- r- proper mapping :

(i) Let F be an r- closed subset of X . To prove that $f(F)$ is an r- closed set in Y , let K be an r- compact set in Y , then $f^{-1}(K)$ is an r- compact set in X , then by Theorem (2.10) ,

$F\cap f^{-1}(K)$ is r- compact in X . Since f is r- irresolute , then $f(F\cap f^{-1}(K))$ is r- compact set in Y . But $f(F\cap f^{-1}(K)) = f(F)\cap K$, then $f(F)\cap K$ is r- compact , thus $f(F)$ is compactly r- closed set in Y . Since Y is T_2- space , then by Theorem (2.15) , $f(F)$ is r- closed set in Y. Hence f is a st- r- closed mapping .

(ii) Let $y\in Y$, then $\{y\}$ is r- compact in Y . Since f is a st- r- compact mapping , then $f^{-1}(\{y\})$ is r- compact in X . Therefore by (i) , (ii) and using Theorem (3.10) , f is a st- r- proper mapping .

Theorem 3.20 : Let X and Y be spaces , such that Y is a T_2 - space and $f : X \rightarrow Y$ is a continuous , r- irresolute , mapping . Then the following statements are equivalent :

(i) f is a st- r- coercive mapping .

(ii) f is a st- r- compact mapping .

(iii) f is a st- r- proper mapping .

Proof :

(i → ii). By Proposition (2.28) .

(ii → iii). By Theorem (3.19) .

(iii → i). Let J be an r- compact set in Y . Since f is a st- r- proper , then by Proposition (3.17) , f is a st- r- compact mapping , then $f^{-1}(J)$ is an r- compact set in X . Thus $f(X \setminus f^{-1}(J)) \subseteq Y \setminus J$. Hence $f : X \rightarrow Y$ is a st- r- coercive mapping .

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التطبيقات السديدة المنتظمة بقوة

الهدف الأساسي من هذا العمل هو تقديم نوع خاص و جديد للتطبيق السديد هو التطبيق السديد المنتظم بقوة . كما قدمنا تعريف جديد للتطبيق المتراص و التطبيق الأضطراري . كما تضمن البحث بعض الخواص و العبارات المتكافئة و كذلك شرحنا العلاقة بين هذه التعريفات .