

Design and Realization of Bireciprocal Lattice Wave Discrete Wavelet Filter Banks

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Abstract

In this paper, designs and efficient realizations of special types of IIR wavelet filter banks are introduced. These special types are the bireciprocal lattice wave discrete wavelet filter banks (BLW-DWFBs). It is believed that these IIR filter banks possess superior band discriminations and perfect roll-off frequency characteristics with respect to their FIR counterparts. 5th and 7th order bireciprocal lattice wave digital filters (BLWDFs) are first derived. They are designed to simulate scaling and wavelet functions of six-level wavelet transform. Such IIR filter banks are then realized as all-pass sections. Computationally-efficient realizations by such sections indicate that the resulting IIR discrete wavelet filter banks can lead to hardware implementations with less-complexity.

Keywords: All-Pass Sections, Bireciprocal Lattice Wave Filters (BLWDFs), Bireciprocal Lattice Wave Digital Discrete Wavelet Filter Banks (BLW-DWFBs), IIR Wavelet Filter Banks, Scaling And Wavelet Functions.

تصميم وتحقيق أجراف مرشحات موجية متقطعة من النوع الموجي المتشابك ثنائي التبادل

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المستخلص

في هذا البحث، قُدمت تصاميم وتحقيقات كفوءة لأنواع خاصة لأجراف مرشحات موجية ذات الدفقة غير المحدودة. هذه الأنواع تشمل أجراف مرشحات موجية متقطعة من النوع الموجي المتشابك ثنائي التبادل (BLW-DWFBs). إن أجراف المرشحات تلك هي من نوع IIR ومعروفة بأنها تمتلك تمييزاً متفوقاً وخصائص تدرج ترددي متقنة بالمقارنة مع نظرائها من نوع FIR. في البداية تم اشتقاق مرشحات رقمية من النوع الموجي المتشابك ثنائي التبادل (BLWDFs) ومن المرتبتين الخامسة والسابعة، ومن ثم صممت تلك المرشحات لمحاكاة الدوال المعيارية والموجية (scaling and wavelet functions) للتحويل الموجي المتقطع بمستويات ستة. تم بعد ذلك تحقيق أجراف المرشحات تلك كمقاطع إمرار كلية. إن التحقيق الكفوء حسابياً لمثل هذه المقاطع تُشير بأن أجراف المرشحات الموجية المقطعة نوع IIR يُمكن أن تقود إلى التنفيذ بأجزاء مادية أقل تعقيداً.

1. Introduction

The main advantage of infinite-impulse response (IIR) filter banks is that a good frequency selectivity and orthonormality are not mutually exclusive properties for IIR filter banks, as they are in a finite-impulse response (FIR) case [1]. In 2004, S. Damjanovic, *et al.* [2] presented the design and characteristics of orthonormal two-band QMF banks, with perfect reconstruction (PR) and linear-phase properties. The corresponding wavelet structures were derived with their wavelet and scaling functions. Five iterations of the analysis filter bank in low-pass branch were used for such purpose with no implementations. In 2004 also, R. Yamashita *et al.* [3] proposed an IIR half-band filter with an arbitrary degree of flatness. The application of such IIR half-band filters in filter banks and wavelets was presented. The stability of such filter was guaranteed. Several design examples were illustrated with no comments on realization.

Many other examples on the designed orthonormal wavelet transform implemented with IIR filter pairs were considered by S. Damjanovic, *et al.* [1] in 2005. Then the frequency transformations of such wavelet IIR filter banks were also presented by S. Damjanovic, *et al.* [4] in 2005. In 2006, low complexity half-band IIR filters were presented and realized by L. Milic, *et al.* [5] using two path polyphase structures utilizing all-pass filters as components. Such realization was accomplished with no-phase linearity. Recently, many attempts have been reported concerning FPGA implementations and other hardware of two-band wavelet IIR filter banks [6]-[8].

In this paper, new solutions are presented for the design problem of generating the wavelet transform by iteration of orthonormal two-band power-complementary IIR wavelet filter banks, with PR properties. The structure of such IIR wavelet filter bank is based on the bireciprocal lattice wave digital filters (BLWDF) to simulate a two channel wavelet filter bank, resulting in a bireciprocal lattice wave discrete wavelet filter bank (BLW-DWFB). The designed lattice structures are composed of two parallel real all-pass sections. 2nd order to 1st order all-pass filter section reduction method is applied via down sampling position alteration in the designed structure. Consequently, the resulting BLW-DWFB structures possess efficient computations. Suggesting, 1st order wave adaptors for the hardware implementations of all 1st order all-pass filter sections in the structure, open the way for efficient FPGA implementations of such BLW-DWFBs. The generations of scaling and wavelet functions concerning these structures are also conducted in this paper.

This paper is divided into five major sections. Besides this introductory section, the next section reviews the design and realization of IIR wavelet filter banks. In section 3, the proposed design methods for 5th and 7th order IIR PR wavelet filter banks are described. Software and hardware efficient realizations are presented in Section 4. Section 5 includes the generation of scaling and wavelet functions after six-level analysis filter banks. Section 6 highlights some conclusions.

2. IIR Wavelet Filter Banks

A very efficient way for representing the QMF bank can be obtained by using polyphase structure [9]. QMF banks, composed of two all-pass filters, are known to be one of the best circuits for building up a multi-channel IIR filter banks. They can completely eliminate the aliasing error and amplitude distortion [10]. Fig. 1 shows a two channel all-pass filter based IIR QMF banks with polyphase realization [11].

In Fig. 1 the polyphase components are the 2nd order all-pass filters $A_0(z^2)$ and $A_1(z^2)$ with the following transfer functions:

$$A_0(z^2) = \prod_{i=2,4,\dots}^{(N+1)/2} \frac{\alpha_i + z^{-2}}{1 + \alpha_i z^{-2}} \quad \dots (1)$$

and

$$A_1(z^2) = \prod_{i=3,5,\dots}^{(N+1)/2} \frac{\alpha_i + z^{-2}}{1 + \alpha_i z^{-2}} \quad \dots (2)$$

where α_i is the value of the multiplier coefficient in the i^{th} all-pass section.

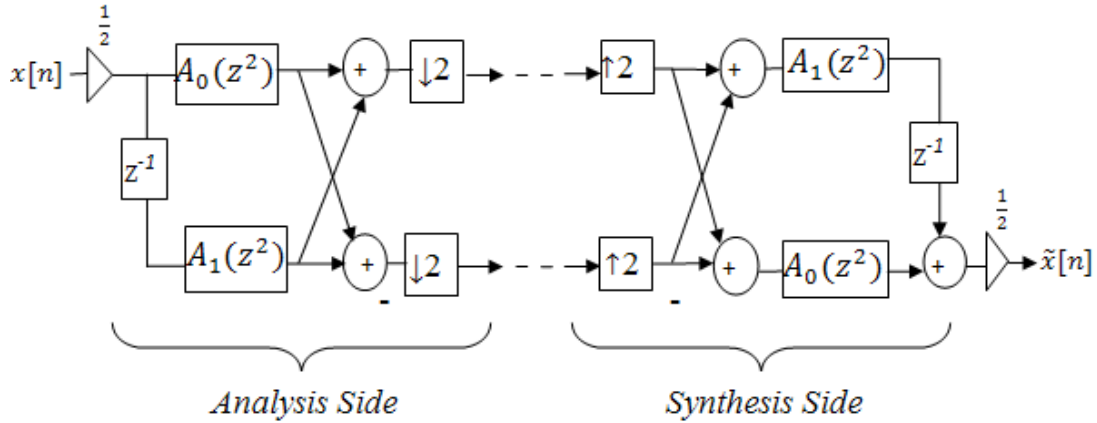


Fig. 1 Polyphase realization of the IIR wavelet filter bank.

Let $H_0(z)$ and $H_1(z)$ denote the transfer functions of the lowpass and highpass filter of the analysis part of the two-channel quadrature-mirror filter (QMF) bank, and let $G_0(z)$ and $G_1(z)$ denote, respectively the transfer functions of the lowpass and highpass filter of the synthesis part. By choosing transfer functions to satisfy the following conditions: $H_1(z) = H_0(-z)$, $G_0(z) = H_0(z^{-1})$ and $G_1(z) = H_1(z^{-1})$, then, the filter bank will possess both perfect reconstruction and orthonormality properties [12]. Analysis filters $H_0(z)$ and $H_1(z)$ can respectively, be written for the low-pass side as

$$H_0(z) = \frac{1}{2} [A_0(z^2) + z^{-1}A_1(z^2)] \quad \dots(3)$$

and for the high-pass side as

$$H_1(z) = \frac{1}{2} [A_0(z^2) - z^{-1}A_1(z^2)] \quad \dots (4)$$

3. The Proposed Design for IIR PR Wavelet Filter Banks

In this section, analytic solutions for the design problems of intermediate filters (whose characteristics are between IIR Butterworth and Daubechies filters) are proposed. The half band filter's poles are placed on an imaginary axis of the complex z -plane, where one of them is placed at the origin and the remaining conjugate complex pole pairs are located between $e^{-j\pi}$ and $e^{j\pi}$. All zeros of such filters are located on the unit circle, where three of them are placed at $z = -1$ for 5th order filter and five of them at $z = -1$ for 7th order filter to meet the flatness condition.

In applications of filter banks and wavelets, an important role is played by the regularity of the low-pass prototype filter; a feature which is closely related to the flatness on the magnitude response of the filter at the nyquist frequency $\omega = \pi$. In constructing orthonormal bases of wavelets from iterated filter banks, a greater number of zeros of the low-pass filter at $\omega = \pi$ results in more regular wavelets [13]. On the other hand, the number of vanishing moments of the wavelets can be obtained by the multiplicity of zeros at $z = -1$. It can be shown that having a maximum number of zeros at $z = -1$, implies a maximally flat characteristic for the filters involved [14]. As an example, the Butterworth filter has a maximally flat magnitude response as it has all zeros at $z = -1$ and the highest possible regularity order, but it has the worst frequency selectivity.

3.1 5th order intermediate IIR filter design

This proposed filter, as shown in Fig. 2, has five zeros and five poles. The values of these poles and zeros can be found out depending on the desired magnitude response that is shown in Fig. 3.

The transfer function of such IIR filter depends on the positions of its poles and zeros as in Fig. 2. Such transfer function is given by

$$H(z) = \frac{k[(z + 1)^3[(z + a)^2 + b^2]]}{z(z^2 + \alpha^2)(z^2 + \beta^2)} \quad \dots (5)$$

where a, b, α and β are constants less than 1, as shown in Fig. 2, with k as a magnitude scaling factor. Equation (5) can be factorized and reordered as

$$H(z) = \frac{k[(z + 1)(z^2 + 2z + 1)(z^2 + 2az + a^2 + b^2)]}{z(z^4 + \beta^2z^2 + \alpha^2z^2 + \alpha^2\beta^2)}$$

or

$$(z) = \frac{k[z^5 + (2a + 3)z^4 + (a^2 + b^2 + 6a + 3)z^3 + (3a^2 + 3b^2 + 6a + 1)z^2 + (3a^2 + 3b^2 + 2a)z + (a^2 + b^2)]}{z^5 + (\alpha^2 + \beta^2)z^3 + \alpha^2\beta^2z} \quad \dots (6)$$

Equation (6) is found as a function of z . In order to be used in terms of z^{-1} , both numerator and denominator of (6) can be multiplied by z^{-5} , yielding

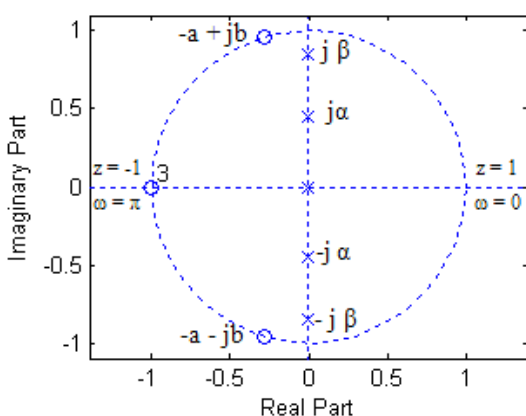


Fig. 2 Pole-zero plots for 5th order intermediate IIR half-band filter.

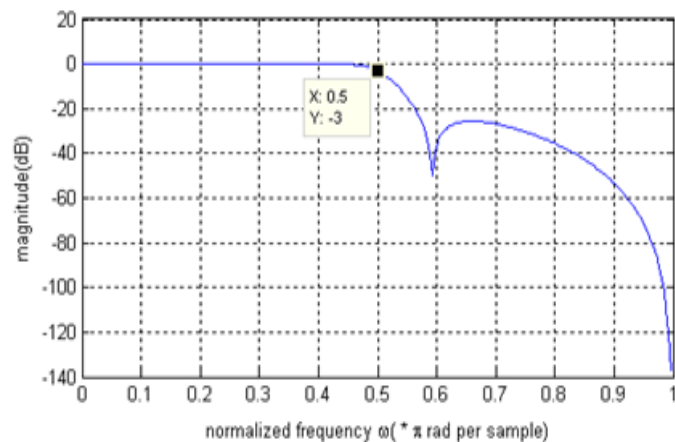


Fig. 3 Magnitude response for 5th order intermediate IIR half-band filter.

$$H(z) = \frac{k[1 + (2a + 3)z^{-1} + (a^2 + b^2 + 6a + 3)z^{-2} + (3a^2 + 3b^2 + 6a + 1)z^{-3} + (3a^2 + 3b^2 + 2a)z^{-4} + (a^2 + b^2)z^{-5}]}{1 + (\alpha^2 + \beta^2)z^{-2} + \alpha^2\beta^2z^{-4}} \quad \dots (7)$$

Finding the transfer function of such 5th order filter means an intermediate IIR filter has been concluded. The corresponding IIR filter can be implemented on BLWDF bases as a parallel connection of two all-pass IIR sections of the type shown in Fig. 1. By Gazsi method [15] that uses an alternative pole technique as illustrated in Fig. 4, the transfer functions of the two all-pass sections $A_0(z^2)$ and $A_1(z^2)$ can then be derived.

In Fig. 4, the poles of the area R_1 can formulate the transfer function of the all-pass section $A_0(z^2)$ and the poles of the area R_2 can formulate the transfer function of the all-pass section $A_1(z^2)$. The pole at the origin represents the delay element z^{-1} in (3).

So,

$$A_0(z^2) = \frac{\alpha^2 + z^{-2}}{1 + \alpha^2 z^{-2}} \quad \dots (8)$$

and

$$A_1(z^2) = \frac{\beta^2 + z^{-2}}{1 + \beta^2 z^{-2}} \quad \dots (9)$$

Substituting (8) and (9) in (3) to form the low-pass transfer function of BLWDF as

$$H_o(z) = \frac{1}{2} \left(\frac{\alpha^2 + z^{-2}}{1 + \alpha^2 z^{-2}} + z^{-1} \frac{\beta^2 + z^{-2}}{1 + \beta^2 z^{-2}} \right) \quad \dots (10)$$

or

$$H_o(z) = \frac{1}{2} \left(\frac{\alpha^2 + \beta^2 z^{-1} + (1 + \alpha^2 \beta^2)z^{-2} + (1 + \alpha^2 \beta^2)z^{-3} + \beta^2 z^{-4} + \alpha^2 z^{-5}}{1 + (\beta^2 + \alpha^2)z^{-2} + \alpha^2 \beta^2 z^{-4}} \right) \quad \dots (11)$$

The filter coefficients can be calculated by equating (11) to the general filter function (7). Referring to Fig. 2, the following equation can be written

$$\alpha^2 + b^2 = 1 \quad \dots (12)$$

and from Fig. 3, the magnitude response at $\omega = 0.5\pi$ is

$$|H(e^{j0.5\pi})| = \frac{1}{\sqrt{2}} \quad \dots (13)$$

Substituting (12) in (7), yields

$$H(z) = \frac{k[1 + (2a + 3)z^{-1} + (1 + 6a + 3)z^{-2} + (3 + 6a + 1)z^{-3} + (3 + 2a)z^{-4} + z^{-5}]}{1 + (\alpha^2 + \beta^2)z^{-2} + \alpha^2\beta^2z^{-4}} \quad \dots (14)$$

Also, substituting $z^{-1} = e^{-j\omega}$ in equation (14), results in

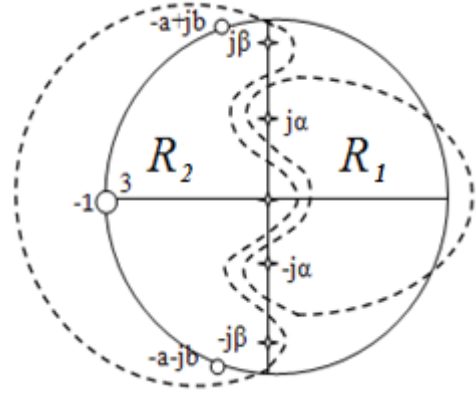


Fig. (4) 5th order alternative pole technique.

$$H(e^{j\omega}) = \frac{k[1 + (2a + 3)e^{-j\omega} + (6a + 4)e^{-j2\omega} + (6a + 4)e^{-j3\omega} + (3 + 2a)e^{-j4\omega} + e^{-j5\omega}]}{1 + (\alpha^2 + \beta^2)e^{-j2\omega} + \alpha^2\beta^2e^{-j4\omega}} \quad \dots (15)$$

Using Euler formula, the magnitude response is then given by

$$|H(e^{j\omega})| = \frac{k\{[1 + (2a + 3)(\cos\omega + \cos4\omega) + (6a + 4)(\cos2\omega + \cos3\omega) + \cos5\omega]^2 + [-(2a + 3)(\sin\omega + \sin4\omega) - (6a + 4)(\sin2\omega + \sin3\omega) - \sin5\omega]^2\}^{\frac{1}{2}}}{\{[1 + (\alpha^2 + \beta^2)\cos2\omega + \alpha^2\beta^2\cos4\omega]^2 + [-(\alpha^2 + \beta^2)\sin2\omega - \alpha^2\beta^2\sin4\omega]^2\}^{\frac{1}{2}}} \quad \dots (16)$$

By assuming initial values for a , b , α , and β . The value of k can be obtained numerically to be equal to 0.1, so that $|H(e^{j0})| = 1$.

Substituting (13) in (16), yields

$$\frac{1}{\sqrt{2}} = \frac{0.565685424 a}{1 - \alpha^2 - \beta^2 + \alpha^2\beta^2} \quad \dots (17)$$

Expressing a in terms of other variables, yields

$$a = 1.25(1 - \alpha^2 - \beta^2 + \alpha^2\beta^2) \quad \dots (18)$$

Comparing numerator of (11) with numerator of (7), results in the followings:

For the absolute terms

$$0.5\alpha^2 = 0.1 \quad \implies \alpha^2 = 0.2 \quad \dots (19)$$

For z^{-1} terms

$$0.5\beta^2 = 0.3 + 0.2a \quad \dots (20)$$

Substituting (18) in (20), yields

$$0.5\beta^2 = 0.3 + 0.25(1 - \alpha^2 - \beta^2 + \alpha^2\beta^2) \quad \dots (21)$$

So, substituting (19) in (21), yields

$$0.5\beta^2 = 0.3 + 0.25 - 0.05 - 0.25\beta^2 + 0.05\beta^2$$

Thus,

$$\beta^2 = 0.714285714 \quad \dots (22)$$

For z^{-2} terms

$$0.4 + 0.6a = 0.5 + 0.5\alpha^2\beta^2 \quad \dots (23)$$

The substitution of (19) and (22) in (23) yields

$$a = 0.285714285 \quad \dots (24)$$

And substituting (24) in (12), yields $0.081632652 + b^2 = 1$, or

$$b^2 = 0.918367347 \quad \dots (25)$$

The transfer functions of these two all-pass filter sections depending on (8) and (9) respectively, can be written as

$$A_o(z^2) = \frac{0.2 + z^{-2}}{1 + 0.2 z^{-2}} \quad \dots (26)$$

and

$$A_1(z^2) = \frac{0.714285714 + z^{-2}}{1 + 0.714285714z^{-2}} \quad \dots (27)$$

The resulting coefficients of (19) and (22) can be substituted in (11) in order to find out the final form of the transfer function of the intermediate IIR filter as a 5th order BLWDF low-pass transfer function with

$$H_o(z) = \frac{0.1 + 0.357142857z^{-1} + 0.571428571z^{-2} + 0.571428571z^{-3} + 0.357142857z^{-4} + 0.1z^{-5}}{1 + 0.914285714z^{-2} + 0.142857142z^{-4}} \quad \dots (28)$$

The corresponding transfer function of the high-pass filter can be written as

$$H_1(z) = \frac{0.1 - 0.357142857z^{-1} + 0.571428571z^{-2} - 0.571428571z^{-3} + 0.357142857z^{-4} - 0.1z^{-5}}{1 + 0.914285714z^{-2} + 0.142857142z^{-4}} \quad \dots (29)$$

3.2 7th order intermediate IIR filter design

This proposed filter as shown in Fig. 5 has seven zeros and seven poles. Their values can be obtained depending on the desired magnitude response that is shown in Fig. 6.

The transfer functions of such filter depends on its pole-zero location of Fig. 5 and can be given by

$$H(z) = \frac{k[(z + 1)^5[(z + a)^2 + b^2]]}{z(z^2 + \alpha^2)(z^2 + \beta^2)(z^2 + \gamma^2)} \quad \dots (30)$$

$$H(z) = \frac{k[z^7 + (2a + 5)z^6 + (a^2 + b^2 + 10a + 10)z^5 + (5a^2 + 5b^2 + 20a + 10)z^4 + (10a^2 + 10b^2 + 20a + 5)z^3 + (10a^2 + 10b^2 + 10a + 1)z^2 + (5a^2 + 5b^2 + 2a)z + (a^2 + b^2)]}{z^7 + (\gamma^2 + \beta^2 + \alpha^2)z^5 + (\beta^2\gamma^2 + \alpha^2\gamma^2 + \alpha^2\beta^2)z^3 + \alpha^2\beta^2\gamma^2z} \quad \dots (31)$$

Equation (31) can be rewritten in terms of z^{-1} as

$$H(z) = \frac{k[1 + (2a + 5)z^{-1} + (a^2 + b^2 + 10a + 10)z^{-2} + (5a^2 + 5b^2 + 20a + 10)z^{-3} + (10a^2 + 10b^2 + 20a + 5)z^{-4} + (10a^2 + 10b^2 + 10a + 1)z^{-5} + (5a^2 + 5b^2 + 2a)z^{-6} + (a^2 + b^2)z^{-7}]}{1 + (\gamma^2 + \beta^2 + \alpha^2)z^{-2} + (\beta^2\gamma^2 + \alpha^2\beta^2 + \alpha^2\gamma^2)z^{-4} + \alpha^2\beta^2\gamma^2z^{-6}} \quad \dots (32)$$

After calculating the transfer function of the 7th order intermediate IIR filter, this IIR filter can be implemented on BLWDF bases as a parallel connection of two all-pass sections as in Fig. 1. The same Gazsi method as shown in Fig. 7 can be used to evaluate the transfer function $A_o(z^2)$ and $A_1(z^2)$ of these all-pass sections.

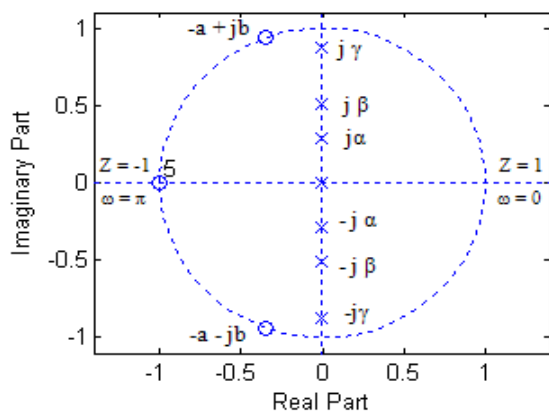


Fig.5 Pole-zero plots for 7th order intermediate IIR half-band filter.

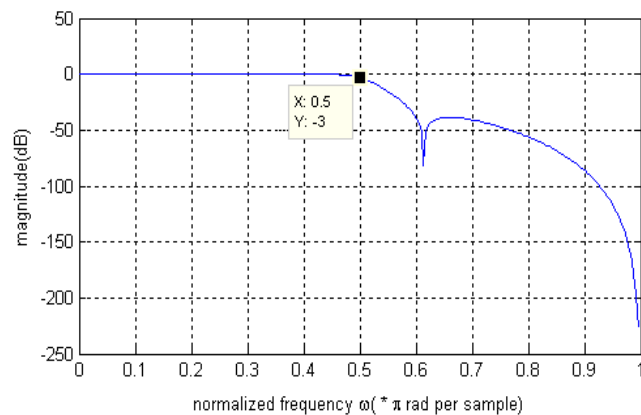


Fig. 6 Magnitude response for 7th order intermediate IIR half-band filter.

In Fig. 7, the poles of the area R_1 and the poles of the area R_2 can formulate the transfer functions of the all-pass filter sections $A_1(z^2)$ and $A_0(z^2)$, respectively. Here, the pole at the origin also represents the delay element z^{-1} in (3). So

$$A_0(z^2) = \frac{\alpha^2 + z^{-2}}{1 + \alpha^2 z^{-2}} \cdot \frac{\gamma^2 + z^{-2}}{1 + \gamma^2 z^{-2}} \quad \dots (33)$$

which can be rewritten as

$$A_0(z^2) = \frac{\alpha^2 \gamma^2 + (\alpha^2 + \gamma^2)z^{-2} + z^{-4}}{1 + (\alpha^2 + \gamma^2)z^{-2} + \alpha^2 \gamma^2 z^{-4}} \quad \dots (34)$$

and

$$A_1(z^2) = \frac{\beta^2 + z^{-2}}{1 + \beta^2 z^{-2}} \quad \dots (35)$$

Substituting (34) and (35) in (3) to form the corresponding low-pass transfer function of BLWDF,

$$H_o(z) = \frac{1}{2} \left(\frac{\alpha^2 \gamma^2 + (\alpha^2 + \gamma^2)z^{-2} + z^{-4}}{1 + (\alpha^2 + \gamma^2)z^{-2} + \alpha^2 \gamma^2 z^{-4}} + z^{-1} \frac{\beta^2 + z^{-2}}{1 + \beta^2 z^{-2}} \right) \quad \dots (36)$$

The final form of (36) can be written as

$$H_o(z) = \frac{1}{2} \left(\frac{\alpha^2 \gamma^2 + \beta^2 z^{-1} + (\alpha^2 + \gamma^2 + \alpha^2 \gamma^2 \beta^2)z^{-2} + (1 + \alpha^2 \beta^2 + \gamma^2 \beta^2)z^{-3} + (1 + \alpha^2 \beta^2 + \gamma^2 \beta^2)z^{-4} + (\alpha^2 + \gamma^2 + \alpha^2 \gamma^2 \beta^2)z^{-5} + \beta^2 z^{-6} + \alpha^2 \gamma^2 z^{-7}}{1 + (\alpha^2 + \beta^2 + \gamma^2)z^{-2} + (\alpha^2 \gamma^2 + \alpha^2 \beta^2 + \gamma^2 \beta^2)z^{-4} + \alpha^2 \gamma^2 \beta^2 z^{-6}} \right) \quad \dots (37)$$

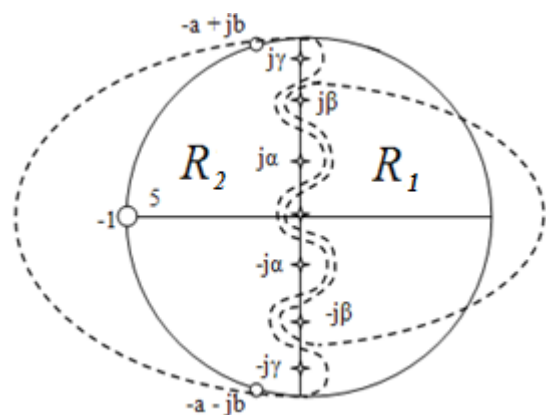


Fig. 7 7th order alternative pole technique.

In order to calculate the filter coefficients, (37) must equal to (32) and from Figs. 5 & 6, one can use a similar method as in the previous 5th order intermediate IIR filter to perform such calculations.

Substituting (12) in (32), yields

$$H(z) = \frac{k[1 + (2a + 5)z^{-1} + (10a + 11)z^{-2} + (20a + 15)z^{-3} + (20a + 15)z^{-4} + (10a + 11)z^{-5} + (2a + 5)z^{-6} + z^{-7}]}{1 + (\gamma^2 + \beta^2 + \alpha^2)z^{-2} + (\beta^2\gamma^2 + \alpha^2\beta^2 + \alpha^2\gamma^2)z^{-4} + \alpha^2\beta^2\gamma^2z^{-6}} \quad \dots (38)$$

Then, substituting $z^{-1} = e^{-j\omega}$ in (38), yields

$$H(e^{j\omega}) = \frac{k[1 + (2a + 5)e^{-j\omega} + (10a + 11)e^{-j2\omega} + (20a + 15)e^{-j3\omega} + (20a + 15)e^{-j4\omega} + (10a + 11)e^{-j5\omega} + (2a + 5)e^{-j6\omega} + e^{-j7\omega}]}{1 + (\gamma^2 + \beta^2 + \alpha^2)e^{-j2\omega} + (\beta^2\gamma^2 + \alpha^2\beta^2 + \alpha^2\gamma^2)e^{-j4\omega} + \alpha^2\beta^2\gamma^2e^{-j6\omega}} \quad \dots (39)$$

Using Euler formula, the magnitude response is then given b

$$|H(e^{j\omega})| = \frac{k\{[1 + (2a + 5)(\cos\omega + \cos6\omega) + (10a + 11)(\cos2\omega + \cos5\omega + (20a + 15)(\cos3\omega + \cos4\omega) + \cos7\omega]^2 + [- (2a + 5)(\sin\omega + \sin6\omega) - (10a + 11)(\sin2\omega + \sin5\omega) - (20a + 15)(\sin3\omega + \sin4\omega) - \sin7\omega]^2\}^{\frac{1}{2}}}{\{[1 + (\gamma^2 + \beta^2 + \alpha^2)\cos2\omega + (\beta^2\gamma^2 + \alpha^2\beta^2 + \alpha^2\gamma^2)\cos4\omega + \alpha^2\beta^2\gamma^2\cos6\omega]^2 + [-(\gamma^2 + \beta^2 + \alpha^2)\sin2\omega - (\beta^2\gamma^2 + \alpha^2\beta^2 + \alpha^2\gamma^2)\sin4\omega - \alpha^2\beta^2\gamma^2\sin6\omega]^2\}^{\frac{1}{2}}} \quad \dots (40)$$

By assuming initial values for a , b , α , β and γ , the value of k can be obtained numerically to be 0.03, so that $|H(e^{j0})| = 1$.

Then substituting (13) in (40), yields

$$\frac{1}{\sqrt{2}} = \frac{0.339411255 a}{1 - \gamma^2 - \beta^2 - \alpha^2 + \beta^2\gamma^2 + \alpha^2\beta^2 + \alpha^2\gamma^2 - \alpha^2\beta^2\gamma^2} \quad \dots (41)$$

Expressing a in terms of other variables, yields

$$a = 2.083(1 - \gamma^2 - \beta^2 - \alpha^2 + \beta^2\gamma^2 + \alpha^2\beta^2 + \alpha^2\gamma^2 - \alpha^2\beta^2\gamma^2) \quad \dots (42)$$

Comparing numerator of (37) with numerator of (32) results in the followings:

for the absolute terms

$$0.5\alpha^2\gamma^2 = 0.03 \quad \dots (43)$$

for z^{-1} terms

$$0.5\beta^2 = 0.15 + 0.06 a \quad \dots (44)$$

for z^{-2} terms

$$0.5\alpha^2 + 0.5\gamma^2 + 0.5\alpha^2\gamma^2\beta^2 = 0.33 + 0.3 a \quad \dots (45)$$

for z^{-3} terms

$$0.5 + 0.5\alpha^2\beta^2 + 0.5\gamma^2\beta^2 = 0.45 + 0.6 a \quad \dots (46)$$

Equations (43) - (46) can be solved numerically to find out that

$$\begin{aligned} \alpha^2 = 0.082369 & \implies \alpha = 0.287 \\ \beta^2 = 0.2616 & \implies \beta = 0.51146847 \\ \gamma^2 = 0.755161 & \implies \gamma = 0.869 \\ a^2 = 0.12117361 & \implies a = 0.3481 \\ b^2 = 0.87890625 & \implies b = 0.9375 \end{aligned}$$

The transfer functions of these two all-pass filters depending on (34) and (35) respectively, can be written as

$$A_0(z^2) = \frac{0.0622 + 0.8375z^{-2} + z^{-4}}{1 + 0.8375 z^{-2} + 0.0622 z^{-4}} \quad \dots (47)$$

and

$$A_1(z^2) = \frac{0.2616 + z^{-2}}{1 + 0.2616z^{-2}} \quad \dots (48)$$

The resulting intermediate IIR 7th order BLWDF low-pass transfer function depending on (37) can be written as

$$H_0(z) = \frac{0.03 + 0.1308z^{-1} + 0.4269z^{-2} + 0.6096z^{-3} + 0.6096z^{-4} + 0.4269z^{-5} + 0.1308z^{-6} + 0.03z^{-7}}{1 + 1.09913 z^{-2} + 0.28129 z^{-4} + 0.01627z^{-6}} \quad \dots (49)$$

The corresponding transfer function of the high-pass filter can then be given by

$$H_1(z) = \frac{0.03 - 0.1308z^{-1} + 0.4269z^{-2} - 0.6096z^{-3} + 0.6096z^{-4} - 0.4269z^{-5} + 0.1308z^{-6} - 0.03z^{-7}}{1 + 1.09913 z^{-2} + 0.28129 z^{-4} + 0.01627z^{-6}} \quad \dots (50)$$

4. Efficient Realizations

Filtration by 2nd order all-pass filter section followed by downsampling is equivalent to having downsampling first, followed by the 1st order all-pass filter section. Because of such lower sampling rate utilization, saving in computations by a factor of 2 will be achieved [16]. In Fig. 1, the polyphase components are the 2nd order all-pass filter sections $A_0(z^2)$ and $A_1(z^2)$ followed by down samplers. Their transfer functions can respectively, take the same forms of (8) and (9). In Fig. 8, the polyphase components are first down sampled followed by all-pass filter sections $A_0(z)$ and $A_1(z)$. The expressions and their parameters for such all-pass filter transfer functions in 5th and 7th orders BLWDFBs are illustrated in Table-1.

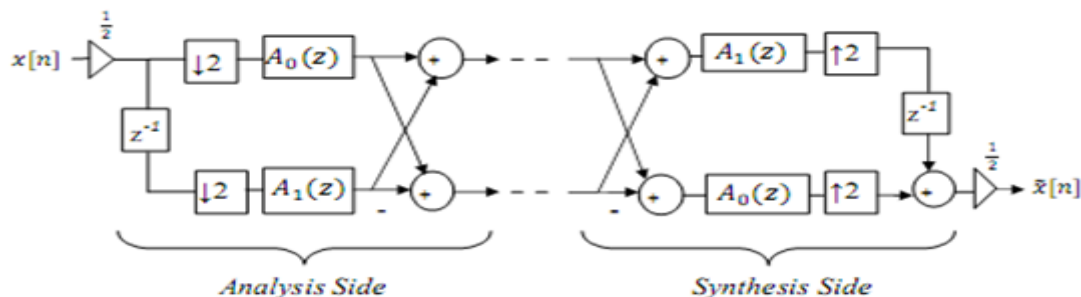


Fig. 8 A computationally efficient realization the IIR wavelet filter bank.

Table-1 Expressions for all-pass sections $A_0(z)$, $A_1(z)$ and their parameters.

All-Pass Section or Parameter	Expressions and values	
	5 th order BLWDFB	7 th order BLWDFB
$A_0(z)$	$\frac{\alpha^2 + z^{-1}}{1 + \alpha^2 z^{-1}}$	$\frac{\alpha^2 + z^{-1}}{1 + \alpha^2 z^{-1}} \cdot \frac{\gamma^2 + z^{-1}}{1 + \gamma^2 z^{-1}}$
$A_1(z)$	$\frac{\beta^2 + z^{-1}}{1 + \beta^2 z^{-1}}$	$\frac{\beta^2 + z^{-1}}{1 + \beta^2 z^{-1}}$
α^2	0.2	0.082369
β^2	0.714285714	0.2616
γ^2	—	0.755161

In addition to the efficient computations of the realization in Fig. 8, the 1st order wave adaptor in Fig. 9 [17] can be utilized to construct $A_0(z)$ and $A_1(z)$ in Fig. 8. Therefore, Fig. 8 becomes also more efficient in hardware implementations than the realization in Fig. 1 because the filter's order is reduced to half. That will minimize the overall hardware complexity and consequently, a reduced chip area is required by such filter bank for implementation. That is why Fig. 8 is a candidate BLWDF bank structure to be implemented as an IIR wavelet transform on an FPGA chip.

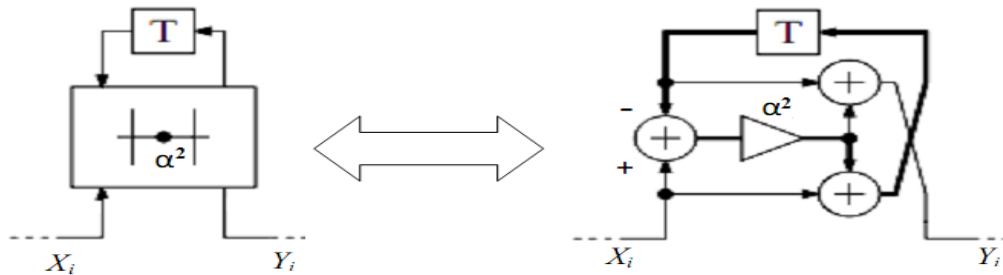


Fig. 9 First-order all-pass section for LWDFs, the multiplier coefficient α^2 may be changed to β^2 or γ^2 according to different all-pass sections.

Analysis filters $H_0(z)$ and $H_1(z)$ of Fig. 8, can, respectively be expressed as

$$H_0(z) = \frac{1}{2} [A_0(z) + z^{-1}A_1(z)] \quad \dots(51)$$

and

$$H_1(z) = \frac{1}{2} [A_0(z) - z^{-1}A_1(z)] \quad \dots (52)$$

The corresponding synthesis filters $G_0(z)$ and $G_1(z)$ can then be obtained according to the following QMF bank conditions: $H_1(z) = H_0(-z)$, $G_0(z) = H_0(z)$ and $G_1(z) = -H_0(-z)$; that because of existence of the mirror image symmetry (about the frequency $\omega = \pi/2$) between $H_0(e^{j\omega})$ and $H_1(e^{j\omega})$ [11].

In this stage, it can be said that, we arrive at well-designed BLW-DWFB structures. A property that indicates the ability of such design procedure to result in wavelet IIR filter banks utilizing reduced order bireciprocal lattice wave filters instead of their original 5th and 7th orders. In addition, these structures can efficiently be realized, since the overall designed 5th (or 7th) order wavelet filter bank can finally be realized only with two (or three) multiplier coefficients.

5. Generations of Scaling and Wavelet Functions

The basis vectors; mother wavelet and scaling functions of orthogonal wavelet transform are obtained from the iteration of the filter bank on its low-pass branch [4]. With the

additional "flatness" constraint, that the low-pass filter should have M zeros at half the sampling frequency. A scaling function is used to approximate an input signal at different levels of approximation. Each approximation is differed by a factor of two from the approximation at the nearest neighboring level. It should be noted that wavelet function is a type of scaling function [18]. Iteration of filter bank generates equivalent band-pass filters of the form [4]

$$\Psi^i(z) = H_0(z) H_0(z^2) H_0(z^4) \dots \dots H_0(z^{2^{i-2}}) H_1(z^{2^{i-1}}) \dots (53)$$

Letting $i \rightarrow \infty$ gives the "mother wavelet" $\psi(t)$:

$$\psi(t) = \lim_{i \rightarrow \infty} \psi^i_{[2^i t]} \dots (54)$$

where $\psi^i[n]$ is the impulse response of $\Psi^i(z)$. The impulse response $\phi^i[n]$ of the equivalent low-pass filters of the form

$$\phi^i(z) = H_0(z) H_0(z^2) H_0(z^4) \dots \dots H_0(z^{2^{i-2}}) H_0(z^{2^{i-1}}) \dots (55)$$

is referred to as scaling function after i iterations.

In this section, a six-level orthogonal IIR wavelet filter bank of the type shown in Fig. 10 is proposed. It is realized by intermediate IIR filter pairs for the approximate generation of the corresponding wavelet (mother wavelet) and scaling functions.

The mother wavelet and scaling functions can be generated after six iterations of the analysis filter banks on its low-pass branch. Iterations of the bank generate equivalent band-pass filter of the form

$$\Psi(z) = H_0(z) H_0(z^2) H_0(z^4) H_0(z^8) H_0(z^{16}) H_1(z^{32}) \dots (56)$$

The impulse response of (56) is referred to as mother wavelet $\psi(n)$, while the impulse response of the equivalent low-pass filter given by the form

$$\phi(z) = H_0(z) H_0(z^2) H_0(z^4) H_0(z^8) H_0(z^{16}) H_0(z^{32}) \dots (57)$$

is referred to as scaling function $\phi(n)$.

When all-pass IIR filter sections $A_0(z)$ and $A_1(z)$ in Fig. 10 are realized as 5th or 7th orders BLWDFs and the Dirac impulse signal is employed as input signal to the system, the output signals will represent the corresponding wavelet and scaling functions. These functions are shown in Fig. 11 for the 5th order filter and in Fig. 12 for the 7th order. They are generated after six iterations of the analysis filter bank on its low-pass branch according to (56) and (57), respectively.

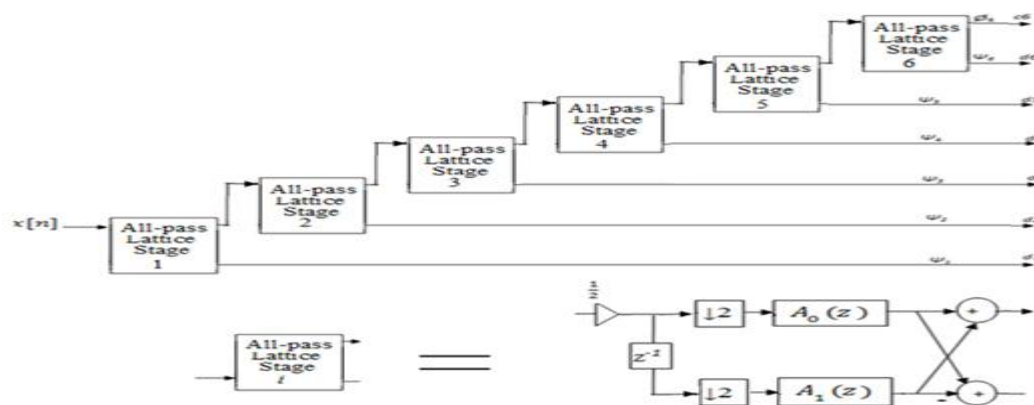


Fig. 10 Six-level wavelet filter bank based on efficient BLWDF realization: Analysis part.

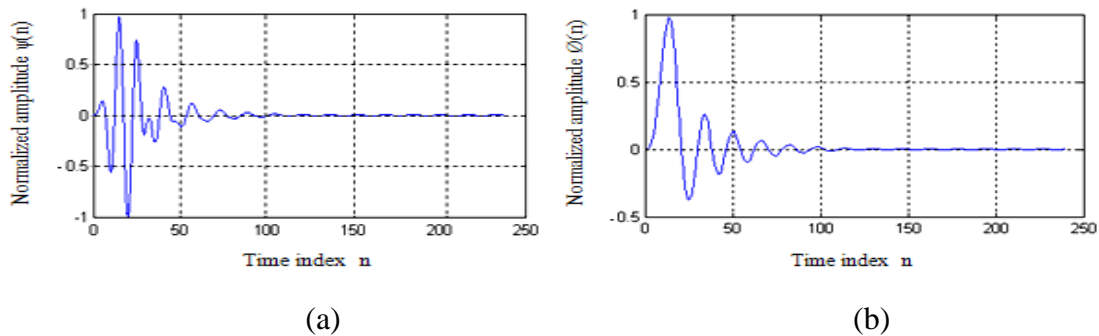


Fig. 11 5th order BLWDF bank: (a) wavelet function (b) scaling function.

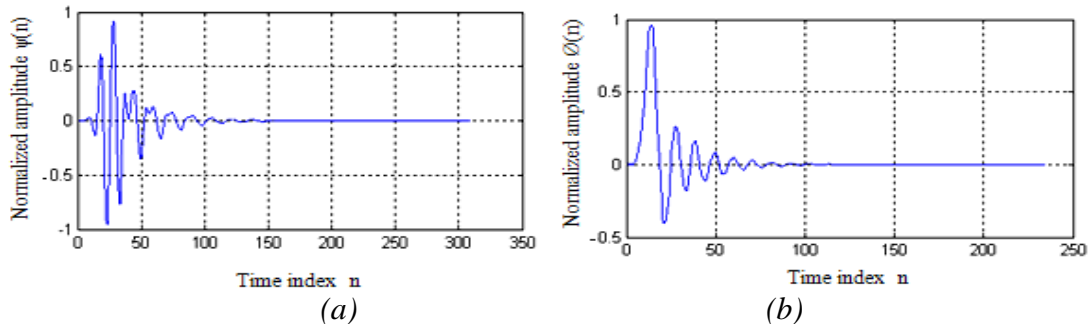


Fig. 12 7th order BLWDF bank: (a) wavelet function (b) scaling function.

6. Conclusions

Designs and efficient realizations of bireciprocal lattice wave discrete wavelet filter banks (BLW-DWFBs) have been introduced. 5th and 7th order bireciprocal lattice wave digital filters (BLWDFs) have been derived and well-designed to suit for wavelet filter bank utilizing all-pass sections with computationally-efficient realizations. The resulting IIR discrete wavelet filter banks can be hardware-implemented with less-complexity.

Moreover, Applying Sum-Power-Two method for the implementations of all multiplier values in different all-pass sections, Promising efficient multiplierless implementations can be obtained for such IIR wavelet filter banks on FPGA. This is to be considered as a future work concerning the FPGA implementations of these IIR wavelet filter banks.

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