Optimum selection of truss joints coordinates

SALAH NOORI ABBOOD
Department of mechanical engineering, College of engineering,
University of Kerbala, Iraq.
jousef_salah@yahoo.ca phone: 07803822664

Abstract

The overall stiffness of trusses can be improved by increasing external supports along the trusses length. In this paper, an improvement in truss stiffness was satisfied by minimizing the strain energy of the loaded truss without adding any external supports or change in elements material or cross sectional areas. This was done by changing some joints coordinates and makes an optimum selection to the angle of the truss model's free end which was found as (61.46^o).

ملخص البحث

يمكن تحسين الجساءة الكلية للهياكل المشبكة باستخدام مساند ثابتة اضافية على طول المنشأ. تم في هذا البحث تحسين جساءة الهيكل المشبك بتقليص طاقة الانفعال الى أقل مايمكن بدون اضافة أية مساند خارجية أو اجراء اي تعديل على معدن الاعمدة المشكلة للمنشأ أو تغيير مساحة المقطع لها. أمكن الوصول الى ذلك من خلال اجراء بعض التعديلات على احداثيات بعض المفاصل والاختيار الأمثل لزاوية النهاية الحرة للمنشأ . حيث تبين أن أفضل زاوية حرة للمنشأ بمقدار (61.46^o).

Keywords: truss, stiffness, strain energy.

Nomenclature:

A	Cross-sectional area of truss element (m^2)	π_p	Overall Potential energy of truss
E	Modulus of elasticity (Mpa)	Δ	Maximum truss deflection (m)
F	External truss force (N)	Δ_x	Maximum horizontal truss deflection (m)
L	Truss element length (m)	Δ_y	Maximum vertical truss deflection (m)
P	Internal force of truss elements (N)	Ω	Potential energy of external forces
U	Strain energy of truss		

1- Introduction:

Truss is a slender member (length is much larger than the cross-section). It is a two-force member i.e. it can only support an axial load and cannot support a bending load. Members are joined by pins (no translation at the constrained node, but free to rotate in any direction). Commonly, the cross-sectional dimensions and elastic properties of each member are constant along its length. Mechanically the element is equivalent to a spring, since it has no stiffness against applied loads except those acting along the axis of the member. However, unlike a spring element, a truss element can be oriented in any direction in a plane, and the same element is capable of tension as well as compression.

The truss is loaded by forces acting on the nodes only. By changing the location of the joints as well as the strength of individual beams, it can be simultaneously optimized the geometry and the mass of structures [1]. An optimization problem is generally recognized to be nondeterministic as well as fuzzy in nature and the nondeterministic condition is not only in the design variables, it can also be in the allowable limits [2].

According to the principle of virtual work the total potential energy will be minimized because of the deformation of an elastic truss under a given load [3]. The strain energy of the deformed truss depends on the applied load and on the data of the truss. If the cross-sectional areas and elastic properties of the members in addition to the applied loads are prescribed, the strain energy of the loaded structure is a function of the nodal coordinates of the truss before deformation. By moving the nodes in an appropriate manner, the strain energy can be lowered and even minimized, thus enhancing the rigidity of the truss as in Fig. (1).

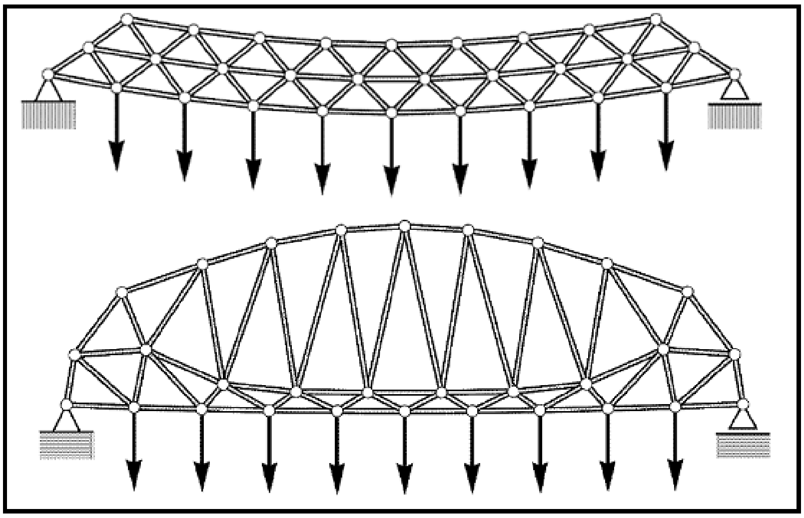

Fig (1) increase the rigidity of the truss by moving its joints

2- Strain energy method in trusses:

The strain energy method provides exact solutions for many structural problems which are statically indeterminate, that is they cannot be analyzed by the application of the equations of statically equilibrium alone [4]. The total potential energy of the truss is equal to the summation of the strain energy and the potential energy of external forces:

$$\pi_p = U + \Omega \quad \text{......... (1)}$$

If the external forces are prescribed, minimizing strain energy will leads to minimizing the potential energy of the truss and as a result the overall truss elastic deflection will be less and truss will be stiffer although the strain energy is not a linear function of the stresses or strains [5].

The strain energy (U) of truss element is a function of element length (L) and the internal force (P) carried by this element and can be written in the following form [6]:

$$U = \frac{P^2 L}{2AE} \quad \text{..................... (2)}$$

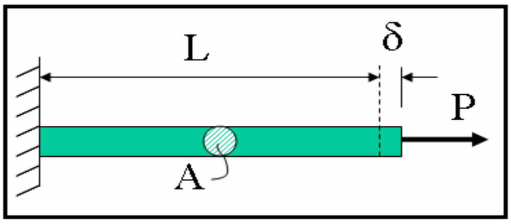

Fig (2) truss member

The total strain energy stored in the system is the sum of the individual strain energies in each of the truss members numbered i=1 to n:

$$U = \sum_{i=1}^{n} \frac{P_i^2 L_i}{2E_i A_i} \quad \text{.................. (3)}$$

The member's axial force and lengths are always a function of external forces and a reference length with respect to angles of inclination of members. So that, equating the partial derivatives of the strain energy with respect to angles of inclination of members to zero gives the optimum selection for these angles that minimize strain energy to the lowest level and as a result, an increasing in truss stiffness will be improved.

The external Work done by forces is equal to the internal strain energy for optimum selection [7]:

External Work = $\frac{1}{2} F\Delta$ .... (4)

Equations (3) and (4) with trusses of constant axial stiffness (EA) gives:

$$\Delta = \frac{1}{FEA}\sum_{i=1}^{n} P_i^2 L \qquad \text{......................} (5)$$

3- The selected truss model for study:

The two dimensional truss shown in Fig. (3) was selected to make an optimum selection for its joints coordinates to increase its stiffness and minimize the vertical deformation of joint B (Δ_B). The main data for this model were as follows:

1.It forms from seven members and five joints.
2.The model is a lightweight aluminum truss with (E=70 Gpa).
3.The main length of its members is (L=1m).
4.It is made of tubular stock with a cross sectional area of (250 mm^2).
5.The only one external force acts on joint (B) with magnitude of (F=20kN).

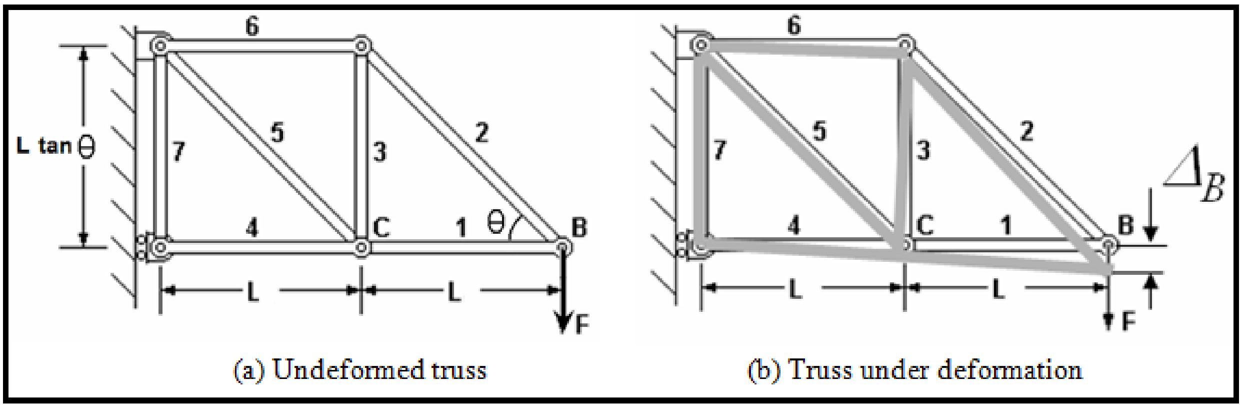

Fig (3) Truss model

4- Solution of the model:

The model under study consists of one angle (θ) that governed the coordinates of the joints of member's number 2, 3, 5, 6, and 7. Using static's equations, member's lengths and internal forces were obtained as in table-1.

The goal of this solution is to minimize the strain energy as possible and it's well noticed from table (1) that the strain energy is a function of the angle of inclination of boom number (2). Equation (3) will be:

$$U = \sum_{i=1}^{n} \frac{P_i^2 L_i}{2E_i A_i} = \frac{1}{2EA}\sum_{i=1}^{7} P_i^2 L_i$$

For (E=70000Mpa), (A=250 mm^2), (F=20kN) and (L=1m), the strain energy for the truss model will be as follows:

$$U = 11.43\left[\frac{6\cos^3\theta + 2 + \sin^3\theta}{\sin^2\theta\cos\theta}\right] \text{..........} (6)$$

Table (1): Lengths and internal forces for truss members

Member	**Internal forces**	Length of members	$P_i^2 L_i$
1	-F/tan θ	L	F^2 L/ $\tan^2$ θ
2	F/sin θ	L/cos θ	F^2 L/($\sin^2$ θ cos θ)
3	-F	L tan θ	F^2 L tan θ
4	-2F/ tan θ	L	$4F^2$ L/ $\tan^2$ θ
5	F/sin θ	L/cos θ	F^2 L/($\sin^2$ θ cos θ)
6	F/tan θ	L	F^2 L/ $\tan^2$ θ
7	0	L tan θ	0

The strain energy as a function of angle of inclination has been plotted as shown in Fig (4).
To find the exact angle (θ) at which the strain energy and truss deflection is minimum, strain energy (U) of equation (6) must be differentiated with respect to the angle (θ) and equating the result to zero:

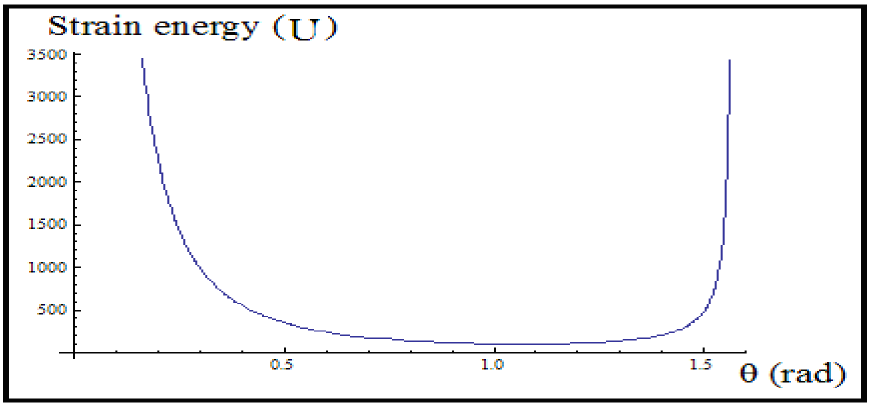

Fig (4) Strain energy of the truss against its free end angle

$$\frac{\partial U}{d\theta}=0$$

$$\left(\sin^2\theta\cos\theta\right)\frac{d}{d\theta}\left(6\cos^3\theta+2+\sin^3\theta\right)-\left(6\cos^3\theta+2+\sin^3\theta\right)\frac{d}{d\theta}\left(\sin^2\theta\cos\theta\right)=0$$

$$\left(\sin^2\theta\cos\theta\right)\left(-18\cos^2\theta\sin\theta+3\sin^2\theta\cos\theta\right)-\left(6\cos^3\theta+2+\sin^3\theta\right)\left(2\sin\theta\cos^2\theta-\sin^3\theta\right)=0$$

To indicate the exact angle of inclination of the member number (2) that yields maximum stiffness for the entire truss, a simple quick basic program (as given in appendix I) was written and the optimum selection of (θ) is found to be (61.46^o) while (103.3J) is the minimum strain energy obtained for the model.

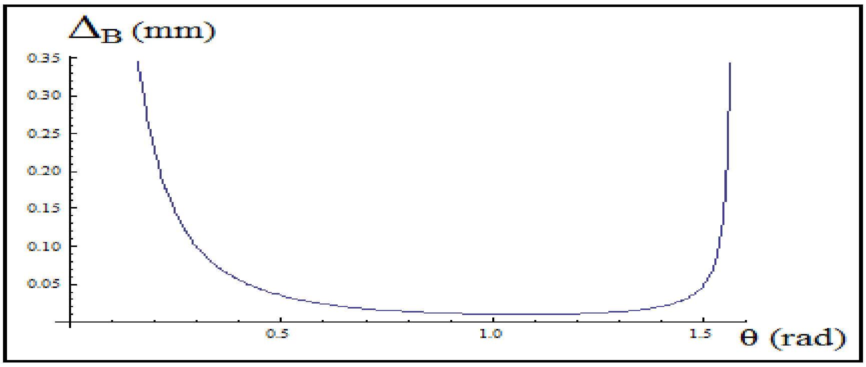

Fig (5) free end deflection of the truss

Figure (5) show the free end deflection against angle (θ) and its well significant that at ($\theta = 61.46^o$), Δ_B will be at its minimum value.

$$\frac{1}{2}F\Delta=\sum_{i=1}^{7}\frac{P_i^2L_i}{2E_iA_i}=103.3\Rightarrow\Delta=10.33mm$$

6- Modeling with use of ANSYS package:

To prove the solution of the truss model under study, finite elements method with the use of ANSYS package was used. Five models were simulated which are similar to that of Fig (3) with free end angles of (59^o, 60^o, 61.46^o, 62^o, and 63^o) and with the same previous data. Figure (6) shows the results of the solution of the model with (62^o) free end angle and the table-2 shows the results of solution of all models numerically.

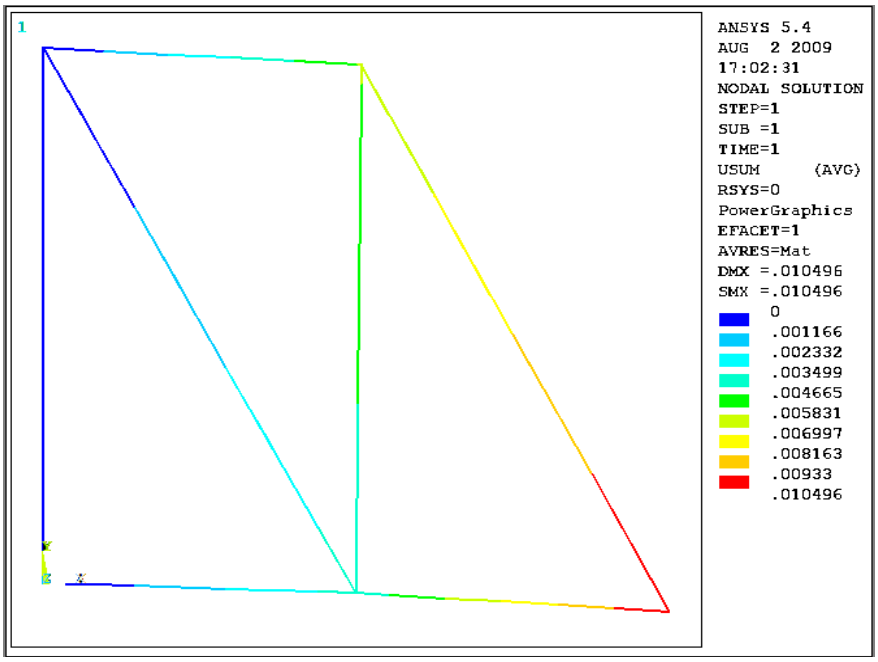

Fig (6) simulation of truss model with ANSYS program

Table-2 : Free end deflections of the simulated models

Free end angle	Δ_x (m)	Δ_y (mm)	Δ (mm)
59^o	-2.06	10.4	10.62
60^0	-1.32	-10.36	10.55
61.46^o	-1.86	-10.3	10.493(minimum)
62^o	-1.82	-10.33	10.496
63^o	-1.75	-10.37	10.51

6- Discussion:

The strain energy of a truss, regarded as a function of nodal displacements, governs the deformation of trusses. Strain energy still depends on the design, especially on the nodal positions in the undeformed configuration. By minimizing the strain energy an optimum shape of the truss is obtained, which stands out for a high rigidity under the prescribed load. Some of the nodes have to be kept fixed for constructional reasons. Otherwise the optimization would make the whole truss shrink to a single node of zero strain energy.

The analysis can be extended to include configuration-dependent loads, such as the self weight of the truss. Also initial stresses could be taken into account, thus providing additional degrees of freedom.

Increasing the angle of the truss model free end up to optimum angle selection didn't increase truss stiffness only but minimize stresses on members since the internal forces were decreased (this is well notified from table-1). Therefore, such truss will be able to carry out additional external forces and the truss will be safer.

7- conclusions:

1.Truss stiffness is a function not only of elastic properties of members but also of truss joints coordinates.

2.The overall stiffness of a truss is optimized by choosing the nodal coordinates of the undeformed truss such that the strain energy of the loaded truss attains a minimum.

3.The optimum angle of free end for hanged truss is exactly equal to (61.46^o) for the given truss length.

4.With use of the optimum truss free end angle, safety factor will be higher.

References

[1] Jeffrey S., Jessica H., Irving Oppenheim, and Andrew W., Creating models of truss structures with optimization, ACM, Volume 21 , Issue 3, July 2002, Pages: 295 – 301.
[2] Omer K. and Mehmet U., Fuzzy Optimization of Geometrical Nonlinear Space Truss Design, Turkish J. Eng. Env. Sci., Volume 29, 2005, pages: 321 – 329.
[3] STEINMANN P. and MAUGIN A., "MECHANICS OF MATERIAL FORCES", volume11, Springer, 2005, page 211.
[4] Megson T. H. G., "Aircraft Structures for engineering students", Third Edition, Butterworth-Heinemann 2003, page 68.
[5] SADD M. H., "ELASTICITY theory, applications, and numerics", Butterworth -Heinemann, 2005, page 105.
[6] Roylance D., "INTRODUCTION TO ELASTICITY", Cambridge, MA 02139, 2000, page 7.
[7] Marghitu D.B."Mechanical Engineer's Handbook", ACADEMIC PRESS, Auburn University, Alabama 2001, page 160.

APPENDIX I

Q.Basic program for optimum selection of the angle (θ)

```
CLS
For x = 0 to 3.14 step 0.000001
Y=(sin(x)^2*cos(x))*(-18*cos(x)^2*sin(x)+3*sin(x)^2*cos(x))-(6*cos(x)^3+2+
sin(x)^3)*(2*sin(x)*cos(x)^2-sin(x)^3)
If Y>-0.00001 and Y<0.00001 then d= x
Next
U = 11.43*(6*cos (d) ^3+2+sin (d) ^3) / (sin (d) ^2*cos (d))
Print d: print U
End
```