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New Technique for Solution Linear Fredholm Integral Equation of The second Kind

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Abstract

In this paper iterated method (Adomain's method) used to evaluate an approximate solution of the Linear Fredholm Integral Equation of the second kind.Moreover a new technique is proposed in applying Aitken's Extrapolation method,the new proposed technique, the convergence of adomain's method is increased. Program associated with above methods is written in MATLAB language , by using method will be shown.

Introduction

Adomain's method is a mathematical method which can be applied to the solution of linear or nonlinear differential equation, deterministic and stochastic operator equation ,and many algebraic equation.In this method, the solution is found as an infinite series, which converge repidly to accurate.This method is well-studied for physical problems, since it makes unnecessary linearization, perturbation and other restricitve methods and assumptions which may change the problem, we know that the Adomain's method can be considered as an extension to the successive approximation method. (Fatima ,2002) used this method to treat Linear Fredholm Integro-differential, equations of the second Kind, and K.Maleknejad and (Hadizadeh & Maleknejad ,1998) proposed this method to treat Non-linear Vollerra Integro – differential Equations of the Second Kind . In the persent paper, we consider the Adomain's method for Linear Fredholm Interal Equation of the second Kind of the form.

$$Y(x) = f(x) + \int_a^b k(x,t)y(t)dt \qquad ...(1)$$

where K(x,t) and F(x) are known functions.

Preliminaries

Let B be a Banach space, and consider the general functional quation (Cherruault,1992)

$$Y=T.x \qquad ...(2)$$

Where T is an operator from B in to B, is a given function in B, and we are looking for $x \in B$ satisfying (2). We assume that (2) has a unique solution for

$x \in B$. not that the banach space B is not necessarity a finite – dimensional space, and it can be A functional space. Throughout this paper, we suppose B is a Banach space of L functions.

We assume that the operator T, the linear term is decomposed in to L+R , where L is easily invertible and R is the remainder of the linear operator.L is taken as the highest order derivative avoiding the difficult integrations which result when complicated Green's functions are involved. Thus, Equation (2) is written:

$$y=L.x+R.x \qquad \ldots(3)$$

Then the solution of (2) or (3) verifies

$$X=L^{-1}.y-L^{-1}.R.x \qquad \ldots(4)$$

Where L^{-1} is the inverse of the linear operator L.

Definition (Adomain's Polynomials)(Al-Ani ,1996)

Let N be an analytical function and $\sum x_n$ convergent series in B .The Adomain's polynomials are defined by

$$A_m=\frac{1}{m!}\ \frac{d^m}{d\lambda^m}[N(\sum_{i=0}^{\infty} \lambda^i x_i)] \qquad m=0,1,2 \qquad \ldots(5)$$

Generally,it is possible to obtain exactly A_N AS A function of $x_0,x_1,\ldots,x_n$ from the linearity N by these method , folmula (4) can be written

$$\sum_{n=0}^{\infty} x_n = L^{-1}y-L^{-1}(\sum_{N=0}^{\infty} A_N) \qquad \ldots(6)$$

taking $X_0=L^{-1}Y$, we can identify the other terms of the series $\sum_{n=0}^{\infty} x_n$ by the following algorithm ,

$$X_1=-L^{-1}A_0$$
$$X_2=-L^{-1}A_1$$
$$\cdot \qquad \cdot$$
$$X_n=-L^{-1}A_{n-1}$$

Thus, all components of X can be calculated once the A_n are given for n=0,1,2…

then we define n-term approximate to the solution, x by $\phi_n(x)=\sum_{i=0}^{n-1} x_i$,with $\lim_{n\to\infty} \phi(x)$ =X .

Aitken Extrapolation Method

This method deals with the Linear Fredholm Integral Equation of the Second Kind having three approximate solutions, $y_0(x)$, $y_1(x)$, $y_3(x)$ .we can extrapolate to an improved and estimate, this can be done by considering the following formula(Al-Gardi,1999):

$$\phi(x) = Y_2(x) - \frac{(Y_2(x) - Y_1(x))^2}{Y_2(x) - 2Y_1(x) + Y_0(x)} \qquad \ldots(7)$$

Where Y_0 (x), Y1 (x) and Y_2 (x) are solution of the Adomain's Method, which used iterated kernel method. And $\phi(x)$ is approximate exact solution, we can simplify if as

$$\phi(x) = \frac{y_0(x) y_2(x) - y_1^2(x)}{y_2(x) - 2y_1(x) + y_0(x)} \qquad \ldots(8)$$

The general from of it

$$\phi(x) = \frac{y_n(x) y_{n+2}(x) - y_{n+1}^2(x)}{y_{n+2}(x) - 2y_{n+1}(x) + y_n(x)} \qquad \ldots(9)$$

Adomian's Method Applied to Linear Fredholm Integral Eqution of the Second Kind (Philips & Taylor,1974).

The Adomian's Method consists of representing Y as a series

$$Y = \sum_{i=0}^{\infty} yi$$

Where y_i are calculated by the following algorithm :

The method is begin by choosing an initial approximation:

$$y_0 = f(x)$$

Then first iteration is

$$y_1 = \int_a^b \mathrm{K}(x,t) y_0(t) dt = \mathrm{A}_0$$

We use $y_1(x)$ to find second iteration $y_2(x)$

$$y_2 = \int_a^b \mathrm{K}(x,t) y_1(t) dt = \mathrm{A}_1$$

Continue this process, We obtain the n^{th} iteration $y_n(x)$

$$y_n = \int_a^b K(x,t) y_{n\_1}(t) dt = A_{n\_1} \qquad \ldots(10)$$

Where A_i is called Adomian's Polynomials

Where i=0,1,……., ∞

Aitken method on Adomian Method (Modification Method) (Mustafa,2002)

In this Section for the first time Aitken method has been used Successfully on Adomian method as extrapolated formula to find exact solution where three values of the an known function $y(x)$ are given to achieve this method we follow the following steps:-

Step 1:

Recall equation (1)

$$y(x) = f(x) + \int_a^b k(x,t)y(t)dt$$

Step 2 :

We find $y_0(x)$, $y_1(x)$, $y_2(x)$ and $y_3(x)$

Using the equation (10):

Step 3:

We find $\phi_0(x)$,$\phi_1(x)$ and $\phi_2(x)$ as following

$$\phi_0(x) = Y_0(x) + Y_1(x) \quad \ldots(11)$$

$$\phi_1(x) = Y_0(x) + Y_1(x) + Y_2(x) \quad \ldots(12)$$

Finally

$$\phi_2(x) = Y_0(x) + Y_1(x) + Y_2(x) + Y_3(x) \quad \ldots(13)$$

We can simplify (11,12,13)

$$\phi_0(x) = Y_0(x) + Y_1(x)$$

$$\phi_1(x) = \phi_0(x) + Y_2(x)$$

$$\phi_2(x) = \phi_1(x) + Y_3(x)$$

Step 4: Suppose that the exact solution is $\phi(x)$

Step 5: Use $\phi_0(x)$, $\phi_1(x)$ and $\phi_2(x)$ as an approximate solution by substituting them in Aitken's formula (9) we get :

$$\varphi(x) = \frac{\varphi_n(x)\varphi_{n+2}(x) - \varphi_{n+1}^2(x)}{\varphi_{n+2}(x) - 2\varphi_{n+1}(x) + \varphi_n(x)} \quad \ldots(14)$$

Numerical Result

In order to show the usefulness of the new technique (Aitken's method on Adomian's method) described in equation (14), we consider some typical integral equations of the linear fredholm integral equation of the second kind.

Example (1): Consider the following Linear Fredholm Integral Equation of the second kind

$$y(x) = f(x) + \int_0^1 -2\exp(x-t)\,y(t)\,dt \qquad 0 \le x \le 1$$

With the equation $f(x) = 2.x.e^x$

The exact solution of this problem is $y(x) = e^x(2x - \frac{2}{3})$

Table (1) gives comparison Adomian's method, Aitken's on Adomian's method and exact solution.

x	Exact	Adomian	Aitken and domain
0	-0.6667	-3.4200000	-0.6667
0.1	-0.5157	-3.7774742	-0.5157
0.2	-0.3257	-4.1723118	-0.3257
0.3	-0.0900	-4.6084180	-0.0900
0.4	0.1989	-5.0901059	0.1989
0.5	0.5496	-5.6221395	0.5496
0.6	0.9718	-6.2097807	0.9718
0.7	1.4768	-6.8588417	1.4768
0.8	1.0772	-7.5757413	2.0772
0.9	2.7876	-8.3675698	2.7876
1.0	3.6244	-9.2421582	3.6244
L.S.E		$4.22299*10^{+6}$	0.0000000

Example (2)Consider the following problem

$$y(x) = f(x) + \int_0^1 x \cdot t \cdot y(t)\,dt$$

$f(x) = 3 - \frac{4}{3}x$ Where

With the exact solution

$$Y(x) = 3 + \frac{1}{4}x$$

Table(2)A comparison of our result and exact solution

X	Exact	Adomian's	Aitken on Adomains
0	2.0000	2.000000	2.0000
0.1	2.0450	2.044444	2.0450
0.2	2.0901	2.088888	2.0901
0.3	2.1351	2.133332	2.1351
0.4	2.1802	2.177776	2.1802
0.5	2.2252	2.222221	2.2252
0.6	2.2702	2.266665	2.2702
0.7	2.3153	2.311109	2.3153
0.8	3.3603	2.355553	2.3603
0.9	2.4054	2.399997	2.4054
1	2.4504	2.444441	2.4504
L.S.E		$3.389159{*}e^{-11}$	0.000000

Conclusion

In this paper, iterated technique (Adomians method, Aitken on Adomians method) had been used to solve linear Fredholm integral equations of the second kinds.

In the practice, we conclude that:

1- The Adomians method is a numerical elegant method that can solve variety types of linear Fredholm integral equations.

2- Numerical computations of this method compared to the numerical schemes are simple and inexpensive.

3- The solution is given by a function, and not only at some grid points as in the projection method

4- Adomians method give arises to disconvergent series, and therefor it is difficult to generate an approximation to the exact Solution. To overcome this drawback we suggest the following modification on Adomians method using Aitken extrapolation.

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التقنية الجديدة لحل معادلات فريدهولم التكاملية الخطية من الدرجة الثانية

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الخلاصة

في هذا البحث استخدمت طريقة تكرارية (طريقة آدمن) لحساب الحل التقريبي لمعـــادلات فريـــدهولم التكامليـــة الخطية مـــن الدرجـــة الثانية.افترضـــت التقنيـــة الجديـــدة فـــي تطبيـــق(Aitken's Extrapolation) علىطريقـــة (Adomain's) وبواسطة التقنية الجديدة المقترحة يزداد تقارب طريقة (Adomain's).وان البرامج المقترحة بالطرق المذكورة أعلاه تكتب بلغة Matlab وذلك باستخدام أمثلة متنوعة وبذلك نلاحظ دقة الطريقة الجديدة .