J. of university of anbar for pure science : Vol.5:NO.2 : 2011

ISSN: 1991-8941

SOME TYPES OF COMPLETELY REGULAR SPACE AND IT'S RELATIONS

Taha Hameed Jasim **Sufyan A. W**

College of computer sciences and Math. University of Tikrit

Received: 30/9/2010 **Accepted:2/5/2011**

Abstract : In this paper , we introduced a new definitions *of semi completely regular space and semi regular space* . And we study some relations among the(s , g , $s\,g$, $g\,s$, g^*, $s\,g^*$, g^*s) *of completely regular spaces* .

1. Introduction

In 1970, Levine [7] introduced a new and significant notion in General Topology, namely the notion of a generalized closed set. A subset A of a topological space $(X,^{2})$ is called generalized closed, (briefly g-closed), if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in $(X,^{2})$. This notion has been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and interesting concepts . This notion has been studied extensively in recent years by many topologists because generalized closed sets are not only natural generalizations of closed sets.

P.Bhattacharya and B.K. Lahiri [8] , S.P.Arya [10] , investigated semi g closed set , g semi closed set respectively . P.Sundaramand A.Pushpalatha [9] , Al-Ddoury A.F.[2] A.I.El-Maghrabi and A.A.Nasef [1]introduced and investigated strongly generalized closed sets ,semi strongly generalized closed set and strongly generalized semi closed , Respectively .We study all definitions was in abstract , relations and some properties .

2. Preliminaries

Definition 2.1: *A subset A of atopological space is said to be* :

(1) Generalized closed (briefly g-closed) if cl(A) ⊆ G whenever A ⊆ G and G is open in X.[6]

(2) Semi generalized closed (briefly sg-closed) if scl(A) ⊆ G whenever A ⊆ G and G is semi open in X.[7]

(3) Generalized semi closed (briefly gs-closed) if scl(A) ⊆ G whenever A ⊆ G and G is open in X.[3]

(4) strongly generalized closed (briefly $g^*closed$) if cl(A) ⊆ G whenever A ⊆ G and G is g *open* in X [4]

(5) semi strongly generalized closed (briefly $s\,g^*$ *closed*) if scl(A) ⊆ G whenever A ⊆ G and G is s g *open* in X [2] .

(6) strongly generalized semi closed (briefly g*s-closed) if scl(A) ⊆ G whenever A ⊆ G and G is g-open in X [9] .

The complements of the above mentioned sets are called their respective open sets .

Definition 2.2 **:** [5]A topological space $(X,^{2})$ is called :

(1) completely regular if for every closed F $\subseteq$ X and x $\in$ X \ F, there is a continuous function f : X $\Rightarrow$ [0,1] , such that f(x) = 0 and f(F) = {1} .

(2) Regular space *if and only if* $\forall x \in X$ and $\forall F$ closed in X , $x \notin F$, $\exists U,V \in {}^{2}$, *such that* $x \in V$ and $F \subseteq V$ $\ni U \cap V = ¤$.

Defention 2.3. [11] *A topological space* $(X,^{2})$ *is said to be* :

(1) *g Complete regular space (briefly g* $[CR]$*) if*

g closed set F *in* X *and* $x \in X$, $x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F) = \{1\}$ *and* $g(x) = 0$.

(2) *g regular space* (*briefly* $g\,[R]$) *if and only if the g closed set* A *and point* $x \notin A$ *there exist disjoint g open sets* $U, V \in$ ² *such that* $A \subseteq U$ *and* $x \in V$ $\ni U \cap V =$ ¤ .

(3) *semi g completely regular* (*briefy* $sg\,[CR]$) *if for every semi g closed set* $F \subseteq X$ *and* $x \in X \setminus F$, *there is a continuous function* $f : X \to [0,1]$, *such that* $f(x) = 0$ *and* $f(F) = \{1\}$.

(4) *semi g regular* (*briefy* $sg\,[R]$) *if and only if the sg closedset* A *and point* $x \notin A$, *There exist disjoint semi g open sets* $U, V \in$ ² *such that* $A \subseteq U$ *and* $x \notin V$ $\ni U \cap V =$ ¤ .

(5) *g semi completely regular* (*briefy* $gs\,[CR]$) *if for every g semi closed set* $F \subseteq X$ *and* $x \in X \setminus F$, *there is a continuous function* $f : X \to [0,1]$, *such that* $f(x) = 0$ *and* $f(F) = \{1\}$.

(6) *g semi regular* (*briefy* $gs\,[R]$) *if and only if the gs closed set* A *and point* $x \notin A$, *There exist disjoint g semi open sets* $U, V \in$ ² *such that* $A \subseteq U$ *and* $x \notin V$ $\ni U \cap V =$ ¤ .

(7) g^* *Complete regular space* (*briefly* $g^*\,[CR]$) *if and only if* *g closed set* F *in* X *and* $x \in X$ $\ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F) = \{1\}$ *and* $g(x) = 0$.

(8) g^* *regular space* (*briefly* $g^*\,[R]$) *if for each* g^* *closed set* A *and point* $x \notin A$ *there exist disjoint* g^* *open sets* $U, V \subseteq X$ *such that* $A \subseteq U$ *and* $x \in V$ $\ni U \cap V =$ ¤

(9) *semi* g^* *completely regular* (*briefy* $sg^*\,[CR]$) *if for every semi* g^* *closedset* $F \subseteq X$ *and* $x \in X \setminus F$, *there is a continuous function* $f : X \to [0,1]$, *such that* $f(x) = 0$ *and* $f(F) = \{1\}$.

(10) *semi* g^* *regular* (*briefy* $sg^*\,[R]$) *if for each* sg^* *closed set* A *and each point* $x \notin A$

There exist disjoint semi g^* *opensets* $U, V \subseteq X$ *such that* $A \subseteq U$ *and* $x \notin V$

(11) g^* *semi completely regular* (*briefy* $g^*s\,[CR]$) *if for every g semi closed set* $F \subseteq X$ *and* $x \in X \setminus F$, *there is a continuous function* $f : X \to [0,1]$, *such that* $f(x) = 0$ *and* $f(F) = \{1\}$.

(12) g^* *semi regular* (*briefy* $g^*s\,[R]$) *if for each* g^*s *closed set* A *and each point* $x \notin A$ *There exist disjoint* g^* *semi open sets* $U, V \subseteq X$ *such that* $A \subseteq U$ *and* $x \notin V$.

3. Some Properties and Relations :

Defention 3.1. A topological space $(X,{}^{2})$ is called :

(1) *semi completely regular* (*briefy* $s[CR]$) *if for every semi closed set* $F \subseteq X$ *and* $x \in X \setminus F$, *there is a continuous function* $f : X \to [0,1]$, *such that* $f(x)=0$ $f(x)=0$ *and* $f(F)=\{1\}$.

(2) *semi regular* (*briefy* $s[R]$) *if for each semi closed set* A *and each point* $x \notin A$ *There exist disjoint semi open sets* $U, V \subseteq X$ *such that* $A \subseteq U$ *and* $x \notin V$.

Theorem 3.2. *Every completely regular space is semi completely regular*

Proof : *Let* $(X,{}^{2})$ *be completely regular space then* F *closed set in* X *and* $x \in X \ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of Complete regular space*), *But every closed is semi closed* [3]. *Then* $(X,{}^{2})$ *is semi completely regular*

Theorem 3.3. *Every semi completely regular is* s g *Complete regular space*

Proof : *Let* $(X,{}^{2})$ *be semi completely regular space then* F *semi closed set in* X *and* $x \in X \ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of semi Complete regular space*), *But every semi closed is* s g *closed* [3]. *Then* $(X,{}^{2})$ *is* s g *Complete regular space*

Theorem 3.4. *Every* s g *Complete regular space is* g s *Complete regular space*

Proof : *Let* $(X,{}^{2})$ *be* s g *completely regular space then* F s g *closed set in* X *and* $x \in X \ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of* s g *Complete regular space*), *But every* s g *closed is* g s *closed* [3]. *Then* $(X,{}^{2})$ *is* g s *Complete regular space*.

Theorem 3.5. *Every completely regular space is* s g *completely regular*

Proof : *Let* $(X,{}^{2})$ *be completely regular space then* F *closed set in* X *and* $x \in X \ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of Complete regular space*), *But every closed is* s g *closed* [3]. *Then* $(X,{}^{2})$ *is* s g *completely regular* .

Theorem 3.6. *Every completely regular space is* g s *completely regular*

Proof : *Let* $(X,{}^{2})$ *be completely regular space then* F *closed set in* X *and* $x \in X \ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of Complete regular space*), *But every*

closed is g s closed [3]. *Then* $(X,^2)$ *is*
g s completely regular .

Theorem 3.7. *Every*
semi completely regular is
g s Complete regular space

Pr*oof* : *Let* $(X,^2)$ *be*
semi completely regular then F
semi closed set in X *and* $x \in X$
$\ni x \notin F$, *Then* $\exists$ *a continuous mapping*
$g : X \to [0,1]$ *such that* $g(F) = \{1\}$
and $g(x) = 0$
(*by definition of*
semi completely regular), *But every*
semi closed set is g s closed [3].
Then $(X,^2)$ *is*
g s Complete regular space .

Theorem 3.8. *Every*
completely regular space is
g^**Complete regular space*

Pr*oof* : *Let* $(X,^2)$ *be*
completely regular space then F
closed set in X *and* $x \in X$
$\ni x \notin F$, *Then* $\exists$ *a continuous mapping*
$g : X \to [0,1]$ *such that* $g(F) = \{1\}$
and $g(x) = 0$
(*by definition of*
Complete regular space), *But every*
closed is g^**closed* [2].
Then $(X,^2)$
is g^**Complete regular space* .

Theorem 3.9. *Every*
g^**Complete regular space is*
g Complete regular space

Pr*oof* : *Let* $(X,^2)$ *be*
g^**Complete regular space then* F
g^**closed set in* X *and* $x \in X$
$\ni x \notin F$, *Then* $\exists$ *a continuous mapping*
$g : X \to [0,1]$ *such that* $g(F) = \{1\}$
and $g(x) = 0$
(*by definition of*
g^**Complete regular space*), *But every*
g^**closed is g closed* [2].
Then $(X,^2)$
is g Complete regular space .

Theorem 3.10. *Every*
g^**Complete regular space is*
g s Complete regular space

Pr*oof* : *Let* $(X,^2)$ *be*
g^**Complete regular space then* F *is*
g^**closed set in* X *and* $x \in X$
$\ni x \notin F$, *Then* $\exists$ *a continuous mapping*
$g : X \to [0,1]$ *such that* $g(F) = \{1\}$ *and*
$g(x) = 0$ (*by definition of*
g^**Complete regular space*), *But every*
g^**closed is g s closed* [2,3]. *Then* $(X,^2)$
is g s Complete regular space .

Theorem 3.11. *Every*
g^**Complete regular space is*
s g^**Complete regular space*

Pr*oof* : *Let* $(X,^2)$ *be*
g^**Complete regular space then is* F
g^**closed set in* X *and* $x \in X$
$\ni x \notin F$, *Then* $\exists$ *a continuous mapping*
$g : X \to [0,1]$ *such that* $g(F) = \{1\}$
and $g(x) = 0$
(*by definition of*
g^**Complete regular space*), *But every*
g^**closed is s* g^**closed* [11].
Then $(X,^2)$
is s g^**Complete regular space* .

Theorem 3.12. Every
g^* *s completely regular* is
g completely regular is

Proof: *Let* $(X,^{2})$ *be* $g^{*}s$ *completely regular then* $g^{*}s$ *closed set* F *in* X *and* $x \in X$ $\ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g \ni g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of* $g^{*}s$ *Complete regular space*), *But every* $g^{*}s$ *closed is* g *closed* [1]. *Then* $(X,^{2})$ is g *completely regular*

Theorem 3.13. Every $g^{*}s$ *completely regular* is $g\ s$ *completely regular* is

Proof:

Let $(X,^{2})$ *be* $g^{*}s$ *completely regular then* $g^{*}s$ *closed set* F *in* X *and* $x \in X \ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g \ni g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of* $g^{*}s$ *Complete regular space*), *But every* $g^{*}s$ *closed is* $g\ s$ *closed* [1,2,3,4]. *Then* $(X,^{2})$ is $g\ s$ *completely regular*

Theorem 3.14. *Every* g *Complete regular space is* $g\ s$ *Complete regular space*

Pr*oof* : *Let* $(X,^{2})$ *be* g *completely regular space then* F *is* g *closed set in* X *and* $x \in X$ $\ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of* g *Complete regular space*), *But every* g *closed is* $g\ s$ *closed* [3]. *Then* $(X,^{2})$ *is* $g\ s$ *Complete regular space*.

Theorem 3.15. *Every semi completely regular is* g *Complete regular space*

Pr*oof* : *Let* $(X,^{2})$ *be semi completely regular then* F *is semi closed in* X *and* $x \in X$ $\ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of semi completely regular*), *But every semi closed is* g *closed* [1,2,3,4]. *Then* $(X,^{2})$ *is* g *Complete regular space*.

Theorem 3.16. *Every semi completely regular space is* g^{*} *s Complete regular space*

Pr*oof* : *Let* $(X,^{2})$ *be semi completely regular space then* F *is semi closed set in* X *and* $x \in X$ $\ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of semi Complete regular space*), *But every semi closed is* g^{*} *s closed* [1]. *Then* $(X,^{2})$ *is* g^{*} *s Complete regular space*

Theorem 3.17. *Every completely regular space is* g *completely regular*

Pr*oof* : *Let* $(X,^{2})$ *be completely regular space then* F *closed set in* X *and* $x \in X$ $\ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F)=\{1\}$ *and* $g(x)=0$ (*by definition of Complete regular space*), *But every closed is* g *closed* [3].

Then $(X,^{2})$ *is* g *completely regular*

Theorem 3.18. *Every completely regular space is g^{*} s Complete regular space*

Pr*oof* : *Let* $(X,{}^{2})$ *be completely regular space then F closed set in X and* $x \in X$ $\ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F) = \{1\}$ *and* $g(x) = 0$ (*by definition of Complete regular space*), *But every closed is* g^{*}*s closed* [11]. *Then* $(X,{}^{2})$ *is* g^{*}*s Complete regular space*

Theorem 3.19. *Every completely regular space is s* g^{*} *Complete regular space*

Pr*oof* : *Let* $(X,{}^{2})$ *be completely regular space then F is closed set in X and* $x \in X$ $\ni x \notin F$, *Then* $\exists$ *a continuous mapping* $g : X \to [0,1]$ *such that* $g(F) = \{1\}$ *and* $g(x) = 0$ (*by definition of Complete regular space*), *But every closed is s* g^{*}*closed* [11]. *Then* $(X,{}^{2})$ *is s* g^{*} *Complete regular space*.

REFRENCE :

[1] A.I. El-Maghrabi and A.A.Nasef " Between semi-closed and GS-closed sets" Journal for science of taibah university,v.2:78-87(2009)

[2] Al-Ddoury A.F.(2009) "On Bitopological Spaces '' M.S.C. Thesis College of Education Tikrit University.

[3] Alhawez,Z.T.(2008) " Generalizations of Slightly Continuous Function between ITS'' M.S.C. Thesis College of Education Tikrit University

[4] Alshamary, A.G. (2008) " On Generalized Homeomorphism between ITS '' M.S.C. Thesis College of Education Tikrit University.

[5] J.Dugungji , topology , library of congress catalog card number :66-1094 printed in the united States of America

[6] Jones and Bartlett "Foundations of topology "second edition.printed in India by sanat printers Kundli-131028,sonepat,Haryara.

[7] N.Levine , Generalized closed sets in topology , Rend.Circ. Mat.Palermo,19, (2) (1970), 89-96.

[8] P.Bhattacharyya and B.K.Lahiri, Semi generalized closed sets in topology, Indian J. Math. , 29 (1987), 375-382

[9] P.Sundaram , A.Pushpalatha, Strongly generalized closed sets in topological spaces, Far East J. Math. Sci. (FJMS) 3(4) (2001), 563-575

[10] S.P.Arya and T.Nour, Characterizations of s-normal spaces, Indian J. Pure Appl. Math., 21 (1990), 717-719.

[11]Taha .H .J. And Sufyan A. W. AND Suham M.A." SOME PROPERTIES OF g* COMPLETELY REGULAR SPACE". Resseved by Sience Jorunal of Kirkuk .

بعض انواع الفضاءات المنتظمة الكاملة وتطبيقاتها

Email: sceanb@yahoo.com

الخلاصة:

في هذا البحث تم تقديم تعريفات جديدة g, sg للفضاء التبولوجي الكامل الأنتظام والمنتظم ودراسة العلاقات بين الفضاءات المنتظمة (g , sg , gs , g^{*}, sg^{*}, $g^{*}s$)