# On Generalized Some Kind of Contra Homeomorphism Functions and Some relations among Them in Intuitionistic Topological Spaces

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## Abstract

In this paper, we introduce the definition of contra homeomorphism functions, contra k-homeomorphism functions, contra strongly k-homeomorphism functions and contra S*-k- homeomorphism functions in intuitionistic topological spaces where k= {semi, α , per , β }, and we give propositions to show the relations among them, some counter examples are given for not implications. We give also a diagram to illustrate these relations.

## Introduction

The notion of homeomorphism, k- homeomorphism, strong-k-homeomorphism and S*-k- homeomorphism functions in intuitionistic topological spaces where k = {semi, α , per , β } was introduced by (Hanna H.Alwan &Yunis J. Yaseen 2007).

The notion of contra continuity was introduced by (Dontchev, 1996), contra semi continuous function was introduced and investigated by (Dontchev & Noiri, 1999), so contra pre continuous was introduced by (Jafari, & Noiri, 2002), and generalized them on intuitionistic topological spaces by (Ali M. Jasem & Yunis J.Yaseen 2009).

In this paper we define some kinds of contra homeomorphism functions, contra semi- homeomorphism, contra α- homeomorphism , contra pre- homeomorphism, contra β- homeomorphism, contra strongly-semi- homeomorphism, contra strongly α- homeomorphism, contra strongly pre-homeomorphism, contra strongly β- homeomorphism, contra S*- semi- homeomorphism, contra S*-α- homeomorphism, contra S*- pre-homeomorphism,contra S*-β-homeomorphism functions in intuitionistic topological spaces, and we study some relation among them.

## Preliminaries

Let X be anon-empty set, an intuitionistic set (briefly IS) A is an object having the form A= $\langle x, A_1, A_2 \rangle$ where $A_1$ and $A_2$ are disjoint subset of X. the set $A_1$ is called a member of A, while $A_2$ is called non- member of A, an intuitionistic topology (briefly IT) on a non-empty set X, is a family T of IS in X containing $\widetilde{\emptyset}, \widetilde{X}$ and closed under arbitrary unions and finitely intersections. In this case the pair(X, T) is called an intuitionistic topological space (briefly ITS), any IS in T is known as an intuitionistic open set (briefly IOS) in X. The complement of IOS is called intuitionistic closed set (briefly ICS), so the interior and closure of A are denoted by int(A) and cl(A) respectively and defined by

$\mathrm{Int}(A) = \cup\{G_i : G_i \in T \text{ and } G_i \subseteq A\text{ ; where } A = \langle x, A_1, A_2\rangle\}$

$\mathrm{cl}(A) = \cap\{F_i : F_i \text{ is ICS in X and } A \subseteq F_i\text{ ; where } A = \langle x, A_1, A_2\rangle\}$

A set A is called:

1. intuitionistic semi - open set (ISOS, for short) if A⊆cl(intA).
2. intuitionistic α - open set (IαOS, for short) if A⊆int(cl(intA)) .
3. intuitionistic per - open set (IPOS, for short) if A⊆int(clA) .
4. intuitionistic β – open set (IβOS, for short) if A⊆cl(int(clA)). (Jeon,J.K.,Jun,Y.B.and Park, J.H.2005)

The complement of ISOS, IαOS, IPOS and IβOS in X is called intuitionistic semi-closed set , intuitionistic α-closed set, intuitionistic pre-closed set and intuitionistic β-closed set in X ( ISCS, IαCS, IPCS and Iβ CS for short) (Thakur & Singh, 1998).

Every IOS (ICS) is ISOS, IαOS, IPOS and IβOS (ISCS, IαCS, IPCS and Iβ CS for short) (Hanna H.Alwan &Yunis J. Yaseen; 2007).

Let (X, T) and (Y,σ) be two ITS's and let $f: X \rightarrow Y$ be a function then f is said to be:

1. An intuitionistic contra continuous (I contra cont., for short) function if the inverse image of each IOS in Y is ICS in X.
2. An intuitionistic contra semi-continuous (I contra semi-cont., for short) function if the inverse image of each IOS in Y is ISCS in X.
3. An intuitionistic contra α-continuous (I contra α-cont., for short) function if the inverse image of each IOS in Y is IαCS in X.
4. An intuitionistic contra pre-continuous (I contra pre-cont., for short) function if the inverse image of each IOS in Y is IPCS in X.
5. An intuitionistic contra β-continuous (I contra β-cont., for short) function if the inverse image of each IOS in Y is IβCS in X.
   (Ali M. Jasem &Yunis J.Yaseen; 2009).

Now we introduce the definition of contra open function and contra homeomorphism function in intuitionistic topological spaces:

**Definition 1:** Let (X,T) and (Y,$\sigma$) be two ITS's and let $f: X \to Y$ be a function then f is said to be an intuitionistic contra open (I contra open , for short) function if the image of each IOS in X is ICS in Y.

**Remark:** f is contra closed function if the image of each ICS in X is IOS in Y.

**Theorem 2**: Let (X, T) and (Y,$\sigma$) be two ITS's and let $f: X \to Y$ be a fictive function then the following statement are equivalent:

1. f is contra open .
2. f is contra closed .

**Proof: (** 1→2)Let A be IOS in X then f(A) is ICS in Y since f is I contra open function , then $f(A^c)$ is IOS in Y and $A^c$ is ICS in X ,i.e : the image of each ICS in X is IOS in Y.Hence f is I contra closed function .

(2→1) by the same way,we can prove them.

**Definition 3:** Let (X,T) and (Y,$\sigma$) be two ITS's and let $f: X \to Y$ be a function then f is called I contra homeomorphism if f is bijective function,I contra continuous function and $f^{-1}$ I contra continuous function

**Theorem 4**: Let (X, T) and (Y,$\sigma$) be two ITS's then $f: X \to Y$ is I contra open function iff $f^{-1}$ is I contra continuous function.

**Proof**: let $f^{-1}: Y \to X$ be contra continuous function then: ∀ A is IOS in X then $(f^{-1})^{-1}(A)$ is ICS in Y; i.e ∀ A is ICS in X then f (A) is ICS in Y. Hence f is I contra open function.

Conversely: let f be I contra open function then: ∀ B is IOS in X then f (B) is ICS in Y ; hence$f^{-1}$ is I contra continuous function since $(f^{-1})^{-1}(B) = f(B)$.

**Corollary 5**: Let (X,T) and (Y,$\sigma$) be two ITS's then $f: X \to Y$ is I contra homeomorphism function if f is bijective function , I contra continuous function and I contra open function .

**proof**: By( theorem 4 ) $f^{-1}$ is I contra continuous function(since f is I contra open function) then by (Definition 3) f is I contra homeomorphism function .

**<u>Definition 6</u> :** Let (X,T) and (Y,$\sigma$) be two ITS's and let $f: X \to Y$ be a function then f is said to be :

1. An intuitionistic contra semi-open (I contra semi-open, for short) function if the image of each IOS in X is ISCS in Y.
2. An intuitionistic contra $\alpha$-open (I contra $\alpha$-open, for short) function if the image of each IOS in X is I$\alpha$CS in Y.
3. An intuitionistic contra pre-open (I contra pre-open, for short) function if the image of each IOS in X is IPCS in Y.
4. An intuitionistic contra $\beta$-open (I contra $\beta$-open, for short) function if the image of each IOS in X is I$\beta$CS in Y.

5. An intuitionistic contra semi- closed (I contra semi- closed, for short) function if the image of each ICS in X is ISOS in Y.
6. An intuitionistic contra α-closed (I contra α-closed, for short) function if the image of each ICS in X is IαOS in Y.
7. An intuitionistic contra pre -closed (I contra pre -closed, for short) function if the image of each ICS in X is IOS in Y.
8. An intuitionistic contra β -closed (I contra β-closed, for short) function if the image of each ICS in X is IβOS in Y.

**Proposition 8:** Let (X, T) and (Y,σ) be two ITS's and let $f: X \to Y$ be function then :

1. If $f$ is I contra open (closed) function then $f$ is I contra k- open (closed) function.
2. If $f$ is I contra α- open (closed) functio then $f$ is I contra semi- open (closed) function.
3. If $f$ is I contra semi- open (closed) function then f is I contra β - open (closed) function.
4. If $f$ is I contra α - open (closed) function then f is I contra pre - open (closed) function.
5. If $f$ is I contra β - open (closed) function then f is I contra pre - open (closed) function.

Where k= {semi, α , per , β }

**Proof:**1.Let A be IOS in X then f(A) is ICS in Y (since f is I contra open function) then f(A) is IKCS in Y (since every ICS is IKCS) hence f is I contra k- open function .Now let A be ICS in X then f(A) is IOS in Y (since f is I contra closed function) then f(A) is IKOS in Y (since every IOS is IKOS) hence f is I contra k- closed function .

2. Let A be IOS in X then f(A) is IαCS in Y (since f is I contra α-open function) then f(A) is ISCS in Y (since every IαCS is ISCS) hence f is I contra semi- open function .Now let A be ICS in X then f(A) is IαOS in Y (since f is I contra α-closed function) then f(A) is ISOS in Y (since every IαOS is ISOS) hence f is I contra semi- closed function .

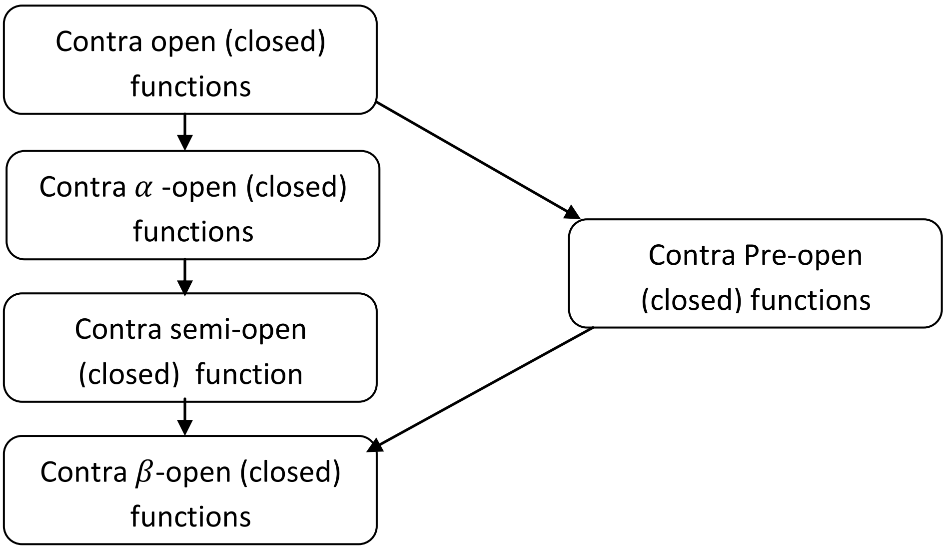
3. Let A be IOS in X then f(A) is ISCS in Y (since f is I contra semi-open function) then f(A) is IβCS in Y (since every ISCS is IβCS) hence f is I contra β- open function .Now let A be ICS in X then f(A) is ISOS in Y (since f is I contra semi-closed function) then f(A) is IβOS in Y (since every ISOS is IβOS) hence f is I contra β- closed function .

4. Let A be IOS in X then f(A) is IαCS in Y (since f is I contra α-open function) then f(A) is IPCS in Y (since every IαCS is IPCS) hence f is I contra pre- open function. Now let A be ICS in X then f(A) is IαOS in Y

(since f is I contra α-closed function) then f(A) is IPOS in Y (since every IαOS is IPOS) hence f is I contra pre- closed function.

5. Let A be IOS in X then f(A) is IβCS in Y (since f is I contra β-open function) then f(A) is IPCS in Y (since every IβCS is IPCS) hence f is I contra pre- open function. Now let A be ICS in X then f(A) is IβOS in Y (since f is I contra β-closed function) then f(A) is IPOS in Y (since every IβOS is IPOS) hence f is I contra pre- closed function.

We summarized the above result by the following diagram.




**Definition 8:** Let (X, T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a bijective function then f is said to be:

1. An intuitionistic contra semi- homeomorphism (I contra semi-hom., for short) function if f is contra semi- continuous function and contra semi- open function .
2. An intuitionistic contra α- homeomorphism (I contra α-hom., for short) function if f is contra α- continuous function and contra α-open function.
3. An intuitionistic contra pre- homeomorphism (I contra pre-hom., for short) function if f is contra pre-continuous function and contra pre - open function.
4. An intuitionistic contra β- homeomorphism (I contra β-hom., for short) function if f is conta β-continuous function and contra β-open function.

5. 6. An intuitionistic contra strongly semi- homeomorphism (I contra S- semi-hom., for short) function if f is contra semi- continuous function and contra open function.
6. An intuitionistic contra strongly α- homeomorphism (I contra S-α-hom., for short) function if f is contra α- continuous function and contra open function.
7. An intuitionistic contra strongly pre- homeomorphism (I contra S-pre-hom., for short) function if f is contra pre-continuous function and contra open function.
8. An intuitionistic contra strongly β - homeomorphism (I contr S-β -hom., for short) function if f is contra β-continuous function and contra open function
9. An intuitionistic contra S* semi- homeomorphism (I contra S*-semi-hom., for short) function if f is contra continuous function and contra semi- open function.
10. An intuitionistic contra S*- α- homeomorphism (I contra S*- α-hom., for short) function if f is contra continuous function and contra α-open function.
11. An intuitionistic contra S*- pre- homeomorphism (I contra S*-pre-hom., for short) function if f is contra continuous function and contra pre - open function.
12. An intuitionistic contra S*- β- homeomorphism (I contra S*- β-hom., for short) function if f is contra continuous function and contra β-open function.

**Proposition 9:** Let k= {semi, α , per , β }and (X,T) , (Y,σ) be two ITS's and let $f : X \to Y$ be a bijective function then :

1. If $f$ is I contra hom. function then $f$ is I contra k- hom. function.
2. If $f$ is I contra hom. function then $f$ is I contra S- k- hom. function.
3. If $f$ is I contra hom. function then $f$ is I contra S*-k- hom. function.
4. If $f$ is I contra α- hom. function then If $f$ is I contra semi-hom.function.
5. If $f$ is I contra semi- hom. function then If $f$ is I contra β- hom. function
6. If $f$ is I contra β- hom. function then If $f$ is I contra pre- hom. function

7. If $f$ is I contra α- hom.function then If $f$ is I contra pre-hom.function.
8. If $f$ is I contra S-α- hom.function then If $f$ is I contra S-semi- hom. function.
9. If $f$ is I contra S-semi- hom.function then If $f$ is I contra S-β- hom. function
10. If $f$ is I contra S-β- hom.function then If $f$ is I contra S-pre- hom. function
11. If $f$ is I contra S-α- hom.function then If $f$ is I contra S-pre- hom. function.
12. If $f$ is I contra S*-α- hom.function then If $f$ is I contra S*-semi-hom. function.
13. If $f$ is I contra S*-semi- hom.function then If $f$ is I contra S*-β-hom. function
14. If $f$ is I contra S*-β- hom.function then If $f$ is I contra S*-pre-hom.function
15. If $f$ is I contra S*-α- hom.function then If $f$ is I contra S*-pre-hom. function.
16. If $f$ is I contra S-k- hom.function then If $f$ is I contra k- hom. function.
17. If $f$ is I contra S*-k- hom.function then If $f$ is I contra k- hom. function.

**Proof:**

1. Let A be IOS in Y then $f^{-1}$ (A) is ICS in X (since f is I contra cont. function) then $f^{-1}$ (A) is IkCS in X, hence f is I contra k- cont. function. Let B be IOS in X then f (B) is ICS in Y (since f is I contra open function.) then f (B) is IkCS in Y then f is I contra k-open function Hence f is I contra k- hom. Function.

2. Let A be IOS in Y then $f^{-1}$ (A) is ICS in X (since f is I contra cont. function) then $f^{-1}$ (A) is IkCS in X, hence f is I contra k-cont. function hence f is I contra S- k- hom. Function**.**

3. Let B be IOS in X then f $^{(}$B) is ICS in Y (since f is I contra open function.) then f $^{(}$B) is IkCS in Y then f is I contra k-open. function. Hence f is I contra S*-k- hom. function.

4. Let A be IOS in Y then $f^{-1}$ (A) is IαCS in X (since f is I contra α- cont. function) then $f^{-1}$ (A) is ISCS in X, hence f is I contra semi-cont. function. Now let B be IOS in X then f $^{(}$B) is IαCS in Y (since f is I contra α-open

function.) then f (B) is ISCS in Y then f is I contra semi-open.function. Hence f is I contra semi- hom.function.

5,6,7 we can prove that by the same way.

8. let A be IOS in Y then $f^{-1}$ (A) is IαCS in X (since f is I contra α- cont. function) then $f^{-1}$ (A) is ISCS in X ,then f is I contra semi-cont. function and f is I contra open function (since f is I contra S-α-hom.function). Hence f is I contra S-semi- hom.function.

9,10,11 we can prove that by the same way.

12. Let B be IOS in X then f (B) is IαCS in Y ( since f is I contra α-open function.) then f (B) is ISCS in Y then f is I contra semi-open function and f is I contra cont. function (since f is I contra S*-α-hom.function). Hence f is I contra S*-semi- hom.function.

13,14,15 we can prove that by the same way.

16. Let B be IOS in X then f(B) is ICS in Y(since f is I contra open function) then f (B) is IkCS in Y then f is I contra k-open function. Hence f is I contra k- hom.function.

17. Let A be IOS in Y then $f^{-1}$ (A) is ICS in X (since f is I contra cont. function) then $f^{-1}$ (A) is IkCS in X ,then f is I contra k-cont. function. Hence f is I contra k- hom.function**.**

**Remark :** the converse of proposition 2.9 is not true **.**

The following examples shows that:

1- I contra semi- hom.function does not I contra hom.function

**Example 1:** Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{\mathrm{b}\}, \{a, c\}\rangle$ and let $Y = \{1,2,3\}$ , $\sigma = \{\tilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y, \{1\}, \{2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2\ and\ f(c) = 3$

Now $f^{-1}(\mathrm{B}) = \langle x, \{\mathrm{a}\}, \{\mathrm{b}\}\rangle$ is ISCS in X since $(f^{-1}(B))^c$ is ISOS in X since $(f^{-1}(B))^c \subseteq clint(f^{-1}(B))^c$ =X then f is I contra semi-cont. function, but not I contra cont.function since $f^{-1}$(B) not ICS in X since $(f^{-1}(B))^c = \langle x, \{b\}, \{a\}\rangle$ not IOS in X . And f is I contra semi-open function since $f(\mathrm{A}) = \langle y, \{2\}, \{1,3\}\rangle$ is ISCS in Y since $(f(A))^c$ is ISOS in Y since $(f(A))^c \subseteq clint(f(A))^c = Y$, but f not I contra open function since $f(A)$ is not ICS in Y since $clf(A) = B^c \neq f(A)$

Hence f is I contra semi-hom. function, but not I contra hom. Function.

2- I contra α- hom.function does not I contra hom.function.

**Example 2:** Let $X = \{a,b,c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{\mathrm{b}\}, \{a,c\}\rangle$ and let $Y = \{1,2,3\}$ , $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y, \{1\}, \{2,3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ *and* $f(c) = 3$
Now $f^{-1}(\mathrm{B}) = \langle x, \{\mathrm{a}\}, \{\mathrm{b}, c\}\rangle$ is IαCS in X since $(f^{-1}(B))^c$ is IαOS in X since $(f^{-1}(B))^c \subseteq intclint(f^{-1}(B))^c$ =X then f is I contra α-cont. function, but not I contra cont.function since $f^{-1}(\mathrm{B})$ not ICS in X since $(f^{-1}(B))^c = \langle x, \{b,c\}, \{a\}\rangle$ not IOS in X . And f is I contra α-open function since $f(\mathrm{A}) = \langle y, \{2\}, \{1,3\}\rangle$ is IαCS in Y since $(f(A))^c$ is IαOS in Y since $(f(A))^c \subseteq intclint(f(A))^c = X$, but f not I contra open function since $f(A)$ is not ICS in Y since $clf(A) = B \neq f(A)$
I contra α- hom.function but not I contra hom.function Hence f is
3- I contra pre - hom.function does not I contra hom.function**.**

**Example 3:** Let $X = \{a,b,c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{\mathrm{b,c}\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$ , $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y, \{2\}, \{1\}\rangle$ Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ *and* $f(c) = 3$ Now $f^{-1}(\mathrm{B}) = \langle x, \{\mathrm{b}\}, \{\mathrm{a}\}\rangle$ is IPCS in X since $(f^{-1}(B))^c$ is IPOS in X since $(f^{-1}(B))^c \subseteq clintcl(f^{-1}(B))^c$ =X then f is I contra β-cont. function, but not I contra cont.function since $f^{-1}(\mathrm{B})$ not ICS in X since $(f^{-1}(B))^c = \langle x, \{a\}, \{b\}\rangle$ not IOS in X . And f is I contra pre-open function since $f(\mathrm{A}) = \langle y, \{2,3\}, \{1\}\rangle$ is IPCS in Y since $(f(A))^c$ is IPOS in Y since $(f(A))^c \subseteq intcl(f(A))^c = Y$, but f not I contra open function since $f(A)$ is not ICS in Y since $clf(A) = Y \neq f(A)$.
Hence f is I contra pre - hom.function but not I contra hom.function.
4- I contra β - hom.function does not I contra hom.function**.**

**Example 4:** Let $X = \{a,b,c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{\mathrm{b,c}\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$ , $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y, \{2\}, \{1\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ *and* $f(c) = 3$ Now $f^{-1}(\mathrm{B}) = \langle x, \{\mathrm{b}\}, \{\mathrm{a}\}\rangle$ is IβCS in X since $(f^{-1}(B))^c$ is IβOS in X since $(f^{-1}(B))^c \subseteq clintcl(f^{-1}(B))^c$ =X then f is I contra β-cont. function, but not I contra cont. function since $f^{-1}(\mathrm{B})$ not ICS in X since $(f^{-1}(B))^c = \langle x, \{a\}, \{b\}\rangle$ not IOS in X . And f is I contra β-open function since $f(\mathrm{A}) = \langle y, \{2,3\}, \{1\}\rangle$ is IβCS in Y since $(f(A))^c$ is IβOS in Y since $(f(A))^c \subseteq cl\,intcl(f(A))^c = Y$, but f not I contra open function since $f(A)$ is not ICS in Y since $clf(A) = Y \neq f(A)$.

Hence f is I contra β - hom.function but not I contra hom.functio

5- I contra S- semi- hom.function does not I contra hom.function.

**Example 5:** Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{\text{b}\}, \{a, c\}\rangle$ And let $Y = \{1,2,3\}$ , $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B, C\}$ where $B = \langle y, \{1\}, \{2\}\rangle$, $C = \langle y, \{1,3\}, \{2\}\rangle$ Define a function $f: X \rightarrow Y$ by $f(a) = 1, f(b) = 2$ $and\ f(c) = 3$ . Now $f^{-1}(\text{B}) = \langle x, \{\text{a}\}, \{\text{b}\}\rangle$ is ISCS in X since $(f^{-1}(B))^c$ is ISOS in X since $(f^{-1}(B))^c \subseteq clint(f^{-1}(B))^c$ =X ,also $f^{-1}(\text{C})$ is ISCS in X then f is I contra semi-cont.function , but not I contra cont.function since $f^{-1}(\text{B})$ not ICS in X since $(f^{-1}(B))^c = \langle x, \{b\}, \{a\}\rangle$ not IOS in X . And f is I contra open function since $f(\text{A}) = \langle y, \{2\}, \{1,3\}\rangle = C^c$ is ICS in Y since C$\in \sigma$ . Hence f is I contra S- semi-hom. Function, but not I contra hom. Function.

6- I contra S- α- hom.function does not I contra hom.function.

**Example 6:** Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$where $A = \langle x, \{\text{b}\}, \{a, c\}\rangle$ and let$Y = \{1,2,3\}$ , $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y, \{1,3\}, \{2\}\rangle$ $B = \langle y, \{1\}, \{2,3\}\rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1, f(b) = 2$and $f(c) = 3$. Now $f^{-1}(\text{C}) = \langle x, \{\text{a}\}, \{\text{b}, c\}\rangle$ is IαCS in X since $(f^{-1}(C))^c$ is IαOS in X since $(f^{-1}(C))^c \subseteq intclint(f^{-1}(C))^c$ =X ,also $f^{-1}(C)$ is IαOS in X, then f is I contra α-cont.function , but not I contra cont.function since $f^{-1}(\text{C})$ not ICS in X since $(f^{-1}(C))^c = \langle x, \{b, c\}, \{a\}\rangle$ not IOS in X .And f is I contra open function since $f(\text{A}) = \langle y, \{2\}, \{1,3\}\rangle$ is ICS in Y since$(f\ (A))^c = B$ is IOS in Y . Hence f is I contra S- α- hom. function but not I contra hom.function .

7- I contra S- pre - hom.function does not I contra hom.function.

**Example 7:** Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{\text{b}\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$ , $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B, C\}$, where $B = \langle \text{y}, \{1\}, \{2\}\rangle$and $\text{C} = \langle y, \{1,3,\}, \{2\}\rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1, f(b) = 2and$ $f(c) = 3$.Now $f^{-1}(\text{C}) = \langle x, \{\text{a, c}\}, \{\text{b}\}\rangle$ is IPCS in X since $(f^{-1}(C))^c$ is IPOS in X since $(f^{-1}(C))^c \subseteq intcl(f^{-1}(C))^c$ =X , also $f^{-1}(\text{B})$ is IPCS in X then f is I contra pre -cont. function, but not I contra cont. function since $f^{-1}(\text{C})$ not ICS in X since $(f^{-1}(C))^c = \langle x, \{b\}, \{a, c\}\rangle$ not IOS in X , and f is I contra open function since $f(\text{A}) = \langle y, \{2\}, \{1\}\rangle$ is ICS in Y since $(f\ (A))^c = B$ is IOS in Y .

Hence f is I contra S- pre - hom.function but not I contra hom.function.

8. I contra S- β - hom.function does not I contra hom.function**.**

**Example 8:** Let $X = \{a,b,c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$where $A = \langle x, \{\text{b}, \text{c}\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$ , $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B, C\}$where $B = \langle y, \{2\}, \{1\}\rangle$ $and$ $C = \langle y, \{1\}, \{2,3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1$, $f(b) = 2$ $and$ $f(c) = 3$

Now $f^{-1}(\text{B}) = \langle x, \{\text{b}\}, \{\text{a}\}\rangle$ is IβCS in X since $(f^{-1}(B))^c$ is IβOS in X since $(f^{-1}(B))^c \subseteq clintcl(f^{-1}(B))^c$ =X ,also $f^{-1}(C)$is IβOS then f is I contra β-cont. function , but not I contra cont. function since $f^{-1}(\text{B})$ not ICS in X since $(f^{-1}(B))^c = \langle x, \{a\}, \{b\}\rangle$ not IOS in X . And f is I contra open function since $f(\text{A}) = \langle y, \{2,3\}, \{1\}\rangle$ is ICS in Y since $(f(A))^c$ =C is IOS in Y. Hence f is I contra S- β - hom. function but not I contra hom.function**.**

9- I contra S*- semi- hom.function does not I contra hom.function **.**

**Example 9:** Let $X = \{a,b,c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{\text{b}\}, \{a,c\}\rangle$ and $\text{B} = \langle x, \{\text{b}\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \{1\}, \{2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1$, $f(b) = 2$ $and$ $f(c) = 3$.Now $f^{-1}(\text{C}) = \langle x, \{\text{a}\}, \{\text{b}\}\rangle$ is ICS in X since $(f^{-1}(C))^c = B$ is IOS in X then f is I contra cont. function. And f is I contra semi- open function since $f(\text{A}) = \langle y, \{2\}, \{1,3\}\rangle$ is ISCS in Y since $(f(A))^c \subseteq clint(f(A))^c = Y$ , also $f(\text{B})$ is ISCS in Y , f is not I contra open function since $f(\text{A})$ is not ICS in Y since cl $f(\text{A}) = \text{C}^\text{c} \neq f(\text{A})$ . Hence f is I contra S*- semi-hom. Function but not I contra hom. Function

10- I contra S*- α- hom.function does not I contra hom. function.

**Example 10:** Let$X = \{a,b,c\}$$and$ $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{\text{b}\}, \{a,c\}\rangle$, $B = \langle x, \{b,c\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \{1\}, \{2,3\}\rangle$.Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$$and$ $f(c) = 3$.Now $f^{-1}(\text{C}) = \langle x, \{\text{a}\}, \{\text{b}, c\}\rangle$ is ICS in X since $(f^{-1}(C))^c = B$ is IOS in X, then f is I contra cont. function. And f is I contra α- open function since $f(\text{A}) = \langle y, \{2\}, \{1,3\}\rangle$ is IαCS in Y since$(f(A))^c$is IαOS in Y since $(f(A))^c \subseteq intclint(f(A))^c = Y$ also $f(\text{B})$ is IαCS in Y, but f is not I contra open function since $f(\text{A})$ is not ICS in Y since $(f(\text{A}))^\text{c}$ is not IOS in Y. Hence f is I contra S*- α- hom. function but not I contra hom. function .

11- I contra S*- pre - hom.function does not I contra hom.function**.**

**Example 11**: Let$X = \{a, b, c\}$and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C, D\}$where $A = \langle x, \{b, c\}, \{a\}\rangle, B = \langle x, \{b\}, \{a\}\rangle, C = \langle x, \{b\}, \{c, a\}\rangle$and D=$\langle x, \{b, c\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, E\}$ where $E = \langle y, \{1\}, \{2,3\}\rangle$ . Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ *and* $f(c) = 3.$ Now f is I contra continuous function since: $f^{-1}(E) = \langle x, \{a\}, \{b, c\}\rangle$ is ICS in X $(f^{-1}(E))^c$=A is IOS in X. and f is I contra pre – open function Since$f(D) = \langle Y, \{2,3\}, \emptyset\rangle is\ IPCS\ \ in\ Y\ since(\ f(D))^c is\ IPOS\ in\ Y$ $f(D))^c \subseteq intcl(f\ (D))^c = Y, also\ f(A), f(B), and\ f(C) are\ IPCS$ in Y but f does not I contra open function since f(D) is not ICS in Y since clf(D)=y≠ f(D). Hence f is I contra S*- p- hom.function but not I contra hom.function

12- I contra S*- β - hom.function does not I contra hom.function

**Example 12:** Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{b, c\}, \{a\}\rangle\ and\ B = \langle x, \{a\}, \{b\}\rangle$ and let $Y = \{1,2,3\}$, $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, C\}$where $C = \langle y, \{2\}, \{1\}\rangle\ and$ . Define a function $f: X \to Y$ by $f(a) = 1,\ f(b) = 2\ and\ f(c) = 3$. Now $f^{-1}$(C) = $\langle x, \{b\}, \{a\}\rangle$ is ICS in X since $(f^{-1}\ (C))^c$=B is IOS in X then f is I contra cont. function. And f is I contra β- open function since $f$(A) = $\langle y, \{2,3\}, \{1\}\rangle$ is IβCS in Y since $(f\ (A))^c$ is IβOS in Y since $(f\ (A))^c \subseteq clintcl(f\ (A))^c = Y$, also $f\ (B)$ is IβCS in Y , but f is not I contra open function since $f$(A) is not ICS in Y since $(f(\text{A}))^c$ is not IOS in Y.

Hence f is I contra S*- β - hom.function but not I contra hom.function**.**

**Remark :**

1. The notion I contra S- semi- hom.function and I contra S*- semi- hom.function are independed notion .

At first we prove that I contra S- semi- hom.function does not I contra S*- semi- hom.function for example : Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{b\}, \{a, c\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B, C\}$ where $B = \langle y, \{1\}, \{2\}\rangle. C = \langle y, \{1,3\}, \{2\}\rangle$ Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2\ and\ f(c) = 3$.Now $f^{-1}$(B) = $\langle x, \{a\}, \{b\}\rangle$ is ISCS in X since $(f^{-1}\ (B))^c$ is ISOS in X since $(f^{-1}\ (B))^c \subseteq clint(f^{-1}\ (B))^c$ =X , also$f^{-1}$(C) is ISCS in X then f is I contra semi-cont.function , but not I contra cont.function since $f^{-1}$(B) not ICS in X since $(f^{-1}\ (B))^c = \langle x, \{b\}, \{a\}\rangle$ not IOS in X . And f is I contra open function since $f$(A) =

$\langle y,\{2\},\{1,3\}\rangle = C^c$ is ICS in Y since C$\in \sigma$ . Hence f is I contra S- semi-hom.function, but not I contra S*- semi hom.function
Now the I contra S*- semi- hom.function does not I contra S- semi-hom.function we shown that by this example :
$let\ X = \{a,b,c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{\text{b}\}, \{a,c\}\rangle$ and B $= \langle x, \{\text{b}\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \{1\}, \{2\}\rangle$ . Define a function $f\colon X \to Y$ by $f(a) = 1, f(b) = 2$ $and\ f(c) = 3.$ Now $f^{-1}(\text{C}) = \langle x, \{\text{a}\}, \{\text{b}\}\rangle$is ICSin X since $(f^{-1}\,(C))^c = B$ is IOS in X then f is I contra cont.function. And f is I contra semi-open function since $f(\text{A}) = \langle y, \{2\}, \{1,3\}\rangle$ is ISCS in Y since $(f\ (A))^c$ is ISOS in Y $(f\ (A))^c \subseteq clint(f\ (A))^c = Y$ , also $f$(B) is ISCS in Y , f is not I contra open function since $f$(A) is not ICS in Y since cl $f(\text{A}) = \text{C}^{\text{c}} \neq f(\text{A})$. Hence f is I contra S*- semi-hom. function but not I contra S-sime- hom.function.
2. The notion I contra S- α - hom.function and I contra S*- α - hom. function are independed notion .
At first we prove that I contra S- α - hom.function does not I contra S*- α - hom.function for this example: Let $X = \{a,b,c\}$ , $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{\text{b}\}, \{a,c\}\rangle$ and let $Y = \{1,2,3\}$ , $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y, \{1,3\}, \{2\}\rangle$ $B = \langle y, \{1\}, \{2,3\}\rangle$ . Define a function $f\colon X \to Y$ by $f(a) = 1, f(b) = 2\ and\ f(c) = 3.$ Now $f^{-1}(\text{C}) = \langle x, \{\text{a}\}, \{\text{b}, c\}\rangle$ is IαCS in X since $(f^{-1}\,(C))^c$ is IαOS in X since $(f^{-1}\,(C))^c \subseteq intclint(f^{-1}\,(C))^c$ =X ,also $f^{-1}\,(C)$ is IαOS in X , then f is I contra α-cont.function , but not I contra cont.function since $f^{-1}(\text{C})$ not ICS in X since $(f^{-1}\,(C))^c = \langle x, \{b,c\}, \{a\}\rangle$ not IOS in X. And f is I contra open function since $f(\text{A}) = \langle y, \{2\}, \{1,3\}\rangle$ is ICS in Y since$(f\ (A))^c = B$ is IOS in Y. Hence f is I contra S- α- hom.function but not I contra S*- α -hom.function
Now the I contra S*- α - hom.function does not I contra S- α - hom.function we shown that by this example: Let$X = \{a,b,c\}and\ T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$where $A = \langle x, \{\text{b}\}, \{a,c\}\rangle, B = \langle x, \{b,c\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \{1\}, \{2,3\}\rangle$ . Define a function $f\colon X \to Y$ by $f(a) = 1, f(b) = 2\ and\ f(c) = 3.$ Now $\langle x, \{\text{a}\}, \{\text{b}, c\}\rangle$ is ICS in X since $(f^{-1}\,(C))^c = B$ is IOS in X , then f is I contra cont.function .And f is I contra α- open function since $f(\text{A}) = \langle y, \{2\}, \{1,3\}\rangle$ is IαCS in

Y since$(f\ (A))^c$is IαOS in Y since $(f(A))^c \subseteq\ intclint(f\ (A))^c = Y$ also $f$(B) is IαCS in Y , but f is not I contra open function since $f$(A) is not ICS in Y since $(f(\text{A}))^{\text{c}}$ is not IOS in Y. Hence f is I contra S*- α-hom.function but not I contra S- α -hom.function.

3. The notion I contra S- pre - hom.function and I contra S*- pre-hom.function are independed notion .

At first we prove that I contra S- pre- hom.function does not I contra S*- pre- hom.function for this example :

Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{\text{b, c}\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B, C\}$, where $\text{B} = \langle y, \{1,3,\}, \{2\}\rangle\ and\ \text{C} = \langle y, \{1\}, \{2,3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2 and\ f(c) = 3. Now f^{-1}(\text{B}) = \langle x, \{\text{a, c}\}, \{\text{b}\}\rangle$is IPCS in X since $(f^{-1}\ (B))^c$ is IPOS in X since $(f^{-1}\ (B))^c \subseteq intcl(f^{-1}\ (B))^c$ =X, also $f^{-1}$(C) is IPCS in X then f is I contra pre -cont. function, but not I contra cont. function since $f^{-1}$(B) not ICS in X since $(f^{-1}\ (B))^c = \langle x, \{a\}, \{b\}\rangle$ not IOS in X . And f is I contra open function since $f(\text{A}) = \langle y, \{2,3\}, \{1\}\rangle$ is ICS in Y since $(f\ (A))^c = C$ is IOS in Y. Hence f is I contra S-pre - hom.function but not I contra S* - pre-hom.function. Now the I contra S*- pre-hom.function does not I contra S- pre- hom.function we shown that by this example : Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C, D\}$ where $A = \langle x, \{\text{b, c}\}, \{a\}\rangle$ , $B = \langle x, \{b\}, \{c\}\rangle, C = \langle x, \{b\}\{a, c\}\rangle\ and\ D\langle x, \{b, c\}, \emptyset\rangle$and let $Y = \{1,2,3\}$and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, E\} where\ E = \langle y, \{1\}, \{2,3\}\rangle\ \ and$ . Define a function $f: X \to Y$ by $f(a) = 1,\ f(b) = 2\ \ and\ f(c) = 3.$ Now f is I contra cont. function $since f^{-1}(\text{E}) = \langle x, \{\text{a}\}, \{\text{b, c}\}\rangle$ is ICS in X since $(f^{-1}\ (E))^c$=A is IOS in X .And f is I contra pre - open function since $f(\text{D}) = \langle y, \{2,3\}, \emptyset\rangle$ is IPCS in Y since $(f\ (D))^c$ is IPOS in Y since $(f\ (D))^c \subseteq intcl(f\ (D))^c = Y, also\ f(B),\ \ f(C) and f(A)$ are IPCSs in Y , but f is not I contra open Function since $f$(D) is not ICS in Y since $clf(\text{D}) = \text{Y} \neq \text{f(D)}$ is not IOS in Y. Hence f is I contra S*-pre - hom.function but not I contra S*-pre -hom.function.

4. The notion I contra S- β- hom.function and I contra S*- β - hom.function are independed notion .

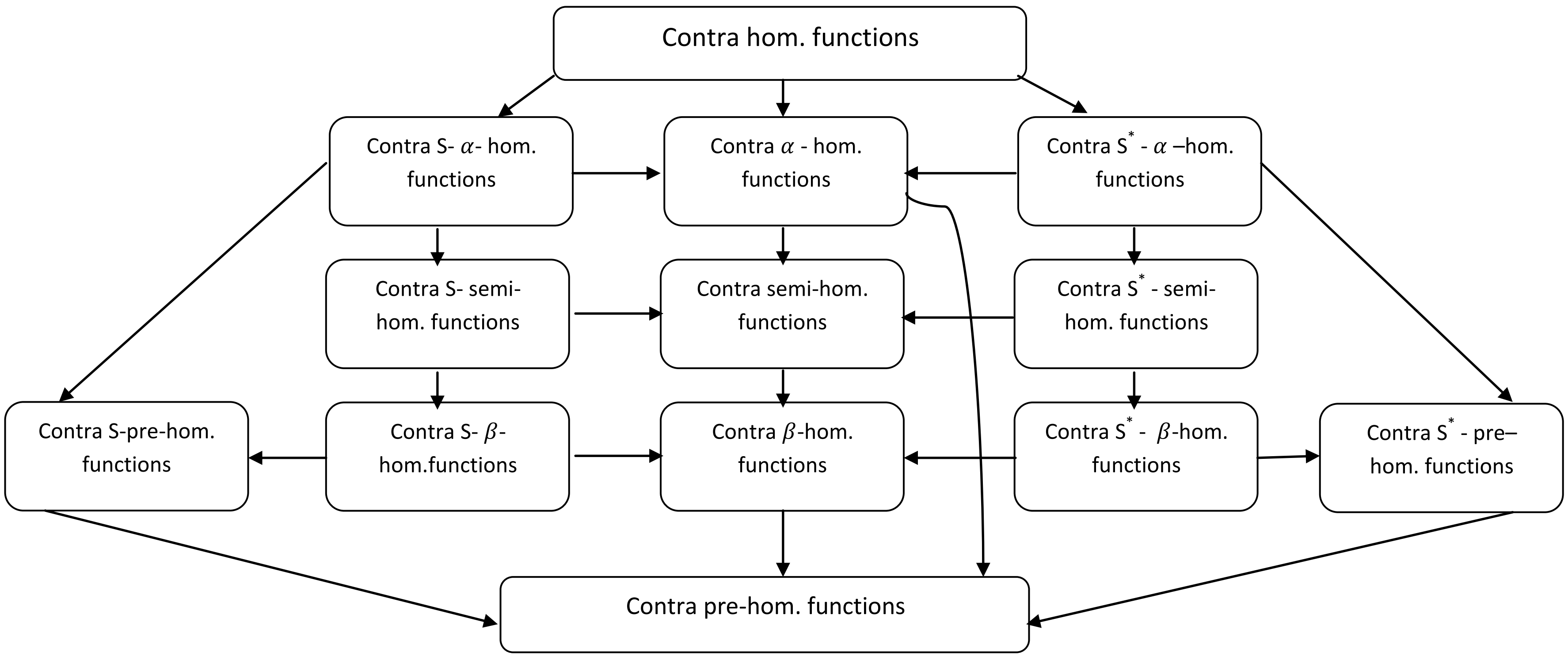
At first we prove that I contra S- β - hom.function does not I contra S*- β - hom.function for this example:Let $X = \{a, b, c\}$ , $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$where

$A = \langle x, \{\text{b}, \text{c}\}, \{a\}\rangle$ and let $Y = \{1,2,3\}$, $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, B, C\}$where $B = \langle y, \{2\}, \{1\}\rangle$ *and* $C = \langle y, \{1\}, \{2,3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1,\ f(b) = 2$ *and* $f(c) = 3$. Now $f^{-1}(\text{B}) = \langle x, \{\text{b}\}, \{\text{a}\}\rangle$ is IβCS in X since $(f^{-1}\,(B))^c$ is IβOS in X since $(f^{-1}\,(B))^c \subseteq clintcl(f^{-1}\,(B))^c$ =X ,also $f^{-1}(C)$is IβOS then f is I contra β-cont.function , but not I contra cont.function since $f^{-1}(\text{B})$ not ICS in X since $(f^{-1}\,(B))^c = \langle x, \{a\}, \{b\}\rangle$ not IOS in X. And f is I contra open function since $f(\text{A}) = \langle y, \{2,3\}, \{1\}\rangle$ is ICS in Y since $(f\,(A))^c$ =C is IOS in Y. Hence f is I contra S- β - hom.function but not I contra S*- β -hom.function.

Now the I contra S*- β - hom.function does not I contra S- β - hom. function we shown that by this example : Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$where $A = \langle x, \{\text{b}, \text{c}\}, \{a\}\rangle$ *and* $B = \langle x, \{a\}, \{b\}\rangle$and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, C\}$where $C = \langle y, \{2\}, \{1\}\rangle$ *and* . Define a function $f: X \to Y$ by$f(a) = 1, f(b) = 2$ *and* $f(c) = 3$.Now $f^{-1}(\text{C}) = \langle x, \{\text{b}\}, \{\text{a}\}\rangle$ is ICS in X since $(f^{-1}\,(C))^c$=B is IOS in X then f is I contra cont.function And f is I contra β- open function since $f(\text{A}) = \langle y, \{2,3\}, \{1\}\rangle$ is IβCS in Y since $(f\,(A))^c$ is IβOS in Y since $(f\,(A))^c \subseteq clintcl(f\,(A))^c = Y$, also $f\,(B)$ is IβCS in Y , but f is not I contra open function since $f(\text{A})$ is not ICS in Y since $(f(\text{A}))^{\text{c}}$ is not IOS in Y. Hence f is I contra S*- β - hom.function but not I contra S- β -hom.function.

We summarized the above result by the following diagram .

**Diagram :** The following implications are true and not reversed .



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# أعمام بعض أنواع من الدوال المتشاكلة المعاكسة وبعض علاقتها مع بعضها بين الفضاءات التبولوجية الحدسية

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**الخلاصة**

سندرس في هذا البحث مفهوم الدوال المتشاكلة المعاكس (contra homeomorphism)وبعض أنواعها contra k-homeomorphism functions ,contra strongly k-homeomorphism functions . and contra S*-k- homeomorphism functions بين الفضاءات التبولوجية الحدسية وكذلك سندرس علاقة هذه الدوال مع بعضها عن طريق بعض المبرهنات والأمثلة وتم وضع مخطط لتلك العلاقة.