Generalization of the radial nuclear two-body potentials and new derivation to find their matrix elements

Aziz H. Fatah
Department of Physics, College of Science_University of Sulaimania
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Abstract

The text of this paper covers a generalization of the radial two-body nuclear potentials and then derivation of a new formula to evaluate their matrix element. A new formula was derived and applied successfully to calculate the single-particle matrix elements of the spin-isospin dependent and spin-isospin independent potentials by using a mathematica code.

Introduction

The basic technical problem in nuclear-shell theory involves the calculation of matrix elements of the physically relevant operators such as one- and two-body operators, pair creation operators, etc., between definite states of an n-particle configuration. The standard technique for the evaluation of such matrix elements makes use of the fractional parentage expansion which is defined as the coefficients for the expansion of antisymmetrical wave - function in terms of the complete set of the vector - coupled parent states with a lower degree of antisymmetry (Kaminski. *et al*, 1975).

From the fact that the atomic nucleus is a quantal and many-body system (containing A-nucleons), the A-body matrix elements of the measurable single- and two- body operators (which are more predominant operators) can be transformed to a spin-isospin included matrix elements corresponding to one-and two-body opertaors. In such transformation the one-body and two-body coefficients of fractional parentage can be used for the one- and two-body operators, respectively (Langanke. *et al*, 1993).
One of the important features of the nuclear interactions from the other types of the interactions is the absence of a perfect mathematical formula of them. Therefore, in the calculations of nuclear shell models many different types of the empirical and phenomenological potentials have been proposed to represent the nuclear force and hence the nuclear interactions between the nucleons. In most nuclear shell model applications the nuclear

interactions were restricted to be two-body interaction (more predominant type) and then the higher order of the interactions were ignored.

In the basics of nuclear shell theory, the isospin-included two-body matrix element corresponding to the operator of a measurable quantity can be classified into two kinds: JJ-coupling and LS-coupling. Each of such types of the couplings (JJ and LS) can also be written either in terms of the two individual nucleon coordinates (position, spin and isospin) correlative-and center of mass- coordinates (Brussaard & Glaudemans, 1977).

From the review of literature one finds that the nuclear two-body interactions with central, spin-orbit, and tensor types, are dependent on the relative coordinates, the Brody-Moshinsky coefficients must be used to transform from the individual coordinate representation to the relative- and center of mass-coordinates then remove algebraically the center of mass coordinates to remain the reduced single-particle (radial) matrix element only.

Actually, the reduced single-particle matrix elements were obtained in two methods; the first one is perfect numerical method by which the reduced matrix element can be treated as integral of an integrand function and then it is obtained numerically. This method was widely used from the works Radhi (Radhi. *et al*, 2008) to calculate single-particle reduced matrix element for study the nuclear structure (electron scattering form factors). In the other hand, the second method represents a mathematical technique by which the single-particle matrix element can be expressed as the Talmi integrals; this method was applied by Ursescu (Ursescu. *et al*, 2005) to determine the reduced matrix elements.

The aim of the present study is to derive a new mathematical formula to compute the single-particle reduced matrix elements. The main text of this paper is divided into three sections. In section two, a general formula will be proposed to represent the nuclear potentials (i.e. central, spin-orbit, tensor, types). In section three we will try to obtain a new derived formula to calculate the single-particle matrix element for some famous two-body interactions. Finally, in the fourth section, a mathematica code has been used to compute the results and compared with thoseinref. (Ursescu *et al*, 2005).

1- General representation of nuclear two-body potentials

Because the nuclear two-body operators depend on the relative coordinates, any two nucleon spin-isospin included state in jj-coupling form, i.e $|n_a l_a j_a, n_b l_b j_b; JT\rangle$, must be transformed to the LS-coupling form $|n_a l_a, n_b l_b; \Lambda, S; JT\rangle$, then from the Brody-Moshinsky coefficients the single-particle coordinates in the LS-state is transformed to the intrinsic and center-of-mass coordinates $|nl, NL; \Lambda, S; JT\rangle$ (Ursescu. *et al*, 2005) and

(Moshinsky, 1959); and from the recoupling process intrinsic coordinates can be separated from the center-of-mass coordinates $|nl, Sj, NL; JT \rangle$. Thus, the matrix element for a nuclear two-body potential is given by van Hees (van Hees & Glaudemans, 1983):

$$\langle n_a l_a j_a, n_b l_b j_b | V | n_c l_c j_c, n_d l_d j_d \rangle_{JT} = \sum_{\substack{\Lambda \Lambda' nln'l' \\ SS'NLj}} B_{nlNL}^{n_a l_a n_b l_b} B_{n'l'NL}^{n_c l_c n_d l_d} \; F_{l_a \frac{1}{2} j_a}^{l_b \frac{1}{2} j_b}(\Lambda, S, J)$$

$$F_{l_c \frac{1}{2} j_c}^{l_d \frac{1}{2} j_d}(\Lambda', S', J) F_{L\,0\,L}^{l\,S\,j}(\Lambda, S, J) F_{L\,0\,L}^{l'S'j}(\Lambda', S', J)$$

$$\frac{[1 - (-1)^{l+S+T}][1 - (-1)^{l'+s'+T}]}{2\sqrt{(1 + \delta_{ab})(1 + \delta_{cd})}} \langle nl, S, j | V | n'l', S', j \rangle_{JT} \qquad \dots (1)$$

Where the coefficients B and F are symbolized to the Brody-Moshinsky brackets and the normalized 9j-symbols, respectively; while $\langle nl, S, j | V | n'l', S', j \rangle_{JT}$ is the spin-isospin matrix element of the potential V in the relative coordinates.

In general, the nucleon-nucleon interaction V may be classified into two classes: central and noncentral (i.e. tensor, two-body spin-orbit…ect interactions). They can conveniently be written in three kinds: Wigner-Majorana-Bartlett-Heisenberg representation with the space and spin exchange operators; spin and isospin representation with dot product of the individual two position-, two spin-, and two isospin- operators; and triplet-singlet even-odd representations with the singlet-odd, singlet-even, triplet-odd, and triplet-even operators (Schiffer & True, 1976).

All the local potentials may have different strengths and radial dependence with exponential, a Yukawa and a Gaussian shapes (Schiffer & True, 1976) and (Negele & Vogt, 2003). Thus, the different proposed types of the nuclear two-body interactions will be the coefficients of the above three mentioned forms including spin-isospin effects, that can be removed the spin-isospin operators by finding their expectation values for two-body spin-isospin states (Bydžovský. *et al*, 2007). Therefore to define a general formula covers exponential, Yukawa and Gaussian potentials, V(r)can be replaced by its generalized formulaV(r, b, m, a, c, μ, τ)that contains m-terms and given by:

$$V(r, m, a, c, \mu, \tau) = - \sum_{i=1}^{m} \frac{a_i \, e^{-(\mu_i r^2 + \tau_i r)}}{r^{c_i}} \qquad \dots (2)$$

Here, a_i, c_i, μ_i, and τ_i are some parameters where as for the Gaussian potentials $\tau_i = 0$ and $\mu_i = 0$ for the exponential types; while a_i is used for the strengths of the interaction.

Serber force is an example which has Gaussian shape with the formula (Negele & Vogt, 2003):

$$V(r) = V_0 e^{-(r/r_0)^2} [W + MP_x] \qquad \text{... (3)}$$

, where $r_0 = 2.8\ f$, $W = M = 0.5$, $V_0 = -40\ MeV$

Knowing that $P_x = -P_\sigma P_\tau$ where P_x, P_σ, and P_τ exchange the position, spin, and isospincoordinates,respectively. Both P_σ and $P_\tau$can be obtained from the inner product of the two Pauli spins and isospins, respectively from the relations

$$\begin{cases} P_\sigma = (1 + \vec{\sigma}_1 . \vec{\sigma}_2)/2 \\ \\ P_\tau = (1 + \vec{\tau}_1 . \vec{\tau}_2)/2 \end{cases} \qquad \text{... (4)}$$

With their eigenvalue in the two-body spin-isospin state $|ST \rangle$(Sitenko &Victor, 1997; Heyde, 1994):

$$\begin{cases} P_\sigma |S \rangle = (-1)^{S-1}|S \rangle \\ \\ P_\tau |T \rangle = (-1)^{T-1}|T \rangle \end{cases} , \quad S, T = 0, 1 \qquad \text{... (5)}$$

In ref. [13] by replacing the inner products by their corresponding operators P_σ and P_τ in eq. (5), we can use the Serber-Yukawa force,that is an example of the Yukawa and hence exponential shapes:

$$V(r) = \left[V_1 \frac{e^{-t_1 r}}{t_1 r} \frac{(1 - P_\sigma)}{2} + V_2 \frac{e^{-t_2 r}}{t_2 r} \frac{(1 + P_\sigma)}{2} \right] \frac{1}{2} [1 - P_\sigma P_\tau]\ MeV \qquad \text{... (6)}$$

knowing that$t_1 = 0.8547\ f^{-1}$, $t_2 = 0.7261\ f^{-1}$, $V_1 = 46.87\ MeV$, $V_2 = 52.13\ MeV$.

By using the eigenvalue equations in eq. (5) and the orthonormality condition$\delta_{SS'} = 0\ or\ 1$, both operators P_σ and P_τ can be re-written in terms of their respective eigenvalues, and hence, both formulae in eqs. (3 & 6) become net radial dependent,$v(r)$, or

$$v(r) = \langle ST|V|S'T \rangle = V_0 e^{-(r/r_0)^2} \langle ST|W - MP_\sigma P_\tau|S'T \rangle$$
$$= [W + M(-1)^{S+T+1}]V_0 e^{-(r/r_0)^2}\ MeV \qquad \text{... (7)}$$

, and,

$$v(r) = \left[V_1 \frac{e^{-t_1 r}}{t_1 r} \frac{[1 + (-1)^S]}{2} + V_2 \frac{e^{-t_2 r}}{t_2 r} \frac{[1 + (-1)^{S+1}]}{2} \right] \frac{[1 + (-1)^{S+T+1}]}{2}\ Me \text{ ... (8)}$$

Comparing eqs. (7 & 8) with eq. (2), we find that the potential in eq. (7) contains one term only (i.e.,$m = 1$) then:

$$m = 1, a_1 = [W + M(-1)^{S+T+1}]V_0 , c_1 = 0, \quad \mu_1 = 1/r_o^2 ,$$
$$\tau_1 = 0 \qquad \qquad \qquad \qquad \dots (9)$$

and similarly, for the potential of eq.(8) $m = 2$, the parameters are given by:

$$a_1 = -V_1[1 + (-1)^S][1 + (-1)^{S+T+1}]/4t_1 , c_1 = 1, \mu_1 = 0, \quad \tau_1 = t_1$$
$$\left. \right\} \dots (10)$$
$$a_2 = -V_2[1 + (-1)^{S+1}][1 + (-1)^{S+T+1}]/4t_2 , c_2 = 1, \mu_2 = 0, \quad \tau_2 = t_2$$

The final potential which is used to calculate its matrix element is the central part of the generalized density-dependent M3Y-interaction that is ST-independent and given by (El-Shal, 2003):

$$V(r) = v(r) = -\sum_{k=1}^{3} \frac{u_k e^{-t_k r}}{t_k r} \qquad \qquad \dots (11)$$

, with the coefficients:

$$a = u = -(1524.25/4 , 518.75/2.5 , 7.8474/0.7072)MeV$$
$$\left. \right\} \dots (12)$$
$$c = (1,1,1), \mu = (0,0,0), \text{ and } \tau = (4, 2.5 , 0.7072)f^{-1}$$

2- Derivation of the reduced matrix element elementof $v(r)$

By introducing the successive transformations mentioned in Sec.2, and knowing that the final matrix element of the nuclear two-body potential $v(r)$ will be in the relative-coordinates, symbolized by $\langle n'l'|v(r)|n\,l \rangle$, then the single-particle state vectors $|n\,l\rangle$ and $|n'\,l'\rangle$ are chosen to be the harmonic oscillator wave functions with $R_{nl}(r,b) = \langle r|nl \rangle$, such that (Suhonen, 2007):

$$R_{nl}(r,b) = N_{nl}(b)e^{-r^2/2b^2} \sum_{k=0}^{n} \frac{(-1)^k \left(\frac{r}{b}\right)^{2k+l}}{k!\,(n-k)!\,\Gamma(l + k + 3/2)} \qquad \dots (13)$$

With

$$N_{nl}(b) = \sqrt{\frac{2\,n!\,\Gamma(n + l + 3/2)}{b^3}} , \qquad \qquad \dots (14)$$

Where b is the length parameter of the harmonic oscillator and it differs for different nuclei; n and l represent the respective principal and orbital quantum numbers; $n = 1,2, ...$ and $l = 0,1,2,..$

Thus, by using eqs. (13-14) in the single-particle matrix element $\langle n'l'|v(r)|n\,l \rangle$, it can be re-expressed as:

$$\langle n'l'|v(r,b,m,a,c,\mu,\tau)|n\,l \rangle = \mathcal{M}_{nln'l'}(b,m,a,c,\mu,\tau)$$
$$= \int_0^{\infty} R_{n'l'}(r,b)R_{nl}(r,b)r^2 V(r,b,m,a,c,\mu,\tau)dr$$

$$= b^2 N_{nl} N_{n'l'} \sum_{k=0}^{n} \sum_{k'}^{n'} \frac{(-1)^{k+k'}}{k!\,k'!\,(n-k)!\,(n'-k')!\,\Gamma(l+k+3/2)\Gamma(l'+k'+3/2)}$$

$$\sum_{i=1}^{m} \int_{0}^{\infty} \left(\frac{r}{b}\right)^{l'+l+2k+2k'+2} e^{-r^2/b^2} V(r,b,m,a,c,\mu,\tau)\,dr \qquad \qquad \dots(15)$$

Now by defining $r = b\,\theta$ and $p = l' + l + 2k + 2k' + 2$, eq. (15) takes the form:

$$\mathcal{M}_{nln'l'}(b,m,a,c,\mu,\tau) = A_{nln'l'}(b) \sum_{k=0}^{n} \sum_{k'}^{n'} \sum_{i=1}^{m} G_{nln'l'}^{kk'i}\, H(t,\alpha,\beta,\gamma) \qquad \dots(16)$$

Here, the coefficients A and G are given by:

$$A_{nln'l'}(b) = -b^2 N_{nl}(b) N_{n'l'}(b) \qquad \qquad \dots(17)$$

$$G_{nln'l'}^{kk'i} = \frac{(-1)^{k+k'}}{k!\,k'!\,(n-k)!\,(n'-k')!\,\Gamma(l+k+3/2)\Gamma(l'+k'+3/2)} \qquad \dots(18)$$

and $H(t,\alpha,\beta,\gamma)$ is used for the integral:

$$H(t,\alpha,\beta,\gamma) = \alpha e^{\frac{\gamma^2}{4\beta}} \int_{0}^{\infty} \theta^t\, e^{-\beta\left(\theta+\frac{\gamma}{2\beta}\right)^2} d\theta \qquad \qquad \dots(19)$$

With the new parameters t, α, β, and γ:

$$\left.\begin{aligned} t &= p - c_i = l + l' + 2k + 2k' + 2 - c_i \\ \alpha &= a_i/b^{c_i-1} \\ \beta &= 1 + \mu_i b^2 \end{aligned}\right\} \qquad \qquad \dots(20)$$

$$\gamma = \tau_i\, b$$

To find the integral in eq. (19), we must change the element of the integration from the relation:

$$s = \beta\left(\theta + \frac{\gamma}{2\beta}\right)^2 \Rightarrow \theta = \frac{1}{\sqrt{\beta}}\left[s^{\frac{1}{2}} - \frac{\gamma}{2\sqrt{\beta}}\right] \qquad \qquad \dots(21)$$

With $d\theta = ds/2\sqrt{s\beta}$ and the new boundaries $s_1 = \gamma^2/4\beta$ and $s_2 = \infty$. Hence, the integral in eq. (19) becomes:

$$H(t,\alpha,\beta,\gamma) = \frac{\alpha e^{-\frac{\gamma^2}{4\beta}}}{2\,\beta^{\frac{t+1}{2}}} \int_{z}^{\infty} \left[\left(s^{\frac{1}{2}} - \frac{\gamma}{2\sqrt{\beta}}\right)\right]^t e^{-s} s^{\frac{-1}{2}} ds \quad, \qquad z = \frac{\gamma^2}{4\beta} \qquad \dots(22)$$

For the case $t \geq 0$ the term $s^{1/2}$ can be separated by the binomial method expansion of the bracket (Znidaric, 2009), or

$$\left(s^{1/2} - \frac{\gamma}{2\sqrt{\beta}}\right)^t = t! \sum_{j=0}^{t} \frac{s^{\frac{t-j}{2}}}{j!\,(t-j)!}\left(-\frac{\gamma}{2\sqrt{\beta}}\right)^j \qquad \qquad \dots(23)$$

and thus, $H(t, \alpha, \beta, \gamma)$ in eq. (22) is given by:

$$H(t, \alpha, \beta, \gamma) = \frac{\alpha \, t! \, e^{\frac{\gamma^2}{4\beta}}}{2 \, \beta^{\frac{t+1}{2}}} \sum_{j=0}^{t} \frac{\left(-\frac{\gamma}{2\sqrt{\beta}}\right)^j}{j! \, (t-j)!} \, I(t, j, z) \qquad \dots (24)$$

With

$$I(t, j, z) = \int_{z}^{\infty} s^{\frac{t-j+1}{2}-1} \, e^{-s} \, ds \qquad \dots (25)$$

The integral in eq. (25) depends on the range-values of the quantity (z). For the cases $z > 1$ the integral must be determined numerically and hence $H(t, \alpha, \beta, \gamma)$ does not have analytical value, for $z = 0$, the integral will be a gamma function of $(t - j + 1)/2$; while for the values $0 < z \leq 1$, the integral can be divided in to two parts which can be evaluated from the definitions of the gamma function (Γ) and the confluent hypergeometric function (F), or (Boros & Moll, 2004):

$$
I(t, j, z)
= \begin{cases}
\Gamma\left(\frac{t-j+1}{2}\right), & z = 0 \\[2ex]
\Gamma\left(\frac{t-j+1}{2}\right) - \frac{2 \, z^{0.5(t-j+1)}}{t-j+1} \, F\left(\frac{t-j+1}{2}, \frac{t-j+3}{2}, -z\right), & 0 < z \leq 1
\end{cases}
\qquad \dots (26)
$$

Finally, by taking into consideration the cases $z > 1$ and ($z = 0$ and $0 < z \leq 1$)in eq. (26), the integral $I(t, j, z)$ in eq. (26). Hence $H(t, \alpha, \beta, \gamma)$ in eq. (24) and the coefficients in eqs. (17 & 18) can be substituted in eq. (16) to find its final formula given by:

$$\therefore \mathcal{M}_{nln'l'}(b, m, a, c, \mu, \tau) = -\sqrt{\frac{4 \, n! \, n'! \, \Gamma(n+l+3/2)\Gamma(n'+l'+3/2)}{b^2}}$$

$$\sum_{k=0}^{n} \sum_{k'}^{n'} \sum_{i=1}^{m} \frac{(-1)^{k+k'} \, H\left(p - c_i, \frac{a_i}{b^{c_i-1}}, 1 + \mu_i b^2, \tau_i \, b\right)}{k! \, k'! \, (n-k)! \, (n'-k')! \, \Gamma(l+k+3/2)\Gamma(l'+k'+3/2)} \qquad \dots (27)$$

Discussion

Practically all microscopic calculations in nuclear structure use the matrix element of the residual (effective) two –body interaction between valence nucleons corresponding to the matrix elements in, either jj- or LS-coupling, that can be abbreviated to some coefficients and a reduced radial matrix element in the relative coordinates, eq. (1). The two-body nuclear potentials tensor- and non-tensor types can be represented in relative coordinates and also their radial component be written in exponential,

Yukawa, and Gaussian functions; they were generalized and expressed in eq.(2).

After generalization the radial component of the nuclear two-body potentials, a new method was proposed to derive a relation to calculate the net radial matrix elements, eq. (27), either will be completely numerical or depends on the confluent hyper geometric and gamma functions, eq.(26).

In general the nuclear two body potentials, tensor and non-tensor types, are spin-and isospin dependent. Thus, to find their matrix elements two ways can be followed. In the first way, the potential has a net radial form because S, T, and their Pauli componentsσ_1,σ_2, τ_1 and τ_2 are absent, eq. (11); while in the second way we must first remove the spin- and isospin effects and then comes the rule of the radial part to be found that is under the scope of this study for finding the radial matrix elements.

To test the derived relation in eq.(27), three types of nuclear potential have been chosen representing a spin-isospin dependence with Gaussian and Yukawa shapes (Seiber and Yukawa potentials, eq.(7 & 8), respectively), spin-isospin independent which also has a Yukawa shape , eq.(11).All three interactions can be distinguished from their parameters used in their mathematical representations of the interactions.

In the present study for the quantum numbers $1 \leq n, n' \leq 3$and$0 \leq l, l' \leq 3$, the radial matrix elements have been found and then the results were compared with those determined from ref. (Ursescu. *et al,* 2005).

Interactions in eqs. (7-8) include both two-particle spin and isospin (S and T respectively), eq. (7) involves coefficient $(-1)^{S+T+1}$; while eq.(8) includes both coefficients $(-1)^S$, $(-1)^{S+1}$, and $(-1)^{S+T+1}$. For the values $S + T = even$ the coefficient$(-1)^{S+T+1} = 0$, accordingly, the parameter a_1 in eq. (9),a_1 and $a_2$in eq. (10) vanish and hence $v(r)$ in eqs. (7 & 8) and the their corresponding radial matrix elements vanish also. Therefore, in determination of the radial matrix element of the interactions in eqs.(7-8) we must use those values of S and T with $S + T = odd, S = 0$ and $T = 0$,which makes one of the terms in eq.(8),$[1 + (-1)^S]$or $[1 + (-1)^{S+1}]$, vanish.

For the Serber interaction with the parameters given by eq.(7),the radial matrix elements has been determinedfor the coefficients in eq.(9) and by the method of ref.[5]then they are listed in the fifth and sixths columns in Table(1).In addition, Yukawa potential with the coefficients in eq.(10) has been used in eq.(27) to find its matrix element by the two proposed methods, the results are tabulated in the seventh and eighth columns in the table; while eq.(11) was used as a spin-isospin independent potential with its parameters, in eq.(12), to find the radial matrix element in both methods that are listed in the two last columns in the same table.

Table (1) shows that the calculated radial matrix elements in the present study for the three proposed potentials were all in full agreement with those that were evaluated by the reference (Ursescu. *et al,* 2005).

Table (1): The calculated radial matrix element for the Serber, Yukawa, and the generalized density-dependent M3Y-interaction comparing with those determined by ref. (Ursescu. *et al,* 2005).
$(S, T \equiv 0, 1)$

n	l	n'	l'	Serber Int. for $S = 0$ and $T = 1$		Yukawa Int. $S = 0$ and $T = 1$		M3Y Int.	
				Present work	[5]	Present work	[5]	Present work	[5]
1	0	1	0	15.338	15.338	15.2208	15.2208	-26.6863	-26.6863
3	0	2	1	1.60268	1.60268	5.14783	5.14783	-8.80296	-8.80296
2	0	3	3	0.209398	0.209398	1.01331	1.01331	-1.19267	-1.19267
1	1	1	1	12.3776	12.3776	5.77754	5.77754	-4.3986	-4.3986
3	1	3	1	8.72025	8.72025	5.97343	5.97343	-6.31827	-6.31827
3	1	2	2	2.92205	2.92205	2.82327	2.82327	-2.38404	-2.38404
2	1	1	3	-0.50517	-0.50517	0.974741	0.974741	-0.63738	-0.63738
3	1	2	3	-0.90364	-0.90364	0.965288	0.965288	-0.81515	-0.81515
1	2	3	0	-3.91017	-3.91017	0.296797	0.296797	-1.45005	-1.45005
3	2	1	0	7.87282	7.87282	5.01769	5.01769	-4.58117	-4.58117
1	2	2	1	3.17136	3.17136	2.63219	2.63219	-1.86797	-1.86797
2	2	2	1	7.3478	7.3478	3.62633	3.62633	-2.59194	-2.59194
3	2	1	1	7.15705	7.15705	4.064	4.064	-2.80461	-2.80461
3	2	3	1	6.07552	6.07552	3.63397	3.63397	-3.02661	-3.02661
1	2	2	2	6.91516	6.91516	2.71456	2.71456	-1.38734	-1.38734
2	2	1	2	6.91516	6.91516	2.71456	2.71456	-1.38734	-1.38734
2	2	3	2	6.6724	6.6724	3.01522	3.01522	-1.78961	-1.78961
3	2	2	2	6.6724	6.6724	3.01522	3.01522	-1.78961	-1.78961
1	2	1	3	7.8882	7.8882	1.92678	1.92678	-0.78829	-0.78829
1	2	3	3	5.98968	5.98968	2.18375	2.18375	-0.99238	-0.99238
2	2	2	3	6.61791	6.61791	2.09904	2.09904	-0.9851	-0.9851
3	2	1	3	0.663707	0.663707	1.04392	1.04392	-0.55978	-0.55978
3	2	2	3	3.32264	3.32264	1.68965	1.68965	-0.87628	-0.87628
3	2	3	3	5.76421	5.76421	2.20393	2.20393	-1.17248	-1.17248
1	3	1	0	0.049863	0.049863	0.976832	0.976832	-0.8075	-0.8075
1	3	2	0	-4.85362	-4.85362	-0.18695	-0.18695	-0.32281	-0.32281
1	3	3	0	-3.12717	-3.12717	-0.4648	-0.4648	-0.17721	-0.17721
2	3	1	0	3.5979	3.5979	1.85559	1.85559	-1.34494	-1.34494
2	3	2	0	-2.69588	-2.69588	0.338994	0.338994	-0.733	-0.733
2	3	3	0	-4.66937	-4.66937	-0.40477	-0.40477	-0.34902	-0.34902
3	3	1	0	5.29179	5.29179	2.55701	2.55701	-1.82847	-1.82847

n	l	n'	l'	Serber Int. for $S=0$ and $T=1$		Yukawa Int. $S=0$ and $T=1$		M3Y Int.	
				Present work	[5]	Present work	[5]	Present work	[5]
3	3	2	0	0.209398	0.209398	1.01331	1.01331	-1.19267	-1.19267
3	3	3	0	-3.68209	-3.68209	-0.06335	-0.06335	-0.67193	-0.67193
1	3	1	1	5.57148	5.57148	1.90234	1.90234	-0.93925	-0.93925
1	3	2	1	-0.50517	-0.50517	0.974741	0.974741	-0.63738	-0.63738
1	3	3	1	-2.13715	-2.13715	0.406343	0.406343	-0.4764	-0.4764
2	3	1	1	7.67609	7.67609	2.57774	2.57774	-1.30606	-1.30606
2	3	2	1	3.2257	3.2257	1.76159	1.76159	-1.09307	-1.09307
2	3	3	1	-0.90364	-0.90364	0.965288	0.965288	-0.81515	-0.81515
3	3	1	1	7.50916	7.50916	2.93002	2.93002	-1.56616	-1.56616
3	3	2	1	5.68758	5.68758	2.41336	2.41336	-1.47711	-1.47711
3	3	3	1	1.88249	1.88249	1.62322	1.62322	-1.22929	-1.22929
1	3	1	2	7.8882	7.8882	1.92678	1.92678	-0.78829	-0.78829
1	3	2	2	3.53163	3.53163	1.49915	1.49915	-0.66553	-0.66553
1	3	3	2	0.663707	0.663707	1.04392	1.04392	-0.55978	-0.55978
2	3	1	2	7.77439	7.77439	2.20438	2.20438	-0.93545	-0.93545
2	3	2	2	6.61791	6.61791	2.09904	2.09904	-0.9851	-0.9851
2	3	3	2	3.32264	3.32264	1.68965	1.68965	-0.87628	-0.87628
3	3	1	2	5.98968	5.98968	2.18375	2.18375	-0.99238	-0.99238
3	3	2	2	7.18421	7.18421	2.40121	2.40121	-1.1648	-1.1648
3	3	3	2	5.76421	5.76421	2.20393	2.20393	-1.17248	-1.17248
1	3	1	3	7.77399	7.77399	1.5677	1.5677	-0.58347	-0.58347
1	3	2	3	5.80934	5.80934	1.52694	1.52694	-0.57653	-0.57653
1	3	3	3	3.32893	3.32893	1.30314	1.30314	-0.53077	-0.53077
2	3	1	3	5.80934	5.80934	1.52694	1.52694	-0.57653	-0.57653
2	3	2	3	7.08831	7.08831	1.80129	1.80129	-0.73436	-0.73436
2	3	3	3	5.76171	5.76171	1.76997	1.76997	-0.74828	-0.74828
3	3	1	3	3.32893	3.32893	1.30314	1.30314	-0.53077	-0.53077
3	3	2	3	5.76171	5.76171	1.76997	1.76997	-0.74828	-0.74828
3	3	3	3	6.58174	6.58174	1.97014	1.97014	-0.88311	-0.88311
3	3	1	0	5.29179	5.29179	2.55701	2.55701	-1.82847	-1.82847
3	3	2	0	0.209398	0.209398	1.01331	1.01331	-1.19267	-1.19267
3	3	3	0	-3.68209	-3.68209	-0.06335	-0.06335	-0.67193	-0.67193
1	3	1	1	5.57148	5.57148	1.90234	1.90234	-0.93925	-0.93925
1	3	2	1	-0.50517	-0.50517	0.974741	0.974741	-0.63738	-0.63738
1	3	3	1	-2.13715	-2.13715	0.406343	0.406343	-0.4764	-0.4764
2	3	1	1	7.67609	7.67609	2.57774	2.57774	-1.30606	-1.30606
2	3	2	1	3.2257	3.2257	1.76159	1.76159	-1.09307	-1.09307
2	3	3	1	-0.90364	-0.90364	0.965288	0.965288	-0.81515	-0.81515
3	3	1	1	7.50916	7.50916	2.93002	2.93002	-1.56616	-1.56616
3	3	2	1	5.68758	5.68758	2.41336	2.41336	-1.47711	-1.47711

n	l	n'	l'	Serber Int. for $S = 0$ and $T = 1$		Yukawa Int. $S = 0$ and $T = 1$		M3Y Int.	
				Present work	[5]	Present work	[5]	Present work	[5]
3	3	3	1	1.88249	1.88249	1.62322	1.62322	-1.22929	-1.22929
1	3	1	2	7.8882	7.8882	1.92678	1.92678	-0.78829	-0.78829
1	3	2	2	3.53163	3.53163	1.49915	1.49915	-0.66553	-0.66553
1	3	3	2	0.663707	0.663707	1.04392	1.04392	-0.55978	-0.55978
2	3	1	2	7.77439	7.77439	2.20438	2.20438	-0.93545	-0.93545
2	3	2	2	6.61791	6.61791	2.09904	2.09904	-0.9851	-0.9851
2	3	3	2	3.32264	3.32264	1.68965	1.68965	-0.87628	-0.87628
3	3	1	2	5.98968	5.98968	2.18375	2.18375	-0.99238	-0.99238
3	3	2	2	7.18421	7.18421	2.40121	2.40121	-1.1648	-1.1648
3	3	3	2	5.76421	5.76421	2.20393	2.20393	-1.17248	-1.17248
1	3	1	3	7.77399	7.77399	1.5677	1.5677	-0.58347	-0.58347
1	3	2	3	5.80934	5.80934	1.52694	1.52694	-0.57653	-0.57653
1	3	3	3	3.32893	3.32893	1.30314	1.30314	-0.53077	-0.53077
2	3	1	3	5.80934	5.80934	1.52694	1.52694	-0.57653	-0.57653
2	3	2	3	7.08831	7.08831	1.80129	1.80129	-0.73436	-0.73436
2	3	3	3	5.76171	5.76171	1.76997	1.76997	-0.74828	-0.74828
3	3	1	3	3.32893	3.32893	1.30314	1.30314	-0.53077	-0.53077
3	3	2	3	5.76171	5.76171	1.76997	1.76997	-0.74828	-0.74828
3	3	3	3	6.58174	6.58174	1.97014	1.97014	-0.88311	-0.88311

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تعميم الجهود النووية الثنائية المركزية واشتقاق معادلة جديدة لإيجاد عناصر المصفوفة لها

عزيز حمه رحيم فتاح

قسم الفيزياء/ كلية العلوم_ جامعة السليمانية

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الخلاصة

نص هذه الورقة تغطي تعميم لجهود الثنائية الشعاعية للنواة و اشتقاق صيغة جديدة لتقييم عناصر المصفوفة المصاحبة لها. تم تطبيق الصيغة الجديدة المشتقة بنجاح لحساب عناصر المصفوفة للجسيم الواحد للجهود المعتمدة على البرم و البرم ألنظيري و ذالك باستخدام برنامج مكتوب بلغة Mathematica.