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Effect of Moment of Inertia and Aerodynamics Parameters on Aerodynamic Coupling in Roll Mode

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ABSTRACT

The influence of moment of inertia and aerodynamic parameters on the aerodynamic coupling in rolling mode has been analyzed for Aircraft F-94A (case study) for different rolling rate in rolling mode , the equations of motion for aircraft has been analyzed to get the required equations of motion for aerodynamic coupling. The stability of these equations has been tested by Routh Discriminate.

The influence of moment of inertia and aerodynamic parameters on Routh Discriminate was clear, for example the wing span was the most positive influence on aerodynamic coupling stability.

تاثيرعزم القصور الذاتي والعناصر الايروديناميكية على الازدواج الحركي في وضع الدوران

الخلاصة

في هذا البحث تم دراسة تاثير عزم القصور الذاتي والعناصر الايروديناميكية الاخرى على ازدواج الطائرة الحركي في وضع الدوران لطائرة (F-94A) لمعدل دوران مختلف في وضع الدوران،حيث تم تحليل المعادلات الحركية للطائرة للاطوار الثلاثة للوصول الـى المعـادلات الحركية المطلوبة للازدواج الحركي التي تم قياس استقراريتها باستخدامطريقة راوث، وقد تبين تاثير كل من هذه العناصر على الاستقرارية ، حيث لاحظنا ان باع الجناح الاكثر ايجابية علـى زيادة استقرارية الازدواج الحركي .

LIST OF SYMBOLS

$\bar{c}$	$gCho$	
		$ug.ft^2$/sec
		$ug/ft.$sec
		ft/s
		ft/s

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p_o	$Roll Rate$	deg/sec
I_{xx}	$X - Axis$ $moment of Inertia$	$slug.ft^2$
I_{yy}	$Y - Axis$ $moment of Inertia$	$slug.ft^2$
I_{zz}	$Z - Axis$ $moment of Inertia$	$slug.ft^2$
ρ	$Air Density$	$slug/ft^3$
c_{m_q}	$Damping in pitch$	$1/radian$
c_{m_α}	$Static longitudinal stability$	$1/rad$
c_{n_r}	$Damping in yaw$	$1/rad$
c_{n_β}	$Directional Stability$	$1/rad$
α	$angle of attack$	deg
α_o	$Initial\ angle of attack$	deg
β	$angle of attack$	deg
r	$angle of attack$	deg
$\dot{\alpha}$	$Rate of angle of attack$	deg/sec
$\dot{\beta}$	$Rate of angle of attack$	deg/sec
$\dot{r}$	$Rate of angle of attack$	deg/sec
$\dot{q}$	$Rate\ of\ Dynamic\ Pressure$	$slug/ft.\sec^3$
C_X	$Coefficient\ of\ X - force$	$-------$
C_T	$Coefficient\ of\ thrust\ force$	$-------$
C_{T_u}	$Coefficient\ of\ Thrust\ in\ X - Axis$	$-------$

INTRODUCTION

During the rolling maneuvers large angles of sideslip may occur as a result of kinematics coupling [1]. The vertical tail may produce large yawing moment that acts in the direction of roll. In such a case, it may not be possible to stop the flying body from rolling, although the lateral control is held against the roll direction. This is known as autorotation rolling. In this situation positive "G" would facilitate recovery [2]. As the angle of attack is increased to a positive value, kinematics coupling will be result in a moment that opposes the original direction of roll, thus alleviating the tendency for autorotation rolling [3].

The divergence experienced during rolling manufacture is complex because it involves not only inertia properties, but aerodynamic as well, [2]. Coupling results when a disturbance about one aircraft axis causes a disturbance created by an elevator deflection during straight and level flight, [4]. The resulting motion is restricted to pitching motion and no disturbance occurs in yaw or roll. An example of couple motion is the disturbance created by a rudder deflection [5]. The ensuing motion will be some combination of both yawing and rolling motion [6]. Although all lateral disturbance motion are coupled, the only motion that ever results in coupling problems large enough to threaten the structural integrity of the aircraft is coupling as a result of rolling motion, [7] .F-94A aircraft was taken as case study , Figure (1) , Table (1) [8].The roll rate has been taken variable according to an equation in ref.[9].

MATHEMATICAL ANALYSIS OF ROLLING DIVERGENCE

The equation of motion for the airplane can be derive from Newton's second law of motion, which states that the summation of all external force acting on a body must be equal to the time rate of change of the momentum of the body, and the summation of the external moment of momentum (angular momentum).The time rates of change are all taken with respect to inertial space .These laws can be expressed by two vector equations.
The overall equation of motion, [2].

$$\sum F_x = \frac{m}{\rho S u_o}\tilde{u} - \left[c_x(u_o,\alpha_o,\dot{q}_o) + \frac{1}{2}c_{x_o}u_o - \frac{1}{2}c_{T_u}u_o - c_T(M_O)\tilde{u}\right] + \left(2\frac{m\alpha_o}{\rho S\bar{c}} - \frac{1}{2}c_{x_q}\right)\tilde{q} + \frac{mg\cos\theta_0}{\rho s u_o^2}\tilde{\theta} + \frac{1}{2}c_{x_\alpha}\tilde{\alpha} = m(\dot{u} + qw - rv) \quad \text{.... (1)}$$

$$\sum F_z = \frac{2m\alpha_o}{\rho s u_0}\dot{\tilde{u}} + \left[-c_{z_u} - 2c_z\left(u_{o,}\alpha_o,\dot{q}_o,\delta_{e_o}\right)\right]\tilde{u} - \left(c_{z_q} + \frac{4m}{\rho s\bar{c}}\right)\tilde{q} + \frac{2mg}{\rho s u_o^2}\sin\theta_o\tilde{\theta} + \frac{2m}{\rho s u_o}\dot{\tilde{\alpha}} - c_{z_\alpha}\tilde{\alpha} = m(\dot{w} + pv - qu) \quad \text{.... (2)}$$

$$\sum \mathcal{M} = -\left(2c_m\left(u_o,\alpha_o,\dot{q}_o,\delta_{e_o}\right) + c_{m_u}u_o\right)\tilde{u} + \frac{4I_{yy}}{\rho s u_o\bar{c}^2}\dot{\tilde{q}} - c_{m_q}\tilde{q} - c_{m_\alpha}\tilde{\alpha} = I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2) \quad \text{.... (3)}$$

Pitching moment Velocity:-

$$\frac{2u_o}{\bar{c}}\tilde{q} = \dot{\tilde{\theta}}$$

$$\sum F_y = \frac{2m}{\rho s u_o}\dot{\tilde{\beta}} - c_{y_\beta}\tilde{\beta} - \left(c_{y_p} + \frac{4m\alpha_o}{\rho s b}\right)\tilde{p} - \frac{2mg}{\rho s u_o^2}\cos\theta_o\tilde{\emptyset}© \quad \text{.... (4)}$$

$$\sum \ell = -c_{\ell_\beta}\tilde{\beta} - \frac{4I_{xx}}{\rho s u_o b^2}\dot{\tilde{p}} - c_{\ell_p}\tilde{p} - \frac{4I_{xz}}{\rho s u_o b^2}\dot{\tilde{r}} - c_{\ell_r}\tilde{r} = I_{xx}\dot{p} - \left(I_{yy} - I_{zz}\right)qr - I_{xz}(\dot{r} - qp) \quad \text{.... (5)}$$

$$\sum \mathbb{N} = -c_{n_\beta}\tilde{\beta} - \frac{4I_{xz}}{\rho s u_o b^2}\dot{\tilde{p}} - c_{n_p}\tilde{p} + \frac{4I_{zz}}{\rho s u_o b^2}\dot{\tilde{r}} - c_{n_r}\tilde{r} = \mathrm{I_{zz}}\dot{\mathrm{r}} - \left(\mathrm{I_{xx}} - \mathrm{I_{yy}}\right)\mathrm{pq} - \mathrm{I_{xz}}(\dot{\mathrm{p}} - \mathrm{qr}) \quad \text{.... (6)}$$

Rolling Velocity:- $\frac{2u_o}{b}\tilde{p} = \dot{\tilde{\emptyset}} - \sin\theta_o\dot{\tilde{\psi}}$

Yawing Velocity:- $\frac{2u_o}{b}\tilde{r} = \cos\theta_o\dot{\tilde{\psi}}$

The approach for solving the autorotation rolling equations was derived based on some necessary assumption to fit into the present analysis of autorotation rolling [1].
I. Velocity remains constant during the roll maneuver$\dot{u} = 0, u = u_o$.

2 .The rate roll rate is constant;$\dot{p} = 0, so that p = po$.
3. v, w, q, r are small therefore their products are negligible.
4. Engine gyroscopic effect is negligible.
5 .Rudder and elevator are fixed in their initial trim position.
6. Aerodynamic coefficients are negligible with the exception of $c_{m_\alpha}, c_{m_q}, c_{n_\beta} and c_{n_r}$.
7. Small angle assumption on $\alpha and \beta$.

When these assumptions are applied to the six equations of motion the following results are obtained:-

$$\sum F_x = 0 \qquad \ldots\ldots (7)$$

$$\sum F_y = mu_o(\dot{\beta} + r - p_o\alpha) = 0 \qquad \ldots\ldots (8)$$

$$(\dot{\beta} + r - p_o\alpha) = 0 \quad \text{from assumption}$$

$$\sum F_z = mu_o(\dot{\alpha} + p_o\beta - q) = 0 \qquad \ldots(9)$$

(Both lift and side force will average zero throughout a roll)

$$\sum \ell = -(\dot{r} + qp_o)\, I_{xz} = 0 \qquad \ldots\ldots(10)$$

This is a reasonable condition because one considers the motion to be pricipally a steady state roll because such a situation the aileron moment and damping in roll exactly oppose one another.

$$\sum \mathcal{M} = \dot{q}I_{yy} + p_o r(I_{xx} - I_{zz}) + p_o^2 I_{xz} = \frac{1}{2}\rho u_o^2 s\bar{c}\left[c_{m_\alpha}\alpha + c_{m_q} q \frac{\bar{c}}{2u_o}\right] \ldots (11)$$

$$\sum \mathbb{N} = \dot{r}I_{zz} + p_o q(I_{yy} - I_{xx}) = \frac{1}{2}\rho u_o s b\left[c_{n_\beta}\beta + c_{n_r} r \frac{b}{u_o}\right] \qquad \ldots (12)$$

Rewriting the equations in nature form:-

$$\dot{\alpha} + p_o\beta - q = 0 \qquad \ldots\ldots (13)$$

$$\dot{\beta} + r - p_o\alpha = 0 \qquad \ldots\ldots (14)$$

$$\frac{1}{2}\rho u_0^2 s\bar{c}c_{m_\alpha}\alpha - \dot{q}I_{yy} - p_o r(I_{xx} - I_{zz}) + \frac{\rho u_o s\bar{c}^2}{4} c_{m_q} q = p_o^2 I_{xz} \qquad \ldots\ldots (15)$$

$$\frac{1}{2}\rho u_0^2 s b c_{n_\beta}\beta - \dot{r}I_{zz} - p_o q(I_{yy} - I_{xx}) + \frac{\rho u_o s b^2}{4} c_{n_r} r = o \qquad \ldots\ldots (16)$$

Note that there are four equations in four unknowns $(\alpha, \beta, q, and r)$. Particular solution to these equations exists because the pitching moment equation is not homogenous. However, the investigation of the particular solution holds only for design interest. On the other hand the homogenous solution represents motion which is indicative of stable or unstable coupling. Accordingly, the equations are Laplace transformed and coefficient matrix determinant becomes.

$$\begin{vmatrix} s & p_o & -1 & 0 \\ -p_o & s & 0 & 1 \\ \frac{1}{2}\frac{\rho u_o^2 s\bar{c}}{I_{yy}} c_{m_\alpha} & 0 & s-\frac{\rho u_o^2 s\bar{c} c_{m_q}}{4I_{yy}} & p_o\frac{(I_{xx}-I_{zz})}{I_{yy}} \\ 0 & -\frac{1}{2}\frac{\rho u_o^2 sbc_{n_\beta}}{I_{zz}} & p_o\frac{(I_{yy}-I_{xx})}{I_{zz}} & s-\frac{\rho u_0 sb^2 c_{n_r}}{4I_{zz}} \end{vmatrix}$$

The determinant must be expanded to solve for the characteristic equation:-

$$AS^4 + BS^3 + CS^2 + DS + E = 0$$

The equation must be tested for stability in several methods such as Routh Discriminate [6] which conditioned for stability:-

$$BCD - B^2E - AD^2 \geq 0$$

The stability derivatives formation which is given below, [2] was helpful in this analysis for determining:-

$$c_{m_\alpha} = c_{L_\alpha}(\bar{x} - \bar{x}_n) \quad \text{.....(17)}$$

$$c_{m_q} = -2\zeta\eta_t\bar{v}\left(\frac{\partial c_L(\alpha_o,M_o)}{\partial\alpha}\right)_t \frac{\ell_t}{\bar{c}} \quad \text{....(18)}$$

$$c_{n_r} = -2\left(\frac{\partial c_L(\alpha_o,\sigma_o)}{\partial\alpha}\right)_F \bar{V}_F \frac{\ell_F}{b} \quad \text{....(19)}$$

$$c_{n_\beta} = c_{L_{\alpha_F}}\left(1-\frac{d\sigma}{d\beta}\right)_F \bar{V}_F + c_{n_{\beta_{fus}}} \quad \text{....(20)}$$

RESULTS AND DISCUSSION

The parameters exits in autorotation characteristic equation were select as effected parameters, which tested with different roll rate[$p_o = 10,20,30$] rad/sec.

From fig.(2,3 and 4)which present the aerodynamic parameters have positive effect toward the cross coupling ,when they increase the stability of aerodynamic cross coupling increases too due to increases in lift.

The height or density have great effect toward the stability of aerodynamic cross coupling and it is clearly shown in fig.(5) as the height increases(density decreases) the stability of aerodynamic cross coupling decreases due to lift decreases.

Fig.(6,7and 8) show the effect of moment of inertia in (x,y,z) axis and all have negative effect toward the stability of aerodynamic cross coupling because large moment of inertia causes uncontrollable pitching moment.

The static longitudinal stability (c_{m_α}) little effect toward the stability fig.(9)because of lift increases , but the damping in pitch (c_{m_q}) have great negative effect toward the aerodynamic cross coupling stability fig.(10),which increases as the distance between horizontal stabilizer and airplane he aerodynamic coupling which becomes uncontrollable.

Directional stability (c_{n_β}) have a positive effect toward the stability of aerodynamic cross coupling fig.(11), but(c_{n_r})which present the rolling stability have great positive effect toward the aerodynamic cross coupling stability fig.(12)),which increases as the distance between vertical stabilizer and airplane he aerodynamic coupling which becomes uncontrollable.

CONCLUSIONS

1. X-plane no effect plane.
2. The increasing of the wing area (S) leading to increase the stability.
3. High height aerodynamic coupling have more stable than low height.
4. The wing area (S) and directional stability (cnβ) have the most powerful parameters.
5. The weight distribution is very important for avoiding the aerodynamic coupling.

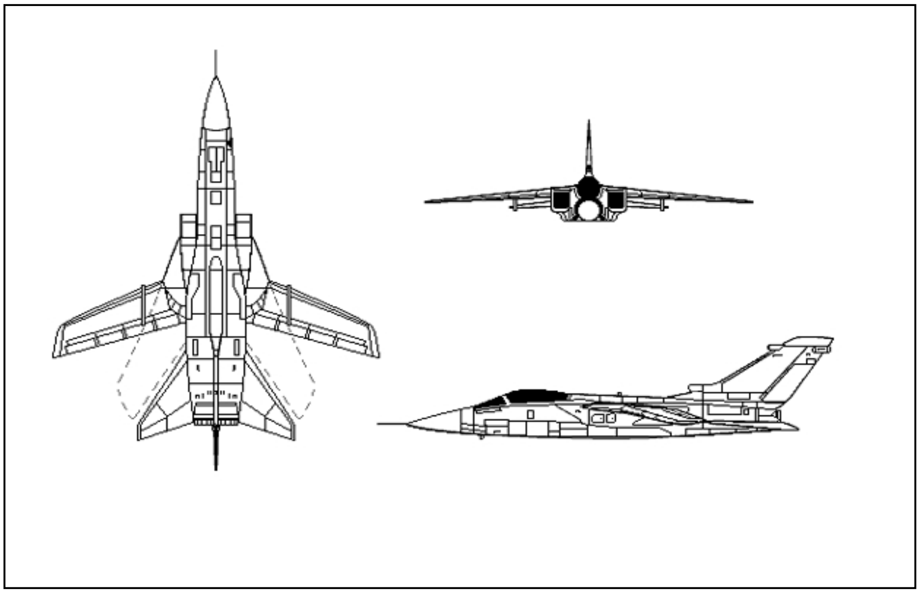

Figure (1) Views of Supersonic Aircraft F- 94A (Case Study)

Aerodynamic Data

Wing Span (ft)	Wing Area (ft^2)	Wing Mean Chord (ft)	Aspect Ratio	Wing Sweep Angle	Taper Ratio	Airfoil Section
130	2400 (ft^2)	20.2	7.04	24(deg)	0.2	NACA 65A004.8

Stability Derivatives

Cm_α	Cm_q	Cn_r	Cn_β
-0.619/rad	-11.4/rad	-0.107/rad	+0.096/rad

Other Data

I_{xx} (Sl-ft2)	I_{yy} (Sl-ft2)	I_{zz} (Sl-ft2)	I_{xz} (Sl-ft2)	Max Speed (ft/sec)	Mass (slug)	Density M=0.8 Alt=20000 ft (Sl/ft^3)	Engine Type
$1.955x10^6$	2.	4	0	440	5900	0.001267	J.33A-33

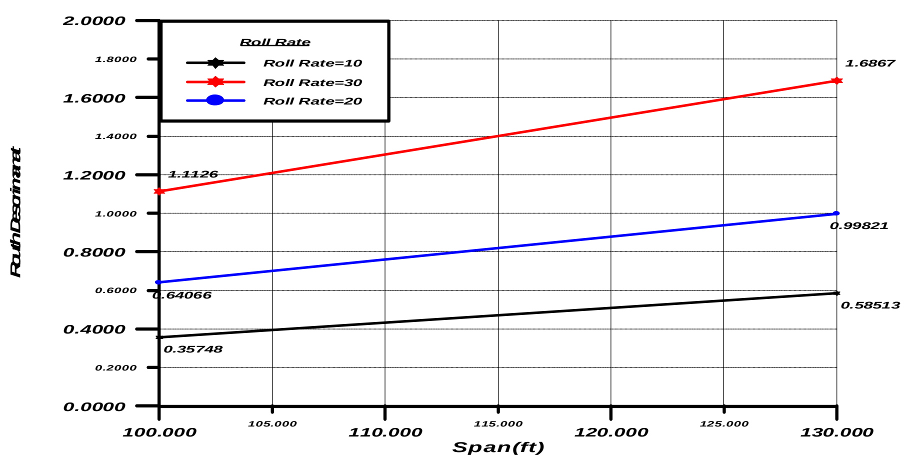

Figure (2) Effect of Wing Span on Air Aerodynamic Coupling

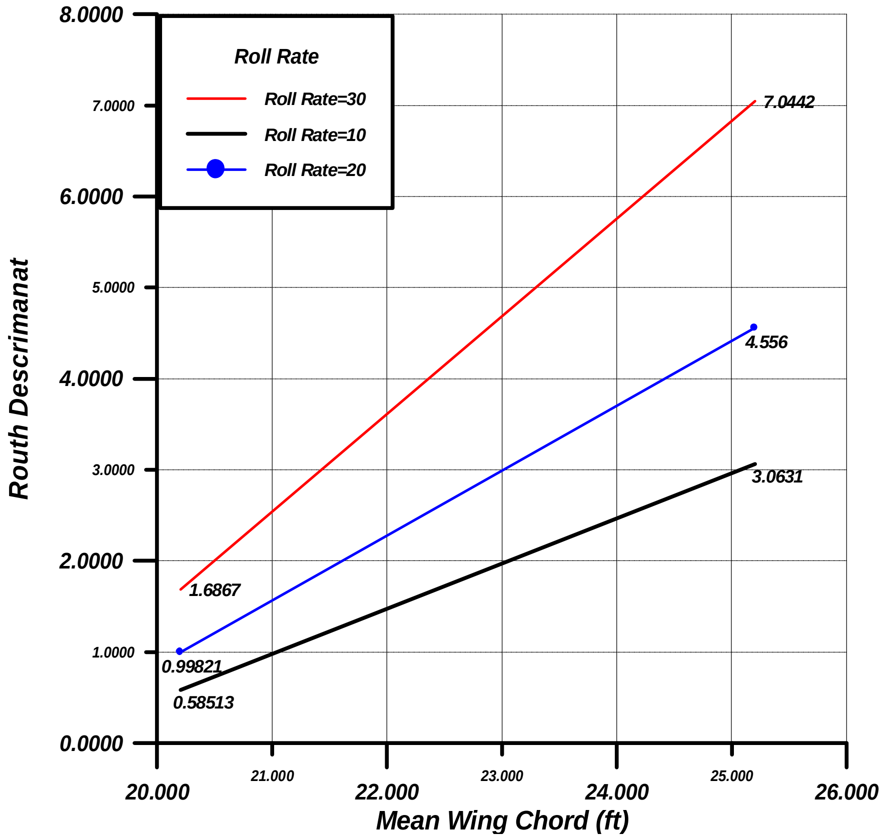

Figure (3) Effect of Wing Mean Chord on Aircraft Aerodynamic Coupling

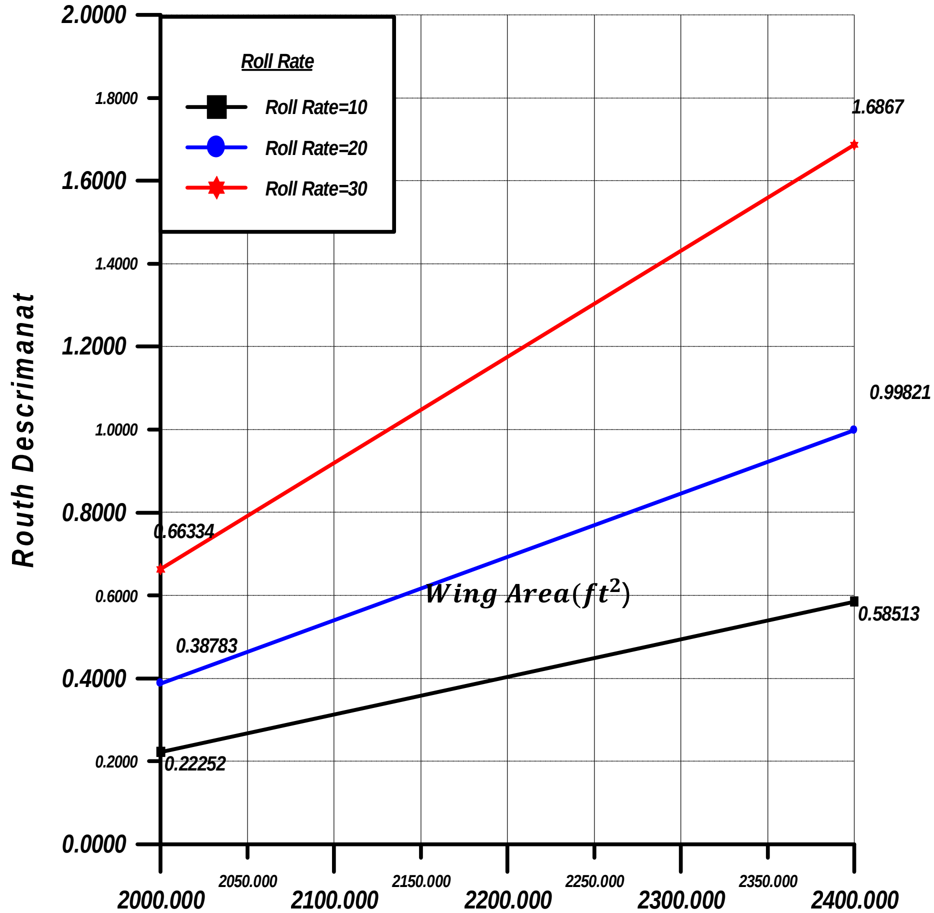

Figure (4) Effect of Wing Area on Aircraft Aerodynamic Coupling

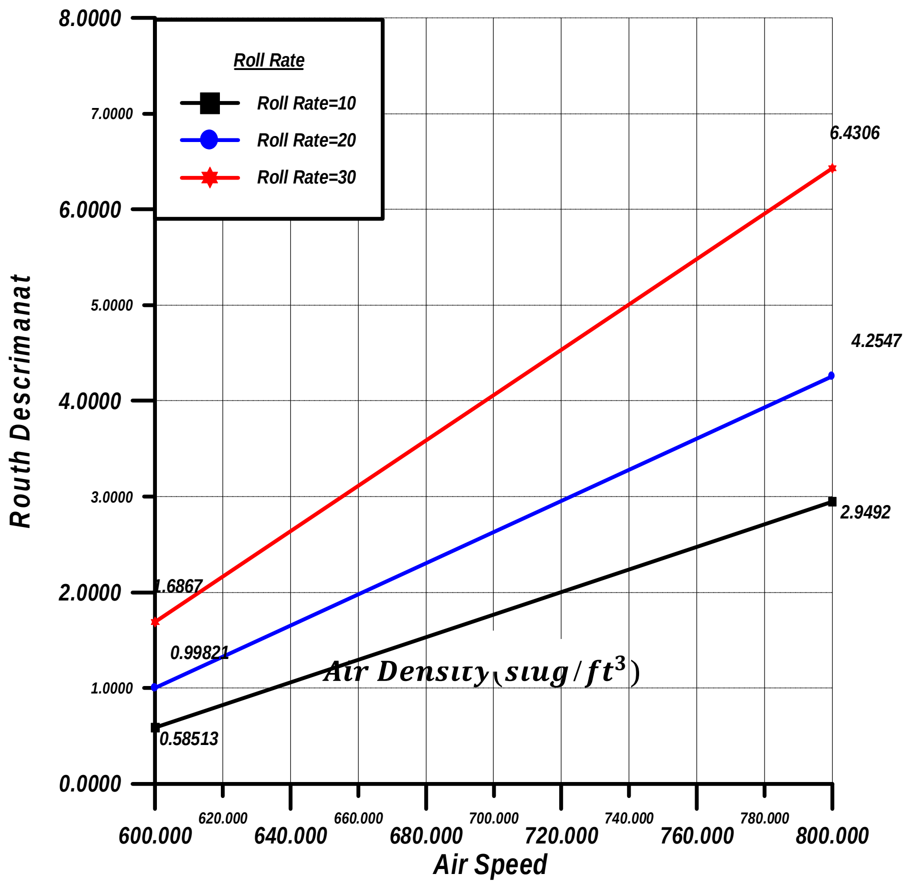

Figure (5) Effect of Air density on Aircraft Aerodynamic Coupling

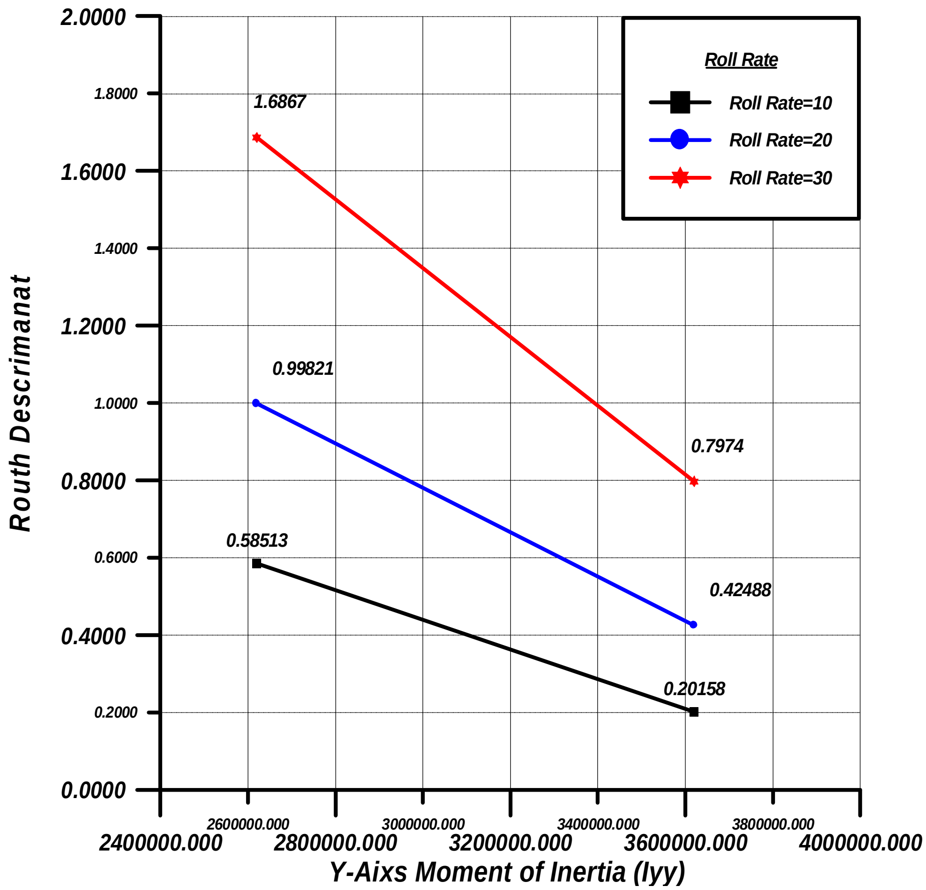

Figure (6) Effect of X-Axis moment of Inertia on Aircraft Aerodynamic Coupling

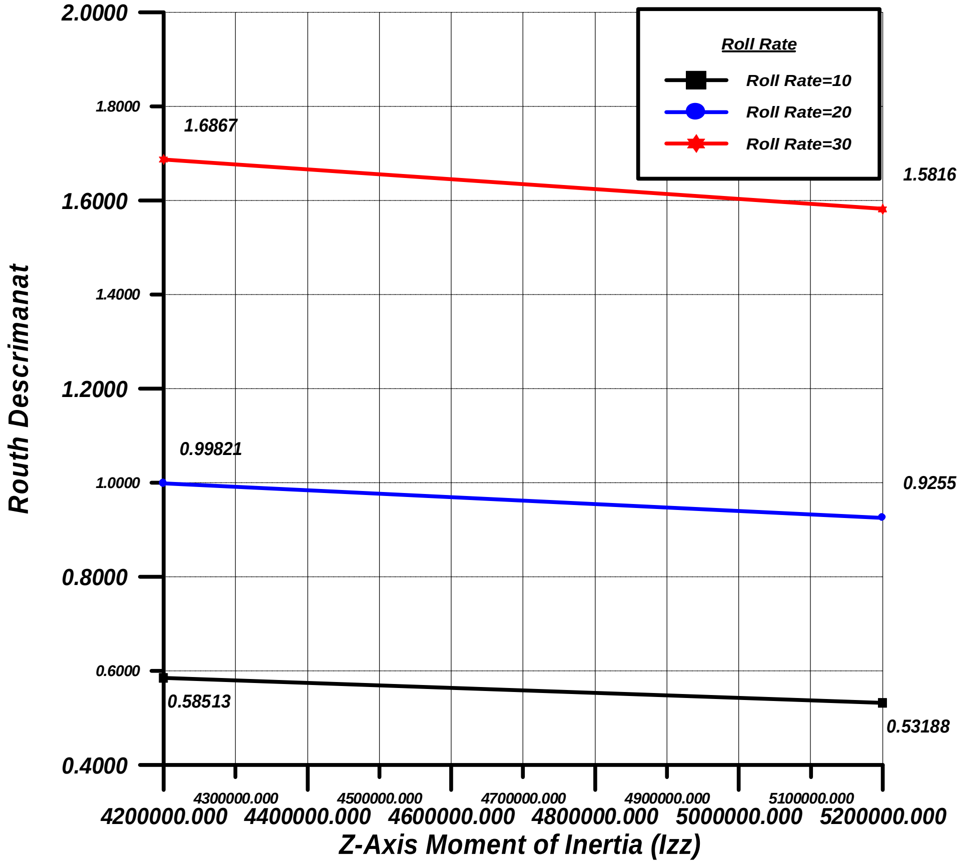

Figure (7) Effect of Y-Axis moment of Inertia on Aircraft Aerodynamic Coupling

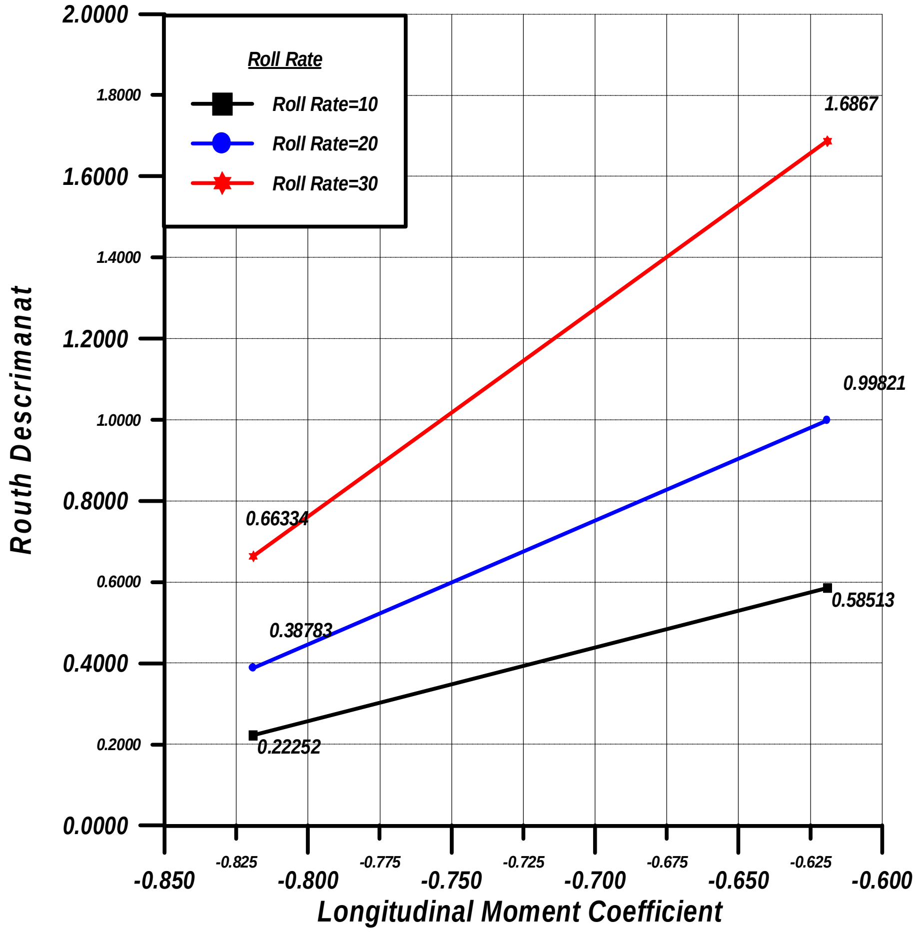

Figure (8) Effect of Z-Axis moment of Inertia on Aircraft Aerodynamic Coupling

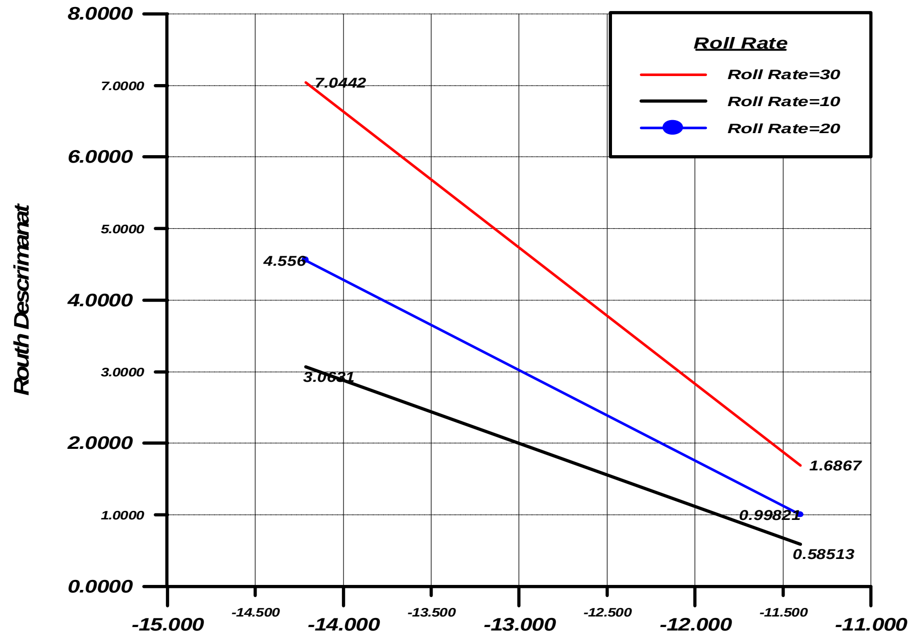

Figure (9) Effect of Static Longitudinal Stability on Aircraft Aerodynamic Coupling

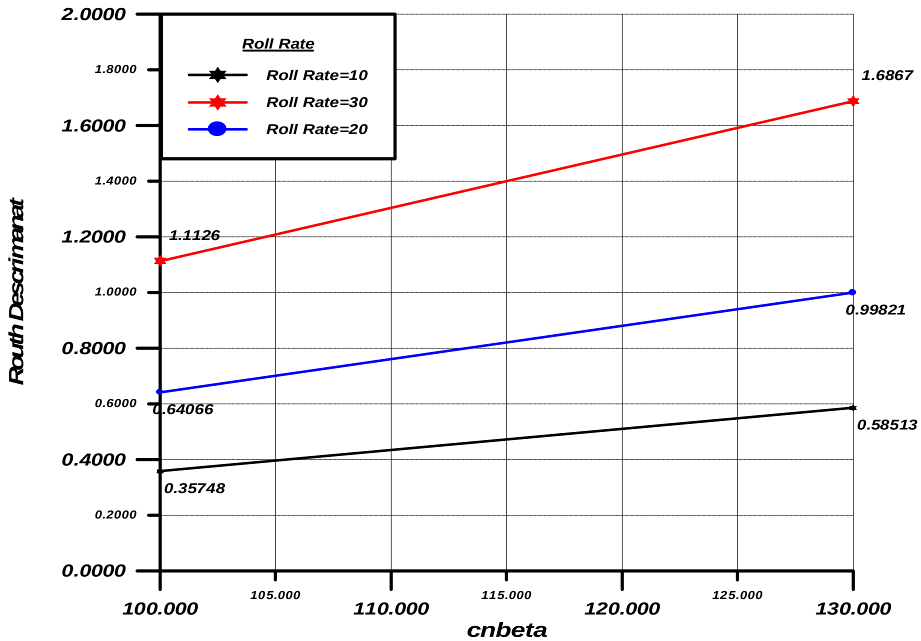

Figure (10) Effect of Damping in Pitch on Aerodynamic Coupling

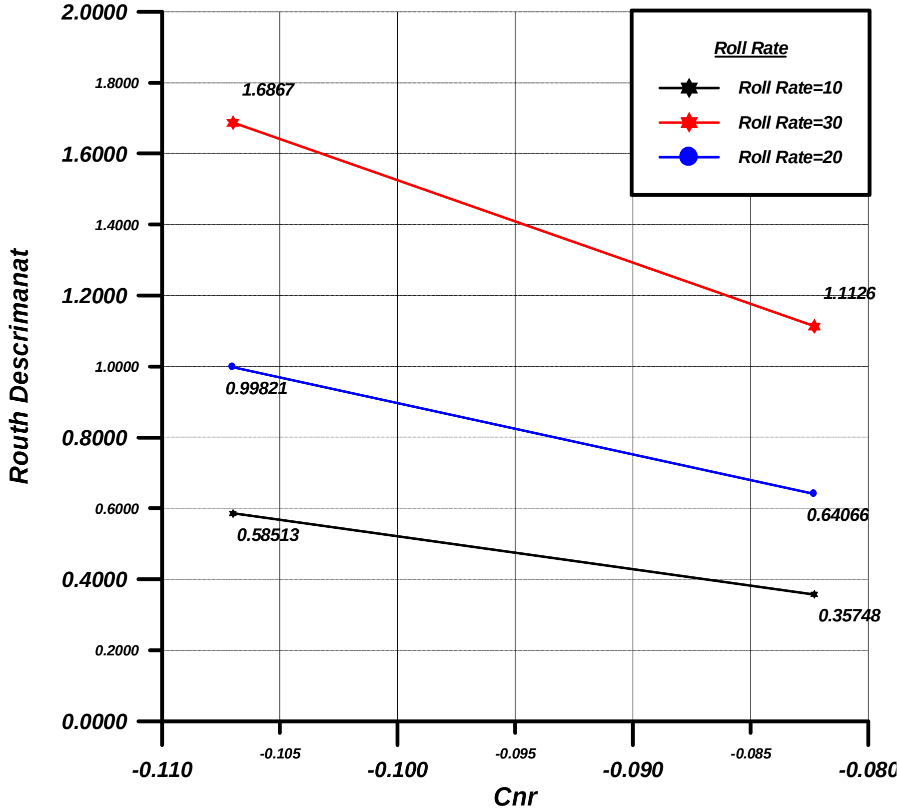

Figure (11) Effect of Directional stability on Aerodynamic Coupling

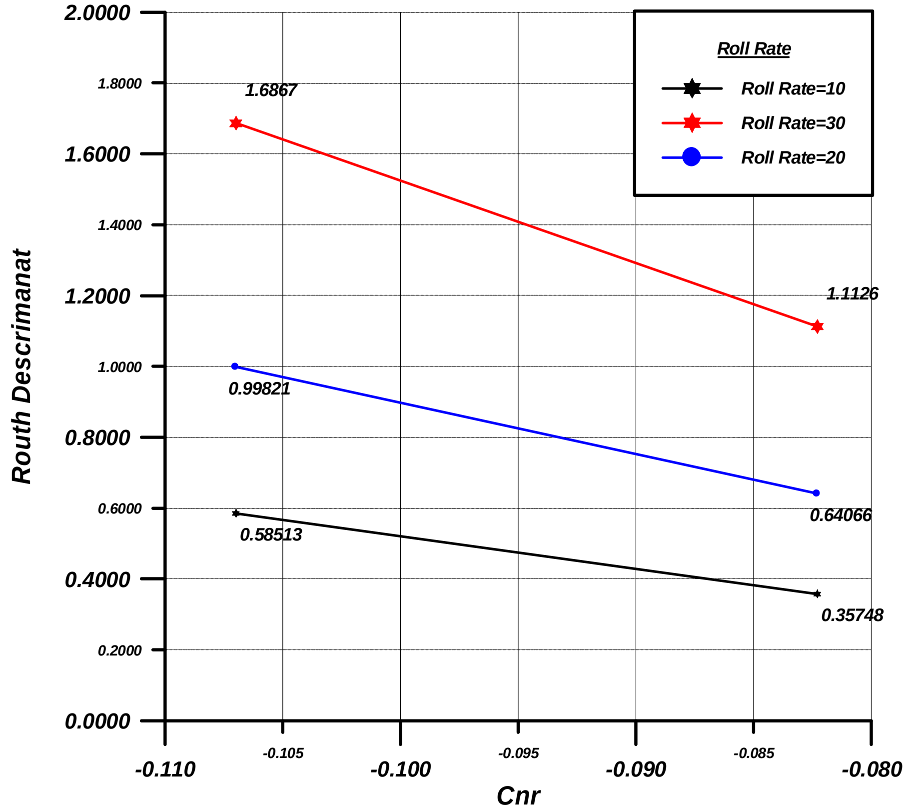

Figure (12) Effect of Rolling Stability on Aerodynamic Coupling

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