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Computation of Odd Magic Square Using a New Approach with Some Properties

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ABSTRACT

Several aspects of magic square studies fall within the computations all universes. Experimented computation has reverted patterns, some of which have lead to analytic insight. With the aid of arithmetic modular and the properties of it, we get magic squares, also we use the algebraic operations of matrices which are addition, subtraction, multiplication, transpose, rotation, and reflection, then discuss the results that we are obtained, the eigenvalues of these magic squares are also computed and we get some magic squares with the same eigenvalues, the inverse of the magic squares which are found by using arithmetic modular are also magic squares. An example with order 3 of magic square is given to explain our results.

Keywords: Magic Square, Odd Magic.

حساب المربعات السحرية الفردية باستخدام اقتراح جديد مع بعض الخصائص

الخلاصة

درست مفاهيم عديدة للمربعات السحرية مع حساباتها بشكل عام .الحسابات المجربة لها نماذج متعددة بعضها ادت الى تحليلات علمية. بمساعدة المعيارالحسابي وخصائصه، تم الحصول على مربعات سحرية . كما تم استخدام العمليات الجبرية على المصفوفات (الجمع، والطرح، والضرب، والمنقول، والتدوير، والانعكاس)، ثم ناقشنا النتائج التي تم الحصول عليها. تم حساب القيم المميزة للمربعات السحرية وحصلنا على بعض المربعات السحرية التي لها نفس القيم المميزة. تم ايجاد المعكوس للمربعات السحرية، معكوس المربعات السحرية التي تم حسابها باستخدام المعيار الحسابي هي ايضا مربعات سحرية. تم توضيح النتائج بمثال لمربع سحري من الرتبة الثالثة.

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INTRODUCTION

Magic squares have been studied for at least three thousand years, the earliest recorded appearance dating to about 2200 B.C., in china. In the 19th century, Arab astrologers used them in calculating horoscopes, and by 1300 A.D. magic square had spread to the west. An engraving by the German artist embedded the date, 1514, in the form of two consecutive numbers in the bottom row, because the concept of a magic square is so easily understood, magic squares have been particularly attractive to puzzlers and a amateur mathematicians[3].

In recreational mathematics, a magic square of order n is an arrangement of n^2 numbers, usually distinct integers, in a square, such that the numbers in all rows, all columns, and both diagonals sum to the same constant.

Many research are discussed the construction of magic squares using different approaches for example [1],[2]. Some of these methods are given in section 2.

In this paper, we give a new method for constructing magic squares depending on the proposition of the arithmetic modular together with the operations of addition, subtraction, multiplication, rotation, and reflection and discussed the results that we obtained within an example on magic square of order 3. The eigenvalues of these magic squares are computed, and the inverse of modular magic square is also found.

CONSTRUCTING OF ODD MAGIC SQUARE

In this section a method for constructing odd magic are given, which is the base of our new method of construction. A method for constructing odd magic square are given which are “The Gamma plus two method ($\Gamma + 2$)” are described as follows [1].

1. Place the number 1 in the cell immediately to the right of the center cell. Go up one cell and then go one to the right and place the number 2
   “Up and to the right” is how the upper case Greek Gamma (Γ) is written, ergo the first part of the name. Continue this “one up and one to the right” process.

2		
		1

2. After inserting M numbers, the “ one cell up and one cell to the right” motion will land you on a cell that already occupied

2		
		1
	3	

3. When this happens, go back to the last entry, move two cells to the right, and insert the next number.

2		
		1
4	3	

4. Continue inserting numbers in cells "one cell up and one cell to one the right" until, after M numbers, you find the next cell that is occupied. At this point you are back to step 3.

2	7	6
	5	1
4	3	

5. Continue the process until every cell of the square is filled.

FORMULATION OF THE NEW CONSTRUCTION METHOD

Our constructing of magic square depends on knowing only one magic square and constructing all others according to it, from section 2 we get the first one.

Now, we discussed our purposed method using the following steps: For example n=3, we have the magic square

S=

2	7	6
9	5	1
4	3	8

With sum equal 15

Step 1: Add to each element in the above magic squares the number 3, 6 (means a multiple of n=3). And then take the modules with respect to n^2 which is equal to $3^2 = 9$, we get S_1 and S_2 respectively

S_1=

5	1	9
3	8	4
7	6	2

S_2=

5	9	1
7	2	6
3	4	8

Step 2: for each magic squares S, S_1, S_2 we take the greatest number and add one to it and subtract from each element in the magic squares, then take the modulation with respect to three, we get S_3, S_4 and S_5

S_3

8	3	4
1	5	9
6	7	2

S_4 =

3	7	5
4	2	9
8	6	1

S_5 =

3	7	5
4	2	9
8	6	1

Step 3: For each of the above magic squares S, S_1, $S_2 S_3$, S_4, S_5 making use of the transpose for each of them, we get S_6, S_7, S_8,S_8, S_9, S_{10} and S_{11} which are

S_6 =

2	9	4
7	5	3
6	1	8

, S_7 =

5	3	7
1	8	6
9	4	2

S_8 =

8	6	1
4	2	9
3	7	5

S_9 =

8	1	6
3	5	7
4	9	2

S_{10} =

5	7	3
9	2	4
1	6	8

S_{11} =

2	4	9
6	8	1
7	3	5

Also taking the reflection (which means fixed the Medill column and change the other one with each other) for S, $S_1, \ldots, S_{11}$ to obtain

S_{12}= .

6	7	2
1	5	9
8	3	4

S_{13}=

9	4	2
1	8	6
5	3	7

S_{14}=

4	3	8
9	5	1
2	7	6

S_{15} =

6	1	8
7	5	3
2	9	4

S_{16} =

1	9	5
6	2	7
8	4	2

S_{17} =

7	6	2
3	8	4
5	1	9

S_{18} =

4	9	2
3	5	7
8	1	6

S_{19} =

5	3	7
1	8	6
9	4	2

S_{20} =

1	6	2
9	8	6
5	3	7

S_{21} =

6	1	8
7	5	3
2	9	4

S_{22} =

9	1	5
4	8	3
2	6	7

S_{23} =

3	4	8
7	2	6
5	9	1

From the above we get the following general results:

1- Add to each element for the n×n magic square the multiplicity of n are n,2n,…, with the condition that the multiplicity is not greater than n^2, or $in < n^2$ (for i=1,2,…,n-1)

2- For each magic squares we take the greatest number and add one to it and subtract from each elements in the magic squares, then take the modulation with respect to n, that is (n^2-i+1) modular 3 where (i) is the position of the terms.

3- For each of the obtained magic square we take the transpose, which is similar to the transposition of the matrices, also we take the reflection.

PROPERTIES OF THE OBTAINED MAGIC SQUARES

1- The number of the obtained magic squares is the multiplicative of 8. Combinatorial speaking, when n=3, the number of magic square is 24 and when n=5, the number of magic squares is 40. First we get n-1 of magic square from step 1 and n magic square from step 2, for the 2n magic squares we compute the 2n transpose, so we have 4n, then after reflection we have 4n, therefore in general the number of the obtained magic squares is 8n, where n is odd.

2- The multiplicative of any magic squares came from modular is equal to squares

1	1	1
1	1	1
1	1	1

All others are also magic squares.

All the obtained magic squares have the same magic values, by using a method for finding eigenvalues and use the properties the magic square we have for n=3, eigenvalues are 15, -4.899, 4.899.

3- The inverse (the same as inverse of matrices) of magic squares comes from the modular is also magic squares but its value are not integers.

All the obtained computations are done using MATLAB software.

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S_0 =

2	7	6
9	5	1
4	3	8

, S_1 =

5	1	9
3	8	4
7	6	2

S_2 =

8	4	3
6	2	7
1	9	5

S_3 =

8	3	4
1	5	9
6	7	2

S_4 =

5	9	1
7	2	6
3	4	8

S_5 =

2	6	7
4	8	3
9	1	5

S_6=

2	9	4
7	5	3
6	1	8

. S_7=

5	3	7
1	8	6
9	4	2

S_8 =

8	6	1
4	2	9
3	7	5

S_9 =

8	1	6
3	5	7
4	9	2

S_{10} =

5	7	3
9	2	4
1	6	8

S_{11} =

2	4	9
6	8	1
7	3	5

S_{12} =

6	7	2
1	5	9
8	3	4

S_{13} =

9	1	5
4	8	3
2	6	7

S_{14} =

3	4	8
7	2	6
5	9	1

S_{15} =

4	3	8
9	5	1
2	7	6

S_{16} =

1	9	5
6	2	7
8	4	3

S_{17} =

7	6	2
3	8	4
5	1	9

S_{18} =

4	9	2
3	5	7
8	1	6

S_{19} =

5	3	7
1	8	6
9	4	2

S_{20} =

1	6	8
9	2	4
5	7	3

S_{21} =

6	1	8
7	5	3
2	9	4

S_{22} =

3	7	5
4	2	9
8	6	1

S_{23} =

9	4	2
1	8	6
5	3	7