Designing robust Mixed H_2/H_∞ PID Controllers based Intelligent Genetic Algorithm

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***Abstract*- It's not easy to implement the mixed H_2/H_∞ optimal controller for high order system, since in the conventional mixed H_2/H_∞ optimal feedback the order of the controller is much than that of the plant. This difficulty had been solved by using the structured specified PID controller. The merit of PID controllers comes from its simple structure, and can meets the industry processes. Also it have some kind of robustness. Even that it's hard to PID to cope the complex control problems such as the uncertainty and the disturbance effects. The present ideas suggests combining some of model control theories with the PID controller to achieve the complicated control problems. One of these ideas is presented in this paper by tuning the PID parameters to achieve the mixed H_2/H_∞ optimal performance by using Intelligent Genetic Algorithm (IGA). A simple modification is added to IGA in this paper to speed up the optimization search process. Two MIMO example are used during investigation in this paper. Each one of them has different control problem.**

I. INTRODUCTION

Many intractable engineering problems, such as mixed H_2/H_∞ optimal control design are characterized by:1) nonlinear multimodal search space; 2) large scale search space; 3) tight constraints; and 4) expensive objective function evaluations [1], [2], [3], [4]. Therefore, it is desirable to develop an efficient optimization algorithm, such that accurate solutions can be economically obtained. The great success for evolutionary programming (EP), and evolutionary strategies (ES), and genetic algorithm (GA), came in the 1980s when extremely complex optimization problems from various disciplines were solved, thus facilitating the undeniable breakthrough of evolutionary computation as a problem solving methodology. The evolutionary algorithm (EA) is a robust search and optimization methodology that is able to cope with ill-behaved problem domains, exhibiting attributes such as multimodality, discontinuity, time variance, randomness and noise [5].

Mixed H_2/H_∞ optimal control design for system with uncertainties and disturbance is an achieve area in many research at presence. There mainly two approaches to dealing with the mixed H_2/H_∞ optimal controller design problem; one is the structure-specified controller, and the other is the output feedback controller. The techniques available in the literature for the output feedback approach include branch and bound, convex upper bounds using semi-definite programming, bilinear matrix inequalities (BMIs) [6]. However, the conventional output feedback design of mixed H_2/H_∞ optimal control are very complicated and not easily implemented for practical industrial applications. Mixed H_2/H_∞ control design are quite useful for robust performance design for systems with parameter perturbation and disturbance effects. In conventional output feedback control is employed to treat the so called mixed H_2/H_∞ control design problem, four Riccati-like equations need to be solved. Therefore, it would be rather complicated design procedure to obtain the mixed H_2/H_∞ controller [7].

The Finding control gains, which minimize or maximize a designated cost function in time domain subject to multiple constrains specified by frequency domain specifications became a complex constrained optimization problem. The problem is so complex that it can't be analytically or numerically, solved. Fortunately, recent applications in genetic algorithm reveal a way to resolve the problem considering many points in the search space, a GA had a reduced chance converging to the local optimum. The GA had been applied in system identification adaptive control of both continuous and discrete time. GA utilizes a collective learning process of a population of individuals. Descendants are generated using randomized processes intended to model mutation and crossover. Mutation corresponds to an erroneous self-replication of individuals, while crossover exchanges information between two or more existing individuals. According to a fitness measure, the selection process favors better individuals reproduce more often than those that are relatively worse. The superiority of GA is achieved by using several search principles simultaneously such as population based heuristics, and balance between global exploration and local exploration. A large number of system parameters would result in very large search space. The performance of the conventional GA would be greatly degraded when applied to large parameter optimization problems. Furthermore, GA had been shown to be efficient on global exploration by finding the most promising regions of the search space, but they suffer from excessively slow convergence to an accurate solution for tightly constrained problems with large scale multimodal search spaces. This may prevent them from being really of practical interest for intractable large scale engineering problems. Generally, GA with a local search heuristics is beneficial to improve the solution accuracy [8], [9].

IGA is an efficient point based optimization technique, which aims at escaping from local optima to find a global optimal solution, and has been widely applied in various engineering problems. Intelligent Crossover (IC) is the main phase in IGA. Based on orthogonal experimental design IC uses a divided and conquer approach, which consists of adaptively dividing two individuals of parents into N pairs of gene segments, economically identify the better one of two gene segments of each pair, and systematically obtaining a potentially good approximation to the best one of all combinations[10], [11].

II. Controller design based on mixed H_2/H_∞ and IGA

Consider a system with n_i inputs and n_o outputs as shown in Figure 1, the plant perturbation $\Delta P(s)$ is assumed to be bounded by a known stable function matrix $W_1(s)$

$$\bar{\sigma}(\Delta P(j\omega)) = \bar{\sigma}(W_1(s)), \ \forall\omega \in [0,\infty) \qquad \dots\dots\dots (1)$$

where $\bar{\sigma}(A)$ denotes the maximum singular value of a matrix A.

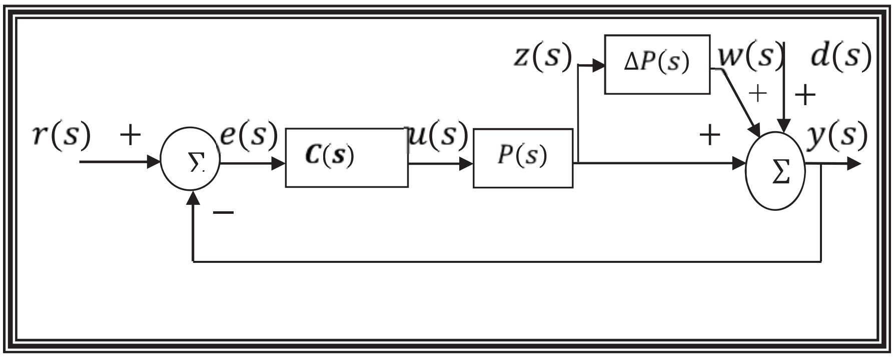

Figure 1 System under test

If a controller $C(s)$ is designed so that 1) the nominal closed loop system ($\Delta P(s) = 0$ and $d(t) = 0$) is asymptotically stable, 2) the robust stability performance satisfies the following inequality

$$J_a = \|W_1(s)T(s)\|_\infty < 1 \quad \ldots\ldots\ldots (2)$$

and 3) the disturbance attenuation performance satisfies the following inequality

$$J_b = \|W_2(s)S(s)\|_\infty < 1 \quad \ldots\ldots\ldots (3)$$

Then the closed loop system is also asymptotically stable with $\Delta P(s)$ and $d(t)$. Where $W_2(s)$ is a stable weighting function matrix specified by the designers. $S(s)$ and $T(s) = I - S(s)$ are the sensitivity functions, and the complementary sensitivity of the system, respectively [11]

$$S(s) = (I + P(s)C(s))^{-1} \quad \ldots (4a)$$

$$T(s) = P(s)C(s)\big(I + P(s)C(s)\big)^{-1} \ldots (4b)$$

A balance performance criterion to minimize $J_\infty = (J_a^2 + J_b^2)^{1/2}$, where J_∞ is the H_∞ performance index. The minimization of tracking error J_2 (i.e., H_2 norm) will be taken into account

$$J_2 = \int_0^\infty e^T(t)e(t)dt \quad \ldots\ldots (5)$$

where e(t)=r(t)-y(t) is the system error.

The handling of constraints (2), and (3) is to recast the constraints as objectives to be minimized and, consequently, a weighted sum approach is conveniently used with a suitable weightings u_1 and u_2 [12], which can be calculated by the designer. Therefore, the objective function of the investigated problem of designing mixed H_2/H_∞ optimal controllers will be as follows

$$\min_C J = u_1 J_2 + u_2 J_\infty \ \ldots\ldots (6)$$

and a suitable structure specified PID controller will be chosen depending on
the number of the inputs and the number of the outputs.

The possibility of our work is come from IGA which consists of many programming steps used to get the optimal mixed H_2/H_∞ controller. A modification is added to IGA in this paper by testing the searching range of parameters during each iterations and find the suitable range for continuity of minimization and the range of parameter and also will be constricted depending on the direction that ensure the minimization will occur. Then in the remaining iterations, the H_∞ performance will be minimize and the range also will be constricted in each iteration. The constriction of parameters technique is performed by replacing the maximum value of the range of each parameter by the last absolute value of it, whereas the minimum value of its range will be replaced by the same value, but with negative signature. Then this new range will be tested if it will lead to minimum or not. The proposed IGA-based design methods is described in Figure 2.

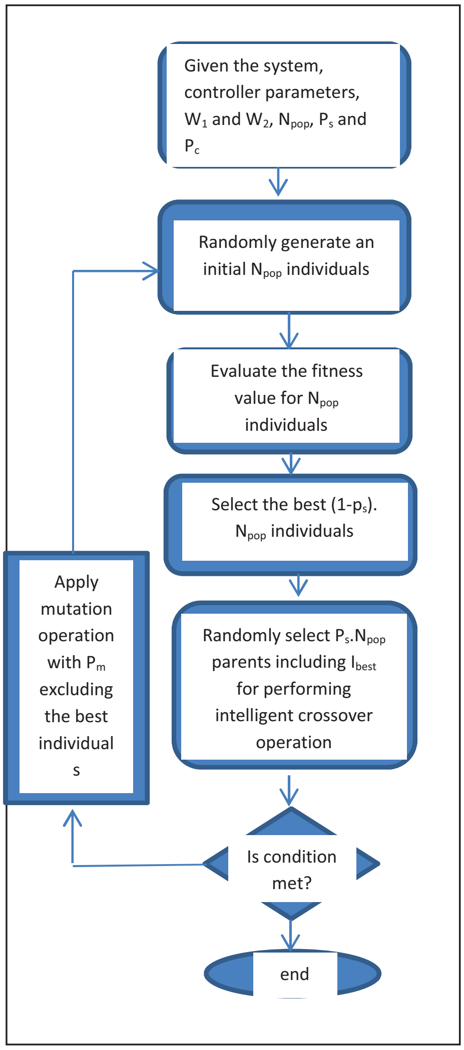

Figure 2 Controller design based- IGA

III. The Structured – Specified Controller

The structured – specified controller has the following form:

$$C(s) = \frac{N_c(s)}{D_c(s)} = \frac{B_m s^m + B_{m-1}s^{m-1} + \cdots + B_0}{s^m + a_{n-1}s^{n-1} + \cdots + a_0} \quad \ldots (7)$$

The controller is assigned with some desired order m and n to where

$$B_k = \begin{bmatrix} B_{k11} & \cdots & B_{k1n_i} \\ \vdots & \ddots & \vdots \\ B_{kn_o1} & \cdots & B_{kn_on_i} \end{bmatrix} \quad \ldots\ldots\ldots (8)$$

For $k = 0,1, \ldots, m$.

Most of the conventional controllers used in industrial control systems have fundamental structures such as PID and lead/lag configurations. Such controllers are special case of the structure-specified controllers[11]. For PID controller, $n = 1, m = 2$ and $a_o = 0$, i.e.

$$C(s) = \frac{B_2 s^2 + B_1 s + B_0}{s} \quad \ldots\ldots\ldots (9)$$

MIMO Examples

This section presents numerical examples to demonstrate the effectiveness of the robust PI or PID structure-specified mixed H_2/H_∞ controller.

Example (1)

Consider the pilot scale distillation column model [13]

$$G_1(s) = \begin{vmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{110.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{vmatrix} \quad \ldots\ldots\ldots (10)$$

The effect of delays is simplified for each transfer function using pade approximation, the system would be

$$G_1(s) = \begin{vmatrix} \frac{12.8(-s+2)}{(16.7s+1)(s+2)} & \frac{-18.9(-3s+2)}{(21s+1)(3s+2)} \\ \frac{6.6(-7s+2)}{(110.9s+1)(7s+2)} & \frac{-19.4(-3s+2)}{(14.4s+1)(3s+2)} \end{vmatrix} \quad \ldots\ldots (11)$$

The problem now is converted from delay to RHS zeros. To find the whole zeros and pole of $G_1(s)$, let $\emptyset = \det|G_1(s)|$, then the zeros and poles of $G_1(s)$ will be equal to that of $\emptyset$ which is equal to

$$\emptyset = (-0.5136s^5 + 2.926\, s^4 - 1.069\, s^3 - 1.069\, s^3 - 0.3799\, s^2 - 0.03553\, s - 0.0008964)/(s^7 + 3.221\, s^6 + 2.915\, s^5 + 1.023 s^4 + 0.1614\, s^3 + 0.01257\, s^2 + 0.0004741\, s + + 0.000006921)$$

The system has two RHS zeros at $s = 5.2743$, and $s = 0.6667$, with no RHS poles. The singular value decomposition for $G_1(s)$ at these two zeros are shown below

$$G_1(5.2743) = \begin{vmatrix} -0.064297 & 0.131158 \\ -0.01011 & 0.19553 \end{vmatrix} = \begin{vmatrix} 0.5884 & 0.8086 \\ 0.8086 & -0.5884 \end{vmatrix} \begin{vmatrix} 0.2397 & 0 \\ 0 & 0.0469 \end{vmatrix} . \begin{vmatrix} -0.1919 & -0.9814 \\ 0.9814 & -0.1919 \end{vmatrix} \quad \ldots\ldots (12)$$

Its excepted that any zero in the system must give zero gain, but this zero didn't satisfied this assumption (the maximum and minimum singular value didn't equal to zero), so it must be taken under consideration during the design of W_2

For the other zero:

$$G_1(0.6667) = \begin{vmatrix} 0.52743 & 0.0000315 \\ -0.035231 & 0.0000458 \end{vmatrix} = \begin{vmatrix} -0.9978 & 0.0666 \\ 0.0666 & 0.9978 \end{vmatrix} \begin{vmatrix} 0.5286 & 0 \\ 0 & 0.000047798 \end{vmatrix} . \begin{vmatrix} -1.0000 & -0.0001 \\ -0.0001 & 1.0000 \end{vmatrix} \quad \ldots\ldots\ldots (13)$$

As shown above the zero 0.6667 has approximately zero gain at the second output so its effect can be ignored during the design of the weighting related to this output, while it must be taken under consideration for the first output. Based on the above enumeration the parameters of weighting matrix will be chosen as follow:

$$W_2 = \begin{vmatrix} \dfrac{0.6s+0.1}{1+0.001} & 0 \\ 0 & \dfrac{0.6s+0.1}{1+0.001} \end{vmatrix} \quad \dots (14)$$

A structure-specified PI controller will be designed in the next steps, and since the system has two inputs and two output so according to (7) and (8), the controller will be

$$C(s) = \frac{\begin{bmatrix} B_1 & B_3 \\ B_5 & B_7 \end{bmatrix} s + \begin{bmatrix} B_2 & B_4 \\ B_6 & B_8 \end{bmatrix}}{s} \quad \dots (15)$$

So the weightings in the optimal problem (6) are assumed as: u_1=o.1 and u_2=0.9, $i.e.$, the H_∞ will take 90% as a portion of the optimal while the H_2 will take 10%. After 3 runs the best results are

$$J_2 = 3.1041, \text{ and } J_b = 0.9080,$$

With following structured specified PI optimal controller:

$$C(s) = \frac{\begin{vmatrix} 0.2205 & 0.0193 \\ -0.0478 & -0.0709 \end{vmatrix} s + \begin{vmatrix} 0.1295 & -0.0311 \\ 0.0599 & -0.0192 \end{vmatrix}}{s} \quad \dots (16)$$

The step response for each one of channel of output of the resultant feedback system are shown in Figure 3.

It's observed from Figure 3 that the resultant system have an overshoot reached to 1.25 in the first channel of output one, this overshoot came from the value of the proportional part of the first transfer function related to the output one. It can be seen from Figure 4 that the overshoot remain in acceptable value even when the delays of the system have been perturbed.

To investigate the disturbance attenuation possibility of the resultant system the singular value of the integral matrix of the controller will help, by finding the minimum singular value of this matrix, which is approximately equal to the value of attenuation. This value had been found equal to 0.0042. This is very good value compared with 0.0131 for [9], 0.118 for [14], and 0.0056 for [15]. Figure 5 articulates the disturbance response for this system. As shown in this figure, the system has very good disturbance attenuation where the overshoot didn't exceed 1.2.

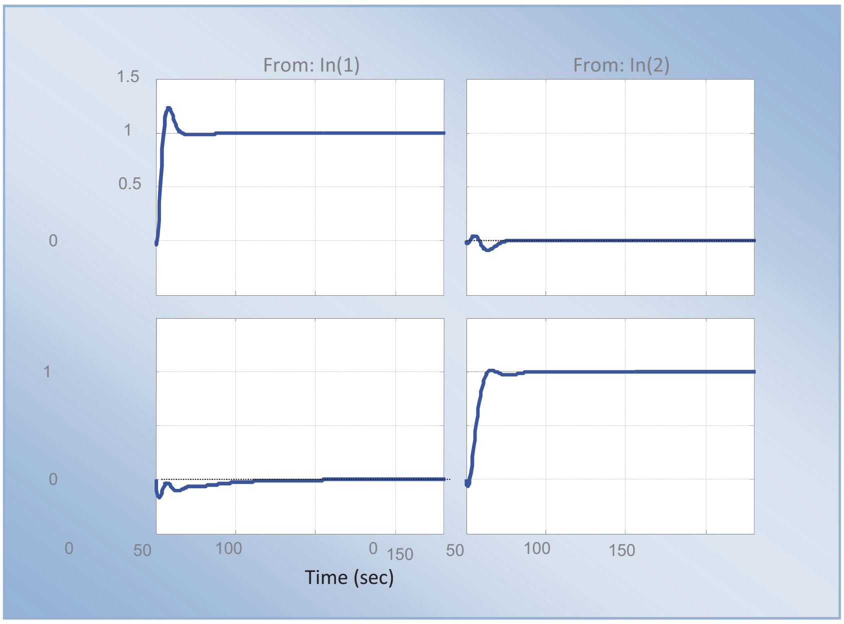

Figure 3 Step response of the resultant feedback system in Example 1

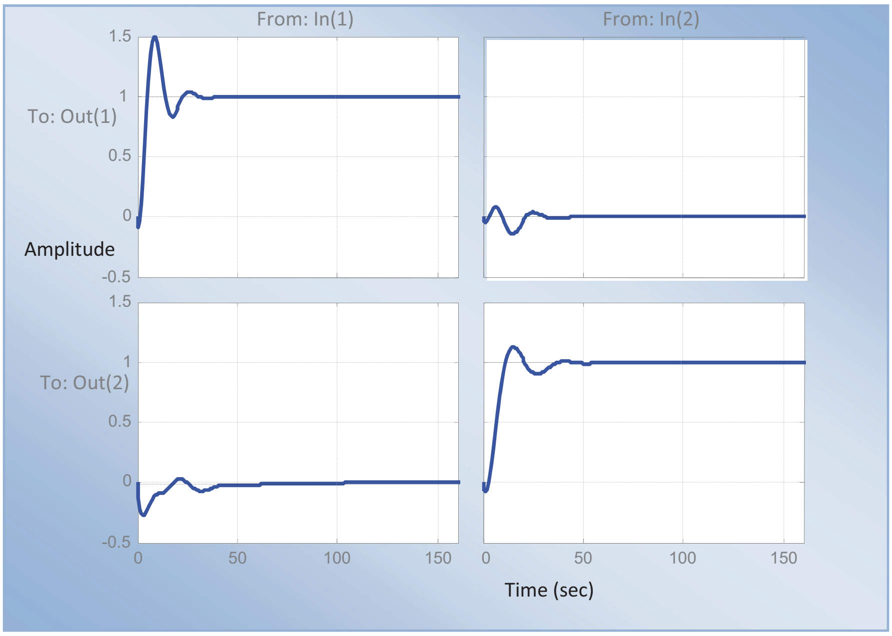

Figure 4 Step response when the delays had been perturbed in Example1

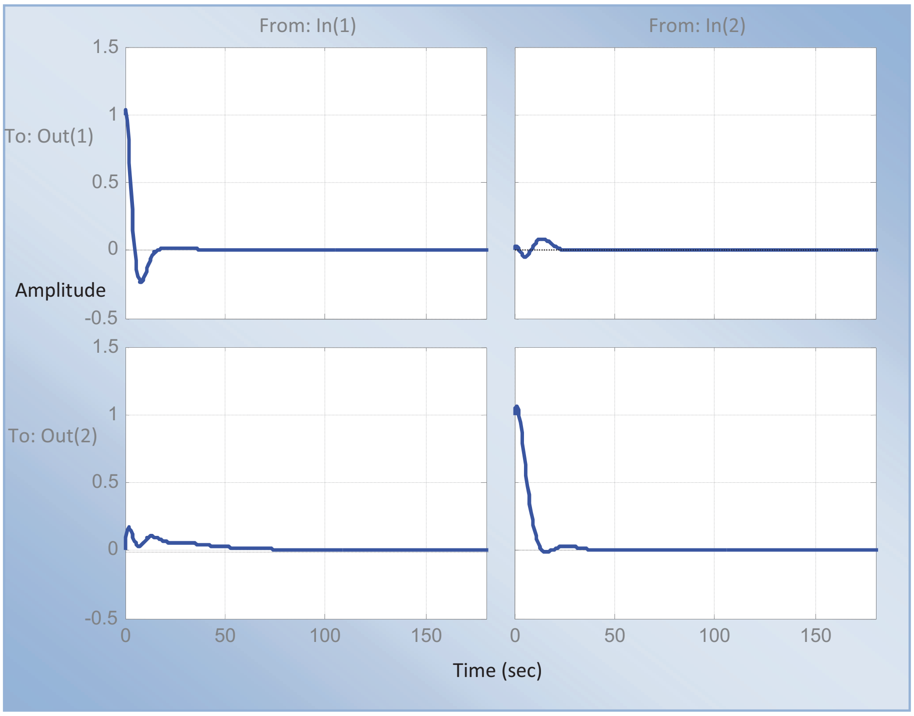

Figure 5 Disturbance response to a signal $d(t) = u(t)$ for Example 1

Example 2

Consider a longitude control system of the supermaneuverable F18/HARV fighter aircraft system [1]:

$$a = \begin{bmatrix} -0.0075 & -24.5 & 0 & -36.16 \\ -0.0009 & -0.1954 & 0.9896 & 0 \\ -0.0002 & -1.454 & -0.1677 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \ldots\ldots (17)$$

The following weightings are chosen:

$$W_1 = \frac{0.00125s^2 + 0.12025s + 0.125}{s^2 + 20s + 100} * I_{3\times3}$$

$$W_2 = \frac{0.25s + 0.025}{s^2 + .4s + 10000000} * I_{3\times3}$$

A structure-specified PID controller would be designed in the next steps, according to (7) and (8), the controller will be:

$$C(s) = \frac{\begin{bmatrix} B_1 & B_3 & B_5 \\ B_7 & B_9 & B_{11} \\ B_{13} & B_{15} & B_{17} \end{bmatrix} s + \begin{bmatrix} B_2 & B_4 & B_6 \\ B_8 & B_{10} & B_{12} \\ B_{14} & B_{16} & B_{18} \end{bmatrix}}{s}$$

After 4 runs the best results were:

$J_2 =$ 4.9998 , $J_a =$ 0.5483 , and $J_b =$ 0.7332 with the following PI controller:

$$C(s) = \frac{10^3 * \begin{bmatrix} 7.0986 & -9.9942 & 5.7322 \\ -11.835 & -10.467 & 05.964 \\ -17.09 & 12.97 & 16.947 \end{bmatrix} s}{s} + \frac{10^3 * \begin{bmatrix} -3.3856 & -6.2088 & -2.1195 \\ -11.141 & -2.097 & -12.559 \\ -14.409 & -2.85 & 4.606 \end{bmatrix}}{s}$$

Figure 6 illustrates the response of the resultant system to the following inputs $r(s) = [0 \quad 1 - e^{-3t} \quad 1 - e^{-6t}]^T$

To test the robust possibilities of the resultant system, some parameters in matrix $\boldsymbol{a}$ of the system is perturbed as follows

$$a = \begin{bmatrix} -0.008 & -30 & 0 & -36.16 \\ -0.00015 & -0.1954 & 0.9896 & 0 \\ -0.0002 & -1.454 & -0.2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

As shown in Figure 7, there is no any palpable change in the system response. The disturbance response is shown in Figure 8.

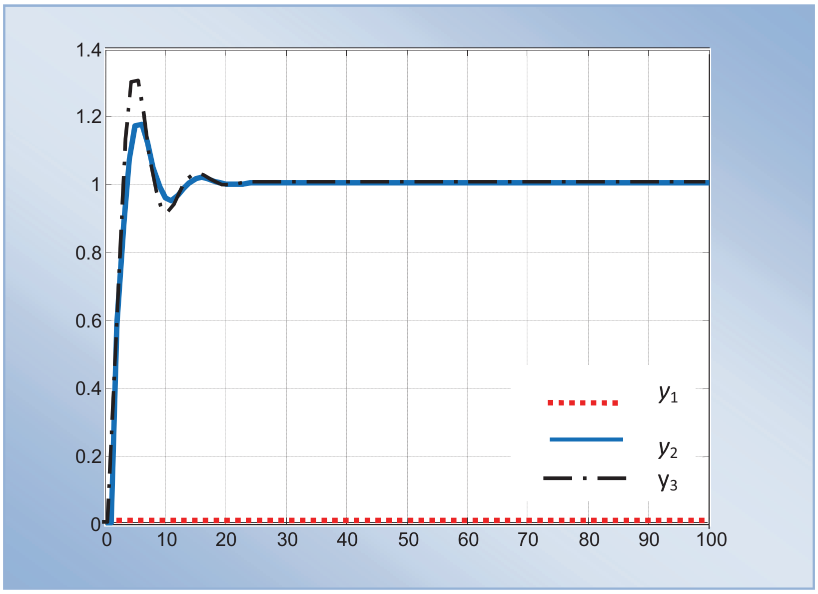

Figure 6 The response of the resultant system to the following inputs $r(s) = [0 \quad 1 - e^{-3t} \quad 1 - e^{-6t}]^T$

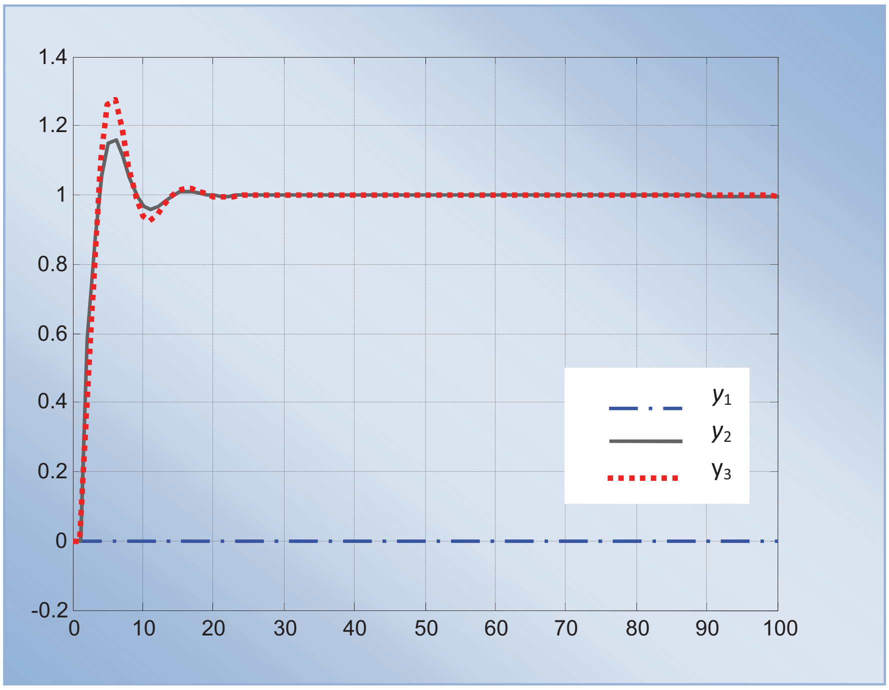

Figure 7 Step response when some parameters of the resultant system in Example 2 is perturbed

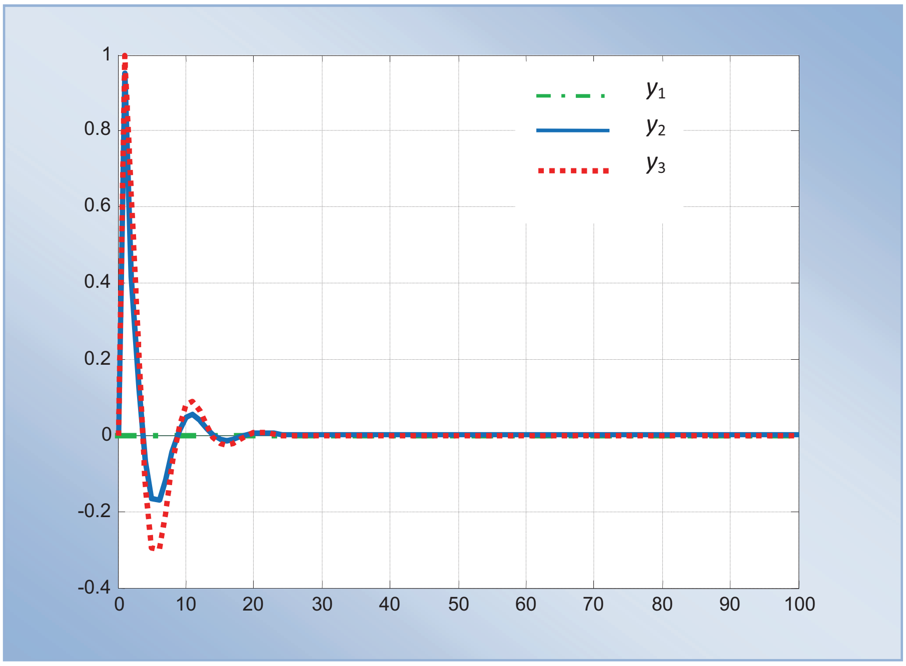

Figure 8 Disturbance response for Example 2

IV. Conclusions

In this paper, the PID controller parameters have been tuned to meet the mixed H_2/H_∞ optimal performance. It's observed for MIMO cases that the optimal operation became very fast when the modified part has been added to IGA. The resulting PID was immune to any perturbation that will occur in the parameters of the plant in fact it's also immune if any small deviation occurs in its parameters. This guarantees that the PID controller will still robust and work properly even if any element in the circuitry is changed according to any change in the physical conditions.

The computing of the H_2 norm for MIMO systems became very easy when the simulation results is used. It's also guarantee to get the real value of H_2 norm more than the other way that was use before such as state space method [5], or using residue [6](for SISO).

H_2 norm was the guidance of the stability during the optimal search process, where IGA follow the minimum value of H_2 norm in each time, $i.e.$, minimum error value which is led to guarantee stability. Finally, simulation results show that a good performance can be achieved by the proposed method for MIMO models under different conditions.

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