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Calibrating the Discharge Coefficient of Semicircular Crested Weir

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Abstract

This study deals with the evaluation of discharge coefficient of semicircle crested weir extending across the full width of the channel (suppressed).It is stable over flow pattern, easy to pass floating debris, and has large coefficient of discharge. Data obtained from laboratory experiments provide information on head – discharge relationship for three models with different radius of curvature. Each model has an empirical head- discharge equation. The momentum equation is applied to derive discharge per unit width. The predicted values of discharge coefficient for the proposed models based on direct discharge measurement & the derived discharge equation. The proper application of this study is an equation show that the discharge coefficient is proportional with upstream head above weir crest & inversely with the radius of curvature.

Keywords: Curvilinear flow, pressure distribution, discharge coefficient semicircular.

معايرة معامل التصريف لسد غاطس ذو حافة نصف دائرية

الخلاصة

يتناول هذا البحث حساب قيم معامل التصريف لسد غاطس نصف دائري الحافة موضوع على عرض القناة (غير مقلص) .إن هذا النموذج ذو نمط جريان مستقر و يمتاز بسهولة تصريف المواد العالقة و له معامل تصريف عال . أظهرت المعادلات التجريبية التي تم استنتاجها من نتائج التجارب المختبرية عن العلاقة ما بين التصريف والشحنة المقاسة لثلاثة نماذج مختلفة الأقطار . تم تطبيق معادلة الزخم في اشتقاق معادلة لحساب التصريف لوحدة العرض .إن قيم معامل التصريف المحسوبة لهذه النماذج معتمدة على قيم التصاريف المقاسة وقيم التصاريف النظرية . أظهرت المعادلة التي تم اشتقاقها ان معامل التصريف يتناسب طردياً مع الشحنة المقاسة و عكسياً مع نصف قطر التقوس للسد الغاطس .

Introduction

A semicircular broad crested weir consists of an obstruction in the form of a raised portion of bed extending across the full width of the channel. This raised portion has a shape of semicircle figure (1). The basic difference between broad crested weir and the present model is the curvature of stream lines. The effect of curvature is to produce appreciable acceleration components or centrifugal forces normal to the direction of flow [3].Another feature of this model that it has larger discharge capacity than the broad crested weir and sharp crested weirs. Related applications include three models of polished wood models

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with different radius of curvature. In the present study the models calibrated to determine the relation between the head and rate of discharge.

Theoretical Background

The two dimensional flow patterns over cylindrical weirs can describe by the Equation of motion in the s-and n-direction [9].Curvilinear flow resulting in diversion of pressure distribution from hydrostatic as the flow occur in the vertical plane[3] .The pressure will be less than hydrostatic pressure in convex flow when centrifugal force acting upward against the gravity action[7]. The rate of departure from hydrostatic pressure distribution is defined by the local centrifugal acceleration (v^2/r). Thus the change of piezometric head in n-direction is [9]:

$$d\left(\frac{P}{\gamma}+Z\right)=\frac{-v^2}{gr}.dn \ldots\ldots(1)$$

The integration of equation (1) from point (1) to point (2) in the n-direction figure (2a) yields:

$$\left[\frac{P}{\gamma}+Z\right]_1-\left[\frac{P}{\gamma}+Z\right]_2=\frac{1}{g}\int_1^2\frac{v^2}{r}.dn(2)$$

Where $\left[\frac{P}{\gamma}+Z\right]$ equals the piezometric head at point (1) and point (2) respectively and $\left[\frac{1}{g}\int_1^2\frac{v^2}{r}.dn\right]$ is the loss of piezometric head due to curvature of surface line. A decrease of piezometric head, which is due to centrifugal acceleration, necessarily a corresponding increase of velocity head, thus:

$$\frac{v_2^2}{2g}-\frac{v_1^2}{2g}=\frac{1}{g}\int_1^2\frac{v^2}{r}.dn \ldots\ldots\ldots(3)$$

Critical Depth

In a rectangular horizontal channel with hydrostatic pressure distribution, the flow depth at critical flow condition equals:

$$d_c=\sqrt[3]{q^2/g} \quad \ldots(4)$$

dc is commonly called the critical flow depth at the crest of semi-circular weir, critical flow occurs but the pressure distribution is not hydrostatic. The stream line curvature implies that the pressure gradient is less than hydrostatic and velocity distribution is rapidly varied [1].Critical depth and its position on weir are governed mainly by frictional effects and stream line curvature. Therefore, curvilinear flow (dc/y1) $>\frac{2}{3}$ while in parallel flow (dc/y1) $=\frac{2}{3}$ [6]. For all these reasons, the depth at the crest of semi-circular weir is expected to differ from equation (4).

Computation of Pressure Distribution

Knowledge of the variation in pressure is particularly important in the design of hydraulic structure, over which and in which flow is a curved nature. The induced pressure forces may cause damage or erosion to the face of the structure. In the vertical plane the effect of streamline curvature is to increase or decrease the pressure distribution. Flow over a convex surface the pressure is reduced below hydrostatic [8]. For curved profiles, the total pressure distribution can be determined analytically by numerical method or graphically by flow – net analysis. The second method is adopted in this study. Figure (3) show the application of flow-net analysis on model (I). The corresponding variation of pressure distribution along the weir surface shown in figure (4). Calculation of pressure forces acting on the downstream weir surface that counteract pressure force acting on upstream weir surface revel that the ratio between these two resultant forces approximately (0.5) and that's mean the upstream force is twice than the down stream force for the three types of models and the pressure force per unit width acting on weir surface which is equal and opposite in direction to force exerted by weir on block of water is calculated from equation:

$$P_w = \frac{1}{2}\gamma(y_1 - R + y_1) * \frac{R}{2} = \frac{\gamma R}{4}(2y_1 - R) \quad \ldots\ldots(5)$$

Where y_1 is upstream flow depth. The accuracy of the last assumption has been checked by flow-net analysis.

Evaluation of Discharge

The discharge per unit width of a broad crested weir across a rectangular channel (q) can be determined by application of the momentum principle. Since it deals only with external forces represented by upstream and downstream hydrostatic and pressure forces acting on weir surface. The assumptions to be made are the frictional forces $(F_{f'})$, $(F_{f''})$ are negligible, where first symbol is the frictional forces on channel bed and the second is frictional forces on weir surface. The longitudinal section of the short length of horizontal channel shown in the figure(5) in which the flow is impeded by semicircular weir located on channel bed.

Let the force exerted by the weir on flow be (Pw) as derived in equation (5).if the momentum equation is applied to the body of water between upstream approach section (1) and downstream section (2) as shown in the figure (5),the following equation may be written:

$$\frac{q\gamma}{g}\left(\frac{q}{y_2} - \frac{q}{y_1}\right) = \frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 - \frac{\gamma R}{4}(2y_1 - R) \quad \ldots.(6)$$

Where (q) the discharge per unit width, y1and y2 are upstream and downstream weir depths. Experimental data have shown that on the average, $y_2 = \frac{y_1 - R}{3.25}$ in that case the above equation can be simplified and solved for q as:

$$q_{th} = \sqrt{0.45g}\left(\frac{y_1}{2.25y_1 + R}\right)^{0.5}\left(1 + \frac{K}{H^3}\right)^{0.5}(H)^{3/2} \quad \ldots\ldots\ldots(7)$$

Where

$K = (0.55R)(2y_1 - R)(y_1 - R)$,

(K) is coefficient has the dimension of (L3), g is gravity acceleration, and (H) is the upstream head above the weir crest. In practice the observed discharge differs from equation (7) because it based on idealized assumptions such as: hydrostatic pressure distribution and uniform velocity distribution in reality these effects do occur and the must be accounted for by introduction of a discharge coefficient (C_d)

Experimental Works

The experiments were performed in a horizontal laboratory flume with (5m) length, (0.076 m) width, and (0.15) depth. Both bed and walls were made of fibber glass to ensure hydraulic smooth condition, figure (1). The flume is supplied with a constant head tank. The water discharge was measured by a volume-time method, the flow depth had been measured using point gages, and the flow of semi-circle were investigated in the laboratory for three models of weir with different radius of curvature (4, 6, 7) cm which are placed along (3.85 m) downstream out let of the tank and fixed directly on side walls (suppressed type). The flow regulating value was adjusted to give the maximum possible discharge with the corresponding depth upstream and downstream of the weir. The discharge was reduced in five steps and series of readings for (q) and (H) were taken for each type of weirs. All experiments done with zero slope channel bed. In addition, photographs were taken during the experiments and used to visualize the flow pattern as shown in the figure (1). Full details of data were reported in appendix (A).

Experimental Results

The variations of (q) with (H) for three models were shown in figures (6), (7), and (8) respectively, with five readings for each model as showing in appendix (A). By plotting (Log q) against (Log H) a straight line relationship obtained its equation of association, ($q = C*H^m$) the experimental relationship between (q) and (H) are:

$q = 1.7648 \times H^{1.5}$…………. (For R = 4 cm)……………….……..…. (8)

$q = 2.1898 \times H^{1.5}$………….. (For R = 6cm)……………….…….….. (9)

$q = 2.1747 \times H^{1.5}$………….. (For R = 7 cm)…………………..……. (10)

Experimental data shows that the practical range of the coefficient to ($H^{3/2}$) is varied as shown in table (1).

Effect of Upstream Head on Discharge Coefficient

The discharge coefficient data are plotted as function of ratio (H/R). The graphical representation show that the discharge coefficient is proportional to the ratio of (H/R) as illustrated in figure (9). It appears that if (R=∞) the stream lines are straight and if they are curved there is a significant flow velocity result is increasing of discharge coefficient.

Effect of Radius of Curvature on Discharge Coefficient.

The effect of radius of curvature was investigated through series of experiments. The average discharge coefficients for each model are plotted versus radius of curvature as shown in the figure (10) which reveal that the discharge coefficient increased as the radius of curvature decreased.

That is for any element of stream line having radius of curvature (r), has a normal component of acceleration (an) which is equal to (v^2/r) [9]. Resulting in non uniform velocity distribution which has maximum value at the inner surface and lower value at the surface of flow as shown in the figure (11).Thus the velocity profile over the crest of circular weir is significantly differs from the theoretical log-law profile due to the strong favorable pressure gradient [2].

Discussion:

Pressure Distribution

The existence of centrifugal forces in the upstream and down stream cross-section bounding the considered zone of acceleration causing reduction in pressure. This reduction of pressure is investigated through the application of flow-net analysis with positive values of water pressure measured along the weir surface. Under those such circumstance where the over flowing jet is in contact with the body of weir and under pressure may not developed near the weir crest. Note that the reduction in pressure due to streamlines curvature increases the discharge coefficient.

Weir Coefficient

In the head-discharge equation of each type of models, there is a weir coefficient which represents the combination of the approach velocity head at measuring station, the breadth of flow at control section, and the streamline curvature. Thus the influences of all these values are included in weir coefficient (C). The practical ranges of the coefficient (C) to (H3/2) are shown in table (1).

Discharge coefficient

The variation of discharge coefficient (Cd) with respect to head normalized to radius of curvature (H/R) show that the discharge coefficient is greatly effected by the upstream head over the crest (H) and the discharge coefficient values increases with increasing values of (H) over the range of the calibration note that the flow over the weir was independent of downstream water level. It's apparent at very low values of (H), the effect of surface tension and viscosity would influence the behavior and results in curves shown in figure (9).

For very high weir(that is R/H large)the velocity of approach becomes small so that (E≈H) but with lower weir and the same flow rate the velocity of approach becomes larger and (E/H) increasing in magnitude resulting in larger values of (Cd) for lower weirs. The actual flow over the weir will be less than the ideal flow because the effective flow area is considerably smaller than (H*L) due to drawdown from the top and contraction of flow from the crest below [4].

In the case in which there is bed curvature, equation will contain additional term due to bed curvature [7]. This effect included in equation (7). Since the flow lie in vertical curved plane, thus subjected to normal acceleration in addition to tangential acceleration and it may used to conclude that a decrease of velocity is to be expected with increase of distance from center of curvature [9]. Which result in decreases in discharge coefficient as radius of curvature increased as shown in figure (10).

The pressure gradient over the crest of semicircle broad crested weir result in increasing discharge coefficients illustrated in equation (1).A decrease of piezometric head due to centrifugal acceleration necessarily induces a corresponding increase of velocity head as given in equation (3).

Conclusions

The main objective of this study is to estimate the discharge coefficient for semicircular crested weir. The experimental results and theoretical discharge equation show that the discharge coefficient increases as the head upstream the weir crest increased, the degree of curvature of over flowing has a significant influence on (Cd) and the discharge coefficient increases if the streamlines curvature increases. Thus the discharge coefficient is depends on shape and type of the measuring structure.

The pressure distribution varies along the weir surface and there is a transition from hydrostatic to hydrodynamic pressure forces as the flow moves in curvilinear vertical plane however the determination of these forces are calculated from flow-net analysis.

From practical experiments there are three different value of (C) related to (H3/2) these values are depend on the shape of approaching channel section and to the power of the head above the crest level. Each of the empirical stage-discharge equations (1, 2, 3) of this type have been derived for that structure only.

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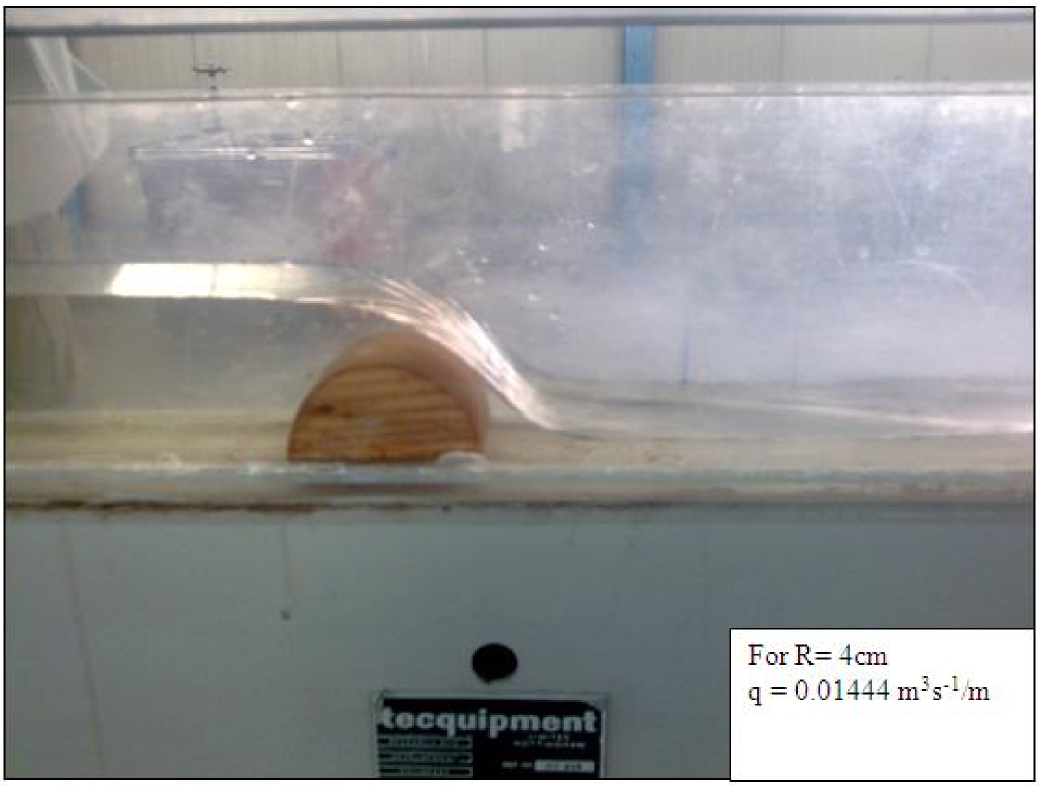
Table (1) coefficient values for models.

Type of model	weir radius (cm)	(C) value
Model I	4	1.7648
Model II	6	2.1898
Model III	7	2.1747



For R= 4cm
$q = 0.01444\ m^3 s^{-1}/m$

Figure (1) Flow pattern over the model. (I)

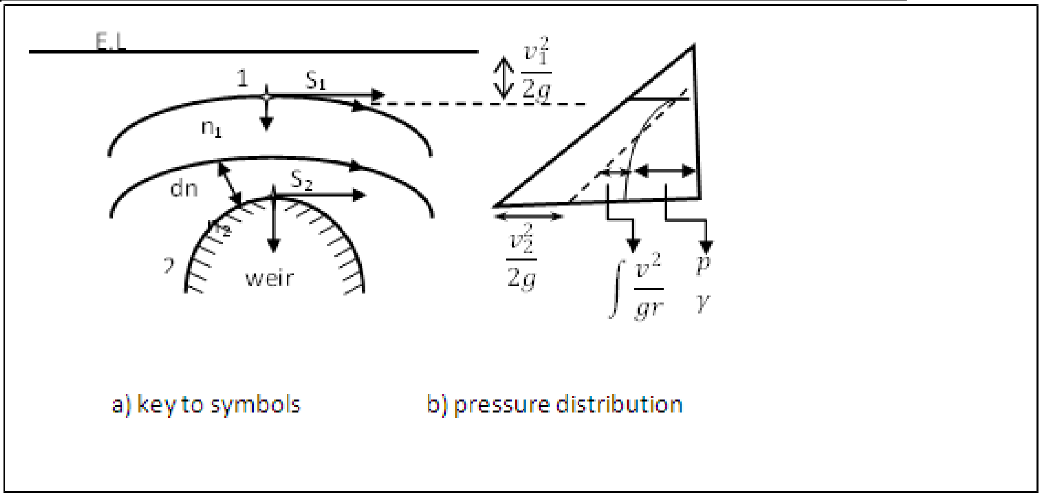


Figure (2) convex flow

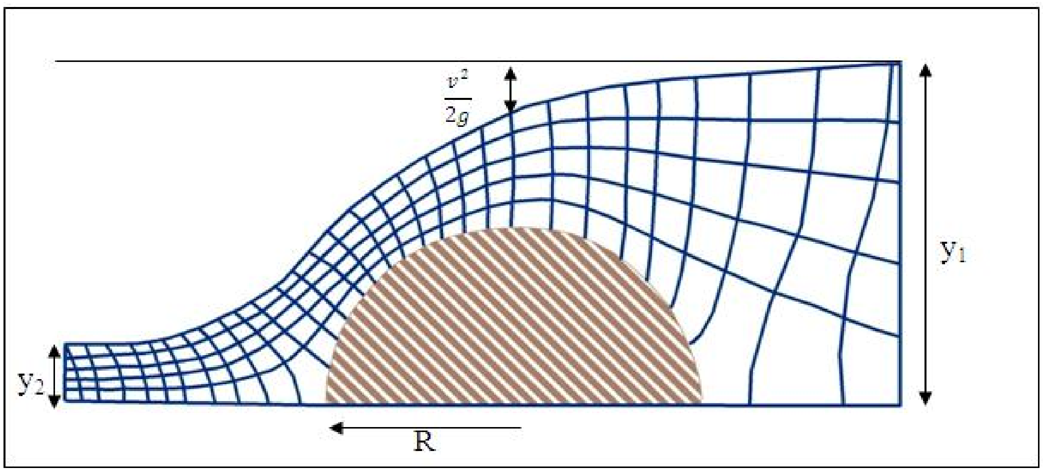


Figure (3) Flow-net diagram for model (I).

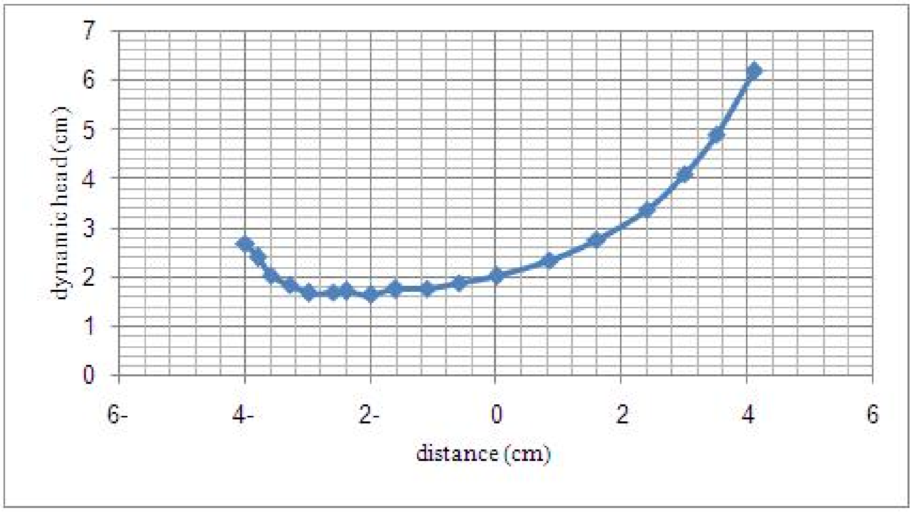


Figure (4) pressure distribution over the weir

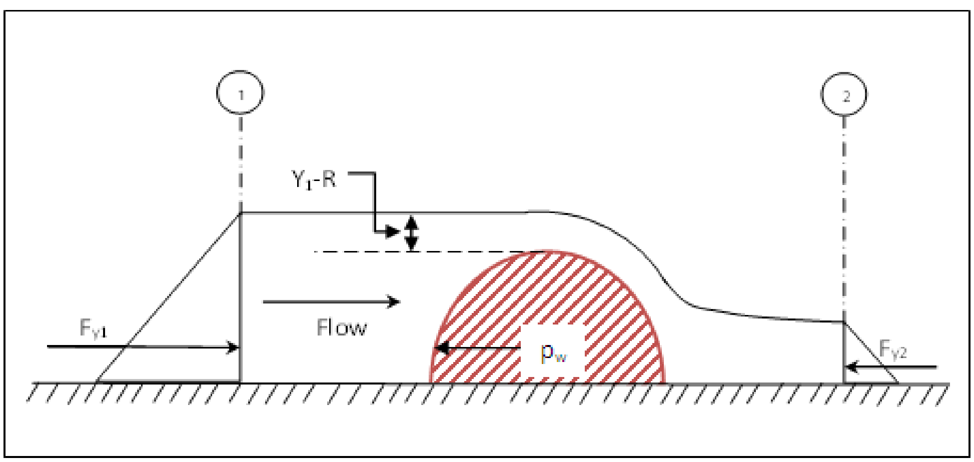


Figure (5) Definition sketch for momentum considerations.

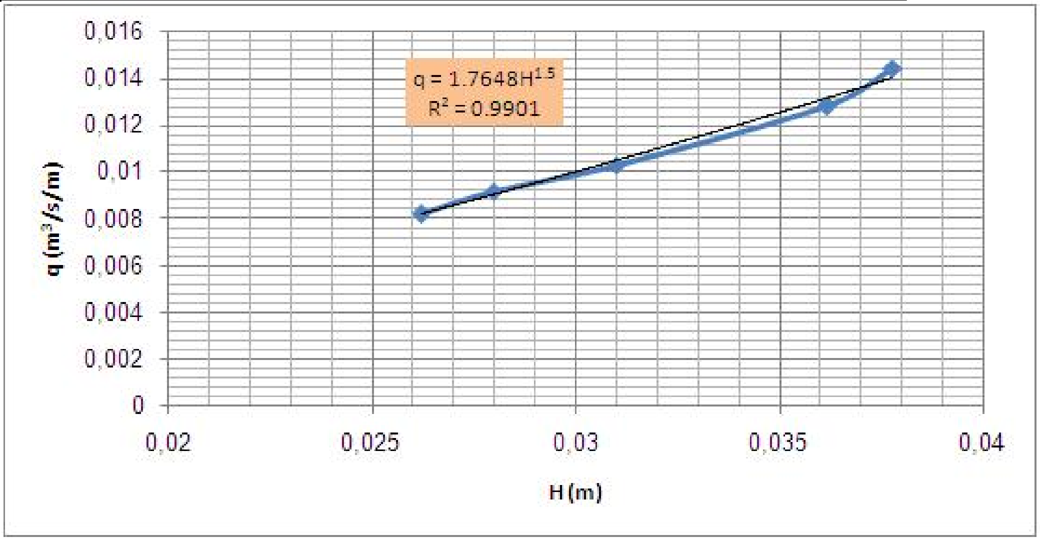


Figure (6) the relationship between (q) with (H) for model (I)

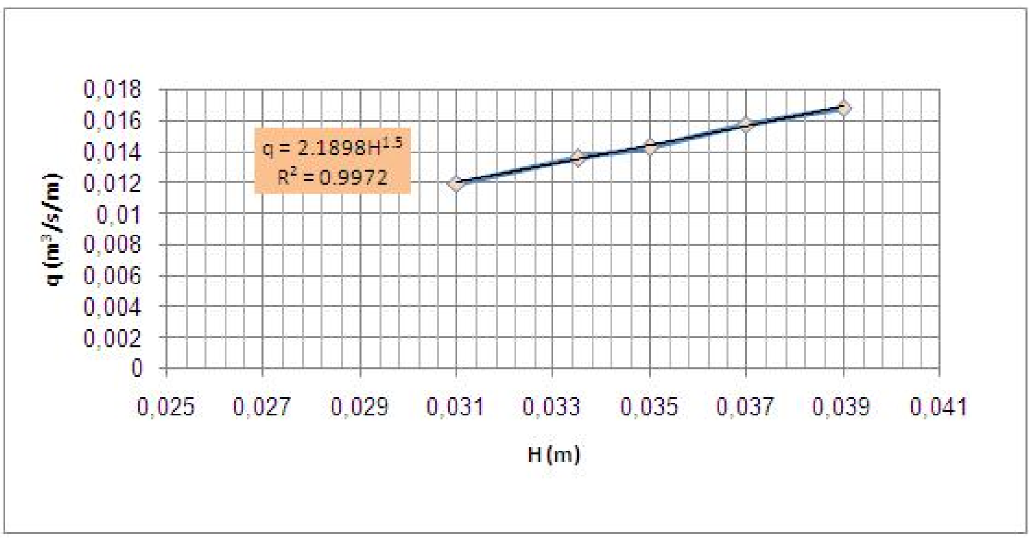


Figure (7) the relationship between (q) with (H) for model (II)

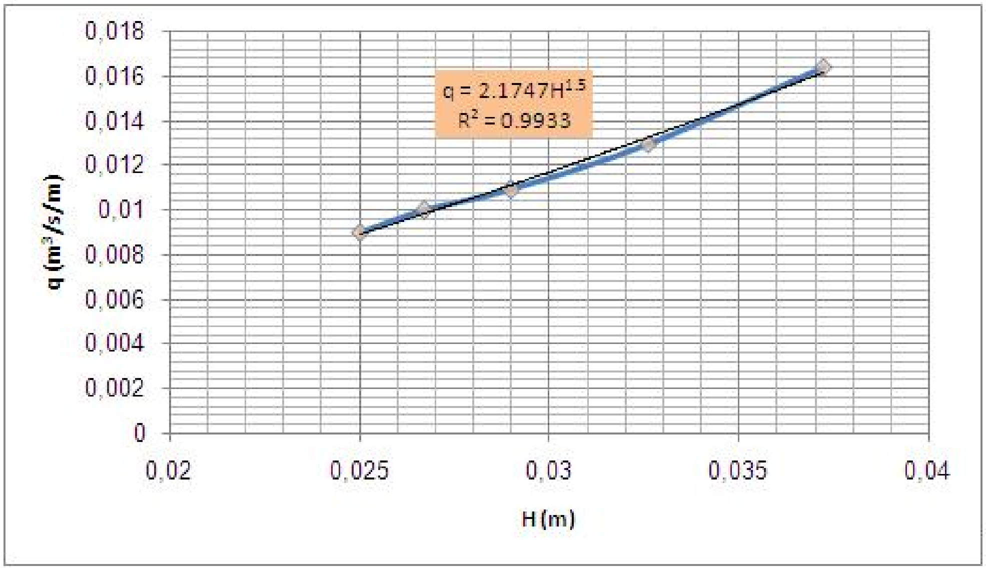


Figure (8) the relationship between (q) with (H) for model (III)

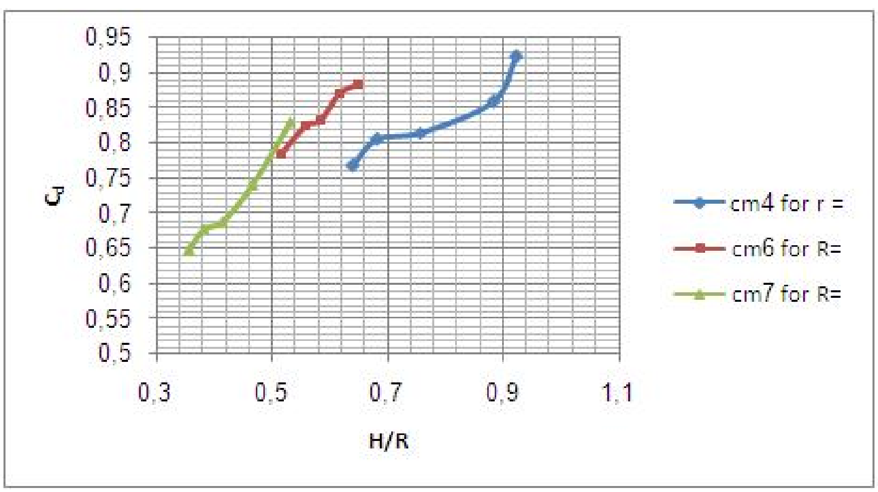


Figure (9) relationship between C_d with H/R

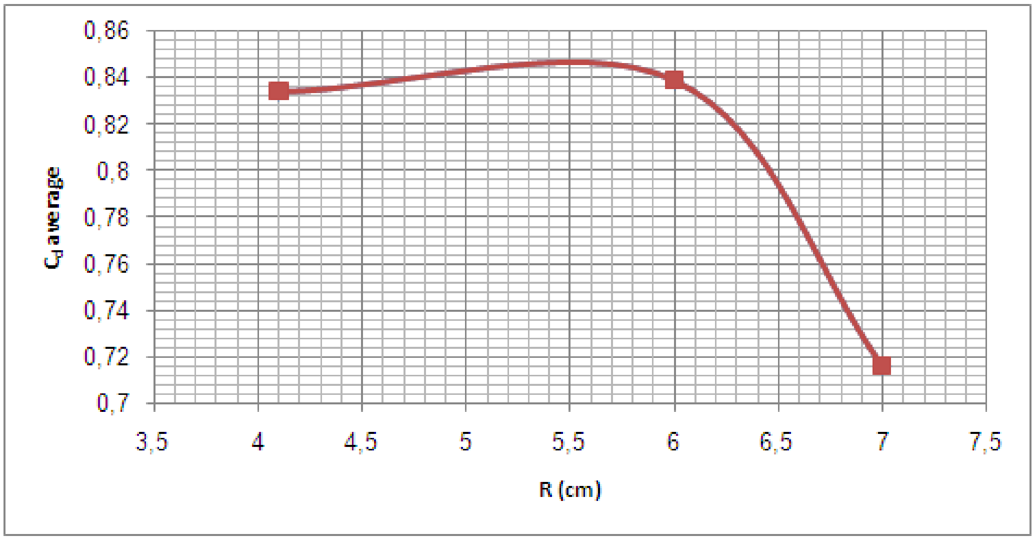


Figure (10) average discharge coefficient VS. radius of curvature

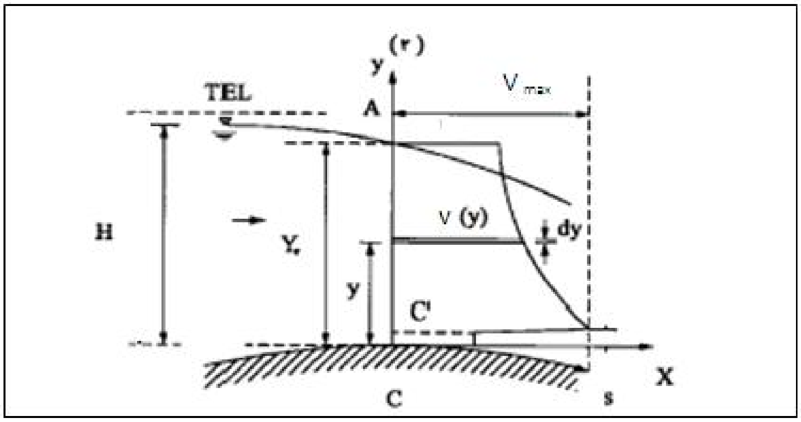


Figure (11) velocity distribution on crest of the model (after M. Heidarpour, 2008)

Appendix (A)

Experimental Data

Laboratory measurement for model (I) with (R = 4 cm) (the measured at equal distance of (1 cm) from the center line)

Q (l/s)	Upstream											C.L	downstream										
1.098	7.88	7.88	7.88	7.88	7.88	7.83	7.7	7.63	7.49	7.35	7.1	6.83	6.27	5.6	4.85	3.94	3.09	2.3	1.8	1.59	1.45	1.4	1.25
0.972		7.72	7.7	7.68	7.6	7.55	7.5	7.4	7.21	7.1	6.89	6.54	6.09	5.4	4.55	3.67	2.67	1.98	1.5	1.27	1.2	1.12	1.12
0.78		7.2	7.2	7.2	7.2	7.2	7.2	7.1	7.04	6.9	6.7	6.35	6.08	5.53	4.86	3.61	3.46	1.68	1.2	1	1	0.92	
0.714			6.9	6.9	6.9	6.9	6.9	6.86	6.8	6.63	6.46	6.19	5.73	5.25	4.5	3.3	2.1	1.36	0.9	0.86			
0.62			6.72	6.72	6.72	6.72	6.72	6.72	6.62	6.46	6.2	6	5.64	5.18	4.32	3.3	1.9	0.99	0.84	0.75			

Laboratory measurement for model (II) with (R = 6 cm) (the measured at equal distance of (1 cm) from the center line)

Q (l/s)	Upstream											C.L	downstream										
1.229	9.9	9.88	9.8	9.76	9.7	9.64	9.6	9.45	9.34	9.14	8.9	8.63	8.25	7.78	7.2	6.5	5.4	4.44	3.15	2.44	1.84	1.54	1.33
1.209	9.7	9.7	9.7	9.7	9.53	9.53	9.4	9.24	9.2	9.05	8.61	8.43	8	7.54	6.9	5.1	3.9	2.99	2.16	1.5	1.3		
1.083	9.5	9.5	9.5	9.5	9.44	9.34	9.3	9.2	9.06	9.84	8.6	8.3	8	7.62	6.84	5.95	4.96	3.68	2.7	1.9	1.54	1.25	
0.980	9.35	9.35	9.35	9.35	9.3	9.2	9.1	9.04	8.9	8.7	8.5	8.24	7.88	7.4	6.77	5.9	4.8	3.5	2.47	1.7	1.2	1	
0.9337			9.14	9.1	9.04	8.98	8.9	8.75	8.6	8.35	8	8.1	7.7	7.2	6.5	5.64	4.96	3.1	2.05	1.34	1.08		

Laboratory measurement for model (III) with (R = 7 cm) (the measured at equal distance of (1 cm) from the center line)

Q (l/s)	Upstream									C.L	Downstream										
1.25	10.72	10.6	10.6	10.5	10.44	10.32	10.2	10	9.92	9.5	9.14	8.6	8.1	7.4	6.5	5.98	4.98	2.98	2.15	1.64	1.32
0.9868	10.26	10.26	10.26	10.17	10.1	10	9.9	9.7	9.51	9.2	8.9	8.45	7.8	7.1	6.26	5.1	3.75	2.6	1.77	1.2	1.1
0.8282	9.9	9.9	9.9	9.9	9.9	9.7	9.6	9.44	9.25	9	8.6	8.2	7.66	6.91	5.96	4.63	3.1	2.9	1.32	1	
0.7587	9.67	9.67	9.67	9.67	9.65	9.5	9.4	9.2	9.08	8.84	8.5	8.09	7.6	6.8	5.7	4.46	2.77	1.75	1.15	0.86	0.75
0.6622	9.5	9.5	9.5	9.42	9.3	9.25	9.2	9.02	8.92	8.62	8.3	7.9	7.34	6.53	5.54	4.4	2.6	1.5	1	0.75	