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Minimizing Error Bounds in Lacunary Interpolation by Spline (0, 2) Case

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Abstract

In this paper, we have changed the boundary conditions and the class of spline functions which are given by (Varma**,** (1973) from first derivative to third derivative, and show that the change of the boundary conditions and the class of spline functions affecting in minimizing the error bounds for lacunary interpolation by spline function.

Introduction

Spline functions are used in many areas such as interpolation, data fitting, numerical solution of ordinary, partial differential equations and also numerical solution of integral equations. Spline functions are also used in curve and surface designing (Siddiqi & Akarm, 2003)Lacunary interpolation by spline appears whenever observation gives scattered or irregular information about a function and it's derivatives. Also, the data in the problem of lacunary interpolation are values of the functions and of it's derivatives but without Hermite condition that only consecutive derivatives be used at each nodes (Varma, 1973). Mathematically, in the problem of interpolating a given data $a_{i,j}$ by a polynomial $p_n(x)$ of degree at most n satisfying:

$$p_n^{(j)}(x_i) = a_{i,j} \quad ,i = 1,2,...,\ n\ ;\ j = 0,1,2,...,\ m \quad ....(1)$$

We have Hermite interpolation if for each *i*, the order *j* of derivatives in (1) form unbroken sequence, if some of the sequences are broken, we have lacunary interpolation Varma, obtained the error bounds for (0,2) lacunary interpolation of certain function by deficient quantic spline (Varma, 1973). In this paper we study the same lacunary interpolation, but the essential difference here being in the boundary condition and the class of spline function, in order to improve the state in which the change of the boundary condition and the class of spline function affect on minimizing the error bounds and this fact is the main object of this work. Similar idea used in (Mishra&Muthur, 1980; Saeed & Jawmer, 2005) for (0,2,3) , (0,2,4) and

(0,1,4) cases. For more about applications of spline functions, see (Bokhari, Dikshit& Sharma, 2000;Lenard, 1999; Siddiqi & Akarm, 2003)and for description our problem, let $\Delta: 0 = x_0 < x_2 < \ldots < x_{2m} = 1$ be a uniform partition of the interval [0,1] with $x_i = \frac{i}{2m}$, i=0, 2, …, 2m and n=2m+1. We define the class of spline function $S_p(5,4,n)$ as follow :

Any element $S_\Delta(x) \in Sp(5,4,n)$ if the following two conditions are satisfied:

$$\begin{cases} \text{(i) } S_\Delta(x) \in C^4[0,1] \\ \text{(ii) } S_\Delta(x) \text{ is a polynomial of degree five} \\ \quad \text{in each } [x_{2i}, x_{2i+2}],\ i = 0,1,\ldots,\ m-1 \end{cases} \quad \ldots(1.0)$$

In the subsequent section, we prove the following:

Theorem1.1:

Given arbitrary numbers $f(x_{2i})$, $f^{(r)}(t_{2i})$, i=0,1,…,m-1; r=0, 2 and $f'''(x_0), f'''(x_{2m})$, there exits a unique spline $S_n(x) \in S_p(5,4,n)$ such that

$$\begin{cases} S_n(x_{2i}) = f(x_{2i}) \ , i = 0,1,\ldots, m \\ S_n^{(r)}(t_{2i}) = f^{(r)}(t_{2i}), i = 0,1,\ldots, m-1; r = 0, 2, \\ S_n'''(x_0) = f'''(x_0), S_n'''(x_{2m}) = f'''(x_{2m}) \end{cases} \quad \ldots(1.1)$$

Theorem 1.2:

Let $f \in C^5[0,1]$ and $S_n(x) \in S_p(5,4,n)$ be a unique spline satisfying the conditions of Theorem 1.1, then

$$\left\| S_n^{(r)}(x) - f^{(r)}(x) \right\| \le 12.0368449931\ m^{r-5} w(f^5; \frac{1}{m}) + 2m^{r-5} \| f^{(5)} \|\ ,\ r = 0,1,2,3,4.$$

Where $w(f^{(5)}; \frac{1}{m})$ denotes the modulus of continuity of $f^{(5)}$ and $\| f^{(5)} \| = \max \{ |f^{(5)}(x)| ; 0 \le x \le 1 \}$.

Technical Preliminaries

If $P(x)$ is a polynomial of degree five on [0,1] {because we want to construct a spline function of degree five}, then we have

$$P(x) = P(0)A_0(x) + P(\tfrac{1}{3})A_1(x) + P(1)A_2(x) + p''(\tfrac{1}{3})A_3(x) + P'''(0)A_4(x) + P'''(1)A_5(x). \quad (2.0)$$

Where

$$\begin{cases} A_0(x) = \frac{1}{15}(-54x^5 + 135x^4 - 70x^2 - 26x + 15), \\ A_1(x) = \frac{1}{10}(54x^5 - 135x^4 + 70x^2 + 11x), \\ A_2(x) = \frac{1}{30}(-54x^5 + 135x^4 - 70x^2 + 19x), \\ A_3(x) = \frac{1}{90}(54x^5 - 135x^4 + 115x^2 - 34x), \\ A_4(x) = \frac{1}{972}(27x^5 - 108x^4 + 162x^3 - 100x^2 + 19x), \\ A_5(x) = \frac{1}{4860}(189x^5 - 270x^4 + 110x^2 - 29x). \end{cases} \quad ...(2.1)$$

In the subsequent section we need the following values: For $f \in C^5[0,1]$ we have the following expansions

$$\begin{cases} f(x_{2i+2}) = f(x_{2i}) + 2hf'(x_{2i}) + 2h^2 f''(x_{2i}) + \frac{4}{3}h^3 f'''(x_{2i}) + \frac{2}{3}h^4 f^{(4)}(x_{2i}) + \frac{4}{15}h^5 f^{(5)}(\lambda_{1,2i}), \\ \qquad x_{2i} < \lambda_{1,2i} < x_{2i+2} \\ f(x_{2i-2}) = f(x_{2i}) - 2hf'(x_{2i}) + 2h''(x_{2i}) - \frac{4}{3}h^3 f'''(x_{2i}) + \frac{2}{3}h^4 f^{(4)}(x_{2i}) - \frac{4}{15}h^5 f^{(5)}(\lambda_{2,2i}), \\ \qquad x_{2i-2} < \lambda_{2,2i} < x_{2i} \\ f(t_{2i-2}) = f(x_{2i}) - \frac{4}{3}hf'(x_{2i}) + \frac{8}{9}h^2 f''(x_{2i}) - \frac{32}{81}h^3 f'''(x_{2i}) + \frac{32}{243}h^4 f^{(4)}(x_{2i}) - \frac{128}{3645}h^5 f^{(5)}(\lambda_{4,2i}), \\ \qquad t_{2i-2} < \lambda_{4,2i} < x_{2i} \\ f(t_{2i}) = f(x_{2i}) + \frac{2}{3}hf'(x_{2i}) + \frac{2}{9}h^2 f''(x_{2i}) + \frac{4}{81}h^3 f'''(x_{2i}) + \frac{2}{243}h^4 f^{(4)}(x_{2i}) + \frac{4}{3645}h^5 f^{(5)}(\lambda_{3,2i}), \\ \qquad x_{2i} < \lambda_{3,2i} < t_{2i} \qquad \ldots(2.2) \\ f'(t_{2i}) = f'(x_{2i}) + \frac{2}{3}hf''(x_{2i}) + \frac{2}{9}h^2 f'''(x_{2i}) + \frac{4}{81}h^3 f^{(4)}(x_{2i}) + \frac{2}{243}h^4 f^{(5)}(\lambda_{5,2i}), \quad x_{2i} < \lambda_{5,2i} < t_{2i} \\ f''(t_{2i}) = f''(x_{2i}) + \frac{2}{3}hf'''(x_{2i}) + \frac{2}{9}h^2 f^{(4)}(x_{2i}) + \frac{4}{81}h^3 f^{(5)}(\lambda_{7,2i}), \quad x_{2i} < \lambda_{7,2i} < t_{2i} \\ f'''(t_{2i}) = f'''(x_{2i}) + \frac{2}{3}h f^{(4)}(x_{2i}) + \frac{2}{9}h^2 f^{(5)}(\lambda_{10,2i}), \quad x_{2i} < \lambda_{10,2i} < t_{2i} \\ f^{(4)}(t_{2i}) = f^{(4)}(x_{2i}) + \frac{2}{3}hf^{(5)}(\lambda_{11,2i}), \quad x_{2i} < \lambda_{11,2i} < t_{2i} \\ f'''(x_{2i+2}) = f'''(x_{2i}) + 2hf^{(4)}(x_{2i}) + 2h^2 f^{(5)}(\lambda_{8,2i}), \quad x_{2i} < \lambda_{8,2i} < t_{2i} \\ f''(t_{2i-2}) = f''(x_{2i}) - \frac{4}{3}hf'''(x_{2i}) + \frac{8}{9}h^2 f^{(4)}(x_{2i}) - \frac{32}{81}h^3 f^{(5)}(\lambda_{6,2i}), \quad t_{2i-2} < \lambda_{6,2i} < x_{2i} \\ f'''(x_{2i-2}) = f'''(x_{2i}) - 2hf'''(x_{2i}) + 2hf^{(4)}(x_{2i}) + 2h^2 f^{(5)}(\lambda_{9,2i}), \quad x_{2i-2} < \lambda_{9,2i} < x_{2i} \end{cases}$$

Proof of Theorem 1.1

The proof depends on the following representation of $S_n(x)$ *for* $2ih \le x \le (2i+2)h$, $i=0,1,\ldots,m-1$, we have

$$S_n(x) = f(x_{2i})A_0(\frac{x-2ih}{2h}) + f(t_{2i})A_1(\frac{x-2ih}{2h}) + f(x_{2i+2})A_2(\frac{x-2ih}{2h}) +$$
$$+4h^2 f''(t_{2i})A_3(\frac{x-2ih}{2h}) + 8h^3 S_n'''(x_{2i})A_4(\frac{x-2ih}{2h}) + 8h^3 S_n'''(x_{2i+2})A_5(\frac{x-2ih}{2h}) \qquad \ldots(3.0)$$

On using (3.0) and conditions

$$S_n'''(0) = f'''(0)\,,\ S_n'''(1) = f'''(1). \qquad \ldots(3.1)$$

We see that $S_n(x)$ as given by (3.0) satisfies(1.0)and is quantic in$[x_{2i}, x_{2i+2}]$, i=0,1,…,m-1. We also need to show that whether it is possible to determine $S_n'''(x_{2i})\ , i = 1,2,\ldots,m-1$ uniquely. For this purpose we use the fact that $S_n^{(4)}(x_{2i+}) = S_n^{(4)}(x_{2i-})\ , i = 1, 2, \ldots, m-1$; with the help of (3.0) and (3.1) reduce to (3.2) $\frac{1}{3}h^3 S_n'''(x_{2i-2}) + 3h^3 S_n'''(x_{2i}) + \frac{2}{3}h^3 S_n'''(x_{2i+2}) =$

$$\frac{81}{4}f(x_{2i}) - \frac{81}{4}f(t_{2i}) + \frac{27}{4}f(x_{2i+2}) - \frac{81}{4}f(t_{2i-2}) + \frac{27}{2}f(x_{2i-2}) - 9h^2 f''(t_{2i-2}) - 9h^2 f''(t_{2i}) \qquad \ldots(3.2)$$

$for\ ,\ i = 1, 2, \ldots, m-1.$

But (3.2) is a strictly tri diagonal dominant system which has a unique solution (Kincaid&Cheney,2002) 177p. Thus $S_n'''(x_{2i})$, i=1, 2…, m-1 can be obtained uniquely by the system (3.2) which establishes Theorem 1.1.

Estimates

In order to prove the Theorem 1.2 we needs the following

Lemma 4.0:

Let us write $E_{2i} = |S_n'''(x_{2i}) - f'''(x_{2i})|$, then for $f \in C^5[0,1]$, we have

(4.0) max $E_{2i} \le \frac{91}{30} h^2 w(f^{(5)}; \frac{1}{m})$,*for i=1,2,…,m-1* .

Proof:

Form (3.2) we have

$$\frac{1}{3}h^3(S_n'''(x_{2i-2}) - f'''(x_{2i-2})) + 3h^3(S_n'''(x_{2i}) - f'''(x_{2i})) + \frac{2}{3}h^3(S_n'''(x_{2i+2}) - f'''(x_{2i+2})) =$$
$$\frac{81}{4}f(x_{2i}) - \frac{81}{4}f(t_{2i}) + \frac{27}{4}f(x_{2i+2}) - \frac{81}{4}f(t_{2i-2}) + \frac{27}{2}f(x_{2i-2}) - 9h^2 f''(t_{2i-2}) - 9h^2 f''(t_{2i}) - \frac{h^3}{3}f'''(x_{2i-2}) - 3h^3 f'''(x_{2i})$$
$$-\frac{2}{3}h^3 f'''(x_{2i+2}) = -\frac{18}{5}h^5 f^{(5)}(\lambda_{2,2i}) + \frac{32}{45}h^5 f^{(5)}(\lambda_{4,2i}) - \frac{1}{45}h^5 f^{(5)}(\lambda_{3,2i}) + \frac{9}{5}h^5 f^{(5)}(\lambda_{1,2i}) + \frac{32}{9}h^5 f^{(5)}(\lambda_{6,2i}) - \frac{4}{9}h^5 f^{(5)}(\lambda_{7,2i})$$
$$-\frac{2}{3}h^5 f^{(5)}(\lambda_{9,2i}) - \frac{4}{3}h^5 f^{(5)}(\lambda_{8,2i}) = \frac{91}{15}h^5 \alpha_1 w(f^{(5)}; \frac{1}{m}),\ \ |\alpha_1| \le 1$$

The result (4.0) follows on using the property of diagonal dominant (Kincaid & Cheney, 2002).

Lemma 4.1 :

Let $f \in C^5[0.1]$ then

(i) $\left|S_n^{(4)}(x_{2i+}) - f^{(4)}(x_{2i})\right| \le \frac{106}{15} h w(f^{(5)};\frac{1}{m})$,

(ii) $\left|S_n^{(4)}(x_{2i-}) - f^{(4)}(x_{2i})\right| \le \frac{31}{3} h w(f^{(5)};\frac{1}{m})$, ,

(iii) $\left|S_n^{(4)}(t_{2i}) - f^{(4)}(t_{2i})\right| \le \frac{95}{27} h w(f^{(5)};\frac{1}{m})$,

(iv) $\left|S_n^{(3)}(t_{2i}) - f^{(3)}(t_{2i})\right| \le \frac{1831}{810} h^2 w(f^{(5)};\frac{1}{m})$,

(v) $\left|S_n'(t_{2i}) - f'(t_{2i})\right| \le \frac{3589}{18225} h^4 w(f^{(5)};\frac{1}{m})$,

Proof:

Form (3.0) we have

$$h^4 S_n^{(4)}(x_{2i+}) = \frac{27}{2} f(x_{2i}) - \frac{81}{4} f(t_{2i}) + \frac{27}{4} f(x_{2i+2}) - 9h^2 f''(t_{2i}) - \frac{4}{3} h^3 S_n'''(x_{2i}) - \frac{2}{3} h^3 S_n'''(x_{2i-2})$$

Hence

$$h^4(S_n^{(4)}(x_{2i+}) - f^{(4)}(x_{2i})) = -\frac{1}{45} h^5 f^{(5)}(\lambda_{3,2i}) + \frac{81}{45} h^5 f^{(5)}(\lambda_{1,2i}) - \frac{20}{45} h^5 f^{(5)}(\lambda_{7,2i}) - \frac{60}{45} h^5 f^{(5)}(\lambda_{8,2i}) - \frac{4}{3} h^3 (S_n'''(x_{2i}) -$$
$$f'''(x_{2i})) - \frac{2}{3} h^3 (S_n'''(x_{2i-2}) - f'''(x_{2i+2})) = \frac{81}{45} h^5 \alpha_2 w(f^{(5)};\frac{1}{m}) - \frac{4}{3} h^3 (S_n'''(x_{2i}) - f'''(x_{2i}))$$
$$-\frac{2}{3} h^3 (S_n'''(x_{2i-2}) - f'''(x_{2i+2})),\ |\alpha_2| \le 1$$

By using (4.0),the Lemma4.1(i) follows, the proof of the Lemma 4.1(ii-v) are similar ,we only mention that

$$h^4 S_n^{(4)}(x_{2i-}) = -\frac{27}{2} f(x_{2i}) + \frac{81}{4} f(t_{2i-2}) - \frac{27}{4} f(x_{2i-2}) + 9h^2 f''(t_{2i-2}) + \frac{1}{3} h^3 S_n'''(x_{2i-2}) + \frac{5}{3} h^3 S_n'''(x_{2i}),$$

$$h^4 S_n^{(4)}(t_{2i}) = \frac{9}{2} f(x_{2i}) - \frac{27}{4} f(t_{2i}) + \frac{9}{4} f(x_{2i+2}) - 3h^2 f''(t_{2i}) - \frac{7}{9} h^3 S_n'''(x_{2i}) + \frac{1}{9} h^3 S_n'''(x_{2i-2}),$$

$$h^3 S_n^{(3(}(t_{2i}) = 6f(x_{2i}) - 9f(t_{2i}) + 3f(x_{2i+2}) - 4h^2 f''(t_{2i}) + \frac{8}{27} h^3 S_n'''(x_{2i}) - \frac{5}{27} h^3 S_n'''(x_{2i-2}),$$

and

$$hS_n'(t_{2i}) = -\frac{28}{15} f(x_{2i}) + \frac{41}{20} f(t_{2i}) - \frac{11}{60} f(x_{2i+2}) + \frac{26}{45} h^2 f''(t_{2i}) - \frac{8}{243} h^3 S_n'''(x_{2i}) + -\frac{16}{1215} h^3 S_n'''(x_{2i-2}).$$

<u>Proof of Theorem 1.2.</u>

For $0 \le z \le 1$, we have

$A_0(z)+A_1(z)+A_2(z)=1$ …(5.0)

Let $x_{2i} \le x \le x_{2i+2}$. On using (5.0) and (3.1) , we have obtain (5.1)

$$S_n^{(4)}(x) - f^{(4)}(x) = (S_n^{(4)}(x_{2i+}) - f^{(4)}(x)) A_0(\frac{x-2ih}{2h}) + (S_n^{(4)}(x_{2i+2}) -$$
$$f^{(4)}(x))A_2(\frac{x-2ih}{2h}) + (S_n^{(4)}(t_{2i}) - f^{(4)}(x)A_1(\frac{x-2ih}{2h}) = L_1 + L_2 + L_3$$

From (2.1) it follows that:

$|A_0(x)| \le 1$, $|A_1(x)| \le 1$ *and* $|A_2(x)| \le 1$

Since $f^{(4)}(x) = f^{(4)}(x_{2i}) + (x - x_{2i}) f^{(4)}(\lambda)$,

$x_{2i} < \lambda < x,$

Therefore $L_1 = (S_n^{(4)}(x_{2i+}) - f^{(4)}(x)) A_0(\frac{x-2ih}{2h})$

$= (S_n^{(4)}(x_{2i}) - f^{(4)}(x_{2i}) - (x - x_{2i}) f^{(4)}(\lambda)) A_0(\frac{x-2ih}{2h}).$

On using Lemma4.1 (ii) and $|x - x_{2i}| \le 2h$, we obtain

$$|L_1| \le \frac{106}{15} h w(f^{(5)}; \frac{1}{m}) + 2h \|f^{(5)}\| \qquad \dots(5.2)$$

Similarly,

$$|L_2| \le \frac{31}{3} h w(f^{(5)}; \frac{1}{m}) + 2h \|f^{(5)}\|, \qquad \dots(5.3)$$

and

$$L_3 = (S_n^{(4)}(t_{2i}) - f^{(4)}(x) A_1(\frac{x-2ih}{2h})$$

$$= (S_n^{(4)}(t_{2i}) - f^{(4)}(t_{2i}) + f^{(4)}(t_{2i}) - f^{(4)}(x_{2i}) - (x - x_{2i}) f^{(5)}(\lambda)). \qquad \dots(5.3)$$

Form (2.2) we obtain

$$f^{(4)}(t_{2i}) - f^{(4)}(X_{2i}) = \frac{2}{3} h f^{(5)}(\lambda_{10,2i})$$

Use this result and Lemma4.1 (iii) in (5.3) and we obtain

$$|L_3| \le \frac{149}{27} h w(f^{(5)}; \frac{1}{m}) \cdot \qquad \dots(5.4)$$

Putting (5.2) – (5.4) in (5.1) we obtain

$$|S_n^{(4)}(x) - f^{(4)}(x)| \le \frac{3094}{135} h w(f^{(5)}; \frac{1}{m}) + 4h \|f^{(5)}\|. \qquad \dots(5.5)$$

This proves (1.2) for r=4. To prove (1.2) for $r=3$,

since $S_n^{(3)}(x) - f^{(3)}(x) = \int_{t_{2i}}^{x} (S_n^{(4)}(t) - f^{(4)}(t)) dt + S_n^{(3)}(t_{2i}) - f^{(3)}(t_{2i}).$ $\dots(5.6)$

On using Lemma 4.1 (i) and (5.6) we obtain

$$|S_n^{(3)}(x) - f^{(3)}(x)| \le \frac{38959}{810} h^2 w(f^{(5)}; \frac{1}{m}) + 8h^2 \|f^{(5)}\|.$$

Which prove (1.2) for r=3, The proof of (1.2) for $r= 2$,since

$$S_n''(x) - f''(x) = \int_{t_{2i}}^{x} (S_n^{(3)}(t) - f^{(3)}(t)) dt + S_n''(t_{2i}) - f''(t_{2i}).$$

and $S_n''(t_{2i}) - f''(t_{2i}) = 0$ .Thus

$$|S_n''(x) - f''(x)| \le \frac{38959}{405} h^3 w(f^{(5)}; \frac{1}{m}) + 16h^3 \|f^{(5)}\|.$$

Which prove (1.2) for r=2.

The proof of (1.2) for $r= 1$, since

$$S_n'(x) - f'(x) = \int_{t_{2i}}^{x} (S_n^{(2)}(t) - f^{(2)}(t)) dt + S_n'(t_{2i}) - f'(t_{2i})$$

On using the result of this theorem for *r=2* and Lemma 4.1(iv) we obtain $|S_n'(x)-f'(x)|\le \frac{3509944}{18225}h^4 w(f^{(5)};\frac{1}{m})+32h^4\|f^{(5)}\|$. This proves (1.2) for *r=1*.To prove for *r=0*, since $S_n(x)-f(x)=\int_{t_{2i}}^{x}(S_n'(t)-f'(t))\,dt+S_n(t_{2i})-f(t_{2i})$

And clearly $S_n(t_{2i})-f(t_{2i})=0$ then

$$|S_n(x)-f(x)|\le \frac{7019888}{18225}h^5 w(f^{(5)};\frac{1}{m})+64h^5\|f^{(5)}\|.$$

since 2mh=1 then h=(1/2m) put it in above result we get:

$|S_n(x)-f(x)|\le 12.036844993\,m^{-5} w(f^{(5)};\frac{1}{m})+2m^{-5}\|f^{(5)}\|$. This completes the proof of Theorem1.2.

Conclusion

In this paper we conclude that some times change of the boundary conditions and the class of spline function affect on minimizing error bounds in the subject of lacunary interpolation by spline functions.

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تأثير تغير الشروط الحدودية لفئات داله السبلاين في تصغير حدود الأخطاء للبيانات الاندراجية للحالة(٠ ،٢)

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الخلاصة

في هذا البحث تم تغير الشروط الحدوديه لفئات داله السبلاين كما في المصدر (Varma, 1973) وتغيــر المشتقه الاولى الى و المشتقه الثالثه . وقد لاحظنا تاثير تغير الشروط الحدوديه لفئات دالة السبلاين في تصغير او تقليل حدود الاخطاء للبيانات الاندراجيه باستخدام دالة السبلاين .