------ Raf. Jour. Sci., Vol. 16, No. 1 Physics, Special Issue, pp. 22-27, 2005 ------

# The Effective Atomic Number of Tissue Equivalent Materials

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(Received 24/8/2003 ; Accepted 22/9/2003)

## ABSTRACT

The photoelectric cross-section for the tissue equivalent materials is in the first place dependent on the effective atomic number calculated on the basis of the exponent m, where m varies with photon energy and the elements involved.

In the present work, a graphical method was used to evaluate the effective atomic number $Z_{eff}$ for any material at any photon energy independent on the value of the parameter m. The results are in good agreement with the calculated values using the known empirical formula.

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# العدد الذري المؤثر للمواد المكافئة للأنسجة

## الملخص

يعتمد المقطع العرضي للظاهرة الكهروضوئية للمواد المكافئة للنسيج بالدرجة الأولى على العدد الذري المؤثر $Z_{eff}$ المحسوب تبعاً للأس m، حيث أن الأس m يختلف باختلاف كل من طاقة الفوتون والمواد الداخلة في تركيب المادة.

لقد تم في هذا البحث استخدام طريقة بيانية يمكن بواسطتها حساب العدد الذري المؤثر لأية مادة عند أية طاقة وبدون الاعتماد على قيمة m. أعطت النتائج توافقاً جيداً مع القيم المحسوبة بالطريقة التجريبية.

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## INTRODUCTION

In dealing with a compound or a mixture of molecules, it is sometimes convenient to describe the mixture by an effective atomic number $Z_{eff}$. The energy region of interest in discussing the effective atomic number is then from about 30 keV to 80 keV, where the photoelectric process is dominant over the compton process. The photoelectric coefficient per atom depends upon $Z^m$ where m varies with photon energy.

The values of the exponent m (per atom) varies from 3.94 (Spiers, 1946) to 4.1 (Hine, 1952). Weber and Van den Berge (1969) calculated that m to be equal to 4.4, also 4.4 have been chosen by Cho et al. (1975) and 4.8 by McCullaugh (1975), 4.6 by White (1977). More recently Denison et al. (1997) suggested 4.8 as a value for m. Finally RyZhikov et al. (2002) suggested value of m equal to 4.

In this study, a graphical method was used to evaluate the effective atomic number, $Z_{eff}$, for any material at any photon energy independent on the value of the parameter m. A comparison of $Z_{eff}$ obtained by the graphical method with the calculated values using the known empirical formula was also made in this work.

**The Calculation of the Exponent:**

The variation of the cross-section of the photoelectric effect per atom, $\tau$, based on the values tabulated by Hubbell (1969) and Storm and Israel (1970), with atomic number Z was analyzed using the empirical formula (Brunner, 1986):

$\tau = \kappa Z^m$

$\kappa$ is constant. The exponent m was calculated by applying the least square method on log $\tau$ and log Z for the elements found in most biological tissuse H, C, N, O, Na, Mg, P, S, Cl, K, Ca, Fe and Zn as in Fig. (1). For energies between 30 and 80 keV the results obtained for the value of the exponent m vary between 4.7 and 4.9 as shown in Fig. (2).

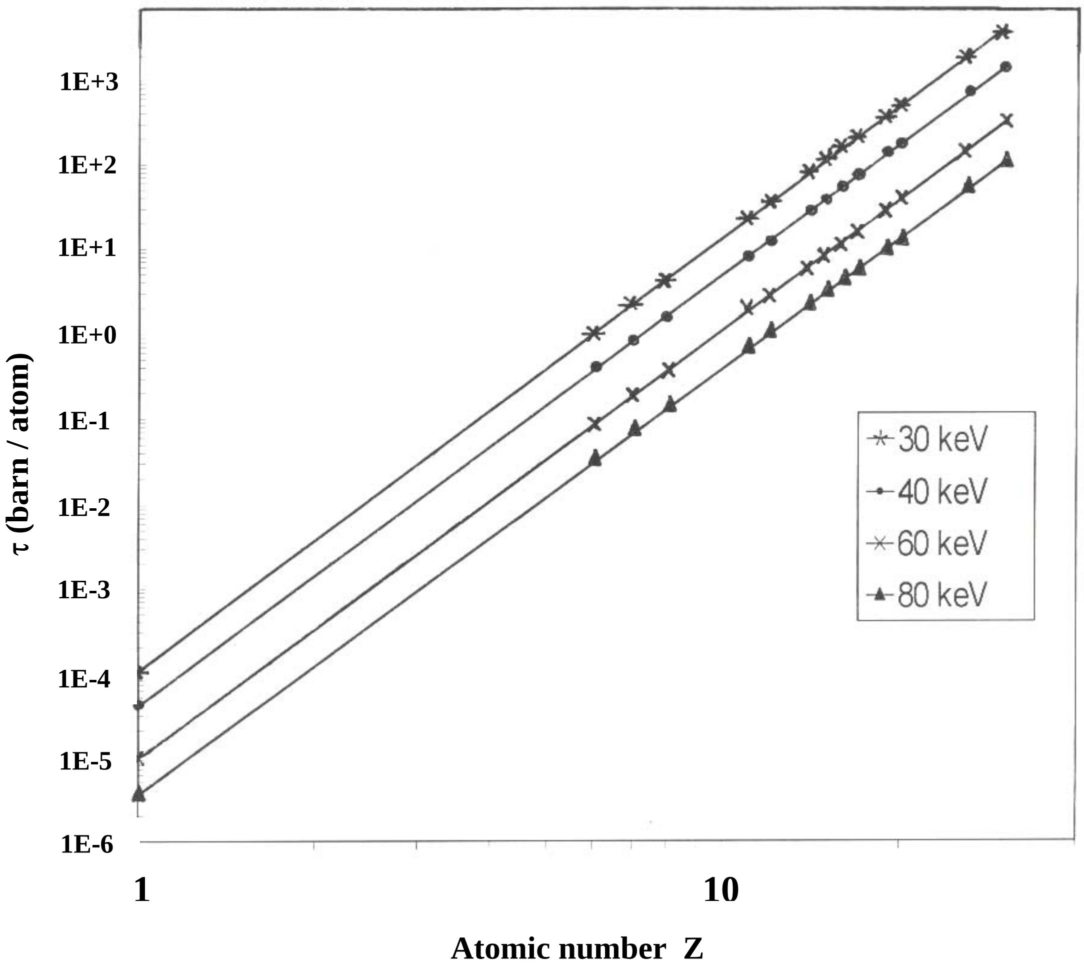


Fig. 1: Cross-section for photoelectric effect $\tau$ as a function of atomic number Z

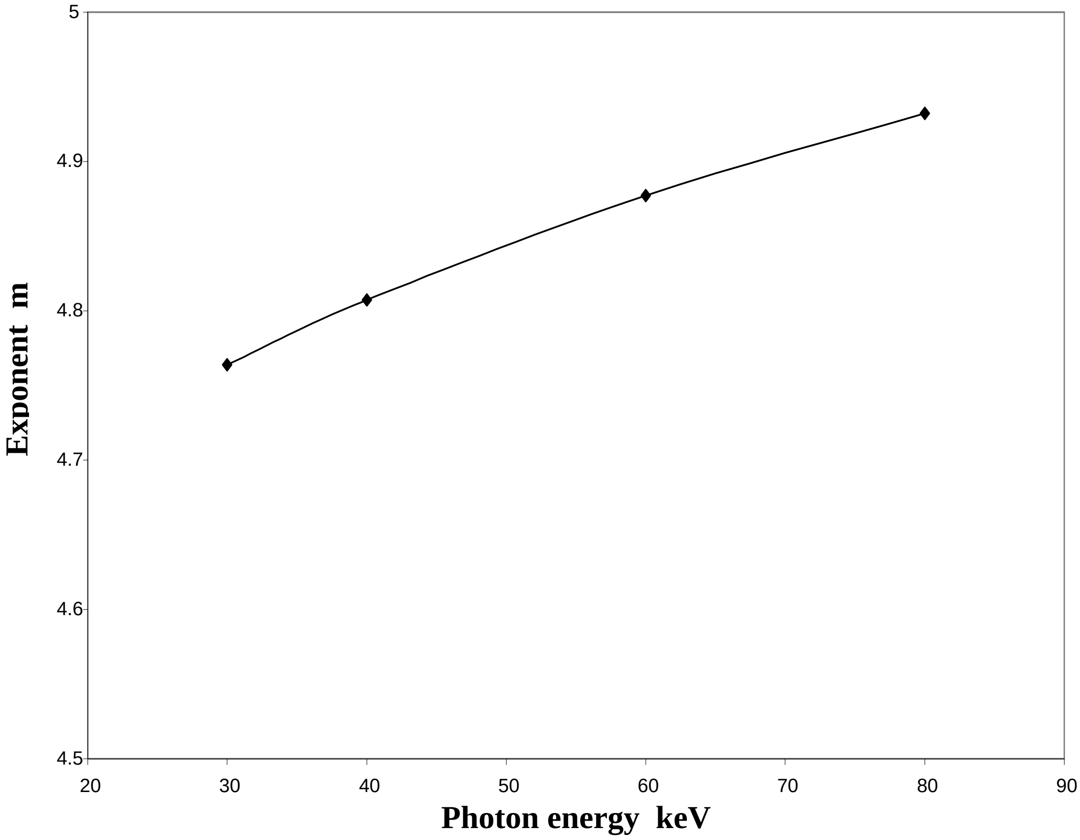


Fig. 2: The variation of the exponent m with photon energy.

**Calculation of $Z_{eff}$ :**

The effective atomic number for a material of known composition was calculated according to equation given by Cho et al. 1975 as:

$$Z_{eff} = \left(\sum_i a_i Z_i^{m}\right)^{1/m} \tag{1}$$

Where $a_i$ is the relative electron fraction of element $Z_i$ given by:

$$a_i = \frac{N_a P_i Z_i}{n_o A_i} \tag{2}$$

And $n_o$ , the electron density (the total number of electrons per gram), is given by:

$$n_o = N_a \sum_i \left( P_i Z_i / A_i \right) \tag{3}$$

$N_a$ is Avogadro's number, $P_i$ is the percentage weight of element i, $A_i$ and $Z_i$ are the atomic weight and atomic number respectively of element i.

For example, $n_o$ for perspex ($C_5H_8O_2$) was calculated according to equation (3) as follows:

$$n_o = N_a \left[ \frac{P_c Z_c}{A_c} + \frac{P_H Z_H}{A_H} + \frac{P_o Z_o}{A_o} \right] \tag{4}$$

The value of $n_o$ was substituted in equation (2) to get the value of $a_i$ for each element (C, H and O).

Finally we calculated $Z_{eff}$ according to equation (1) by using the new values for the exponent m obtained in this work as follows:

$$Z_{eff} = \left[a_c Z_c{}^m + a_H Z_H{}^m + a_o Z_o{}^m\right]^{1/m}$$

The same calculation was done for Muscle.

**Graphical Method for Evaluating $Z_{eff}$ :**

The energy absorption coefficient per electron were plotted versus the atomic number Z using data derived from Hubbell (1982) Fig. (3).

As the most biological materials have effective atomic number, for all photon processes, between 5 and 13 (White, 1977), Boron to Aluminum were chosen to represent this Z-limit.

The elemental composition of tissue and tissue equivalent materials such as muscle and Perspex are given in Table 1. Using these and tables of Hubbell (1982) and by using the Bragg additivity rule for mixing elements which is based on the idea that radiation interacts with atoms individually, and that the atoms do not influence each others interaction probability (Mahrok et al. 1994), we can calculate the energy absorption coefficient of a gram of the mixture and dividing this by the number of electrons per gram to give the energy absorption coefficient in $cm^2$ per electron for the mixture. These are represented by the horizontal line in Fig. (3).

At 30 keV, for example, muscle has an energy absorption coefficient of $0.4768 \times 10^{-24}$ $cm^2$ per electron. This coefficient is represented by point P and shows that muscle at 30 keV has an effective atomic number of 7.62. Similarly at 60 keV, the location of $\bar{P}$ yields an effective atomic number for muscle of 7.67. For Perspex the effective atomic numbers at these two energies are 6.53 and 6.6 respectively. By this procedure one can evaluate $Z_{eff}$ for any material at any photon energy independent on the value of the exponent m.

Elementary composition of tissue and tissue equivalent material (fraction by weight). (Jassim et al. 1993).

| **Elements** | **Muscle** | **Perspex $C_5H_8O_2$** |
|---|---|---|
| H | 0.100637 | 0.080538 |
| C | 0.107830 | 0.599848 |
| N | 0.027680 | |
| O | 0.754773 | 0.319614 |
| Na | 0.000750 | |
| Mg | 0.000190 | |
| P | 0.001800 | |
| S | 0.002410 | |
| Cl | 0.000790 | |
| K | 0.003020 | |
| Ca | 0.000030 | |
| Fe | 0.000040 | |
| Zn | 0.000050 | |

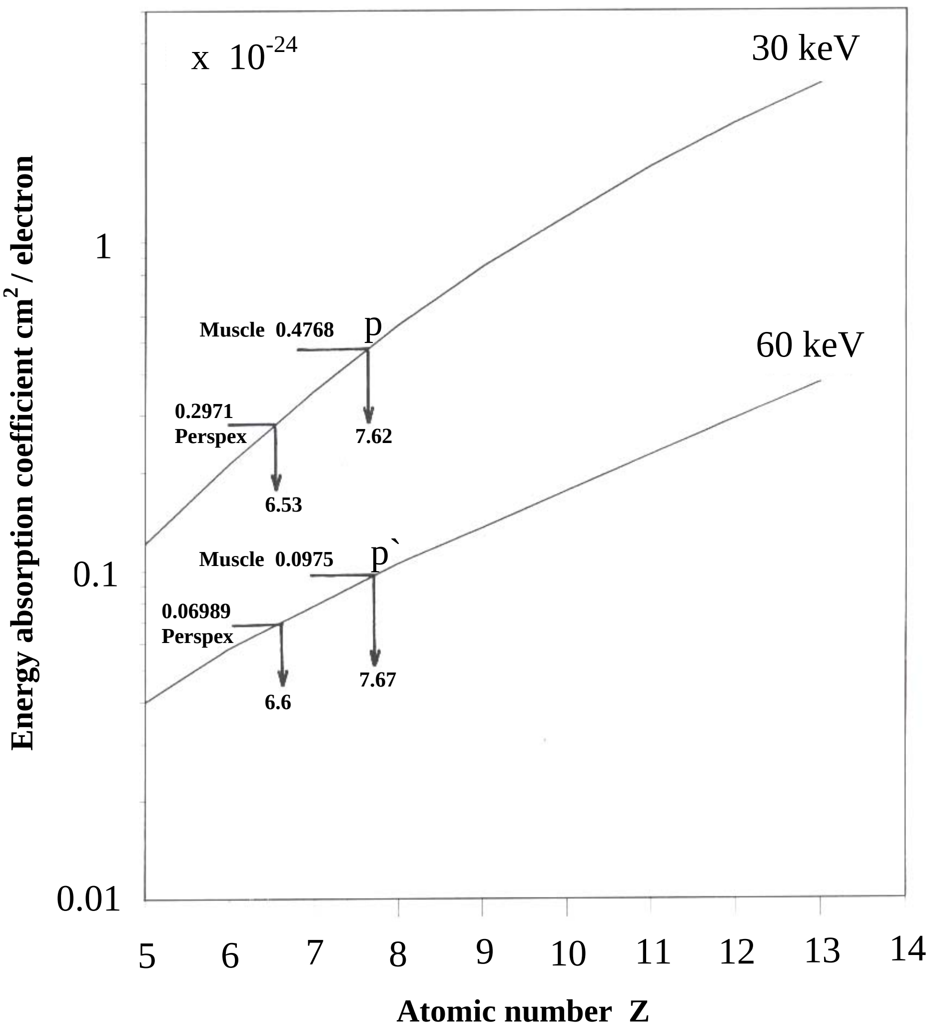


Fig. 3: Plot of energy absorption coefficient in $cm^2$ per electron as a function of atomic number Z for 30 and 60 keV photons.

## RESULTS AND DISCUSSION

The strong dependence of the photoelectric cross-section upon the photon energy is evident in Fig. (1).

The values of the exponent m for photoelectric cross-section (per atom) for a range of photon energy are summarized in Table (2). The results indicated that the exponent m depends upon the photon energy Fig. (2).

Table (2) also shows the variation in the effective atomic number for muscle and Perspex based upon the exponent m and by the graphical method. The graphical method shows good agreement with the calculated $Z_{eff}$ by using the empirical formula with deviation less than 1%*.

The graphical method showed that $Z_{eff}$ is independent on photon energy and so is a meaningful concept.

We have chosen to use the energy absorption coefficient in Fig. (3) because it is the most useful one in dealing with dosimetry problem.

Table 2: Shows the values of $Z_{eff}$ of Muscle and Perspex at different energies.

| Photon Energy keV | Value of m (per atom) present work | Effective atomic number, $Z_{eff}$ | | | |
|---|---|---|---|---|---|
| | | Muscle | | Perspex | |
| | | Calculated (empirical formul | Graphical (present work) | Calculated pirical formula) | Graphical resent work) |
| 30 | 4.7639 | 7.6624 | 7.62 | 6.5996 | 6.53 |
| 40 | 4.8072 | | | | |
| 60 | 4.8771 | 7.6903 | 7.67 | 6.6122 | 6.6 |
| 80 | 4.9321 | | | | |

$$* \; Percentage \;\; deviation = \left( \frac{Z_{eff\ calc.} - Z_{eff\ Graph.}}{Z_{eff\ calc.}} \right) X100$$

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