Influence Of Wall Axial Heat Conduction On The Forced Convection Heat Transfer In Rectangular Channels

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Abstract:

In this paper the conjugate heat transfer in rectangular channel is numerically investigated, where the effect of both axial heat conduction and entrance region on the internal forced convection in rectangular channels are studied. With decreasing the dimensions of channels the thickness of walls become large and in order of the channels dimensions as in microchannels. As a results the heat conduction in the walls especially in the axial direction can not be ignored, since it lead to decrease in the efficiency of heat transfer process. Also the effect of entrance region is taken into consideration where the flow is assumed developing hydro dynamically and thermally. A finite volume method is used to numerically solve the conjugate heat transfer in both the fluid and wall simultaneously. The results obtained shows that the existing of axial heat conduction lead to reduction in the heat transfer and it's effect increased with increasing the thickness of walls and Reynolds number.

In this paper a correlation has been developed to calculate the value of axial heat conduction in channel's walls based on most of the affecting parameters. This correlation can be used accurately to compute the value of axial conduction in rectangular channels.

تأثير التوصيل الحراري المحوري في الجدار على انتقال الحرارة بالحمل القسري خلال القنوات المستطيلة

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الخلاصة:

في هذا البحث تم دراسة انتقال الحرارة المترافقة عدديا حيث تم دراسة تأثير كل من التوصيل الحراري المحوري ومنطقة الدخول على انتقال الحرارة بالحمل القسري خلال القنوات المستطيلة. مع نقصان قطر القناة كما في القنوات المايكروية فأن سمك الجدار يصبح كبير مقارنة بأبعاد القناة وكنتيجة لذلك فان التوصل المحوري خلال الجدران خاصة بالاتجاه المحوري (اتجاه الجريان) لا يمكن إهماله حيث انه يودي إلى تقليل كفاءة عملية انتقال الحرارة.

كذلك فأن تأثير منطقة الدخول قد اخذ بنظر الاعتبار خلال البحث حيث أن الجريان المدروس هو جريان متطور هيدرو ديناميكيا وحراريا. النتائج التي تم التوصل إليها تشير إلى أن وجود التوصيل الحراري المحوري يودي إلى تقليل عملية انتقال الحرارة وان تأثيره يزداد بزيادة سمك الجدران وزيادة عدد رينولد. تم أيضا" في هذا البحث بناء علاقة رياضية لحساب قيمة التوصيل الحراري المحوري بالاعتماد على معظم العوامل المؤثرة فيه. هذه العلاقة يمكن استخدامها بدقة لحساب قيمة التوصيل الحراري المحوري في القنوات المستطيلة.

2. Introduction:

The rapid development of microfabrication enabled the usage of devices having dimensions of micron, such as, micro heat sink, micro biochips, micro reactors, micromotor, micro valve and micro fuel cells which are all called microfluidic devices [1]. These devices have found their applications in micro-electronics, micro scale sensing and measurement, spacecraft thermal control, micro-electro-mechanical-systems (MEMS), and medicals [2 &3]. Most of these applications include fluid flow in microchannels with dimensions in micron size in which the thickness of walls is large and comparable to the channels dimensions. Therefore the heat transfer problem cannot be solved by the conventional methods used for conventional channels. Since the heat transfer in these channels is conjugated process where the heat conducted in the walls as well as in fluid, and the axial heat conduction cannot be ignored. Therefore the energy equation must be solved in both the fluid and solid walls.

There are limited number of researchers in literature studied the axial conduction and its effects.

Tiselj et al (2004) [4] studied experimentally and numerically the heat transfer characteristics of water flowing through triangular silicon microchannel with hydraulic diameter of 160 μm in the range of Re from 3.2 to 64. The numerical analysis was done by using a software package CFX5. It was shown that for thermal boundary condition of constant heat flux subjected on the bottom wall of channels the bulk water temperature, as well as the temperature of the heated wall does not change linearly along the channel. Both water and heated surface temperature do not change monotonously and there are significant changes in the temperature gradient in the flow direction. The non monotonous behavior of fluid and heated wall temperatures is due to high values of axial heat flux which has a maximum value near the inlet and decrease in the flow direction up to zero.

Zhigang et al (2007) [5] studied experimentally the effect of axial conduction on the convective heat transfer in stainless steel micro tube with 168 μm inner diameter. Water and nitrogen gas were used as the working fluids. A correlation between the axial wall heat conduction and the convective heat transfer is obtained by theoretical analysis based on the experimental results. The investigated results clearly show that the axial heat conduction can reduce the convective heat transfer in the stainless steel micro tube by 2.1 % when the working fluid is nitrogen gas, however, the decrement can be neglected for distilled water as the working fluid.

Haji-Sheikh et al (2008) [6] studied the asymptotic variation of wall heat flux values adjacent to the thermal entrance location for parallel plate ducts. The acquired results show interesting variations for different values of the Peclet number near the thermal entrance location.

Wei Guo et al (2009) [7] experimentally investigated the effect of the axial conduction through the pipe wall on the performance of a thermosyphon. They tested two 2-phase closed thermosyphones each had the same dimensions, materials and partially filled with R134a. The thermosyphones were heated by a constant-temperature hot bath and cooled by water via a concentric heat exchanger. They found that the axial conduction through the pipe wall caused an increase in the overall heat transfer coefficient.

In this paper a numerical investigation is made to study the effect of axial heat conduction in solid wall on the heat transfer process in rectangular channels.

3-Mathematical model:

For conventional channels when the wall thickness is very small compared to the dimensions of channel, the heat conduction in the solid wall is negligible and the thermal boundary conditions are applied directly on the fluid as shown in fig. 1 and equations below:

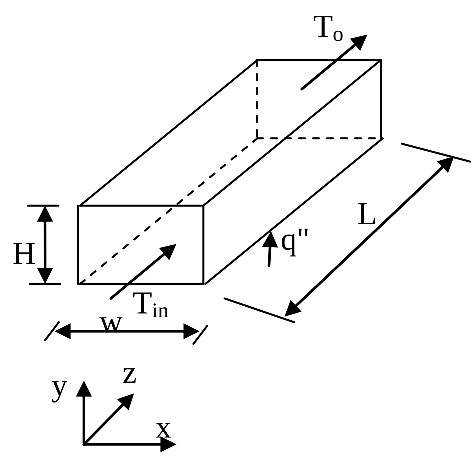

Fig. 1 schematic of conventional channel

$q''=q_1/A_{surface}$ -------(1)

Where q_1 is the heat carried by fluid flow inside the channel.

$$q_1 = \dot{m}\, cp \Delta T \qquad \text{-------(2)}$$

In microchannels where its dimensions are very small, the thickness of channel's walls become large and in order of the channels dimensions as shown in figures 2 and 3 below:

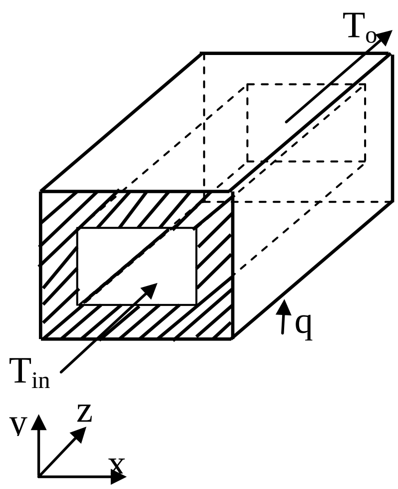

Fig. 2 schematic of microchannel

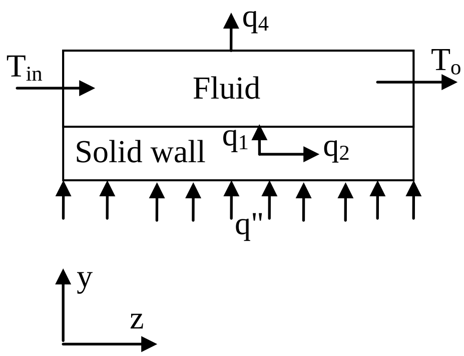

Fig. 3 Two dimensional view of channel

As a result, the heat conduction in the wall cannot be ignored and the heat flux applied on the outer surface of channel cannot be assumed the same for heat received by fluid, since part of heat flux flow in the axial direction as can be seen from fig. 3 and equations below:

$q''=q_1+q_2+q_3+q_4$ ----------(3)

Where q_2 is the axial heat conduction in the longitudinal direction, q_3 is the heat conducted in third direction with x coordinate and q_4 is the heat lost by radiation from channel outer surfaces.

q_3 and q_4 can be neglected due to small value of channel width compared with length and small value of heat lost by radiation. So the equation becomes:

$q''=q_1+q_2$ ---------(4)

In case of considerable wall thickness, the heat transfer is a conjugate process and to do the numerical solution the energy equation must be solved in both the fluid and solid wall zones. The entrance region is taken into consideration since the flow is assumed simultaneously developed (hydrodynamically and thermally).

For steady state, 3-D, incompressible and laminar flow the following equations are solved to calculate the distributions of velocity and temperature throughout the fluid and walls [8]:

Continuity equation:

$$\nabla V = 0 \quad \text{-------(5)}$$

Momentum equation:

$$\rho(V\nabla V) = -\nabla P + \mu \ (\nabla^2 V) \quad \text{--------(6)}$$

Energy equation:

$$\rho C_p (V\nabla T) = k\nabla^2 T \quad \text{--------(7)}$$

For solid wall:

$$k_s \nabla^2 T_s = 0 \qquad \text{--------(8)}$$

Where: V is the velocity vector ($u_i + v_j + w_k$)

The boundary conditions used to complete the model are:

At the channel inlet the finite values of velocity and temperature were used

T = T_{in} , w = w_{in} , u = v = 0

At the channel outlet the flow assumed to be fully developed:

$$\frac{\partial w}{\partial x} = v = u = 0 \ , \ \frac{\partial T}{\partial x} = 0$$

At the lower surface of the channel the constant heat flux boundary condition is used.

$$-k_s \frac{\partial T_s}{\partial y} = q'' \qquad \text{at y=o}$$

The other outer surfaces of the solid model (right, left and top walls) are assumed adiabatic walls:

$$\frac{\partial T_s}{\partial z} = \frac{\partial T_s}{\partial x} = \frac{\partial T_s}{\partial y} = 0$$

All the inside surfaces that separates the solid wall and the fluid regions are used as a coupled wall with conjugate heat transfer:

$$-k\frac{\partial T}{\partial y} = -k_s \frac{\partial T_s}{\partial y} \ \text{and} \ -k\frac{\partial T}{\partial x} = -k_s \frac{\partial T_s}{\partial x}$$

Reynolds number is:

$$\mathrm{Re} = \frac{\rho\, w_i D_h}{\mu} \qquad \text{--------(9)}$$

Where D_h is the hydraulic diameter:

4-Numerical solution:

The above system of equations and boundary conditions are numerically solved using finite volume method. The flow is developing and the heat transfer is conjugated process therefore the 3D continuity and 3D Navier stock equations are solved in flow region while the 3D energy equation is solved in both the fluid and solid wall regions.

Therefore, the equations 5 to 8 are solved simultaneously in two zones (fluid and solid wall).

SIMPLE algorithm is used to solve the problem of velocity-pressure coupling and the upwind scheme is used to discritize the equations [9].

A mesh has been chosen in accepted size since a mesh refinement has been made to find out the appropriate mesh size that gives highly accurate solution.

The convergence criteria used to control the numerical solution for both momentum and energy equations is 10^{-6}.

A software has been used to do the numerical solution.

The fluid used is water with constant properties selected according to mean fluid temperature and the solid wall material is silicon with thermal conductivity of 149 ($W/m^2.K$).

5-Results and discussion:

5.1: Validation:

To check the validity of the built numerical model, verification was made by solving the experimental model presented in [10] and then the results are compared. The experimental model presented in [10] is a microchannel heat sink consists of rectangular microchannels with hydraulic diameter 348.9 μm, channel height 713 μm, channel width 231 μm and length 4.48 cm. Temperature was measured in four points along the channel bottom wall, and experiment was made with inlet velocity of 1 m/s, inlet temperature of 288 K and thermal boundary condition is a constant heat flux of 100 W/cm^2 subjected on the bottom wall of the substrate.

Fig. 4 shows the comparison between results of present numerical model and the experimental data of [10] for temperature distribution along the bottom wall of the channel. From this figure it can be seen that the agreement between numerical and experimental results is accepted since the maximum error is 1.41% which may be due to the end effect. Therefore the present numerical model is reliable and can be accurately used to do the numerical investigation.

5.2. Results:

Fig.5 shows the distribution of lower wall temperature along wall thickness (in y-direction) (average with respect to x) in three locations along channel for Re=100. From this figure it can be seen that the temperature of wall increased from lower surface of constant heat flux to the surface that separate the solid and fluid regions for all selected values of applied heat flux. This mean that the value of temperature reached the fluid is not the same for lower surface i.e. there is a decrement in the temperature of solid wall which means the conduction in thick solid wall cannot be ignored. In addition, it can be seen from the figure that the temperature of solid wall increased with distance along flow direction due to increase the rate of heat transferred.

Fig. 6 represents the distribution of average temperature for wall along thickness (in y-direction) for three values of applied heat flux. From this figure, one can see that as in figure 5, also the average temperature of wall decreased from lower to upper surface of bottom solid wall of the channel for all selected values of heat flux. In addition, it can be seen that the temperature of wall increased with increase the value of subjected heat flux.

Fig. 7 indicates the variation of bulk fluid temperature in flow direction (z-direction) for three values of heat flux. From this figure, the temperature of fluid increased along the channel for all values of heat flux due to heat transfer from bottom wall to the fluid. In addition, the temperature of fluid increased with increase heat flux due to increasing in the amount of heat transferred.

Fig. 8 shows the distribution of bottom wall average temperature in flow direction (z-direction) for three values of heat flux. From this figure, it can be seen that the temperature of lower wall increased along channel due to the existing of axial conduction in the solid wall i.e. accumulation of heat transfer along the channel. As can be seen the wall temperature increased to considerable values which depict the considerable effect of axial heat conduction in the solid wall. Also it can be seen that the temperature increased with increase applied heat flux. In addition, the increment in wall temperature is larger for larger values of heat flux.

In fig. 9 the distribution of axial heat conduction in lower wall ($q_2 = -k_s \dfrac{\partial T_s}{\partial z}$) along z-direction is shown for three values of subjected heat flux. From this figure it can be noted that the sharp increase in axial heat flux occurred in the beginning of the channel due to the existing of entrance region since the entrance region have a valuable effect on the axial conduction, beyond the distance affected by entrance region there is a small increase in axial heat flux with length. Also it can be seen that the axial heat flux increased with increase the subjected constant heat flux. The increment in axial heat conduction also increased with increase the subjected heat flux.

Fig. 10 represent the variation of axial heat conduction value (q_2) with wall thickness for three values of applied constant heat flux for Re =100, L=0.01 m and ks =149 W/m^{2}K. From this figure one can see that the value of axial heat conduction increased with increase wall thickness due increase the cross-sectional area in which the axial heat conduction transferred also due to increasing the conduction resistance for heat transferred to the fluid and as a results increasing the fraction of heat transferred in the longitudinal direction (axial heat conduction). Also as in previous figure, the axial heat conduction increased with increase the applied heat flux.

In fig. 11 the variation of axial heat conduction with channel length is presented for three values of applied heat flux for Re =100, wall thickness t=100 µm, and ks =149 W/m^{2}k. From this figure, it can be seen that, there is a little decrease in axial heat conduction with increasing channel length due to decreasing the effect of entrance region for larger lengths. In general from this figure there is inconsiderable influence of length on he axial heat conduction for small heat flux values and this effect increased with increasing the applied heat flux.

The variation of axial heat conduction value with Reynolds number is indicated in fig. 12 for L = 0.01 m, ks =149 W/m^{2}k, and thickness= 100 µm. From this figure the axial heat conduction increased with increase Reynolds number for all values of applied heat flux due to increase the entrance region and its effect.

Fig. 13 shows the variation of axial heat conduction with wall thermal conductivity (ks) for three values of applied heat flux for L = 0.01 m, Re = 100 and thickness = 100 µm. From this figure, it can be seen that there is a little decrease in axial heat conduction with increasing the conductivity. This is may be due to that, increasing wall thermal conductivity lead to increase the conduction in the up ward direction (y-direction) which lead to decrease the conduction in the longitudinal direction (axial conduction).

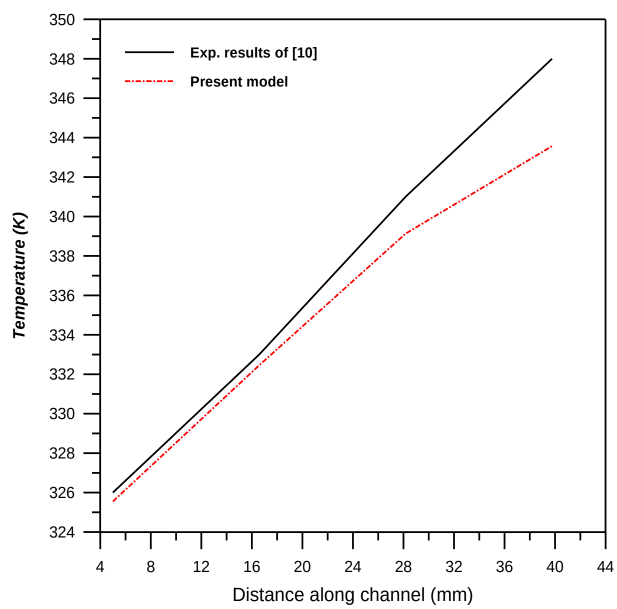

Fig. 4 Comparison between experimental results of [10] and present numerical model

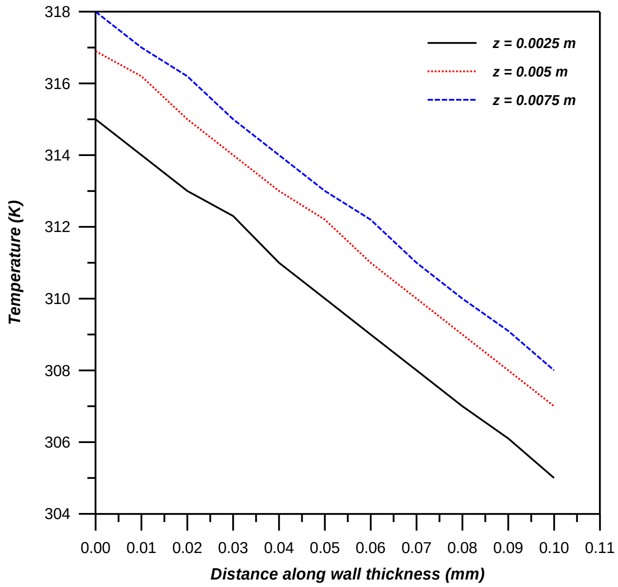

Fig. 5 Distribution of temperature in y-direction in three locations for (q'' = 100000 W/m^2)

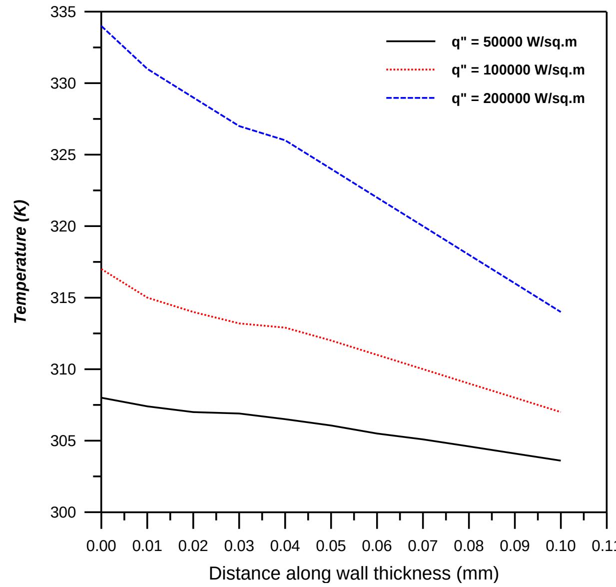

Fig. 6 Distribution of wall average temperature In y-direction for different values of heat flux

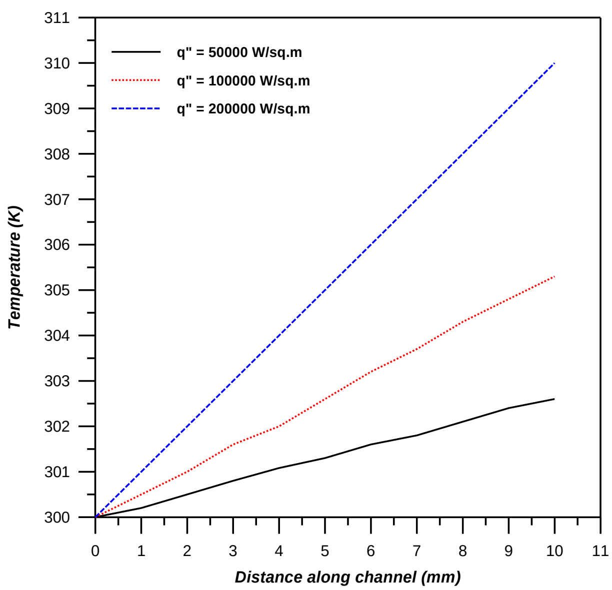

Fig. 7 Variation of fluid temperature along channel for different values of heat flux

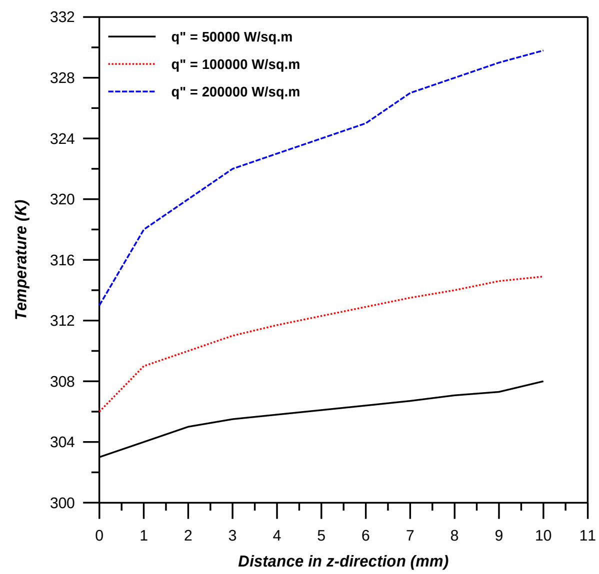

Fig. 8 Distribution of bottom solid wall temperature in flow direction for different values of heat flux

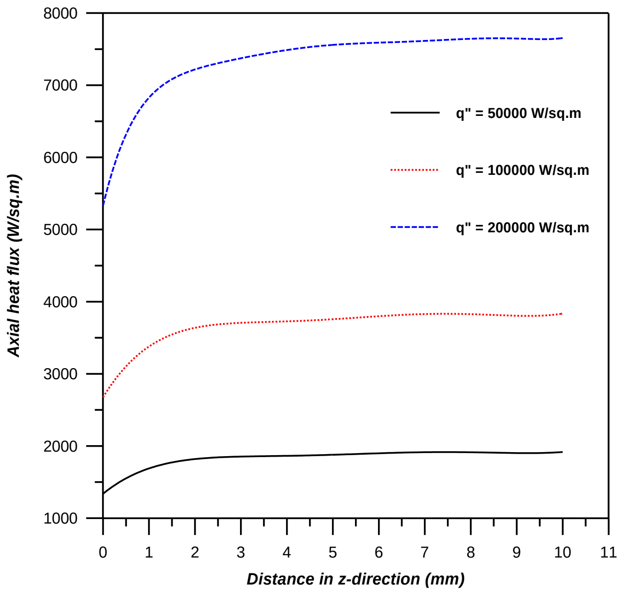

Fig. 9 Distribution of axial heat flux along lower wall for different values of heat flux

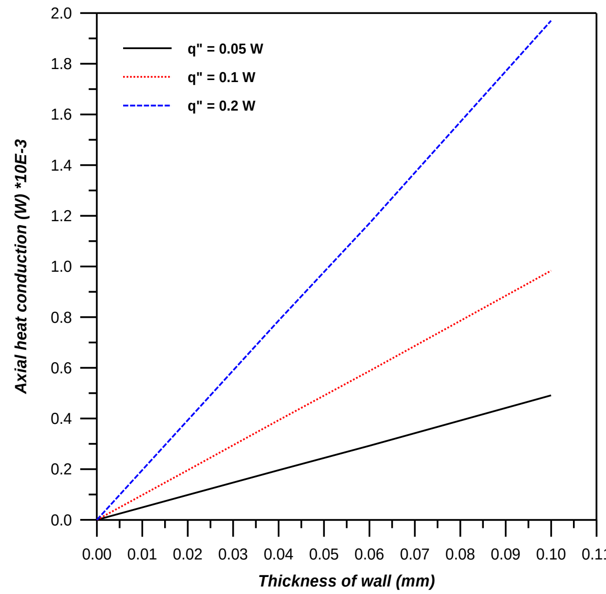

Fig. 10 Variation of axial heat conduction with wall thickness for different values of heat flux

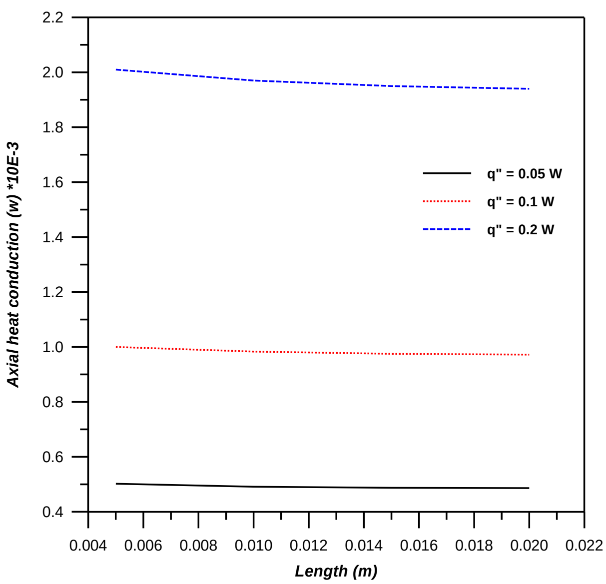

Fig. 11 Variation of axial heat conduction with channel length for different values of heat flux

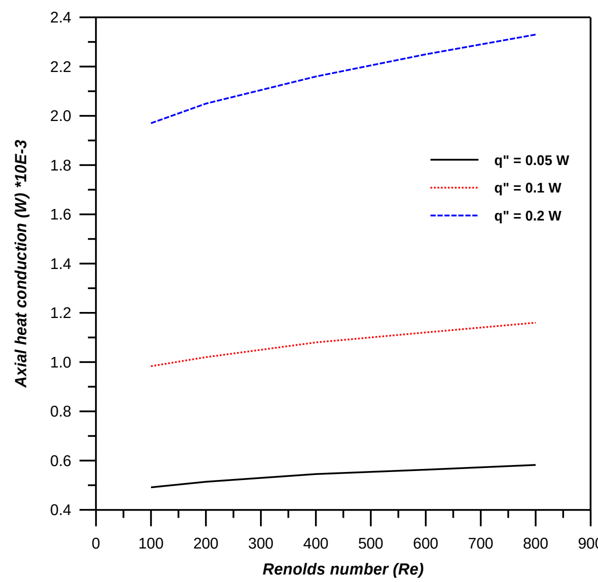

Fig. 12 Variation of axial heat conduction with Re for different values of heat flux

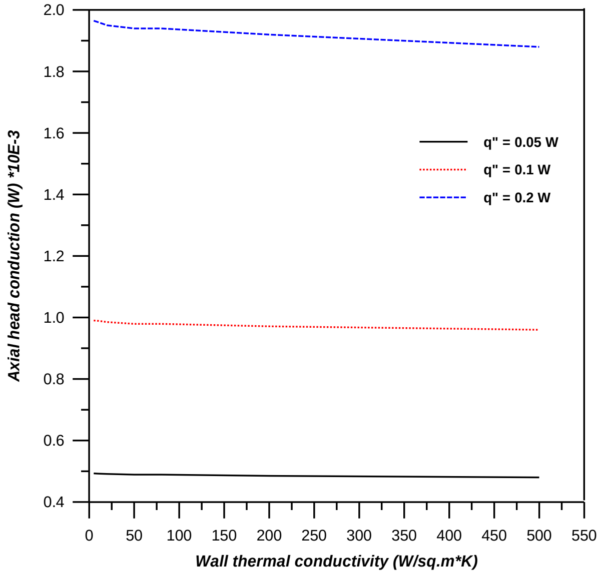

Fig. 13 Variation of axial heat conduction with thermal conductivity for different values of heat flux

5.3. Correlation:

A correlation has been developed to calculate the value of axial heat conduction. In this correlation, most of the parameters, which may affect on the axial conduction, have been taken into consideration.

From results obtained, the affecting parameters are, wall thickness, Re, length of channel, thermal conductivity of the wall and the value of applied heat flux i.e.:

q_2 = f (t, Re, L, ks, q")

As can be seen from results (figure 13) the effect of thermal conductivity is very small, therefore this parameter will not be considered in the correlation and all other parameters are considered.

In non-dimensional form:

qr = f (tr, Re)

Where:

qr : ratio of heat transfer rate = $q_r = \dfrac{q_2}{q''}$

tr : ratio of wall thickness to the channel length = $t_r = \dfrac{t}{L}$

The following correlation is developed to calculate the value of axial heat conduction based on the affecting parameters (wall thickness, Reynolds number, channel length and applied heat flux).

$$q_r = 0.6855\ t_r^{1.0005}\ \mathrm{Re}^{0.0771}$$

The above correlation can be used accurately to calculate the axial heat conduction in rectangular channels.

6-Conclusions:

From the results obtained in this paper it can be concluded the following:

1-In conventional channels with wall thickness small compared with channel dimensions the axial heat conduction can be neglected. While in small channels with walls thickness in order ofo the channel dimensions the axial heat conduction cannot be ignored since it lead to decrease the heat transferred to the fluid.

2- Axial heat conduction have considerable values in entrance region and increasing the length of entrance region lead to increase the axial conduction.

3- Axial heat conduction increased with increasing the thickness of solid walls.

4- Length of channels have no valuable effect on the axial heat conduction.

5- Axial heat conduction increased with increase Reynolds number due to increase the effect of entrance region.

6- Wall thermal conductivity have inconsiderable influence on the axial heat conduction.

7- References:

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8. Nomenclature:

Symbol	Description	SI Unit
A	Cross-sectional area	m^2
c_p	Specific heat	J/(kg K)
D_h	Hydraulic diameter	m
H	Channel height	m
k	Thermal conductivity	W/m K
L	Channel length	m
m	Mass flow rate	kg/s
q	Heat transfer rate	W
T	Temperature	K
u	Fluid x-component velocity	m/s
v	Fluid y-component velocity	m/s
w_{in}	Average inlet velocity	m/s
w	Fluid z-component velocity	m/s
W	Channel width	m
x	Axial coordinate	m
y	Vertical coordinate	m
z	Horizontal coordinate	m
ρ	Density	Kg/m^3
μ	Dynamic Viscosity	m^2/s