Two Stage Kalman Estimators with Probabilistically Weighted Average

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Abstract

With spherical coordinate, the adaptive estimation using multiple model filtering is enhanced in this paper. The enhancement is achieved by using just two depended parallel Kalman filters, instead of multiple models, with the probabilistically weighted average, which provides the adaptive mechanism. The first filter is constant velocity filter (CVF) which is used to estimate the position and velocity of the moving target in non maneuvering course. The second filter calculates the acceleration and the new adjustment for the CVF. The second filter is referred as variable velocity filter (VVF). Monte Carlo computer simulation results are included to demonstrate the effectiveness of the proposed algorithm in enhancement the multiple model adaptive filtering.

Keywords: Estimation, Kalman filter, two stages, probabilistically weighted average

مرحلتين من مخمن كالمان مع المعدل الاجمالي للاحتمالية

الخلاصة

البحث يتضمن تحسين التخمين الفعال باستخدام مرشحات النماذج المتعددة بحيث تم استخدام مرشحان كالمان (Kalman filter) احدهما يعتمد على الاخر مع معدل احتمالية تجهزنا بميكانيكية التحديث . المرشح الاول يسمى بمرشح السرعة الثابتة والذي يستخدم لتخمين السرعة و الموقع للهدف المتحرك في حالة عدم وجود مناورة بينما المرشح الاخر يستخدم لحساب التعجيل في حالة حدوث المناورة . طريقة مونتيكارلو(Monte Carlo) تم استخدامها لتوضيح فعالية الاداء .

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I. Introduction

Target tracking systems operating in a track-while- scan mode have great difficulty maintaining contact with targets performing unpredictable maneuvers methods for solving this problem there are variety of approaches. One of these methods utilizes a bank of N Kalman filter each designed to model different maneuver [1, 2, 9,11]. The estimate of the state of the target is either the output of the filter having an innovation sequence closet to a white noise sequence or is a weighted sum of all N filter outputs. Computational time constraints generally limit the usefulness of this method. Maybeck and Suize [3] also addressed the problem and solved it by using multiple module filtering. The multiple are created by tuning filters for best performance. Bar-Shalom [4] present interacting multiple module algorithms. However, in these

algorithms two or three Kalman filters are operating in parallel. By deriving a transition probability matrix with conditional probability, from the innovation sequence of each filter, the output will be the [i]probability weight sum of each filter. These algorithm gives an improvement over other algorithms such that Bogler algorithm [5]. Djouadi and Berkany[10] use the IMM algorithm with the unscented Kalman filter to deal with non linear model.

This paper investigates adaptation algorithm, which is enhancement of the two multiple adaptive filtering [3]. It is named as enhancement multiple adaptive filtering [EMAF]. In this algorithm, two parallel stage Kalman filters are used to generate state estimates from the shared sensor. Adaptive expansion is attained by generating the probabilistically weighted average of the two filter state estimates.

II. Two Stage Kalman Estimator and the Proposed Algorithm

The idea of using a two stage filter to implement an augmented state filter was introduced in[8]. The idea is to decouple the central filter into two parallel filter. The first filter, the “bias-free” filter, is based on the assumption that the bias is nonexistent. The second filter, the bias filter, produces an estimate of the bias vector. The output of the first filter is then corrected with the output of the second filter [6].

in this paper, the proposed tracking algorithm (Fig.1) is consist of two parallel filter worked together and from the

$$E\{W(k)\} = 0$$
$$E\{W(k)W^T(j)\} = Q(k)d(k-j)$$

property of the innovation sequence and state estimates of these filters, the adaptation detector switch can be worked depending on the probabilistically weighted average.

moreover, the first filter is two state Kalman (constant velocity) filter that used to best performance for estimating the position and velocity of the target in case of non-maneuvering targets tracking, while the second filter(the acceleration filter) which depended on the residual sequence of the first filter, is single Kalman filter and it is used in parallel of the first Kalman filter to estimate the acceleration and the new estimate to the position and velocity of the maneuvered targets without modifying the operation of the first Kalman filter.

The details about the operations and the simulation examples for the proposed tracking algorithm is given in the next sections

III. The Constant Velocity Filter

In the absence of maneuver, the target is modeled as a constant velocity object in a plant with some process noise that models slight changes in the velocity. The target process model, discretized over time interval of length T is [6].

$$X(k+1) = \Phi X(k) + W(k) \quad \text{..... (1)}$$

Where, using spherical co ordinates and for one-dimensional range tracking situation, the state vector is given as

$$X(k) = [r(k) \quad v(k)]^T, \left| \begin{matrix} r(k) = range \\ v(k) = velocity \end{matrix} \right|$$

And

$$\Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

With Φ as the transition matrix and W(k) is white Gaussian noise sequence with, The covariance matrix *Q(k)* is defined by [7],

$$Q(k) = q^T \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \quad \text{..... (2)}$$

where

$$q = s_m^2 T$$
$$q_{11} = T^2/3$$
$$q_{12} = q_{21} = T/2$$
$$q_{22} = 1$$

Where T is the sampling time, q is the spectral density of the continuous white noise change in acceleration process and s_m^2 is the variance of the change in acceleration noise. It is assumed that only range measurement is a variable

$$Y(k) = HX(k) + V(k) \quad \text{..... (3)}$$

where

H=[1 0] and $V(k)$ is white Gaussian noise with,

$$E\{V(k)\} = 0$$
$$E\{V(k)V^T(k)\} = s_r^2 d(k-j)$$

And s_r^2 is the variance of the observation channel noise (the error of the measured range). The noise process $W(k)$ & $V(k)$ are uncorrected.

The recursive two state Kalman filter equations are:

Filter state estimate:

$$\hat{X}(k+1/k) = \Phi \hat{X}(k/k) \quad \text{..... (4)}$$

$$\hat{X}(k/k) = \hat{X}(k/k-1) + K(k)[Y(k) - HX(k/k-1)] \quad \text{.... (5)}$$

Filter gain:

$$K(k) = P(k+1/k)H^T[HP(k+1/k)H^T + s_r^2]^{-1} \quad \text{..... (6)}$$

Error covariance matrices

$$P(k+1/k) = \Phi P(k/k)\Phi^T + GQ(k)G^T \quad \text{..(7)}$$

$$P(k+1/k+1) = [I - K(k+1)H]P(k+1/k) \quad \text{....(8)}$$

$$u(k/k) = u(k/k-1) + k_u(k)[R(k) - S(k)u(k/k-1)] \quad \text{.....(11)}$$

The two state Kalman is initialized as $r(0/0) = y(0)$ and $v(0/0) = [y(1) - y(0)]/T$

where y(0) and y(1) are, respectively, the first and second received sensor measurements

$$P_u(k/k-1)=P_u(k-1/k-1)+GQ_n(k)G^T \quad ...(12)$$

$$k_u(k)=P_u(k/k-1)s^T(k)[s(k)P_u(k/k-1)s^T(k)+HP(k/k-1)H^T+G_r^2]^{-1} \quad .....(13)$$

$$P_u(k/k)=[I-k_u(k)S(k)]P_u(k/k-1) \quad .....(14)$$

The initial estimation error covariance for this coordinate is then

$$p(0/0)=\begin{bmatrix} s_r^2 & s_r^2/T \\ s_r^2/T & 2s_r^2/T^2 \end{bmatrix}$$

$$S(k)=HU(k)+C \quad .....(15)$$

$$U(k)=fV(k-1)+G_u \quad .....(16)$$

$$V(k)=[I-k(k)H]U(k-1)-(k(k)C \quad .....(17)$$

$$X(k+1)=fX(k)+GW(k)+Bu(k) \quad ...(9)$$

IV. The Second(Acceleration) Kalman Filter

When a maneuver occurs, the target process model in Eq.(1) Becomes [6]

Where B is acceleration transition vector defined by,

B=[T^2/2 T]^T

u(k) is the unknown acceleration term, this term can be deemed as either a random signal or a deterministic signal. Here we consider it as a random signal

that influences the system dynamics. The

$$E\{W_u(k)W_u^T(j)\}=Q_u(k)$$

dynamics governing this random maneuver term is,

$$u(k)=u(k-1)+G_uW_u(k) \quad .....(10)$$

The $W_u(k)$ is uncorrelated white Gaussian sequence with zero mean and variance given by [6]

The state measurement model which define in Eq.(2) is become,

$$Y_u(k)=HX(k)+Cu(k)+V(k)$$

Where H & V(k) is define in previous section. while C is defined by,

$$C=T^2/2$$

The Equations for the acceleration filter and the correction steps are[6]:

Where

and R(k) is the residual of the first two-state filter.

$$R(k)=Y(k)-H\hat{X}(k/k-1)$$

…..(18)

The algorithm for compensating the output of the two state Kalman filter with the output of single Kalman filter is given by

$$X_n(k/k)=X(k/k)+V(k)u(k/k) \quad …..(19)$$

$$P_n(k/k)=P(k/k)+V(k)P_u(k/k)V^T(k) \quad …(20)$$

IV. Weighting Coeficients Computation

As given in [1], let b denote the vector of uncertain parameters in a given model; here it is composed of the strength of the white noise driving the target acceleration model. In order to make identification of b tractable, its continues range values is discretized into L representative values. If we define the hypothesis conditional probability $\boldsymbol{P_j}$ (for j=1,2,…………..,L) conditioned on the observed measurement history to time k, i.e.

$$P_j(k_i) = \frac{F_{y(k_i)/b_i y(k_{-1i})}(Y_i / b_j, Y_{i-1}) P_j(k_{i-1})}{\sum_{l=1}^{L} f_{y(k_i)/b,y(k_{-1i})} Y_i / b_j, Y_{i-1}) P_j(k_{i-1})} \quad \text{….. (21)}$$

$$F_{y(k_i)/b_i y(k_{-1i})}(Y_i / b_j, Y_{i-1}) = \frac{\exp\{.\}}{(2p)^{m/2} \left|A_j(k_i)\right|^{1/2}} \quad \text{…..(22)}$$

$A_j(k_i)$ is calculated in the jth Kalman filter as

$$A_j(k_i) = H_j(k_i) P_j(k_i) H_j^T(k_i) + R_j(k_i) \quad \text{….. (23)}$$

and m is measurement dimension the Bayesian estimate of the state is the probabilistically weighted average:

$$\hat{X}(k_i) = E\{X(k_i)/Y(k_i) = Y_i\} = \sum \hat{X}\left(^{+}_{k_i}\right) P_j(k_i) \quad \text{…. (24)}$$

Where $\hat{X}\left(^{+}_{k_i}\right)$ is the state estimate generated by a Kalman filter based on the assumption that parameter vector equals b_j. Thus the multiple model filtering algorithm is composed of a bank of L separate Kalman filter, each based on a particular value of the parameter vector. When the measurement Y_i becomes available at k_i, the residuals $v_j(k_i)$…………$v_j(k_i)$ are generated in the L filter (for our algorithm L=2) and used to compute $b_j(k_i)$…………$P_j(k_i)$ via (21). However $b_j(k_i)$ will converge to unity for the coefficient corresponding to the true process and to zero for the others.

V. Simulation Results

The performance of EMAF algorithm is compared with other tracking methods. The filter tracking performance is evaluated by doing a Monte Carlo simulation of 50 runs and then tacking the average of these runs this is called ;time average root mean square(TARMS) error and given by [7]

$$s_{est}(r) = \frac{1}{N} \sum_{k=1}^{N_1} \left\{ \left(\frac{1}{N^2} \right) \sum_{i=1}^{N_2} \left[r(k) - r^{\wedge}(k/k \right]^2 \right\}^{1/2} \quad \text{…..(25)}$$

Where N_1 is the number of samples for the trajectory, N_2 is the number of Monte Carlo Runs.

The EMAF is compared with the single CVF and two stage Kalman filter estimator.

Two maneuver scenarios considered for performance analysis. In the first maneuver scenarios, we assume that target is on a constant course and velocity until time t=120 second, when it maneuvers a slow 90° turn with acceleration input 30 m/sec^2 . It completes a turn at t=150 sec. Remaining course is constant velocity. The track filter parameters for the two scenarios are given by the following:

Sampling interval T: 1 [sec]
The standard deviation of the observation additive white Gaussian noise σ_r: [100 m]
The anticipated standard deviation of the plant noise disturbance σ_m: 5/sec^2 .
The constant target radial velocity V: 300m/sec.

A Monte Carlo simulation of 50 runs was obtained for each algorithm and the roots mean square (rms) values of the estimation errors were computed. Probability for each filter is shown in Fig. 2, while the rms range and velocity errors for two stage Kalman filter and EMAF are shown in Fig. 3 &Fig.4. It can be seen from the simulation results that EMAF not only yields improved performance during the maneuvering period, but also provides for better estimation during the non maneuvering period than the tracking filter using two stage Kalman filter.

The second maneuver scenarios is considered to test and compare performance of the EMAF with performance of Two-stage Kalman filter. In this scenarios, we assume that the target moves in a plane on constant course with constant velocity until time t=50 sec, when it maneuvers a 90° turn with acceleration 20 m/sec. It completes the turn at t=80 sec. Then non maneuvering course continuous for 40 sec, followed by the second 90° turn which stars at y = 120 with acceleration of 50 m/sec^2 and is completed at t = 150 sec.

Fig.5 shows the probability for each filter and Fig. 6& Fig. 7, shows the rms estimation errors over 50 Monte Carlo runs. This figure show that both of the methods have similar performance, the EMAF has low computation load than the two-stage Kalman filter and it produces small estimation errors especially in velocity.

VI. Conclusions

The enhanced multiple model adaptive filter EMMAF algorithm for tracking the non maneuvering and maneuvering targets has been presented and illustrated. The proposed algorithm is improve and enhance the MMAF because it used two depended parallel Kalman filter instead of multiple model with the probabilistically weighted average, which provide the adaptation mechanism.

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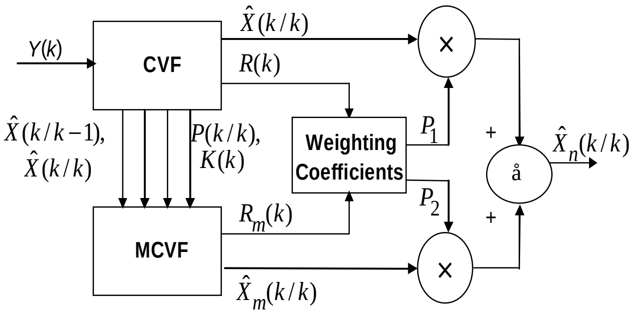

Fig. 1: The proposed tracking system for one coordinates (range).

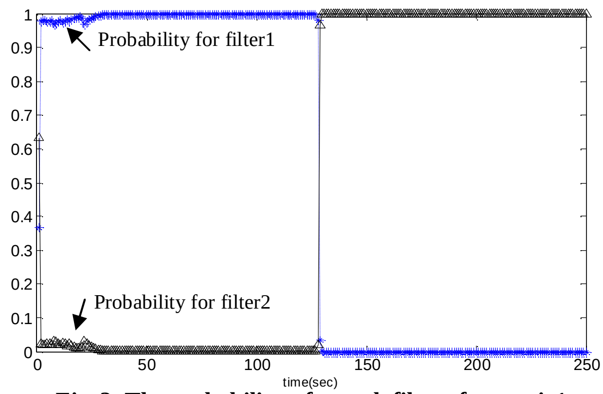

Fig. 2: The probability y for each filter of scenario1

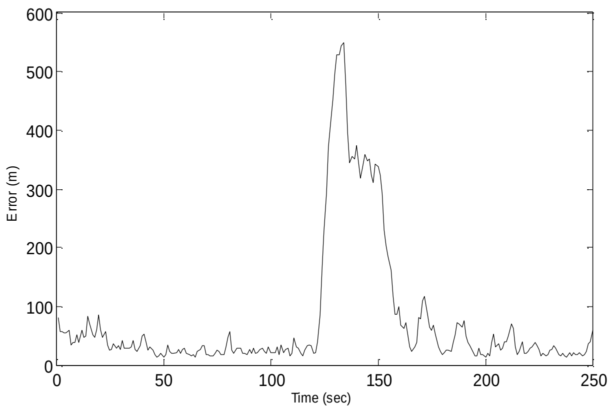

Fig.3a: The rms error of the CVF of scenario1.

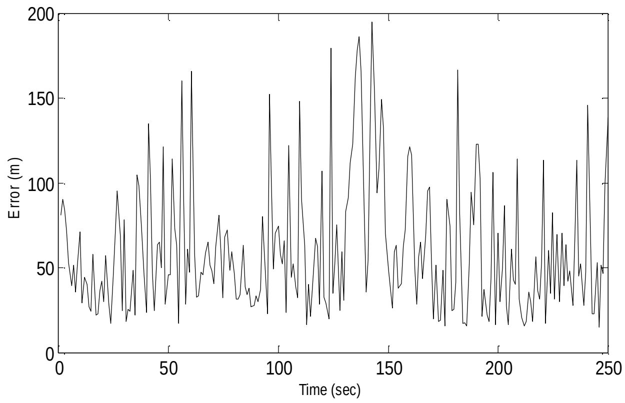

Fig.3b:The rms error of the two stage Kalman filter of scenario 1 .

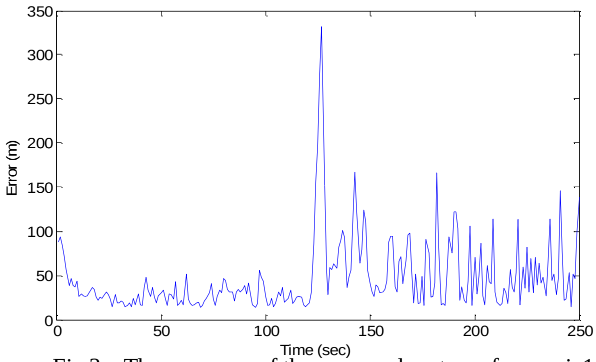

Fig.3c: The rms error of the proposed system of scenario1

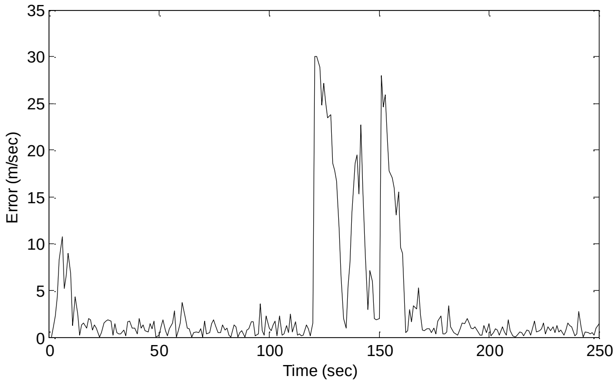

Fig.4a: The rms range rate error of the CVF of scenario1.

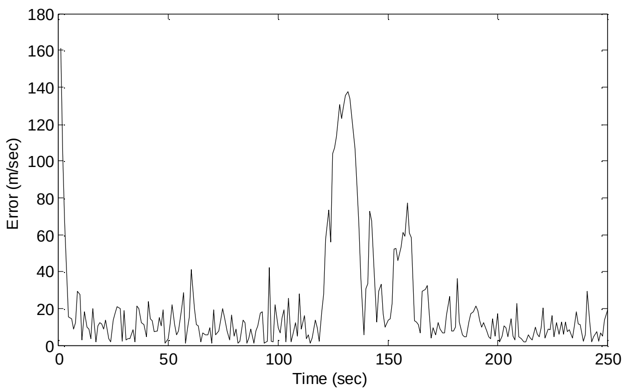

Fig.4b: The rms range rate error of two stage Kalman filter of scenario1.

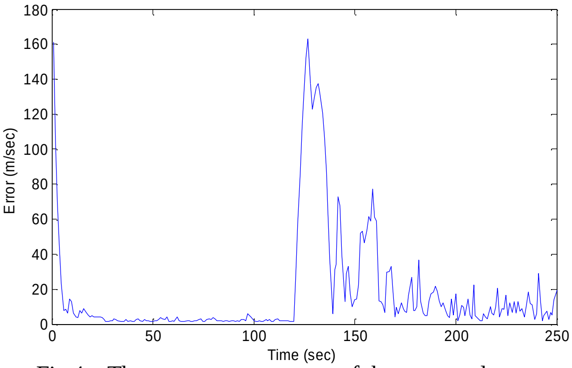

Fig.4c: The rms range rate error of the proposed system of scenario 1

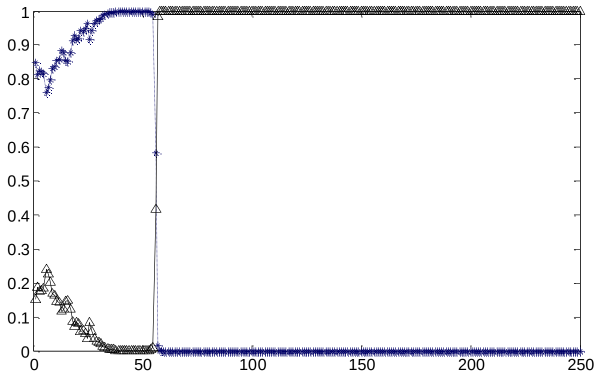
Fig. 5: The probability y for each filter of scenario2.

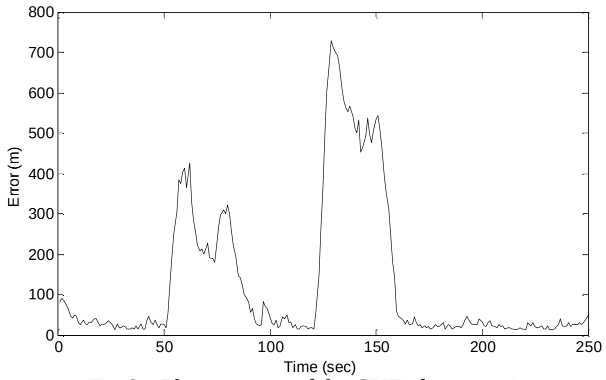

Fig.6a: The rms error of the CVF of scenario2

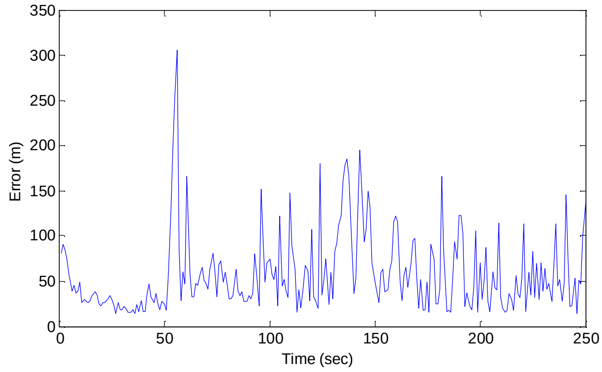

Fig.6c: The rms error of the proposed system of scenario2

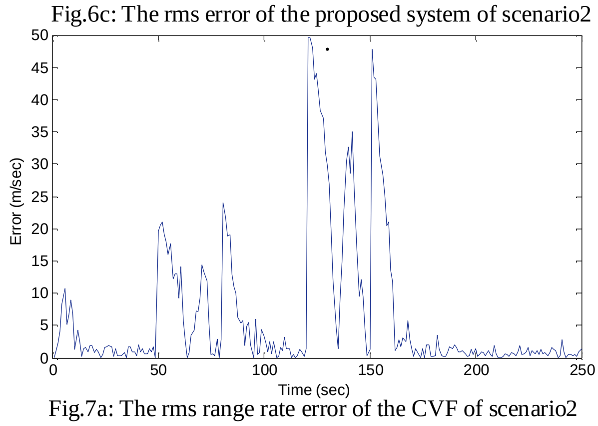

Fig.7a: The rms range rate error of the CVF of scenario2

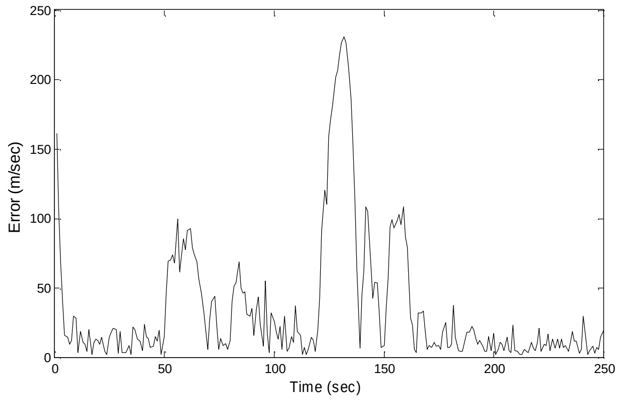

Fig.7b: The rms range rate error of two stage Kalman filter of scenario2

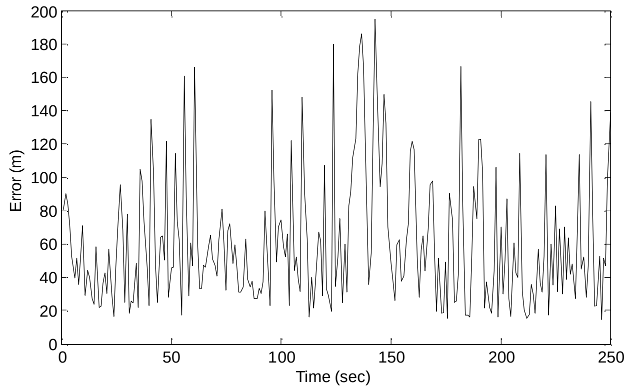

Fig.6b: The rms error of the two stage Kalman filter of scenario2.

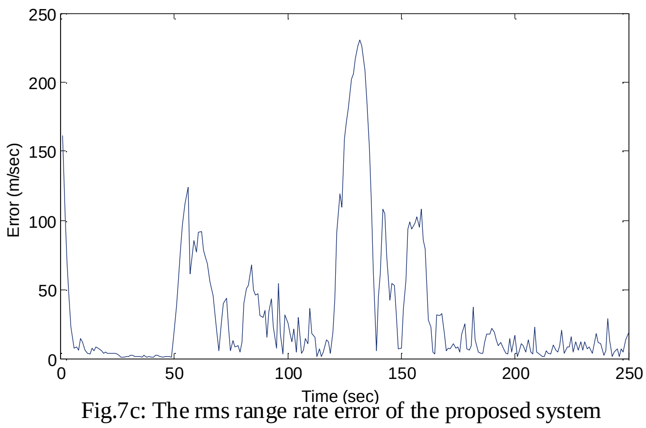

Fig.7c: The rms range rate error of the proposed system of scenario2