Determination of Fold Profiles and Fold Functions, a Mathematical Approach

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ABSTRACT

Two mathematical approaches dealing with fold profiles and their clips of computer programs are adopted in this paper. The first is a mathematical method proposed for determining fold profiles instead of the graphical Busk method. The second approach is the presentation of fold profiles by mathematical functions. These functions can be represented by quarter wavelengths of the fold waves. It can be used to interpolate or extrapolate points within or outside the given fold profile. A known example from Shaikhan Anticline was tested. The result of this test shows accurate and perfect coincidence with the graphical construction.

إيجاد منحني ومعادلة الطية بأسلوب رياضي

الملخص

أسلوبان رياضيان قد اتبعا في تحليل منحني الطية مع مقاطع برمجية أعدت لهذا الغرض . الأسلوب الأول هو إعداد طريقة رياضية لإيجاد منحني الطية بدلا من الطريقة البيانية (طريقة بصك) . الأسلوب الثـاني هو تمثيل منحني الطية بمعادلة رياضية ، هذه المعادلة تمثل ربع الطول الموجي لموجة الطية ومن هـذه المعادلة يمكن إيجاد اندراج أي نقطة تقع ضمن منحني الطية المعلوم أو استكمال نقطة تقع خارج نطـاق هذا المنحني . وقد اخذ نموذج لاختبار هذين الأسلوبين من طية الشيخان أثبتت النتائج دقتهما .

INTRODUCTION

Geometrical description of fold shape seeks to define their profile. Several graphical methods dealing with the determination of fold profiles are known. Among these are Busking or Busk method (Busk, 1929; cited by Ramsay and Huber, 1987) which deals with Parallel folds (Class 1B) that have constant thickness of strata and dip isogon lines perpendicular to both the outer and inner arcs. The Fold reconstruction method that uses dip isogons and the characteristic isogonal pattern to reconstruct folded multilayered rocks (Ramsay andHuber ,1987). The Higgins method uses concentric arcs to represent the profile of fold (Higgins, 1962; cited by Ragan, 1985).

Recent trends in fold shape analysis consider quarter wavelength of the fold wave and use different mathematical approaches that are necessary for this analysis. The profiles are so far extrapolated separately by graphical methods and are then incorporated into the mathematical analysis (Stabler.1968 , Hudleston, 1973, Ramsay and Huber, 1987, Stowe, 1988,Bastida et al., 1999). This work deals with the fold profile in two ways. The first is a proposed mathematical method that is device for drawing fold profiles which can be readily incorporated into the mathematical analysis. The second is an approach for determining fold functions. Two numerical methods were selected for the determination of these functions. These two methods allow the hidden or eroded part of the folded surface to be interpolated or extrapolated.

Mann and Vita-Finzi, (1982) suggested a method for interpolating a concealed part of sinusoidal fold by mathematical procedure. Stowe (1988) stated that " a limb is approximated by a mathematical function." And he stressed the benefit of the mathematical methods; nevertheless, the functions are very complicated, particularly when a large number of coefficients are involved.Whereas Bastida et al.(1999) presented a geometrical analysis of folded surface profile based on their approximation by simple functions.

METHODOLOGY

Most of the modern analytical methods apply the quarter wavelength (QW) of the fold as a smaller unit defining the fold shape geometry. This type of analysis was done in this way because it becomes more complicated and not accurate when it covers the half wavelength of a fold wave by single function and small number of coefficients. This is obvious, particularly, in the asymmetrical fold where the left and right quarter wavelengths are not similar. For these reasons, the derived functions in the methods proposed below deal with the individual quarter wavelength QW, each quarter wavelength represent a limb of an anticline.

A-Fold profile construction:

In this proposed method, it will be assumed that the fold is of Parallel type (Class 1B of Ramsay, 1967). So that, it is based on the properties of this type of fold and the principles of Busk method but it differs in its mathematical and computer application.

The method needs simple data to represent the fold profile. These data may be taken directly from the field or from a geological map. They are the distance between the reference point (A) and the point involved and the dip angle at each point (Fig. 1). The application of this method is explained by the following:

1. Plotting locations of the points, mentioned above, on the x-axis starting from the reference point (Fig. 2a). Considering this point as the origin of the coordinate axes.
2. Drawing a line representing the dip angle at each point, and then draw a perpendicular to that line indicating the dip isogons of the parallel fold (Fig.2b).
3. Determining the positions of (Xi ,Yi) of the fold profile on the perpendicular lines (in 2). These positions are determined by the following trigonometric relationships (Fig.2c and 3).

fig 1,2

Fig 3

The first point (X_1,Y_1) is a reference point (the origin of the coordinate axes), so $X_1 = 0$ and $Y_1 = 0$.

For determining the position of the second point (X_2 ,Y_2):

From Fig.(2)

$\tan \theta_1 = R_2 / S_2$ So, $R_2 = S_2 \tan \theta_1$ ------------------- 1

$\tan \alpha_2 = R_2 / (A_2 - S_2)$ So, $R_2 = (A_2 - S_2) \tan \alpha_2$ ---------------2

when A_2 = the distance between the origin and the second point.

$S_2 \tan \theta_1 = A_2 \tan \alpha_2 - S2 \tan \alpha_2$

$S_2 = A_2 \tan \alpha_2 / (\tan \theta_1 + \tan \alpha_2)$

So,

$X_2 = X_1 + S_2$ and $Y_2 = Y_1 + S_2 \tan \theta_1$

The point (X_3 ,Y_3) in Fig.(3) and the others are determined by :

$\tan \theta_2 = R_3 / S_3$ So $R_3 = S_3 \tan \theta_2$----------------- 3

$\tan \alpha_3 = R_3 / A_3 - S_3 - X_2 - T_3$

$R_3 = \tan \alpha_3 (A_3 - S_3 - X_2 - T_3)$

$R_3 = A_3 \tan \alpha_3 - S_3 \tan\alpha_3 - X_2 \tan \alpha_3 - Y_2$ ------------------- 4

Where $T = Y_2 / \tan \alpha_3$

Because equation 3 equal equation 4, then

$S_3 \tan \theta_2 = A_3 \tan \alpha_3 - S_3 \tan \alpha_3 - X_2 \tan \alpha_3 - Y_2$

Therefore,

$S_3 = A_3 \tan \alpha_3 - X_2 \tan \alpha_3 - Y_2 / (\tan \theta_2 + \tan \alpha_3)$

So,

$X_3 = X_2 + S_3$ and $Y_3 = Y_2 + R_3$

And so on for the other points.

4. For determining more reliable curvature between each two successive points, the curve must gradually change from one point to another. For this purpose, the distance between successive points was divided by (10). The curvature must be determined at each (S/10) distance and tan è at each stage must be calculated (Fig.2d). Elevations against amount of S at each stages give the positions of the minor points (the points between any two given points of the profile), therefore, the curvature would be determined (Fig.2d and Appendix).
5. By repeating these calculations between each two points, fold profile can be obtained. For the sake of simplicity, a clip of computer program PROFILE was written to determine a fold profile, which is based on this method (Appendix).

An accurate result is obtained as shown in figure (3). This figure shows a perfect fit between the profiles drawn using this procedure and that of the graphical Busk method.

B- Determination of Fold Function:

Four methods of numerical analyses are taken into consideration for determining the fold function. These methods are The Lagrangian polynomial interpolation, the Divided difference by Newton formula, The Newton method for solution of nonlinear equations, and the curve fitting by least squares method. Empirically, the most adequate methods are the Lagrangian polynomial and the curve fitting by least squares. The first method is suitable for interpolating a point within the given curve; whereas the second one is accurate for extrapolating a point outside the wavelength (QW) or on a half wavelength (HW) of the studied fold, but the application of QW procedure gives more accurate result.

1- The Lagrangian polynomial :

The theory of this method is described in Gerald and Wheatly (1984). Suppose that points (which may be the output of the latter proposed method) represent a fold profile and have the values of X and Y. These points can be considered as values of X and their functions F(x). They can be written as in the followings:

X: X_0 X_1 X_2 ----------X_n

F(X): $F(X_0)$ $F(X_1)$ $F(X_2)$--------$F(X_n)$

Therefore, the Lagrange's polynomial of degree n can be define as:

$$Pn(X) = F(Xo)\,Lo\,(X) + F(X_1)\,L_1\,(X) + F(X_2)\,L_2\,(X) + \cdots + F(X_n)\,L_n\,(X) \quad \text{---- 5}$$

$$\text{OR, } P_n(X) = \sum_{i=o}^{n} F(X_i)\,Li(X)$$

Where Li (X) is the polynomial of any (X) and it calculated by the following:

$$Li\,(X) = \frac{(X-Xo)\,(X-X_1)\,(X-X_2) \cdots (X-X_{i-1})\,(X-X_{i+1}) \cdots (X-X_n)}{(Xi-Xo)\,(X_i-X_1)\,(X_i-X_2) \cdots (X_i-X_{i-1})\,(X_i - X_{i+1}) \cdots (X_i-X_n)} \quad \text{----- 6}$$

It must be noted that the best degree of the Lagrange's polynomial is less one of the n values. After the substitution of X values in equation (6), and the substitution of Li (X) values in equation (5) and finding Pn (X). Therefore, the curve. These two methods can be applied on a quarter function of fold profile is determined. This function in this form is very complicated, so a clip of a computer program LAG (the first three letters of the ward "Lagrange", Appendix) is responsible for determining the values of (Y) for any given value of (X). This means that any point on the profile can be interpolated, even if this point is located on an eroded or concealed part of the profile.

2-The Curve Fitting by least squares method:

In this type of fitting nonlinear curves, the field data must be converted to matrix form. It can be 3*3 or 5*5 matrix or more. Empirically, the first one is simple and accurate enough to be used for geologic purposes. The field data needed to apply in this method are the X and Y coordinate values of fold profile (may taken from the proposed method for profile construction). After the application of this method the fold function can be obtained and this function can readily extrapolate any point including those falling outside the given profile. A computer program clip LEAST (the first word of "Least squares method", Appendix) is responsible for the determination of this function.

The theory of this method is given in Gerald and Wheatly(1984), and its application can be done as follows:

1- Finding the summations of:

$\Sigma Xi \quad \Sigma Xi^2 \quad \Sigma Xi^3 \quad \Sigma Xi^4 \quad \Sigma Yi \quad \Sigma Xi\, Yi \quad \Sigma Xi^2\, Yi$

2- Making equations from these parameters according to Least squares method.

$$a_0 N + a_1 \Sigma Xi + a_2 \Sigma Xi^2 = \Sigma Yi$$
$$a_0 \Sigma Xi + a_1 \Sigma Xi^2 + a_2 \Sigma Xi^3 = \Sigma XiYi$$
$$a_0 \Sigma Xi^2 + a_1 \Sigma Xi^3 + a_2 \Sigma Xi^4 = \Sigma Xi^2 Yi$$

3- Putting these equations in a form of matrix as shown below:

$$\begin{vmatrix} N & \Sigma Xi & \Sigma Xi^2 \\ \Sigma Xi & \Sigma Xi^2 & \Sigma Xi^3 \\ \Sigma Xi^2 & \Sigma Xi^3 & \Sigma Xi^4 \end{vmatrix} * \begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix} = \begin{vmatrix} \Sigma Yi \\ \Sigma XiYi \\ \Sigma Xi^2 Yi \end{vmatrix}$$

4- Converting this matrix to the upper triangular form:

$$\begin{matrix} b_{1,1} & b_{1,2} & b_{1,3} : b_{1,4} \\ b_{2,1} & b_{2,2} & b_{2,3} : b_{2,4} \\ b_{3,1} & b_{3,2} & b_{3,3} : b_{3,} \end{matrix}$$

$$\begin{matrix} b_{1,1} & b_{2,1} & b_{1,3} : b_{1,4} \\ 0 & b_{2,2} & b_{2,3} : b_{2,4} \\ 0 & 0 & b_{3,3} : b_{3,4} \end{matrix}$$

5- Finding the values of a - coefficients a_1 , a_2 and a_3 by back substitution .

$b_{3,3} * a_2 = b_{3,4}$ so $a_2 = b_{3,4} / b_{3,3}$

$b_{2,2} * a_1 + b_{2,3} * a_2 = b_{2,4}$ so $a_1 = (b_{2,4} – b_{2,3} * a_2) / b_{2,2}$

and,

$b_{1,1} * a_0 + b_{1,2} * a_1 + b_{1,3} * a_2 = b_{1,4}$

So $a_o = (b_{1,4} – b_{1,2} * a_1 - b_{1,3} * a_2) / b_{1,1}$

6-Formulating the function of this profile by the following formula:

$Y = a_o + a_1 X + a_2 X^2$

Function in this form represents the fold profiles and can extrapolate any point outside the curve of fold profile.

THE TESTED EXAMPLE

An example was taken from Shaikhan Anticline. This anticline is located in the northeastern part of Iraq, and within the foreland fold belt. It is an asymmetrical anticline, whose vergence is towards the northeast, it is double plunging and trends in a northwest-southeasterly direction (Ahmad, 1980). The data were taken from the geological map given in Ahmad (1980). The northeastern limb is considered for the QW analysis. The fold profile for this anticline was drawn graphically by Busk method using the above mentioned data (Fig.4). The same data were processed by the proposed method using the computer program PROFILE. The computer output (Table-la) showed exact coincidence with the construction of Busk (Fig.4). The output data of table (la) was applied in the Lagrangian polynomial using the computer program clip LAG. Then the result is shown in table (1b). The data in this table reveal excellent result for the interpolation of missing part of the profile (Fig.4). The data of table (la) were also analyzed by the curve fitting of least squares method using computer program clip LEAST. The result is listed in table (1c). It indicates a good extrapolation of points outside the profile (Fig.4).

Table 1: show the field data and the output of each method for Shaikhan Anticline.

The Field Data		**The proposed Method Output**		**Lagrange Interpolation Output**		**Least Squares Method Output**	
A		**B**		**C**		**D**	
X		X	Y	X	Y	X	Y
0	40	0	0	1	0.84	-0.1	-0.11
2	40	1.17	0.98	2	1.57	-0.2	-0.21
3.75	25	2.69	1.98	3	2.12	-0.3	-0.31
4.7	13	4.0	2.47	4	2.47	-0.4	-0.41
5.5	0	5.5	2.65	5	2.62	-0.5	-0.52

Fig 4

CONCLUSION

Recently, most of the analytical methods consider the quarter wavelength as a suitable unit for analyzing the geometry of fold shape. Also most of these methods use mathematical and computerized techniques as a new approach for analyzing folds. So it has been necessary to propose a mathematical method for accurate determination of fold profiles that is harmonious with the other mathematical methods of fold shape analysis. The proposed method is preferred to the graphical approach because it is faster and more accurate. Also this method can be advanced further to determine the important parameters of fold geometry.

The mathematical presentation of fold profile is also considered as a new approach for analyzing the fold shape. In this paper, there are two numerical methods selected for the presentation of fold profile as mathematical functions. These method are the Lagrangian polynomial and the curve fitting by least squares. Empirically, both of these methods are good for interpolating points within the fold profile although the Lagrangian method is more accurate (Fig-5 and table-2a). On the other hand, when the two methods are applied to extrapolate points outside the given fold profile, especially, when the inflection point is unknown in the field, the least squares method showed a more reliable result than that of the Lagrangian method (Fig-5 and table-2b).

Tab 2

Fig 5

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Appendix

PROFILE:

```
10 REM A COMPUTER PROGRAM CLIP USED FOR THE CONSTRUCTION OF FOLD
PROFILE WRITTEN BY NABEEL K. AL-AZZAW1 - DECEMBER 2001
20 REM INPUT THE NO. OF POINTS USED IN THE CONSTRUCTION OF FOLD PROFILE. .
30 READ P
40 DIM X (P) ,Y (P) ,A (P), TETA (P) , ALFA (P) ,TH ETA (P), N(P)
50 X(1)=0:Y(1)-0
60 REM INPUT THE DATA USING TO CONSTRUCT THE PROFILE
70 FOR 1=1 TOP
80 READ A(1),THETA(I)
90 IF 1= 1 THEN 110
100 N(I)=THETA(I-1)-THETA(1)
110 ALFA(I) =90-THETA(I)
120 TETA (I) = THETA(I) *3.142857 / 180
ISOALFAt^ALFAQ)^. 1412857/180
140 NEXT I
150 DATA 5,0,40,2,40,3.75,25,4.7,13,5.5,0
160 REM THE MATHEMATICAL EQUATIONS USED TO CONSTRUCT THE FOLD PROFILE.
170 FOR 1-2 TO P
180 S(I) - (A (1)*TAN(ALFA(I))-X(I-1)*TAN(ALFA(I))-Y(I-1))/(TAN(TETA(I-1))+ TAN (ALFA(I))
190 X(I)-X(I-])+S(1)
200 IF N (I)= 0 THEN R (I) = S(l) * TAN (TETA (1-1) : GOTO 250
205 IF N(I) <0 THEN 242
210 FOR J=0 TO N(I)-1
220 JJ-J* 3.142857/180
230 R (I) = R (I) + S (I) / N(I) * TAN (TETA (1-1) - JJ)
240 NEXT J
241 GO TO 250
242FORJ-OTO ABS(N(I))-1
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```
243JJ-J * 3.142857 / 180
244 R (I) = R (I) + S (I) / ABS(N(I)) * TAN (TETA (I-1) + JJ)
250Y(1)-Y(I-1)+R(I)
260 NEXT I
270 CLS
280 PRINT ^
290 PRINT " X ", " Y "
300 PRINT "—","—"
310 PRINT " THE X and Y VALUES OF THE DETERMINED PROFILE"
320 FOR I - 1 TO P
330 PRINT X(I),Y(I)
340 NEXT I
350 END
```

```
10 RFM ************* LAG *****************
20REM A PROGRAM CLIP FOR INTERPOLATING A POINT WITHIN THE FOLD
PROFILE.
30 REM WRITTEN BY NABEEL K. AL - AZZAWI -DECEMBER 2001
40PRINT "INPUT NO. OF POINTS USING FOR THE DETERMINATION OF FOLD
PROFILE"
50 INPUT K
60 PRINT'1 INPUT...X...FOR INTERPOLATING ...Y ..."
70 INPUT XX
80 DIM X(K), FX(K), LU (K), LD(K)
90 FOR 1-0 TO K.-1
100 READ X(I),FX(1)
110 NEXT I
120 DATA 0,0, 1.17 , 0.98 , 2.69 , 1.98 , 4.0 , 2.48 , 5.5 , 2.65
130 FOR 1=0 TO K.-1
140 LU(I)= I : LD(1)=1
150 FOR J= O TO K-l
160 IF I-J THEN 190
170 LU (I) = LU (I) * (XX - X (J))
180 LD (I) = LD (1) * (X (I) -X(J) )
190 NEXT J
200 LX (I) - LU (I) / LD (I) * FX (I)
210 TLX = TLX + LX (I)
220 NEXT I
230 PRINT " Y= "; TLX
240 END
```

```
1 REM ****************** LEAST *********************
5 REM A COMPUTER PROGRAM USED TO EXTRAPOLATE A POINT OUTSIDE
THE FOLD
PROFILE.
6 REM WRITTEN BY NABEEL K. AL-A ZZAWI -- DECEMBER 2001
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7 REM INPUT NO. OF POINTS USED TO DETERMINED THE FOLD PROFILE
20 READ NN
30 DIM X( NN) , Y ( NN) , B (6,7 )
40 FOR 1=1 TO NN
45 REM INPUT THE POINTS USED TO DETERMINE THE FOLD PROFILE.
50 READ X(I) , Y (I)
60 NEXT I
70 FOR I - 1 TO NN
80 XI-XI+X(I)
90 XI2 = XI2 + (X(I) ^ )
\QOXl3=Xi3+(X(l)/\3~)
110 X\4-Xl4+(X(}.)A4)
120 YI=YI+Y(I)
130 YX1-YX1+X(1)*Y(I)
140 YX2 = YX2 + (X(I) A 2 ) * Y (I)
150 NEXT I
160 REM THE ARRENGEMENT OF MATRIX USED IN LEAST SQUARES
170 B(1,1)=NN : B(1,2)=XI   : B(1,3)-XI2 ; B(1,4)-YI
180 B(2,1)=XI   : B(2,2)=XI2 : B(2,3)=XI3 : B (2,4) = YXI
190 B(3,1)=XI2 : B(3,2)=XI3 : B ( 3,3 ) = XI4 : B (3,4 ) = YX2
200 PRINT'1*******'1"
210 FOR 1=1 TO 3
220 FOR J = 1 TO 4
230 PRINT B(I,J);
240 NEXT J
245 PRINT
250 NEXT I
260 PRINT"*********
270 REM THE UPPER TRIANGULAR MATRIX
280 INPUT MM
290 FOR N - 1 TO 3
200 FOR M = 1 TO 4
210 IF N-< M THEN 270
220 I=N ; C = M
230 Q ( N, M ) = B ( N, M ) / B (M,M)
240 FOR J = 1 TO 4
250 B (I, J) - B( I, J ) - Q ( N , M ) * B (C , J )
260 NEXT J
270 NEXT M
280 NEXT N
290 FOR I- 1 TO 3
300 FOR J - 1 TO 4
310 PRINT B(I,J);
320 NEXT J
330 PRINT
340 NEXT I
```

```
350 PRINT "****************************
360 INPUT MM
370 REM THE DETERMINATION OF LEAST SQUARES PARAMETERS.
380A2:=B(3,4)/ B (3 , 3 )
390A1=:(B(2,4)-B(2,3)'1•A2)/B(2,2)
400AO-(B( 1 ,4)-B( 1 ,2)*AI -B(l , 3 ) * A2 ) / B ( 1, 1 )
410 PRINT "AO'VAl " , "A2 lt
420 PRINT 't—-(', "-—" , "-—"
430 PRINT AO , Al , A2
440 pRINJT1'*********************************************'!
450 INPUT MM
460 PRINT " THE GENERAL FORMULA OF THIS FOLD IS;"
470 PRINT " Y - (" ; A2 ;lt ) + X A 2 + ( " ; Al ; " ) X + ( " ; AO ;(t) "
480 CLS
490 PRINT " X " , " Y "
500 PRINT "—^ , "-—"
510 FOR X=-0.5 TO -0.1 STEP 0.1
520Y-A2*XA2+A1 * X + AO
530 PRINT X,Y
540 NEXT X
550 DATA 5,0,0, 1.17,0.98,2.69, 1.98,4.0,2.48, 5.5,2.65
560 END
```

ION	B- CURVE EXTRAPOLATION					
Y- values by Least Squares	X- Input Values for Extrapolation	Y- Values by Lagrangian Method	Y-Values by Least Squares	X-Input Values for Extrapolation	Y-Values by Lagrangian Method	Y-Values by Least Squares
-0.01	-1	-0.83	-1.07	6	2.65	2.63
1.00	-2	-1.50	-2.32	7	2.68	2.44
1.96	-3	-1.78	-3.74	8	2.87	2.08

2.47	2.46	-4	-1.42	-5.34	9	3.41	1.54
2.65	2.65	-5	-0.12	-7.12	10	4.57	0.82

Table 2: Interpolation and Extrapolation of points in and outside the profile by Lagrange and Least squares method.