

## Load Line Direction and Bearing Length Effects on The Tilting 4-Pad Bearing Performance

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### Abstract

This paper presents the tilting 4-pad bearing performance under dynamic load, the journal center velocity is calculated by mobility method. Finite element method was used to solve the Reynolds equation in two dimensions from this solution the pressure distribution can be obtained over pad surface. Study of the relationship of the load line angle ( $\beta$ ) with the pressure distribution changing due to the angular position taking four angles  $0^\circ$ ,  $5^\circ$ ,  $40^\circ$  and  $80^\circ$ . Also the effects of many parameters were studied such as length-diameter ratios, oil flow rates and finally dynamic coefficients (stiffness and damping coefficients) of journal bearing. The results of this paper shows that the load line angle ( $\beta$ ) when equal  $40^\circ$  and length-diameter ratio ( $L/D=0.5$ ), gives the best bearing performance.

**Keywords:** Tilting pad bearing, Load line direction, Oil flow rate, Dynamic coefficients

### تأثيرات اتجاه خط الحمل وطول المسند على أداء مسند ذو أربعة وسائد قابلة للإمالة

#### الخلاصة

يستعرض هذا البحث أداء مسند قابل للحركة حول مرتكز ذو أربعة وسائد تحت الحمل الديناميكي، سرعة مركز المحور محسوبة بـ (mobility method). طريقة العنصر المحدودة كانت تُستعمل لحل معادلة رينولد للبعدين ومن حل هذه المعادلة تحصل على توزيع الضغط على سطح الوسادة. دراسة علاقة زاوية خط الحمل ( $\beta$ ) مع تغير توزيع الضغط على الوسائد بسبب تغير موقع الدوران، تأخذ أربع زوايا  $0^\circ$ ,  $5^\circ$ ,  $40^\circ$  وكذلك  $80^\circ$ ، أيضاً تأثيرات العديد من المتغيرات ومنها تغير نسب طول المسند لقطره، معدل تدفق الزيت التزيت وأخيراً المعاملات الديناميكية للمسند (المرونة والتخميد). تبين نتائج هذا البحث عندما زاوية خط الحمل ( $b$ ) تساوي  $40^\circ$  ونسبة الطول لقطر المسند تساوي  $0.5$ ، تعطي هذه القيم أفضل أداء للمسند.

**الكلمات المرشدة:** مسند قابل إمالة الوسائد، اتجاه خط حمل، معدل التزيت، معاملات ديناميكية.

#### Notations

$C_r$	Radial clearance, $C_r=R-r$	m
$C_{xx}, C_{xz}, C_{zz}, C_{zx}$	Fluid film damping coefficients	N.sec/m
D	Bearing diameter	m
e	Eccentricity	m
f	Pressure force	N
h	Oil film thickness	m
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	Fluid film stiffness coefficients	N / m
P	Generated pressure in the oil film	$N/m^2$
Q	Total oil flow rate	$m^3/sec$
$q_x$	Oil flow rate in x-direction	$m^3/sec$

$q_z$	Oil flow rate in z-direction	$m^3/sec$
LN	Load number	-
R	Bearing radius	m
r	Journal radius	m
$t_h$	Liner thickness	m
W	Load	N
$\beta$	Load line angle	degree
$\phi$	Attitude angle	degree
$\mu$	lubricant viscosity	$N.sec/m^2$
$\omega$	Angular velocity of journal	rad/sec
$\rho$	Lubricant density	$Kg/m^3$
$\theta$	Angular coordinate taken from centers lines	degree

**Introduction**

Tilting-pad journal bearings are applied in high speed rotating machines operating at small and mean stationary loads and the peripheral speeds of journal reaching 150 m/s. These bearings have good stability at high speeds, are less sensitive to shaft misalignment compared to the multilobe bearings.

Biao, and Sawicki (2000), [1] compared the mobility method with the rigorous method in a study of dynamically loaded journal bearings. This comparison led to the following conclusions: Mobility method is a very efficient tool to obtain a quick solution in a process of dynamically loaded journal bearing design.

Malik (1983), [2] presented a study that concerned with the performance analysis of the symmetric and tilted four-lobed journal bearing configurations. The need for this study arises for the reason that unlike two- and three-lobed bearings, the four lobed bearing has not received much attention from the lubrication engineers. The difference in a symmetric and tilted configurations may be a symmetric configuration the line joining the bearing geometric centre and the centre of each lobe passes centrally through the lobe, while in a tilted configuration, the

The objective of this paper is to contribute to a better understanding of the load line angle ( $\beta$ ) and bearing length effects on the tilting pad bearing performance with four pads each having an angular dimension of  $80^\circ$ .

**Dynamically Loaded Analysis**

Figure (1) show the tilting 4-pad bearing and the load line position, table (1) shows the tilting pads bearing details used in this work, for these basic dimensions add three values of (L/D) ratios and eccentricity ratios.

The Reynolds equation is the governing equation for the distribution of pressure in dynamically loaded bearings. A local coordinate system for Reynolds equation can be set up in the region between the pad bearing and the journal surfaces.

$$\frac{1}{6\mu} \left[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial P}{\partial x} \right) \right] + \frac{1}{6\mu} \left[ \frac{\partial}{\partial z} \left( h^3 \frac{\partial P}{\partial z} \right) \right] = u \frac{\partial h}{\partial x} + 2v \dots (1)$$

$$u = \omega r \frac{de}{dt} + \sin\theta - e \frac{d\phi}{dt} \cos\theta \dots (2)$$

$$v = \omega \cdot r \frac{\partial h}{\partial x} + \frac{de}{dt} \cos\theta + e \frac{d\phi}{dt} \dots (3)$$

Where:

$u$  = the surface velocity of sliding surface.

$v$  = the radial velocity of the journal surface causing squeeze film action in direction opposite to the Y direction.

$\frac{de}{dt}$  = the radial component of the instantaneous journal center velocity .

$\frac{d\phi}{dt}$  = the tangential component of the instantaneous journal center velocity .

The motion of journal center in radial direction ( $\frac{de}{dt}$ ) is more than the motion in tangential direction ( $\frac{d\phi}{dt}$ ).

Now, estimated the value of oil film thickness equation for pad in spherical pivot type tilting pad bearing,[3].

$$h_{oil} = C_p - C_p.P.F.\cos(\theta - \theta_p) + (r + t_p)d.\sin(\theta - \theta_p) + e.\cos(\theta - \theta_c)$$

$$\frac{1}{6\mu} \left[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial P}{\partial x} \right) \right] + \frac{1}{6\mu} \left[ \frac{\partial}{\partial z} \left( h^3 \frac{\partial P}{\partial z} \right) \right] = \left[ \omega r + \frac{de}{dt} \sin\theta \right] \frac{\partial h}{\partial x} + 2\omega r \frac{\partial h}{\partial x}$$

..... (4)

The pressure distribution for dynamically loaded journal bearing can be obtained by solving the Reynolds equation (4) using finite difference method. During the operation of journal bearing system, the center of journal move at an arbitrary position inside the eccentricity circle from the initial position and let time pass until the journal center position is steady. Mobility method is used to study the effect of this motion on journal bearing behavior by mobility map that is the loci of familiar attitude-eccentricity curves.

The major equation for predicting the motion of journal center is introduced by [4]:

$$\frac{de}{dt} = \frac{WM (C_r/R)^3}{2\mu L} + \omega_{av} \times e \quad \dots (5)$$

When the mobility vector (M), initial position vector (e) and the load (W) are known, it is easy to obtain (e) in current time step from equation (5). Hence, equation (5) can be written in matrix form as follow:

$$\frac{d}{dt} \begin{bmatrix} e_x \\ e_y \end{bmatrix} = \frac{WC_r^3}{2\mu\mu L^3} \begin{bmatrix} M_x \\ M_y \end{bmatrix} + \omega_{av} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$

..... (6)

$$\frac{d}{dt} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} = \frac{WC_r^2}{2\mu\mu L^3} \begin{bmatrix} M_x \\ M_y \end{bmatrix} + \omega_{av} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix}$$

.....(7)

$$M = \sqrt{M_x^2 + M_y^2} \quad \dots (8)$$

Where:

**M** = mobility vector of journal center

**e** = the eccentricity vector

**W** = external load

$\omega_{av}$  = the average angular velocity for journal and bearing

Reynolds equation (4) can be solved over the domain of each pad surface in the new bearing to give the pressure distribution by using finite difference method as follows,[5].

$$\left[ \frac{\partial}{\partial x} \left( \frac{h^3}{\mu} \frac{\partial P}{\partial x} \right) \right] + \left[ \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial P}{\partial z} \right) \right] = \left[ \omega r + \frac{de}{dt} \sin\theta \right] \frac{\partial h}{\partial x} + 2\omega r \frac{\partial h}{\partial x}$$

..... (9)

The dimensionless friction coefficient (Df), defined as the total drag force on the journal surface in the bearing divided by the total load, is given by, [6]:

$$Df = \frac{\mu\omega D}{P_T} \frac{\pi^2(2+\varepsilon)}{Cr(1+\varepsilon)(1-\varepsilon^2)^{1/2}} \quad \dots (10)$$

$$P_T = \frac{1}{LD} \sum_{i=1}^{N_{pads}} W \quad \dots (11)$$

**Oil Flow Rates**

The bearing has a gap in the pad through which oil could leave the bearing. Therefore, the oil flow in direction, z and x must be estimated. Taking into account that the axial length to circumferential width ratio of each pad. It is expected that the circumferential oil flow will be considerably larger than the axial oil flow, particularly when the journal speed is high,[7].

$$q_x = \sum_{i=1}^{N_{pads}} \int_0^L \int_0^h [u \, dy \, dz]_{pad} \quad \dots(12)$$

The oil flow rate in the Z-direction,  $q_z$  can be calculated using the same principles:

$$q_z = \sum_{i=1}^{N_{pads}} \int_0^h w \, dy \, r \, d\theta \quad \dots (13)$$

$$Q = q_z + q_x \quad \dots (14)$$

**Dynamic Coefficients of Bearing**

Dynamic coefficients of journal bearing play a key role in determining the rotor-dynamic characteristics of rotating machinery.  $(e_\epsilon)$  and  $(e_\phi)$  are the unit vectors in the radial and tangential directions respectively. Referring to the fluid force ( $f$ ) is [8]:

$$f = f_\epsilon e_\epsilon + f_\phi e_\phi = [e_\epsilon \ e_\phi] \begin{bmatrix} f_\epsilon \\ f_\phi \end{bmatrix} \dots(15)$$

Taylor series expansions of the radial component and tangential component of the fluid force ( $f$ ) separately as follows:

$$f_\epsilon = (f_\epsilon)_s + \begin{bmatrix} \frac{\partial f_\epsilon}{\partial \epsilon} & \frac{\partial f_\epsilon}{\partial \phi} & \frac{\partial f_\epsilon}{\partial \xi} & \frac{\partial f_\epsilon}{\partial \zeta} \end{bmatrix} \begin{bmatrix} \Delta \epsilon \\ \Delta \phi \\ \Delta \xi \\ \Delta \zeta \end{bmatrix} \quad \dots (16)$$

$$f_\phi = (f_\phi)_s + \begin{bmatrix} \frac{\partial f_\phi}{\partial \epsilon} & \frac{\partial f_\phi}{\partial \phi} & \frac{\partial f_\phi}{\partial \xi} & \frac{\partial f_\phi}{\partial \zeta} \end{bmatrix} \begin{bmatrix} \Delta \epsilon \\ \Delta \phi \\ \Delta \xi \\ \Delta \zeta \end{bmatrix} \quad \dots (17)$$

The above equations can be written in the following form:

$$\begin{pmatrix} f_\epsilon - (f_\epsilon)_s \\ f_\phi - (f_\phi)_s \end{pmatrix} = \begin{bmatrix} \frac{\partial f_\epsilon}{\partial \epsilon} & \frac{\partial f_\epsilon}{\partial \phi} \\ \frac{\partial f_\phi}{\partial \epsilon} & \frac{\partial f_\phi}{\partial \phi} \end{bmatrix} \begin{pmatrix} C_r \Delta \epsilon \\ C_r \epsilon \Delta \phi \end{pmatrix} + \begin{bmatrix} \frac{\partial f_\epsilon}{\partial \xi} & \frac{\partial f_\epsilon}{\partial \zeta} \\ \frac{\partial f_\phi}{\partial \xi} & \frac{\partial f_\phi}{\partial \zeta} \end{bmatrix} \begin{pmatrix} C_r \Delta \xi \\ C_r \epsilon \Delta \zeta \end{pmatrix} \quad \dots(18)$$

The definition of the stiffness coefficients  $(K_{ij})$  and damping coefficients  $(C_{ij})$  from the Taylor series expansions .

$$\begin{pmatrix} f_\epsilon - (f_\epsilon)_s \\ f_\phi - (f_\phi)_s \end{pmatrix} = \begin{bmatrix} K_{\epsilon\epsilon} & K_{\epsilon\phi} \\ K_{\phi\epsilon} & K_{\phi\phi} \end{bmatrix} \begin{pmatrix} C_r \Delta \epsilon \\ C_r \epsilon \Delta \phi \end{pmatrix} + \begin{bmatrix} C_{\epsilon\xi} & C_{\epsilon\zeta} \\ C_{\phi\xi} & C_{\phi\zeta} \end{bmatrix} \begin{pmatrix} C_r \Delta \xi \\ C_r \epsilon \Delta \zeta \end{pmatrix} \quad \dots (19)$$

$$\begin{bmatrix} K_{\varepsilon\varepsilon} & K_{\varepsilon\phi} \\ K_{\phi\varepsilon} & K_{\phi\phi} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \mathbf{f}_\varepsilon}{C_r \partial \varepsilon} & \frac{\partial \mathbf{f}_\varepsilon}{\varepsilon C_r \partial \phi} - \frac{\mathbf{f}_\phi}{C_r \varepsilon} \\ \frac{\partial \mathbf{f}_\phi}{C_r \partial \varepsilon} & \frac{\partial \mathbf{f}_\phi}{\varepsilon C_r \partial \phi} + \frac{\mathbf{f}_\varepsilon}{C_r \varepsilon} \end{bmatrix} \dots(20)$$

$$\begin{bmatrix} C_{\varepsilon\varepsilon} & C_{\varepsilon\phi} \\ C_{\phi\varepsilon} & C_{\phi\phi} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \mathbf{f}_\varepsilon}{C_r \partial \varepsilon} & \frac{\partial \mathbf{f}_\varepsilon}{C_r \varepsilon \partial \phi} \\ \frac{\partial \mathbf{f}_\phi}{C_r \partial \varepsilon} & \frac{\partial \mathbf{f}_\phi}{C_r \varepsilon \partial \phi} \end{bmatrix} \dots(21)$$

$$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} K_{\varepsilon\varepsilon} & K_{\varepsilon\phi} \\ K_{\phi\varepsilon} & K_{\phi\phi} \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \dots(22)$$

$$\begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} C_{\varepsilon\varepsilon} & C_{\varepsilon\phi} \\ C_{\phi\varepsilon} & C_{\phi\phi} \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \dots(23)$$

**Discussion of Results**

The results of this paper are illustrated as follows. Figures (2), (3) and (4) show the effects of load line angle and the change in length – diameter ratios (L/D), for the case  $\beta = 0^\circ$ , L/D=0.2 as shown in figure (2) we found the maximum generated pressure 1085 N/m<sup>2</sup> and increase reached to 1230 N/m<sup>2</sup> when  $\beta = 5^\circ$  and the generated pressure increase reached to 15830 N/m<sup>2</sup> when  $\beta = 40^\circ$ . When comparison between Figures (2), (3) and (4), we found when length diameter ratios increased the generated pressure over each pads increased also. The large increment in generated pressure occurs at  $\beta = 0^\circ$  as shown in figures (3) and (4). Finally from these figures, the length-

diameter ratio (L/D=0.5) is better than another values.

Figure (5) shows the relation between variation of the total oil flow and the journal speed, when eccentricity (e) is equal 0.01 mm, the total oil flow will decrease gradually when speed is increased, the sensitiveness of the oil flow will extend the change in journal speed. when eccentricity (e) is equal 0.025 mm the total oil flow will increase when speed is increased but the amount of increase in total oil flow is higher. Finally when eccentricity (e) is equal 0.031mm the total oil flow will regularly increase with the increase of the journal speed.

Figure (6) shows when L/D =0.5, e =0.01 mm and  $\beta = 11.082^\circ$ , the value

of stiffness coefficients change with the load line angle ( $\beta$ ) and the sensitiveness of the  $K_{zx}$  will increase with the change of ( $\beta$ ), ( $K_{zx}$ )<sub>max</sub> at  $10^\circ$ , ( $K_{zz}$ )<sub>max</sub> at  $0^\circ$ , ( $K_{xz}$ )<sub>max</sub> at  $80^\circ$  and ( $K_{xx}$ )<sub>max</sub> at  $0^\circ$ . Figure (7) shows the relationship between the damping coefficients and the angle of load line ( $\beta$ ) when L/D =0.5. as shown ( $C_{xz} = C_{zx}$ )<sub>max</sub> at  $\beta = 22.5^\circ$ , ( $C_{xx}$ )<sub>max</sub> at  $\beta = 10^\circ$  and ( $C_{zz}$ )<sub>max</sub> at  $\beta = 0^\circ$ .

**Conclusions**

From the results of this work the following conclusions can be obtained:

- 1- Generated pressure increased because the length of bearing increases also to.
- 2- The load line angle at about  $0^\circ$ - $23^\circ$  give maximum damping coefficients, in this range of angles gives more stability of bearing performance, same behavior for stiffness coefficients when the load line angle at about 0-10 degrees.
- 3- The total oil flow consists of axial oil flow and circumferential oil flow but the

axial oil flow decreases with journal speed and circumferential oil flow increases with journal speed.

4- For moderate and high eccentricities the side leakage flow rate increases as the journal speed increases.

5- These results suggest that the load line angle at about  $0^{\circ}$ - $10^{\circ}$  gives maximum stiffness coefficients.

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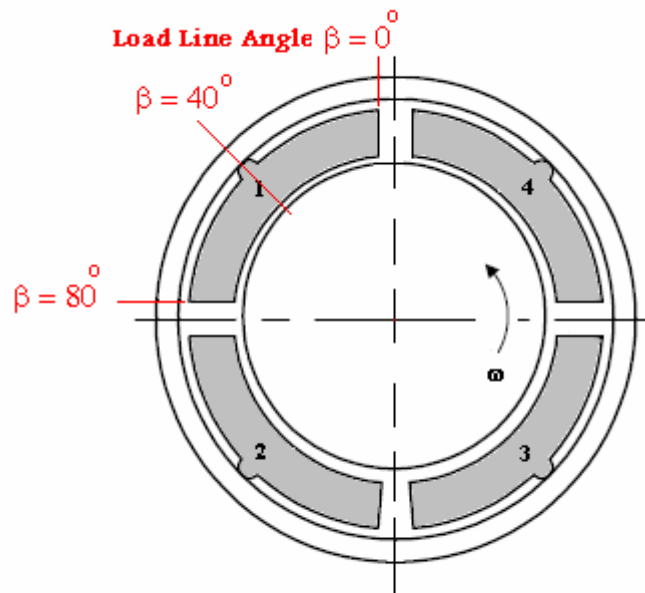
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**Table (1) Basic Dimensions of the Tilting Pad Bearing**

Parameter	Value
Number of pads	4
Radius of bearing	6.6 035 cm
Radius of journal	6.5 5 cm
Pad thickness	1.5 cm
Angular dimension of pad	80°
Pad tilt angle	0.43°
Preload factor	0.45
Bearing load	7 kN to 7.5 kN
Lubricant viscosity	0.04 N/m <sup>2</sup> .sec.
Journal speed	12000 r.p.m



**Figure (1) Tilting Pad Bearing Configuration**

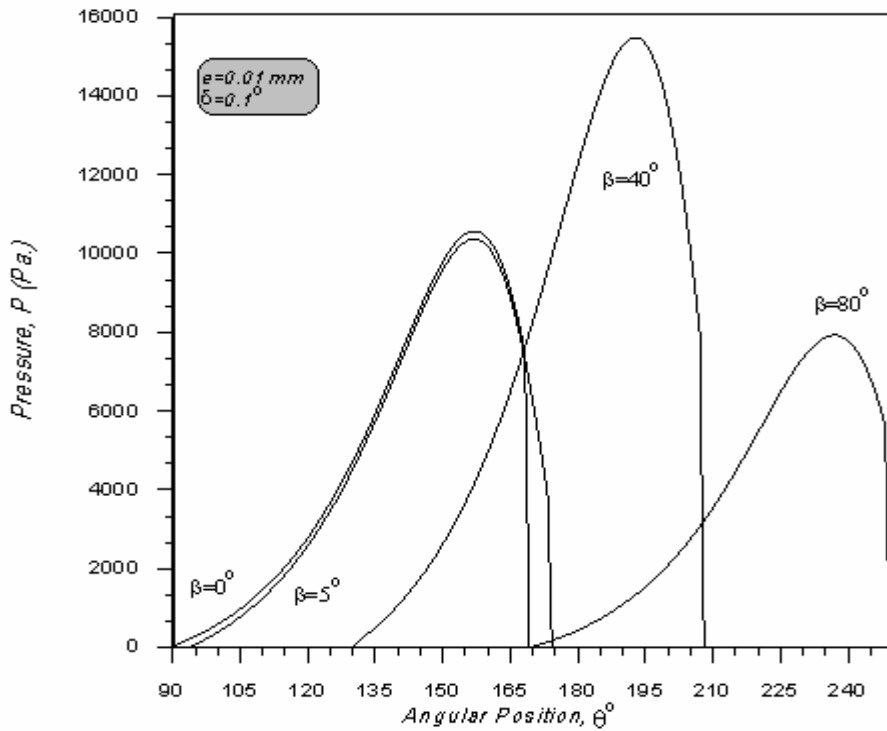


Figure (2) Change Pressure with Angular Position for versus Load Line Angle over Pad No.1 (L/D=0.2)

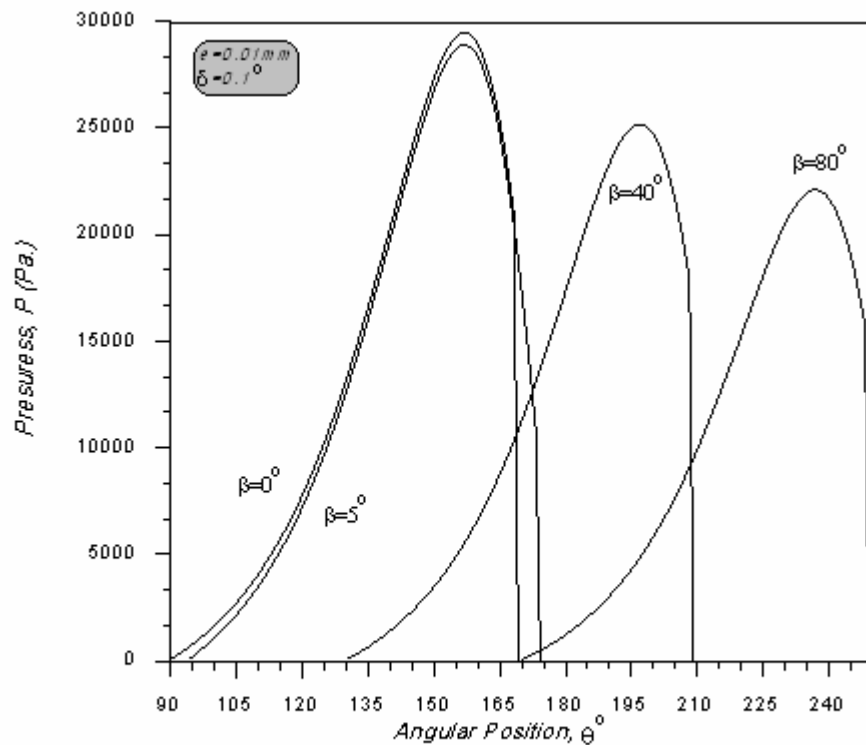


Figure (3) Change Pressure with Angular Position for versus Load Line Angle over Pad No.1 (L/D=0.5)



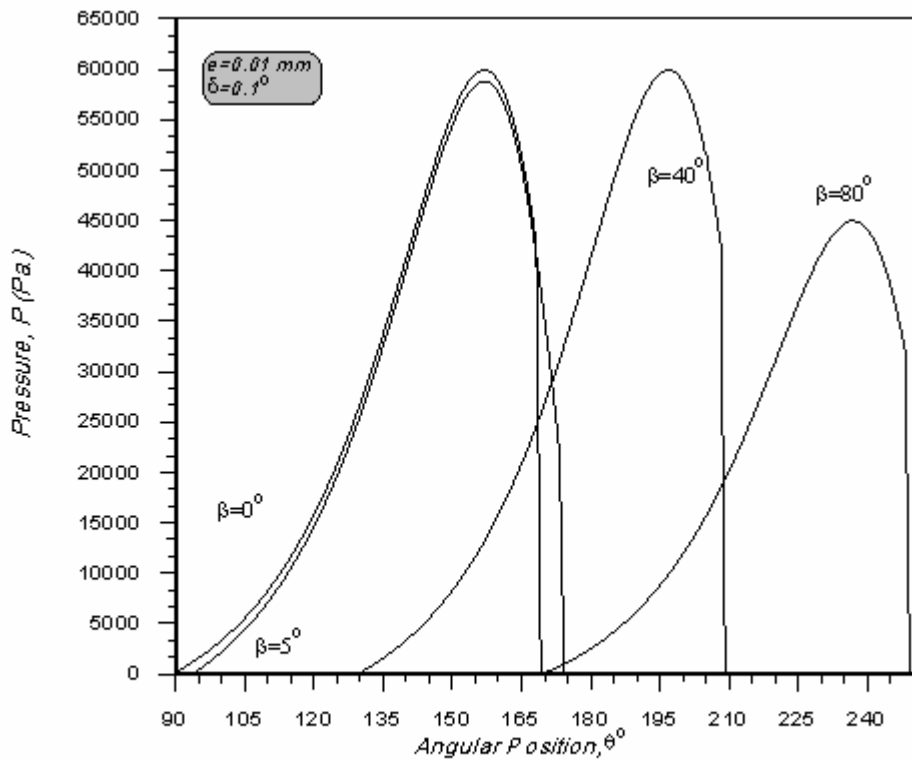


Figure (4) Change Pressure with Angular Position for versus Load Line Angle over Pad No.1 (L/D=1)

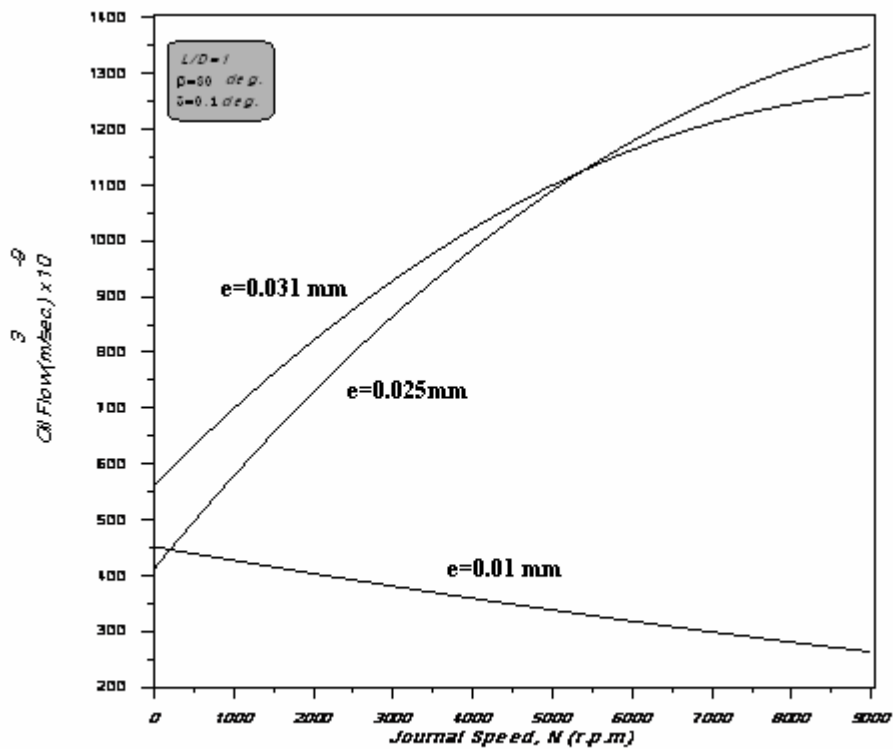


Figure (5) Change The Oil Flow with Journal Speed for Different Eccentricity values

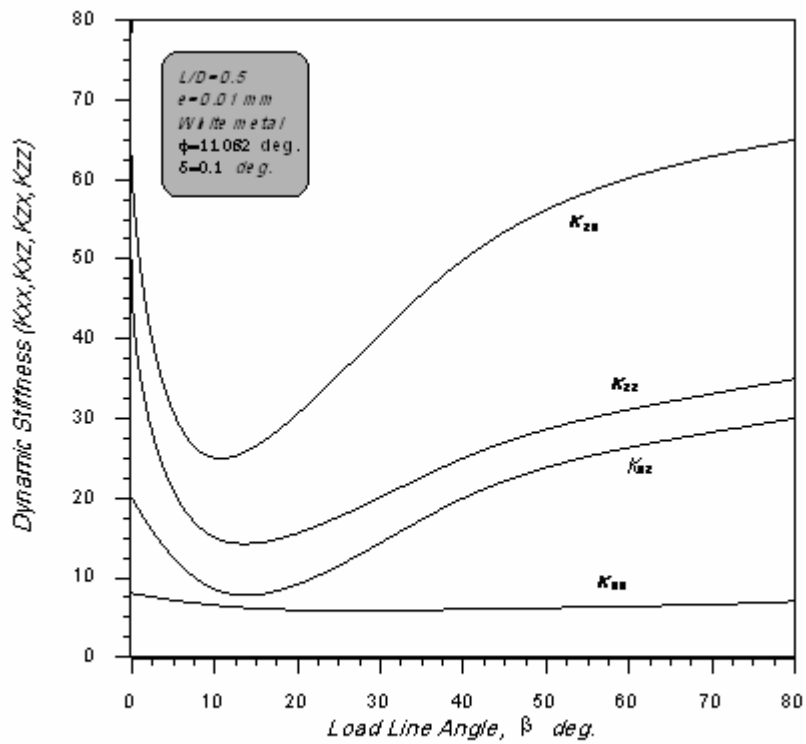


Figure (6) Effect of The Load Line Angle on The Dynamic Stiffness Coefficients

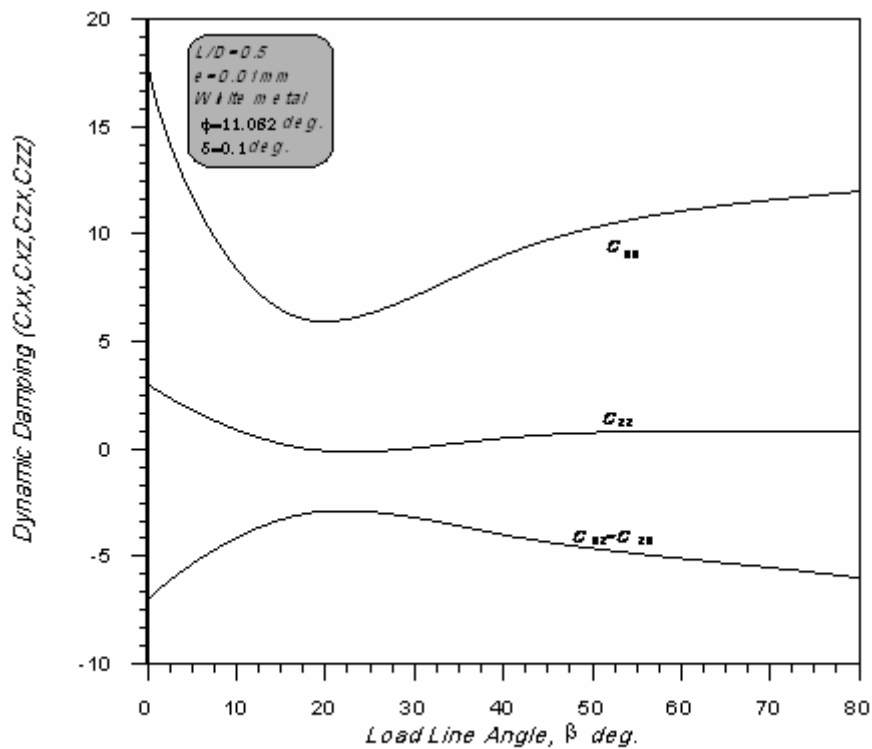


Figure (7) Effect of The Load Line Angle on The Dynamic Damping Coefficients