Eng. & Tech. Journal, Vol.28, No.15, 2010

New Short Term Planning and Scheduling Mathematical Model for Flexible Batch Manufacturing Systems

Dr. Alla El-Din H. Kassam* & Maha Naji Ali**

Received on: 3/6/2009
Accepted on: 6/5/2010

Abstract

In this paper a new approach is developed to tackle a real problem for short term production planning and scheduling work shop. A comprehensive binary mathematical model for a flexible batch manufacturing system based on the JIT philosophy and the group technology is developed. Each job is consisting of a batch of homogeneous parts and the ready time with the due date is determined. There are a number of machines in each work station process different jobs and the setup time is independent on the sequence of processing.

There are nineteen constraints imposed on the formulation of the model. Some of these constraints are relating to the number of tools available and the time required for each job not to be exceed from what was specified. The objective function is composed of three main components which expressed as a function of the profit gained by production of each job, tardiness penalty cost, and setup penalty cost. The objective function is maximized such that the production of neither one of the jobs exceeds its demand, and also the available processing time and the tool magazine capacity at each work station are not exceeded. The developed model is tested by example which shows the effects of all model parameters and constraints. WinQSB software is used to implement the mathematical model.

This research can be considered as the first attempt approach to solve a real short term planning and scheduling problem facing the advanced workshops for Flexible Manufacturing Systems.

نموذج رياضي جديد للتخطيط والجدولة قصير الامد لانظمة التصنيع المرنة

الخلاصة

في هذا البحث تم تطوير وبناء اسلوب جديد لمعالجة المشكلة الحقيقية التي تواجه المعامل في جدولة تخطيط الانتاج القصير الامد للانظمة الانتاجية الحديثة. حيث تم تصميم نموذج رياضي ثنائي شامل يستخدم في انظمة تصنيع الدفعات بالاعتماد على فلسفة JIT و نظام Group Technology. في هذا الموديل كل امر عمل يتكون من عدد من الاجزاء المتشابهة اي ان كل امر عمل يتألف من اكثر من قطعة واحدة, بالاضافة الى تحديد وقت التسليم ووقت الجاهزية له. وفي كل محطة عمل يوجد عدد من المكائن لها القدرة على تشغيل مختلف الاعمال والتي تتطلب ان يكون لها وقت تهيئة مستقل وقد حدد اربعة عشر قيد يوثر على دالة الهدف حيث ان قسم من هذه القيود تحدد العدد من الاحتياج الى العدد المطلوبة, وكذلك تحدد وقت التشغيل المطلوب لكل عملية انتاجية على كل ماكنة.ان دالة الهدف في هذا النموذج الرياضي تتألف من ثلاثة مكونات رئيسية وتشمل مكونات

*Production and Metallurgy Department, University of Technology/ Baghdad
** Applied Science Department, University of Technology/ Baghdad

https://doi.org/10.30684/etj.28.15.5
2412-0758/University of Technology-Iraq, Baghdad, Iraq
This is an open access article under the CC BY 4.0 license http://creativecommons.org/licenses/by/4.0

الكلفة والتي يعبر عنها كدالة الى الربح الناتج من إنتاج كل أمر عمل, وكلفة الخسارة (الجزاء) بسبب التاخر في اكمال انتاج امر العمل وكلفة الخسارة الناتجة من تهيئة الماكنة. وعليه فان دالة الهدف تهدف الى تعضيم الربحية بحيث ان القيود لانتاج كل امر عمل تحدد بكمية إنتاج لا تزيد عن الكمية المطلوبة و وقت التشغيل المتاح والسعة المحددة للادوات المطلوبة في كل محطة عمل. وقد تم اختبار الموديل الرياضي على مثال يوضح استخدام القيود والعوامل الموثرة عليه. واستخدمت برامجية WinQSB الجاهزة في تطبيق النموذج الرياضي.ويعتبر هذا البحث اول محاولة علمية لمعالجة مشكلة حقيقية في تخطيط وجدولة الانتاج ذات المدى القصير في الورش الصناعية الحديثة التي يتم فيها استخدام فلسفة JIT و Group Technology في انظمة التصنيع المرنة. جسيم صغير و/أو خفيف يكون غير قادر على اختراق الفقاعة ويقذف بعد التصادم. وجسيم متوسط يستطيع اختراق الفقاعة ولكن قد لا يكسرها.. وكل هذه الظواهر كانت متوقعة للموديل المقترح.

An Overview

In general a manufacturing system consist of sets of work stations, loading and unloading stations, a material handling system and an inventory system. Each work station consisting of set processors (e.g. direct numerically controlled machines, robots, etc.) machine tools and possibly tool magazine. The material handling system could consist of transporters (e.g. an automatic guided vehicle system, forklifts, cranes, etc.) and line or closed loop conveyors (e.g. belt or chain conveyors).

The inventory system could consist of the pre processed and post processed inventories of raw materials, semi finished products and finished products (Sliver et al., 1998). In modern manufacturing systems, different components of the system are often integrated to facilitate the flows of parts from one location to another, such system are called integrated manufacturing system (Lee, 1993).

A flexible manufacturing system (FMS) is an integrated manufacturing system operating under a central computer control system. Such systems start to become more famous than the classical one. FMSs are often adopted in low to medium volume manufacturing environments, but they can also be implemented in high volume manufacturing systems (Foulds and Wilson, 2002, Zygmont, 1986).

Creation of a computer support system for FMS is a complex and difficult task. Often decision support systems are suggested to assist operator’s of the system to respond to its operational problems in real time. Development of an appropriate support system for a FMS requires that the hierarchical structure of the decision makes process be initially designed, and then appropriate decision models for addressing variety of related operational issues be developed (Nof, et al, 1980, Suri and Hildebrant, 1984).

PDF created with pdfFactory Pro trial version www.pdffactory.com

The basic idea of the Just in time (JIT) manufacturing philosophy is to eliminate the inventory of processed products. For this purpose, diversified products are manufactured just in time to meet the demand of the markets. A ticketing system which is known as Kanban is used in JIT to keep track of the orders and the components of the orders which ought to be manufactured. The work in process inventory can not be greater than the actual number of Kanban. The inventory of manufactured products can be eliminated, as each completed job is already for a known customer (Sly, 1995).

The main focuses of Group Technology are on selecting the families of parts which can be grouped together for processing at compatible work stations, and identify the sets of compatible machines at each work station. Moreover the layout of the machines at each work station can be identified (Smed and Johnson 1999, Ham, Hitomi, and Yoshida, 1985)

Problem Description

As the equipments and machines are becomes more complicated and the market competition increased causes a scheduler facing a real dynamic problem as the situation inside the workshop changes and he must response quickly to such dynamic environments.

The problem description is based on actual nowadays workshop manufacturing systems and can be outlined as follows:

There are *n* jobs each one is a batch of homogeneous parts, the due date for each job is specified by customer. The ready time of each job can be determined. Each job can be split among all the compatible work stations which are individually capable of processing all of its necessary stages of operation. Each job upon its completion is delivered to the customer so that, the formation of the inventory of processed products is not allowed. Job sequencing is specified using the following sequencing dispatching order:

a. A job with an earlier due date is processed first at each work centre.
b. Among all jobs with a common due date, a job with an earlier ready time is considered first.
c. All jobs with a common ready time are arbitrarily sequenced.

Each work station could have more than one machine, may process each one of the jobs, may require a sequence independent setup time prior to performing a specified set of operations on each job and may have a limited tool magazine capacity. Furthermore it is assumed that, each work station is designed based on the group technology concept. Hence, there should exist at least a work station with a set of compatible machines which are collectively

PDF created with pdfFactory Pro trial version www.pdffactory.com

capable of processing all of the necessary stages of operation of each job. So, instead of keeping track of the processing times for all the necessary stages of operation of each job at each compatible work station, the only necessary information is the estimate of the total required processing time of each job at each compatible work station.

In addition, each work station can not be used more than a specified length of time. Also, the number of each type of required tools for each job at each work station and maximum number of each type of available tools at each work station can be specified. The maximum and the minimum acceptable batch sizes for each job at each work station may be specified due to the capacity limitation of the material handling system and the size limitation of the inventory of the semi finished jobs at each work station.

The production of each job may not exceed its demand. This will help the model to get optimal feasible solution. But the resulted solution may indicate that, some jobs should either not be produced because they require additional unavailable resources, or if they are produced then their produced quantity are less than the demands.

A remarkable point is raised here, i.e. there are two different selling prices introduced, one is associated with demand ordered and the other is for market price. The reason is that if manufacturing system is not fully utilized during the idle times of machines different products can be manufactured with assorted batch sizes which can be sold at the market price different from that of ordered price. This will eliminate the idle times of machines and system utilization is increased.

The following costs and profit components are identified:

1. The cost of processing each unit of each job at each work station.
2. The tardiness penalty cost which results from the completion time exceeds its due date.
3. The penalty cost associated with setting up each work station.
4. The market price of each unit of each job.
5. The extra value gained by delivering a complete job to its respective customer, in addition to the market value of the same job.

The Developed Model

The following notations are used for parameters and variables of the developed model:

J_{ijk}	***Job i with the k^{th} due date to be processed at the j^{th} work station***
N_{ik}	***No. of units in job i with k^{th} due date***
A_{ik}	***Availability time of job i with k^{th} due date***
T_{ij}	***Required Time to perform all of the necessary stages of operation of each unit of job i at the j^{th} work station***
D_{ik}	***K^{th} due date of job i***
S_{ij}	***Setup time for j^{th} work station to process job i***

PDF created with pdfFactory Pro trial version www.pdffactory.com

L_j	*Maximum allowable length of time for using work station j*
F_{jt}	*Total no. of available fixtures of type t at the j^{th} work station*
R_{ijt}	*No. of fixtures of type t required for processing job i at the j^{th} work station*
V_{ik}	*Extra value gained by producing i^{th} job with the k^{th} due date if its production equals its demand*
$\mathcal{V}_i$	*Market price of each unit of job i*
C_{ij}	*Cost of producing one unit of job i at the j^{th} work station*
$\check{C}_{ij}$	*Penalty cost for setting up the j^{th} work station for processing i^{th} job*
$\acute{C}_{ik}$	*Penalty cost for the resulted tardiness of i^{th} job with k^{th} due date*
α_{ijk}	*A sufficient large constant(e.g. $=N_{ik}$)*
β_{ijk}	*A sufficient small constant (e.g. =0)*
Φ_{ik}	*A sufficient large constant(e.g. $=D_{ik}$)*
Ψ_{ik}	*A sufficient large constant(e.g. $=N_{ik}$)*
X_{ijk}	*No of units of job i with k^{th} due date to be produced at j^{th} work station*
Y_{ijk}	*Proportion of units of job i with the k^{th} due date which are routed to j^{th} work station*
Z	*Total net operational profit*
E_{ijk}	*Completion time of job i with k^{th} due date at the j^{th} work station*
η_{ijk}	*Binary decision variable*

ϵ_{ik}	*Binary decision variable*
Ω_{ik}	*Binary decision variable*

The mathematical model developed is formulated as a mixed binary linear programming model (Taha, 2008). The objective function is in a maximization form of the following four costs terms,

i. The profit gained by production of each unit of job *i* with the k^{th} due date from the market prices is,

$$\sum_{i=1}^{n} \mathcal{V}_i \sum_{j=1}^{m} \sum_{k=1}^{K} X_{ijk} - \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} \sum_{k=1}^{K} X_{ijk} \quad ...(1)$$

ii. Additional amount gained by producing those jobs in which production are equal to their demand, and this is achieved by introducing binary decision variable ϵ_{ik}, as follows:

$$\sum_{i=1}^{n} \sum_{k=1}^{K} V_{ik}\,(1-\epsilon_{ik}) \quad ...(2)$$

iii. The penalty costs for those tardy jobs are represented by the following term with the binary decision variable Ω_{ik}

$$\sum_{i=1}^{n} \sum_{k=1}^{K} \acute{C}_{ik}\,\Omega_{ik} \quad ...(3)$$

iv. The setup penalty costs are

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \check{C}_{ij} \sum_{k=1}^{K} \eta_{ijk} \quad ...(4)$$

Where the following indices are used:

i for job from 1 to n

PDF created with pdfFactory Pro trial version www.pdffactory.com

j for work station from 1 to m
k for due date from 1 to K

Then the objective function with the above four cost terms can be represented as follows:

Maximize ***Z***

$$\sum_{i=1}^{n} V_i \sum_{j=1}^{m} \sum_{k=1}^{K} X_{ijk} + \sum_{i=1}^{n} \sum_{k=1}^{K} V_{ik} (1- \epsilon_{ik}) - \sum_{i=1}^{n} \sum_{k=1}^{K} \mathrm{C}_{ik} \Omega_{ik} - \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} \sum_{k=1}^{K} X_{ijk} - \sum_{i=1}^{n} \sum_{j=1}^{m} \check{C}_{ij} \sum_{k=1}^{K} \eta_{ijk} \quad ...(5)$$

The objective function is subject to the following constraints:

1) The production of any job can not exceed its demand. The reason for not wanting the production of each job necessarily being equal to its demand is that, the FMS has a limited processing capability and the equality constraint my result in infeasibility of achieving the targeted production levels, is represented as follows:

$$\sum_{j=1}^{m} X_{ijk} \le N_{ik} \quad \text{for } i = 1 \text{ to } n,\ k=1 \text{ to } K \quad ...(6)$$

2) To keep track of the jobs in which their production are not equal to their demand. This achieved by introducing ϵ_{ik} binary decision variable.

$$N_{ik} - \sum_{j=1}^{m} X_{ijk} \le \Psi_{ik}\, \epsilon_{ik} \quad \text{for } i = 1 \text{ to } n,\ k=1 \text{ to } K \quad ...(7)$$

3) To ensure that the j^{th} work station is not used more than the maximum allowable length of time for using work station *j* (L_j) units of time which may be used also to consider the available working time at each work station, and can also implicitly be used to balance the allocated load at each work station during the planning horizon, the following constraints is used:

$$\sum_{i=1}^{n} \sum_{k=1}^{K} (T_{ij} X_{ijk} + S_{ij}\ \eta_{ijk}) \le L_j \quad \text{for } j = 1 \text{ to } m \quad ...(8)$$

4) To guarantee that there are sufficient number of tools available for processing the jobs which are allocated to each work station. This is formulated as follows:

$$R_{ijt}\ \eta_{ijk} \le F_{jt} \quad ...(9)$$

For i=1 to n, j=1 to m, k=1 to K,
Where t for tool type from 1 to T

5) The following constraints (10-15) are particularly important because they incorporate the scheduling aspects of the problem into the model. Here the basic idea is to identify those jobs in which their completion times exceed their due dates, and then associate a penalty cost for those tardy jobs in the objective function. This is achieved by finding the completion time of each portion of each job at

PDF created with pdfFactory Pro trial version www.pdffactory.com

each work station. For this purpose the following constraints are defined:

$E_{ij1} - (T_{ij}X_{ij1} + S_{ij}\,\eta_{ij1}) \geq A_{i1}$ for $i=1$ to n, $j=1$ to m ...(10)

$E_{ij1} - (T_{ij}X_{ij1} + S_{ij}\,\eta_{ij1}) \geq E_{i-1,j,1}$ for $i=2$ to n, $j=1$ to m ...(11)

$E_{1jk} - (T_{1j}X_{1jk} + S_{1j}\,\eta_{1jk}) \geq A_{1k}$ for $j=1$ to m, $k=2$ to K ...(12)

$E_{1jk} - (T_{1j}X_{1jk} + S_{1j}\,\eta_{1jk}) \geq E_{n,j,k-1}$ for $j=1$ to m, $k=2$ to K ...(13)

$E_{ijk} - (T_{ij}X_{ijk} + S_{ij}\,\eta_{ijk}) \geq A_{ik}$ for $i=2$ to n, $j=1$ to m, $k=2$ to$K$ ...(14)

$E_{ijk} - (T_{ij}X_{ijk} + S_{ij}\,\eta_{ijk}) \geq A_{i-1,j,k}$ for $i=2$ to n, $j=1$ to m, $k=2$ to$K$ ...(15)

6) To keep track of the tardy jobs, by using the constraint below which gives the reason for introducing the binary decision variable (Ω_{ik}):

$Eijk - Dik \leq \Phi ik\ \Omega ik$ ...(16)

7) To ensure that if and only if the *jth* work station is used for production of the i^{th} job with k^{th} due date, then the required associated setup time at thej^{th} work station is taken into consideration in constraint:

$B_{ijk}\,\eta_{ijk} \leq X_{ijk} \leq \alpha_{ijk}\,\eta_{ijk}$ ...(17)
for $i=1$ to n, $j=1$ to m, $k=1$ to K

Also by appropriate selection of (α_{ijk}) could be used to implicitly take the size of the pre processed inventory of parts at each work station into consideration. Furthermore the value of (B_{ijk}) can appropriately be selected to avoid allocation of unwanted small batches at each work station. (B_{ijk}) can also be used to implicitly consider the limitations of the material handling system.

8) Non negativity and binary constraints are introduced:

$Ei,j,k \geq 0$ ...(18)
$\eta_{ijk}, \epsilon_{ik}, \Omega_{ik} = 0$ or 1 ...(19)

This model is a mixed binary linear programming and can be solved using WINQSB (Lawrence and Pasternack, 1998) software

Implementation

An example is represented to provide more insight for the model. Consider a FMS consisting of two work station, and there are four orders for production of two batches of different products with two distinct due dates. At each work station there are a limited number of tools available for production of each type of product. The values of all the required parameters are arbitrarily specified it Table 1.

For demonstration of the model three values of ready time A_{ik} are used. These times are (0,120,240).

The net profit Z and the variables quantities X_{ijk} produced from solving the mathematical model are given in Table 2:

Referring to Table 1 and 2 which shows that by increasing A_{i2} from 0 to 120 only the starting time of j_{112} has changed from 100 to 120 while the net profit remained unchanged. But by changing A_{i2} from 0 to 240 in addition to the change of schedule, the net profit decreased from 3792 to 2494. This is due to $J_{2.2}$ not produced.

PDF created with pdfFactory Pro trial version www.pdffactory.com

.

Advantages of the Developed Model

A short term planning and scheduling model is developed to identify among potential orders for different products, which orders to manufacture and in what quantities. The solution to the model gives the operational schedule for efficient utilization of the capacity of the manufacturing system, such that the profitability of the total operation during the planning horizon is maximized. Then the developed model based on the characteristic is achieved to integrate several aspects of the operational planning and scheduling of such manufacturing systems.

The developed model can be used as a decision model for selecting among the received orders which order to manufacture and in what quantities, such that the profitability of the operation will be maximized during the planning horizon.

Furthermore, the model can be used to simultaneously study the effects of scheduling, loading, routing, dispatching, material handling, and also the size of the local pre processes inventory of semi finished parts at each work station, setup times, ready times, lead times, marginal selling prices, marginal processing costs, the tardiness penalty costs, and the setup penalty costs on the total profitability of the system during the planning horizon.

These advantages can assist the scheduler to do his job efficiently, effectively and economically and more accurate. So that, it can be considered as a decision model can be used by the scheduler in every minutes of his work. And to tackle the problems raised by the dynamic situations facing the modern industries and to cope more reliable with such cases which enables him to withstand against the wind of globally competition in the market.

Conclusions

A comprehensive model for short term production planning and scheduling of a flexible batch manufacturing system is developed. The basic idea of this model was based on the JIT manufacturing philosophy for low to medium production systems as well to high volume.

It is possible that if the flexible batch manufacturing system is designed based on the group technology concept, it is reasonable to assume the setup times are significantly small, and can be disregarded.

Another significant point can be raised here is that even though the sequencing dispatching rules must be specified in but those rules depending on the specific applications may be enhanced and be differently restated i.e. sequencing rules other than those which are used in the model which enables to consider the effect of different sequencing rules on the net profit of the operation during the planning horizon.

The developed model can be also used as an approximation decision making tool for studying the effects of various interactions which exist among different operational aspects of other

PDF created with pdfFactory Pro trial version www.pdffactory.com

similar manufacturing systems on their overall profitability.

References

[1] Foulds, L.R., and Wilson, J.M., (2002) "Approaches to the General Cell Formation Problem", Business School Research Series , ISBN 185901178, Loughborough University.

[2] Ham, I., Hitomi, K., and Yoshida, T., (1985) "Group Technology: Applications to Production Management", Int. Series in Management Science/ Operations Research, Kluwer-Nijhoff Publishing.

[3] Lawrence, J. JR., and Pasternack, B. A., (1998) "Applied Management Science", John Wiley & Sons. Inc.

[4] Lee, C. Y., (1993) "A recent Development of the Integrated Manufacturing System: A Hybrid of MRP and JIT", Int. Journal of Operations and Production Management, Vol. 13, No. 4, pp.3-17.

[5] Nof, S., Whinston, A. B., and Bullers, W.I., (1980) "Control and Decision Support in Automatic Manufacturing Systems", AIIE Transaction, vol. 12, No. 2, pp.156-167.

[6] Silver, E. A., Pyke, D. F. and Peterson, R., (1998) "Inventory Management and Production Planning and Scheduling", John Wiley & Sons Inc.

[7] Sly. D. (1995) "Computerized Facilities Design and Management: An Overview", IIE Solutions, August, pp. 43-51.

[8] Smed, J., and Johnson, M., (1999) "Group Technology in Electronic Assembly", Turku Centre for Computer Science and Department of Mathematical Science, University of Turku, FIN-20014, Finland.

[9] Suri, R., and Hildebrant, R. P., (1984) "Modelling Flexible Manufacturing Systems Using Value Analysis", SME Journal of Manufacturing Systems 3, pp. 27-38.

[10] Taha, H. A., (2008) "Operations Research: An Introduction", Eighth Edition, Macmillan Publishing Company.

[11] Zygmont, J., (1986) "Flexible Manufacturing Systems: Curing the Cure-All", High Technology 6, pp. 22-27

PDF created with pdfFactory Pro trial version www.pdffactory.com

.

Table 1: The input parameters

Indices *i j k t*	*No. Of units* N_{ik}	*Required time* T_{ij}	*Setup time* S_{ij}	*Max time* T_j	*Total no. Tools* F_{it}	*No. Tools req* R_{ijt}	*Cost* C_{ij}	*Penalty cost* $\acute{C}_{ik}$	*Penalty cost* [illegible]$_{ij}$	*Market price* [illegible]$_i$	*extra value* V_{ik}	*Due date* D_{ik}
1 1 1 1	50	1	50	420	5	4	3	2	25	15	300	200
1 2 2 2	100	1	50	420	10	3	3	2	25	15	300	400
1 2 1 1	50	2	20	420	5	2	3	2	25	15	300	200
1 1 2 2	100	2	20	420	10	2	3	2	25	15	300	400
2 1 1 1	100	2	100	420	5	2	3	2	25	15	300	200
2 2 2 2	150	2	100	420	10	5	3	2	25	15	300	400
2 2 1 1	100	1	40	420	5	4	3	2	25	15	300	200
2 1 2 2	150	1	40	420	10	3	3	2	25	15	300	400

PDF created with pdfFactory Pro trial version www.pdffactory.com

.

Table 2: The output results

Index I	Ready time A_{i2}	Net profit Z	Quantity produced			
			X_{1j1}	X_{2j1}	X_{1j2}	X_{2j2}
1	0	3792	50	0	100	0
2	0		0	100	0	150
1	120	3792	50	0	100	0
2	120		0	100	0	150
1	240	2494	0	0	100	O
2	240		50	100	0	O

PDF created with pdfFactory Pro trial version www.pdffactory.com