EFFECT OF TAPERING OF RATIO ON GEOMETRICAL NONLINEARITY OF THIN STEEL PLATE UNDER TRANSVERSE LOAD

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ABSTRACT

The primary objective is to determine whether the structural efficiency of plates can be improved with variable thickness. The large displacement analysis of steel plate with variable thickness at x-direction is obtained numerically, using finite differences. The effects of boundary condition, tapering ratio, type of tapering equation and plate aspect ratio on large deflection behavior of rectangular plates are investigated. Numerical results for rectangular steel plate are presented for the different effects. This study showed that the large deflection behavior is very sensitive for thickness variation (tapering ratio) where the maximum deflection will increase about 5% for slenderness ratio (b/t=100) and tapering ratio (1-2) of simply supported plate.

KEYWORDS: **Finite differences, Large deflection, Nonlinear analysis, Rectangular plates, Tapered plates**

تأثير نسبة تغير السمك في أللاخطية الهندسية على الصفائح الفولاذية الرقيقة تحت تأثير الأحمال المستعرضة

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الخلاصة

الهدف الرئيسي للدراسة هو تحديد الا   الإنشائية للصفائح ذات السمك المتغير.تم الحصول على النتـــائج العدديــة الإزاحة الكبيرة لصفائح الفولاذية ذات السمك المتغير باتجاه المحور (x باستخدام طريقة الاختلافات المحددة. تم فحص تاثير ظروف الاسناد   تغير السمك نوع معادلة التغير ونسبة الابعاد على تصرف الانحراف الكبير في الـصفائح المـستطيلة. قدمت النتائج العددية في الصفائح الفولاذية المستطيلة بتاثيرات مختلفة. لقد اوضحت هذه الدراسة بــان تصرف الانحــراف الكبير حساس لتغير السمك (نسبة النحافة) بحيث يزداد اعلى انحراف بنسبة (5%) عند نسبة النحافة (b/t=100) ونسبة تغير السمك من(-) للصفائح ذات الإسناد البسيط.

NOTATIONS

a, b = Plate dimension in x and y- directions respectively.

c = clamped edge.

$c_t = (t_a - t_o)/a t_o$ =Slope coefficient of the Tapered plate.

$D = E t_o^3/12(1 - v^2)$ Modulus of Rigidity.

E =Modulus of Elasticity.

N_x, N_y, N_{xy} = In-plane forces (per unit width).

q =Transverse load (per unit area).

s = Simple supported edge.

t = Plate thickness.

t_a = Thickness at the side $x=a$.

t_{av} = Average thickness $((t_a+t_o)/2)$.

t_o = Thickness at the side $x=0$.

v =Poisson's ratio.

w = Out-of-plane displacement (or deflection).

INTRODUCTION

With the increasing use of rolled, machined, or them-milled skins in aircraft and missile designs, analyses of plates tapered in thickness are becoming the rule rather than the exception. Design data for the large displacement of such plates are limited[2]. Nonlinear structural problems usually fall into one of the following main categories: (a) large deflections; (b) finite strains; (c) nonlinear material properties; (d) deformation-dependent interactions between structural parts; (e) combination of nonlinear material behavior with one of the other categories.

The problem of geometrical nonlinearity is of considerable practical interest for aerospace engineers and naval architects. In civil engineering, hanging roofs, suspension bridges, etc. constitute the most important class of structures which display pronounced geometrical nonlinearities[2].

The function of a thin plate element is generally to withstand a distributed lateral pressure, or to act with the adjoining structure in sustaining in-plane forces, or both.

Although the equations of the large deflection behavior of plates were first derived by Von Karman, it is only through recent advances in the development of numerical methods that the general problem of plates has been treated satisfactorily. The early investigators used infinite double Fourier series (Levy[12]) and finite differences (Basu and Chapman[3]). They studied the large deflection behavior of thin plate under uniformly distributed loading. Aalami and Chapman[1] used the finite difference method to obtain solutions for a number of isolated plates under uniformly distributed loading and with simple boundary conditions. Ueda, et al.[4] studied the large deflection behavior of a rectangular plate by an efficient semi-analytic method. An incremental form of the governing differential equations of plates and stiffened plates with initial deflection had been derived. For each load increment, these equations were solved by Galerkin method with special consideration of simple supported boundaries. Recently, Jayachandran, et al.[5] derived incremental matrices for thin initially imperfect plates with a small out-of-flatness by using minimum potential energy principles. Explicit coefficients of the displacement gradient tensor had been evaluated. These matrices were used in combination with any thin plate element. The formulations were incorporated in software plot-cold.

FORMULATION AND SOLUTIONS

The basic concepts of three methods (or steps) for the solution of the large deflection behavior of plates together with a description of the boundary conditions of the cases examined are summarized in the following.

FORMULATION

Starting from the equilibrium of compatibility of a thin plate element and expressing the strains and curvatures as functions of the stress resultants, the following equations are presented[2]: -

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_x}{\partial x \partial y} + \frac{\partial^2 w_o}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 w_o}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y}$$

$$+ \frac{\partial^2 w_o}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} = 0 \qquad (1)$$

The stress-strain relationships become as follows: -

$$\varepsilon_x = \frac{(N_x - \nu N_y)}{Et_x}$$

$$\varepsilon_y = \frac{(N_y - \nu N_x)}{Et_x} \qquad (2)$$

$$\gamma_{xy} = \frac{2(1+\nu)N_{xy}}{Et_x}$$

where $t_x = t_o \left(1 + c_t x^n \right)$; in which $c_t = \left(t_a - t_o \right) / a t_o$; t_o and t_a denote the thickness at the sides $x = 0$ and $x = a$, n denote to type of equation.

After derivation of strains and substitute the derivations into Equation (1) and express it as a stress function resultant where the compatibility equation becomes:

$$\nabla^4 \Phi - F \left(\frac{\partial^3 \Phi}{\partial x^3} + \frac{\partial^3 \Phi}{\partial x \partial y^2} \right) + Z \left(\frac{\partial^2 \Phi}{\partial x^2} - \nu . \frac{\partial^2 \Phi}{\partial y^2} \right) = Et_x \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2$$

$$- Et_x \left(\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right) \qquad (3)$$

By similar algebraic steps it is possible to write the equilibrium equation in terms of w and , thus: -

$$D_{(x)} . \nabla^4 w + 2 \frac{\partial^2 D_{(x)}}{\partial x} \left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2} \right) + \frac{\partial^2 D_{(x)}}{\partial x^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) =$$

$$\left[q + \frac{\partial^2 \Phi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \Phi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 \Phi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right] \qquad (4)$$

Where

$$F = 3 c_t x^{(n-1)} / \left(1 + c_t x^n \right)^2$$

$$Z = 3 c_t x^{(n-2)} / \left(\left(1 + c_t x^n \right) \left((n-1) + (3n-1) c_t x^n \right) \right) \qquad (5)$$

And the variable Φ is Airy's stress function and is defined such that: -

$$N_x = \frac{\partial^2 \Phi}{\partial y^2}; \quad N_y = \frac{\partial^2 \Phi}{\partial x^2}; \quad N_{xy} = \frac{\partial^2 \Phi}{\partial x \partial y} \tag{6}$$

All the other quantities related to the bending and membrane actions of the plate can be expressed in terms of two variables (w) and (Φ)[2].

The boundary conditions considered for $x=0$, a, as shown in Figure (1), are as follows: -

1-Boundary on rigid supports

$$w = 0 \tag{7}$$

2- For rotationally free cases (on edge parallel to y-axis)

$$\frac{\partial^2 w}{\partial^2 x} = 0 \tag{8}$$

For rotationally fixed cases(on edge parallel to y-axis)

$$\frac{\partial w}{\partial x} = 0 \tag{9}$$

For membrane action cases of fully free is considered.

3-For fully free condition of zero membrane direct stress (on edge parallel to y-axis)

$$\frac{\partial^2 \Phi}{\partial^2 y} = 0 \tag{10}$$

Zero membrane shear stress (on edges parallel to y or x-axis)

$$\frac{\partial^2 \Phi}{\partial x \partial y} = 0 \tag{11}$$

Equations (10) and (11) refer to the boundary condition of an isolated free plate[2].

For solution, the plate is subdivided into a graded mesh. The grading is chosen to be fine under distributed loading and for the fixed boundary conditions for increased accuracy. Equations (3) and (4), together with the boundary conditions are expressed at the nodes in terms of central finite difference expressions, as shown in Figure(2).

The resulting equations may then be arranged into the following coupled matrix form: -

$$[A]\{\Phi\} = [B]\{w\} \tag{12}$$

$$[C]\{w\} - [D(\Phi)]\{w\} = \{q\} \tag{13}$$

in which $[A]$ = a square matrix with constant coefficients depending on Φ; $[B]$ = a square matrix depending on w; $[C]$ = a square matrix for bending effect obtained from the left hand side of Equation (13); $[D(\Phi)]$ = a square matrix depending on Φ; $\{w\}$ and $\{\Phi\}$ = column matrices of the unknown variables w and Φ, respectively; $\{q\}$ = the applied transverse loading. For numerical evaluation, it is convenient to rewrite Equation (10) as follows: -

$$\{\Phi\} = [A]^{-1}[B]\{w\} \tag{14}$$

Now for any specified conditions, Equations (12) and (13) can be solved for w and Φ using an iterative procedure. There are various schemes used for the solution of the present type of coupled equations. In the present study, the successive iteration procedure is considered.

The solution to be obtained for a given applied loading $\{q\}$ is achieved by assigning a value to Φ (which may be assumed equal to zero for first loading, and equal to the previous values for subsequent loading). Thus Equation (12) can be solved for $\{w\}$, from which $[B]$ is evaluated, and subsequently used in Equation (13) to get new values of Φ. The procedure is repeated until a desired degree of accuracy is reached. There are certain refinements in the iteration, which are employed to ensure a rapid convergence.

COMPUTER PROGRAMMING

A computer program was written by Amash [2] and developed to take the out of plane loading and multi equations of variable thickness. The program is written in FORTRAN 90 language.

RESULTS

All the solutions presented were obtained with a specified degree of accuracy (ε_r) of 0.1% in the iterative procedure. In most cases the plate was divided into 16 divisions in any direction. This was considered adequate for obtaining deflections which are accurate enough for practical purposes.

COMPARISON WITH OTHER WORK

Ideally it would be desirable to compare the theoretical predictions of the program with the results of carefully controlled experiments and other theoretical results.

The accuracy of the results of the present study in the analysis of real panels is compared with the theoretical results obtained by Levy [11] [1942] on simply supported panels. The numerical analysis of Levy based on the infinite double Fourier series for the non-linear analysis of general steel-plate.

In the present study, this plate is analyzed based on the prescribed procedure and it is divided into (16×16) divisions.

Figure (3) shows a comparison between the theoretical results of Levy's study and the present study for the out-of-plane displacements.

The curves shown relate the applied load ($qa^4 \big/ Et_{av}^4$) on the vertical side to the non-dimensional maximum deflection ($w \big/ t_{av}$) on the horizontal side. A similar format is adopted for all other figures. It is clear from this figure that good agreement between the results by the present method and the experimental results and the theoretical results is obtained for a simply supported thin plate.

APPLICATION AND DISCUSSION OF RESULTS
1-Simply Supported Tapered Plate under Transverse Uniform Load

Figure (4) presents the load-deflection curve of a square simply supported plate under transverse uniform load. The effect of a variation in thickness is considered. The values of tapering ratio are taken to be (t_a/t_o=1.0, 1.5, and 2.0). So, the modulus of elasticity for all plate in this study was taken equal to 200 GPa. This figure shows that the effect of tapering ratio on large deflection behavior is considerable when the applied load is increasing. The increase in the effect of tapering ratio will be more appearing when the effect of membrane action becomes more appearing.

2-Clamped Tapered Plate under Uniform Load

Figure (5) presents the load-deflection curve of a square thin plate with all edges clamped. This figure shows a comparison between the constant thickness and the variable thickness in perfect plate. The effect of variation in thickness is very clear on large deflection behavior of thin plates. This effect is appearing in the beginning of loading of plate by transverse uniform load. The effect

of material nonlinearity is neglected in the present study.   These results are compared for a plate with constant thickness (ordinary plate) and with average thickness which is equivalent thickness of ordinary plate.

3-Square Thin Tapered Plate with Two Edges Simply Supported and Other Edges Clamped

Figure (6) presents the load deflection curve of a square plate with two edges simply supported and other edges clamped and under transverse uniform load. This figure shows the effect of boundary condition and tapering ratio on the large deflection behavior of a square plate under uniform lateral load.

4-Effect of Aspect ratio (a/b) on the Large Displacement Analysis of Simply Supported Plate

Figures (7) and (8) presents the load-deflection curve of a rectangular simply supported thin plate under uniform lateral load with aspect ratios (a/b=2.0 and 0.5), respectively. These figures show the effect of tapering ratio on the large deflection behavior of a rectangular thin plate with increasing and decreasing in the length in x-direction (parallel to the variation in thickness).

From Figures (7) and (8), the following is noticed:

1- The values of the deflection of the thin plate with aspect ratio (a/b=2.0) and under uniform load are less than the values of the deflection of the thin plate with aspect ratio (a/b=0.5). This difference is due to the effect of tapering ratio on the plate where the deflection becomes larger for the plate with aspect ratio (a/b=0.5).

2- The effect of tapering ratio on large deflection behavior of a thin plate with aspect ratio (a/b=0.5) is more than the effect of tapering ratio on large deflection behavior of a thin plate with aspect ratio (a/b=2.0).

5-Effect of Order of Tapering Equation on the Large Displacement Analysis of Simply Supported Plate

Figures (9) presents the load-deflection curve of a rectangular simply supported thin plate under uniform lateral load with aspect ratios (a/b=1.0), slenderness ratio (b/t_{av}=100), tapering ratio (t_a/t_o=1.5) and varying values of tapering equation (n from 1-4). This figure shows the effect of tapering equation on the large displacement behavior of a rectangular thin plate. from this figure, can be noticed that the effect of tapering decrease with increasing the order of tapering equation.

CONCLUSIONS

A simplified computational procedure is used to study the large deflection analysis of a rectangular thin tapered plate under lateral uniform load. Approximate values can be obtained with a good accuracy when compared with other works. The effects of boundary condition, tapering ratio and plate aspect ratio on the large deflection behavior of rectangular thin tapered plates are studied and presented in graphs. The tapering ratios are taken to be (t_a/t_o=1.0,1.5,2.0). It is shown that the large deflection behavior is dependent on the tapering ratio. So the effect of order of tapering ratio shows that the deflection of steel plate increases with increase the order of tapering ratio. The effect of tapering ratio is dependent on plate aspect ratio. In the present study, the material nonlinearity, initial imperfection, free and rotationally boundary condition are not considered.

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(a) Plate with axes load.   **(b)** Plate cross section.
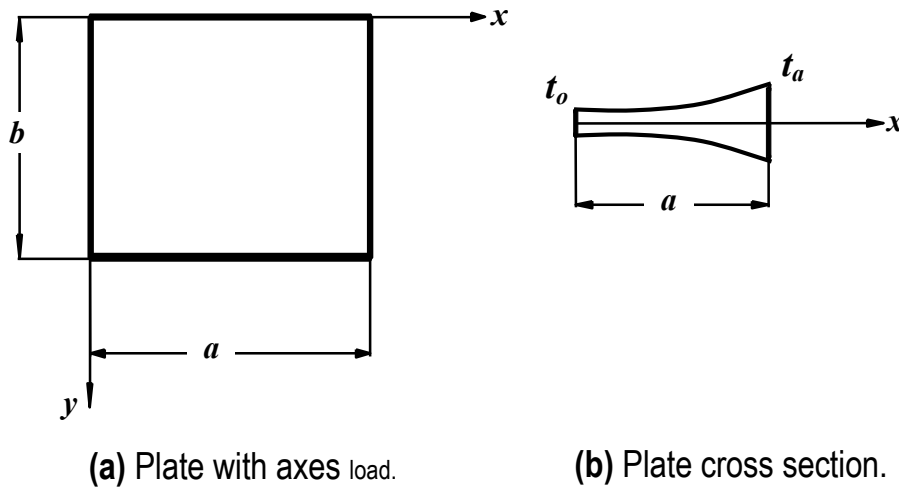
Figure (1): Rectangular thin tapered plate under distributed load
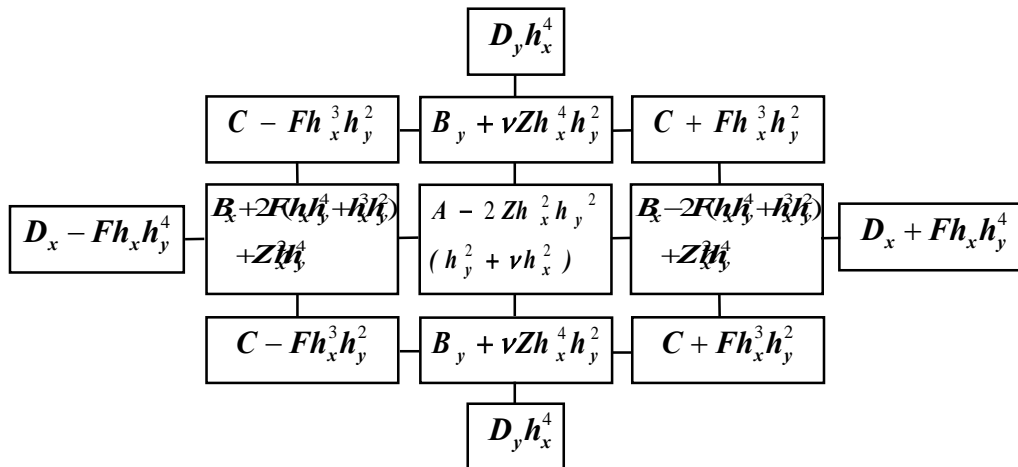
Figure (2): Plate equation in finite difference molecule form[2]
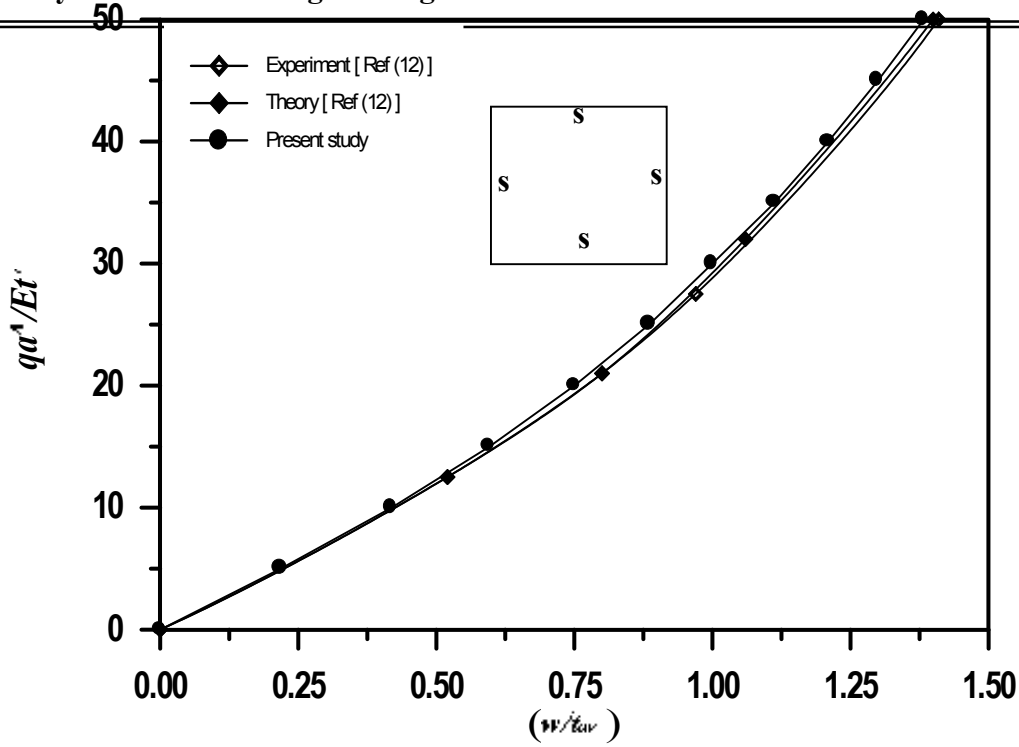
Figure (3): Central deflection of simply supported square plate versus uniform lateral load
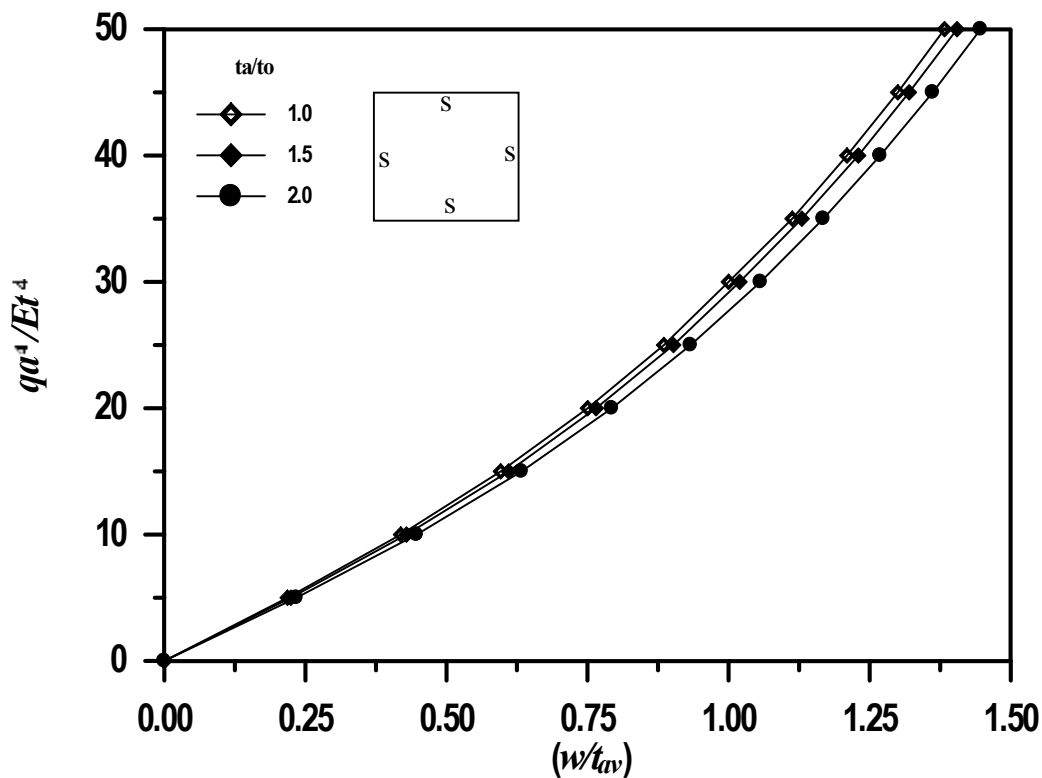

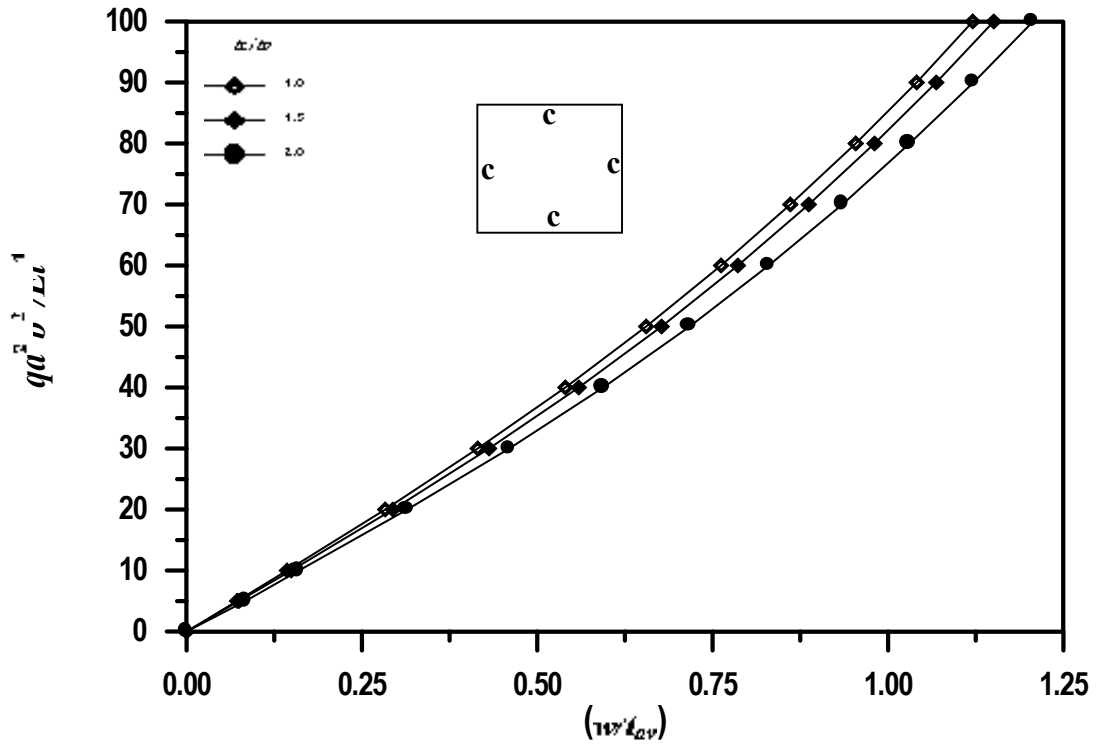**Figure (4):** Load-deflection curve of a square simply supported thin plate under transverse uniform load

Figure (5): Load-deflection curve of a square thin plate with all edges clamped and under transverse uniform load
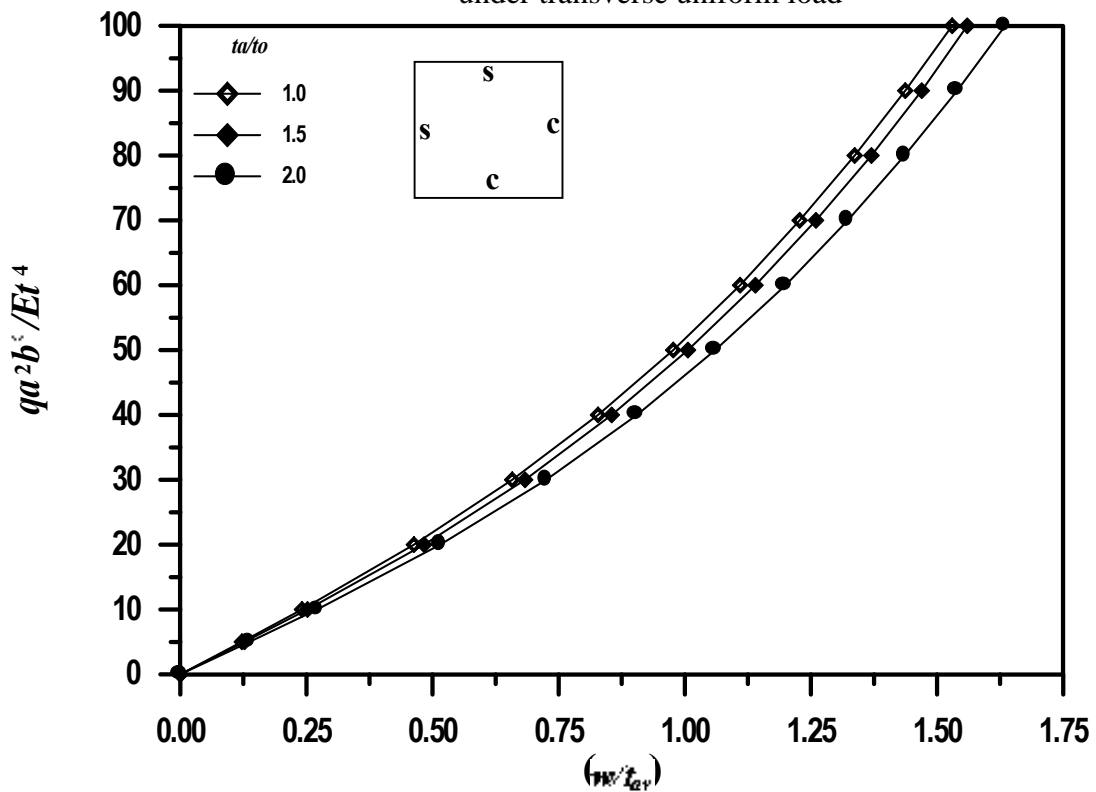

Figure (6): Load-deflection curve of a square thin plate with two edges simply supported and other edges clamped under transverse uniform load
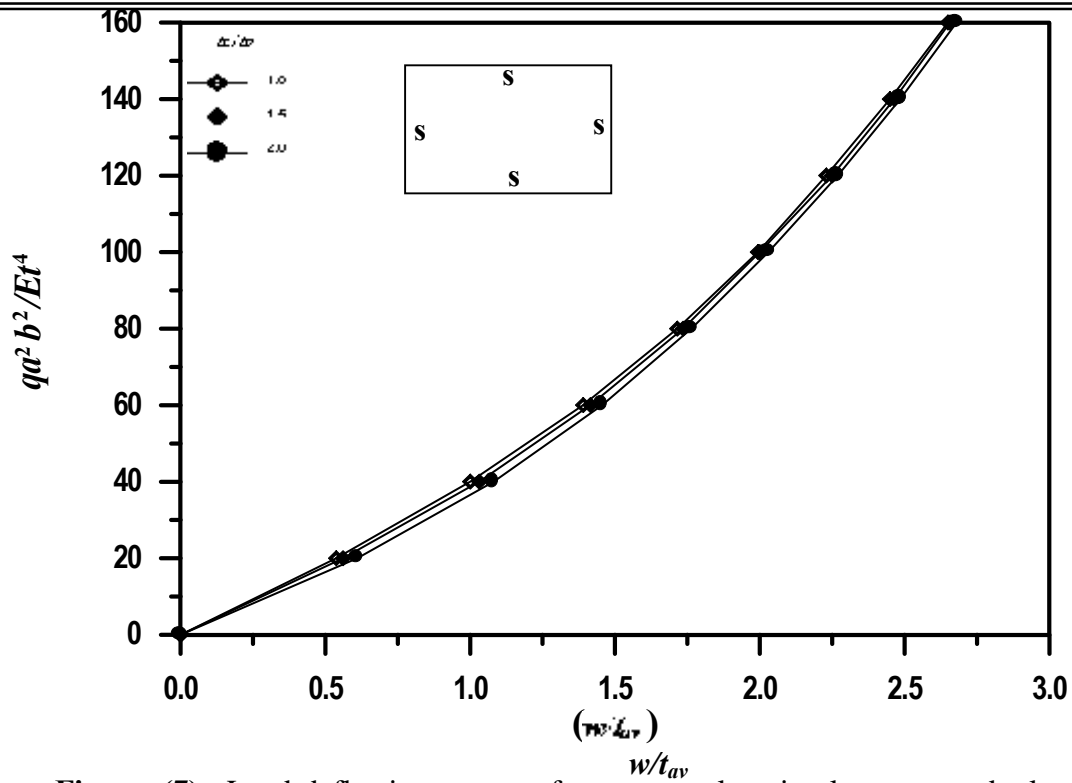
Figure (7): Load-deflection curve of a rectangular simply supported plate with aspect ratio (*a*/*b*=2.0) and under transverse uniform load
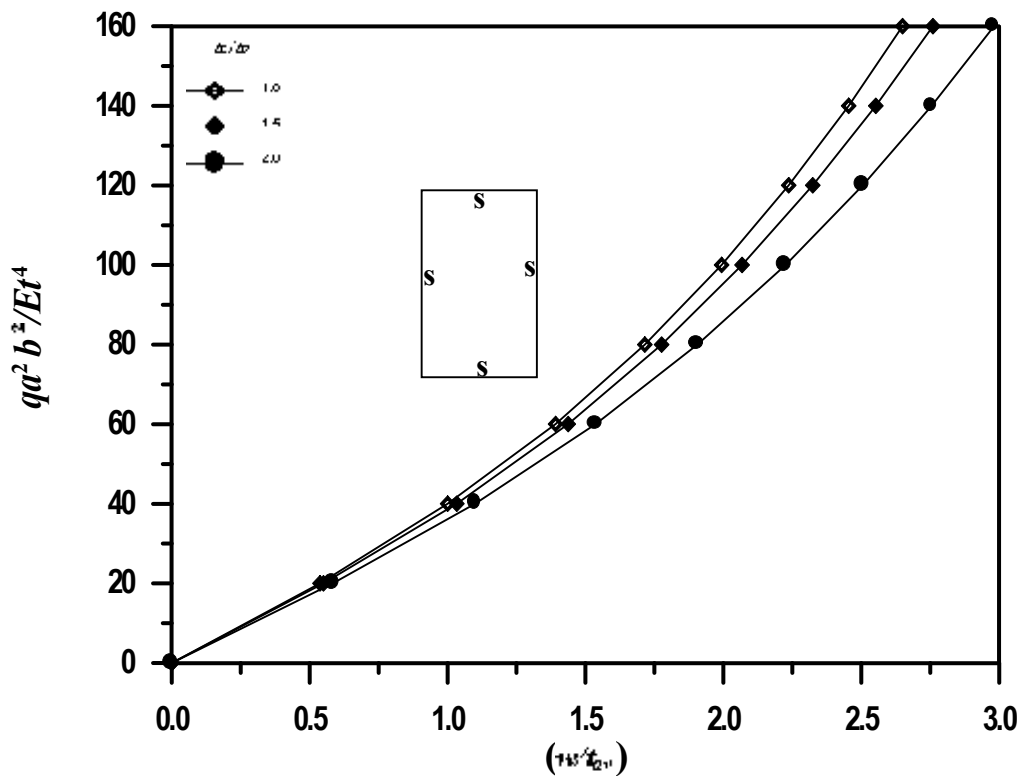

Figure (8): Load-deflection curve of a rectangular simply supported plate with aspect ratio (*a*/*b*=0.5) and under transverse uniform load
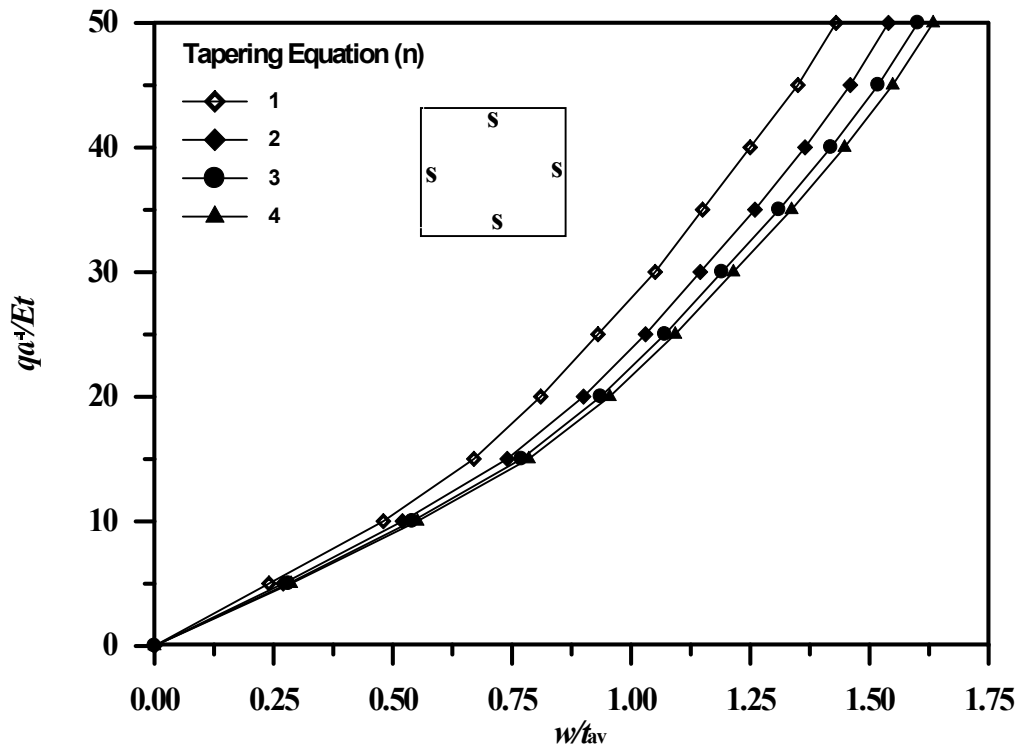
Figure (9): Load-deflection curve of a rectangular simply supported plate and under transverse uniform load with different values of order tapering equation