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Chaotic Analysis of Radial Basis Model

ABSTRACT:

In this paper, a simulation has been done for the radial basis model of sunspot time series . Chaotic analysis of the simulated data from model, and the study of the chaotic behavior for such a model indicates that the model is chaotic and the sunspot time series is chaos .

(Fractal Dimension)

(To Break) , (Fractus)

(Chaos Deterministic) (Chaos)

(Fixed Point) (Random Noise)

(Limit Cycle)

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/ / **

2005/ 10 /5 :

2005/ 7 /24 :

$$C(\epsilon, d) = \frac{2}{N(N-1)} \sum_{j=1}^N \sum_{i=j+1}^N \theta(\epsilon - \|x_i - x_j\|) \quad (2)$$

(Heavisido Function) θ

$$\theta(x) = \begin{cases} 1 & \text{If } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

C(ε , d)

(Monotone Increasing Function) C(ε) (ε)

D_c C(ε) ε

$$C(\epsilon) \approx \epsilon^{D_c} \quad (4)$$

Grassberger and Procaccia,)

ε N (1983)

D_c

(Correlation Dimension)

D_c

$$D_c = \lim_{\epsilon \rightarrow 0} \lim_{N \rightarrow \infty} \frac{\ln C_N(\epsilon, d)}{\ln \epsilon} \quad (5)$$

(Dhamal et al., 2001)

(d)

: (1)

(Chaos) .1

(Noise) .2

(Linear) .3

(Corana et al., 2002)

Test of BDS Statistic (BDS): .2

(C) 1988 BDS
 .Brock, Dechert, Scheinkman 1996

BDS

(Residual Diagnostic)

. (Brock, 1992) ()

4. اختبار الاستقلالية: A test for Independence:

(Brock et al.(1996))

$\{u_t\}$

. F (Strictly Stationary)

$$u_t^d = (u_t, u_{t+1}, \dots, u_{t+d-1})$$

$\{u_t\}$ F_d

$$F_d(x_1, \dots, x_d) = \prod_{k=1}^d F(x_k)$$

$$G_i^j = \delta - \{u_i, u_{i+1}, \dots, u_j\} \quad 1 \leq i < j \leq \infty$$

Denker and Keller) $\{u_t\}$

((1983)

(6)

$$B_k = \sup_{n \geq 1} \left\{ E \left[\sup \left\{ \left| p(A/G_1^n) - p(A) \right| \mid A \in G_{n+k}^\infty \right\} \right] \right\}$$

((Max norm)) $x \in \mathbb{R}^d$. (6)

$$\|x\| = \max_{1 \leq k \leq d} \{ |x_k| \}$$

$A = [0, \varepsilon)$. χ_A A

$$\begin{aligned}
 & \text{() } \quad \varphi: \mathbb{R}^d \rightarrow \mathbb{R}^l \quad \chi_\varepsilon \\
 & \quad \quad \quad x \in \mathbb{R}^d \quad \varphi \quad (D\varphi)_x \in \mathbb{R}^d \\
 & \quad \quad \quad (v) \quad x \quad \varphi \quad (\text{Derivative Directional}) \\
 (D\varphi)_{x,v} &= \lim_{\varepsilon \rightarrow 0} \frac{\varphi(x + \varepsilon v) - \varphi(x)}{\varepsilon} \quad (7)
 \end{aligned}$$

4. تكامل الارتباط: The Correlation Integral

BDS

(Identical Independent Distributed) (iid)

$H_0 : X_t, \text{ is } \{\text{iid}\}$

Grassberger)

$H_0, \{X_t\}$

. (and Procaccia, 1983

(d) (Embedding Dimensions)

$$C_{d,n}(\varepsilon) = \frac{1}{\binom{n}{2}} \sum_{1 \leq s < t \leq n} x_\varepsilon \left(\|u_s^d - u_t^d\| \right) \quad (8)$$

$$C_d(\varepsilon) = \lim_{n \rightarrow \infty} C_{d,n}(\varepsilon) \quad (9)$$

(9)

$$C_d(\varepsilon) = \iint \chi_\varepsilon(\|u - v\|) dF_d(u) \cdot dF_d(v) \quad (10)$$

$$\chi_\varepsilon(\|u - v\|) = \prod_{i=1}^d \chi_\varepsilon(|u_i - v_i|) \quad (11)$$

(4.8)

$$C_d(\varepsilon) = C_1(\varepsilon)^d \tag{12}$$

(Brock et al., 1996)

$$d \geq 2 \quad K(\varepsilon) > C(\varepsilon)^2 \quad (\text{iid } \{u_t\}) \quad : \mathbf{(1)}$$

(Brock et al. (1996))

$$W_{d,n}(\varepsilon) = \sqrt{n} \frac{C_{d,n}(\varepsilon) - C_1(\varepsilon)^d}{\sigma_d(\varepsilon)} \tag{13}$$

$N(0,1)$

$$\frac{1}{4} V_d^2 = d(d-2)C^{2d-2}(K - C^2) + K^d - C^{2d} + 2 \sum_{j=1}^{d-1} [C^{2j}(K^{d-j} - C^{2d-2j} - dC^{2d-2}(K - C^2))] \tag{14}$$

: **(2)**

(Distribution Free Statistic)

$W_{d,n}(\varepsilon)$

. (Brock et al., 1996)

Taken's Method :Taken's .5

$$(y_t \in \mathbb{R}) \quad \{y_t\}$$

$$(k) \quad y_t = y_1, y_2, \dots, y_{\tilde{T}}, \quad t = 1, 2, \dots, \tilde{T}$$

$$d \quad Z_t \quad .(\text{Taken's, 1981})$$

d

$$t = 1, 2, 3, \dots, T \quad (y_t \in \mathbb{R}^d) \quad (\text{Embedding Dimension})$$

$$T = \tilde{T} - (d-1)h \quad (h \geq 1) \quad h$$

$$Z_t = (y_t, y_{t-h}, y_{t-2h}, \dots, y_{t-(d-1)h})$$

(Mees, 1994)

-2 :

(1)

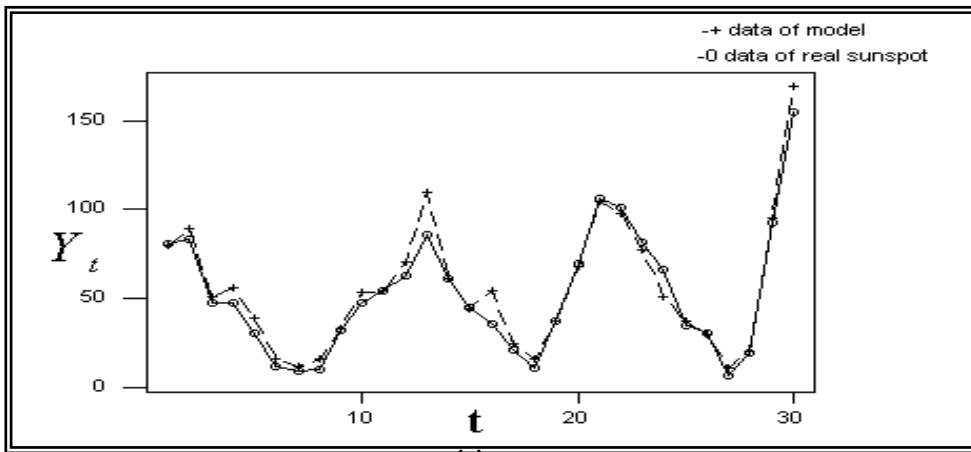
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(2) (3)

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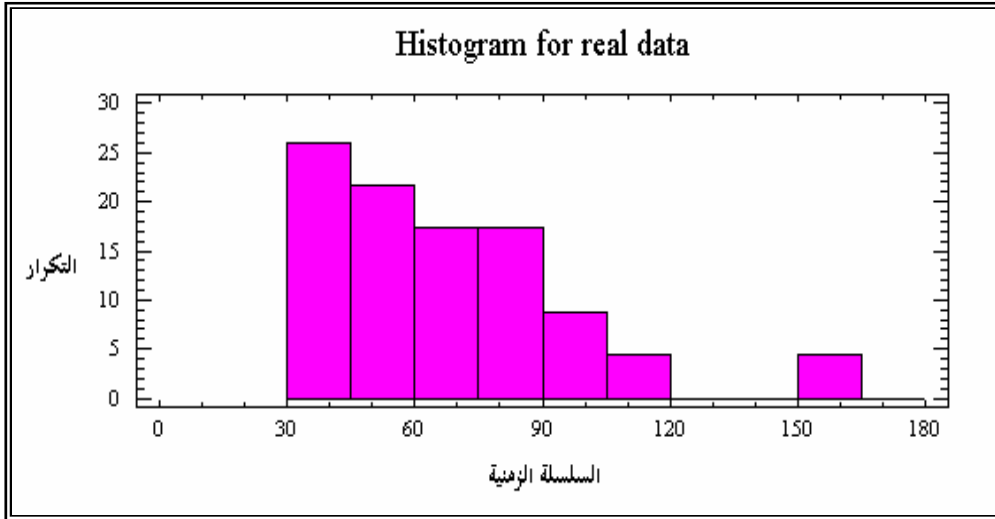
- 1749

.1979

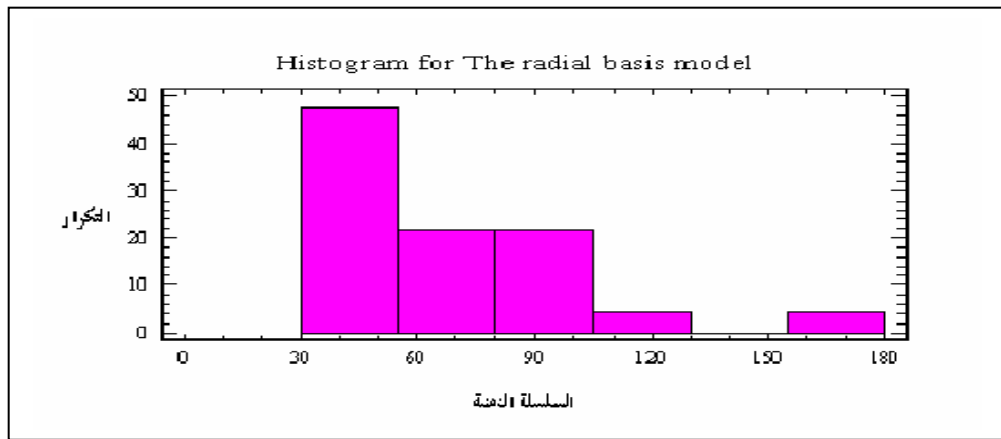


(1)

()



(2)



(3)

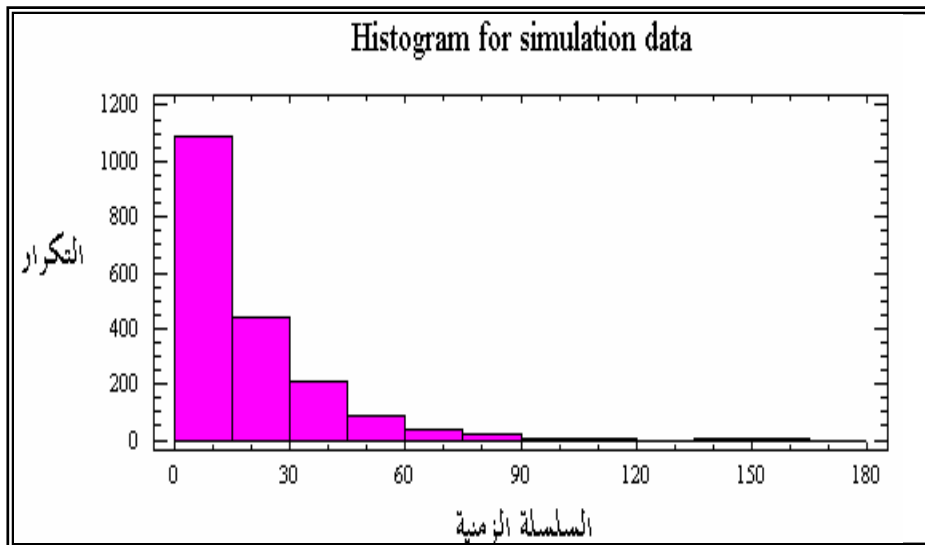
0.05 0.01

(1)

(1)

				Chi- square
0	13	2	6.15	2.8
13	26	5	4.89	0.0025
26	39	4	3.867	0.0046
39	52	3	3.093	0.0028
52	65	6	2.46	5.09
65	78	3	1.95	0.565
78	91	2	1.55	0.131
91	104	2	1.23897	0.467
104	117	2	0.99	1.030
117	130	0	0.771	0.771
130	143	0	0.62265	0.623
143	156	0	0.49455	0.494
156	169	0	0.3933	0.3933
169	182	1	0.3135	1.5033
Total		30		13.8775
$\chi^2_{12,0.01} = 26.217$				Accept H_0^*
$\chi^2_{12,0.05} = 21.0261$				Accept H_0

H_0^* : the data of model have exponential distribution

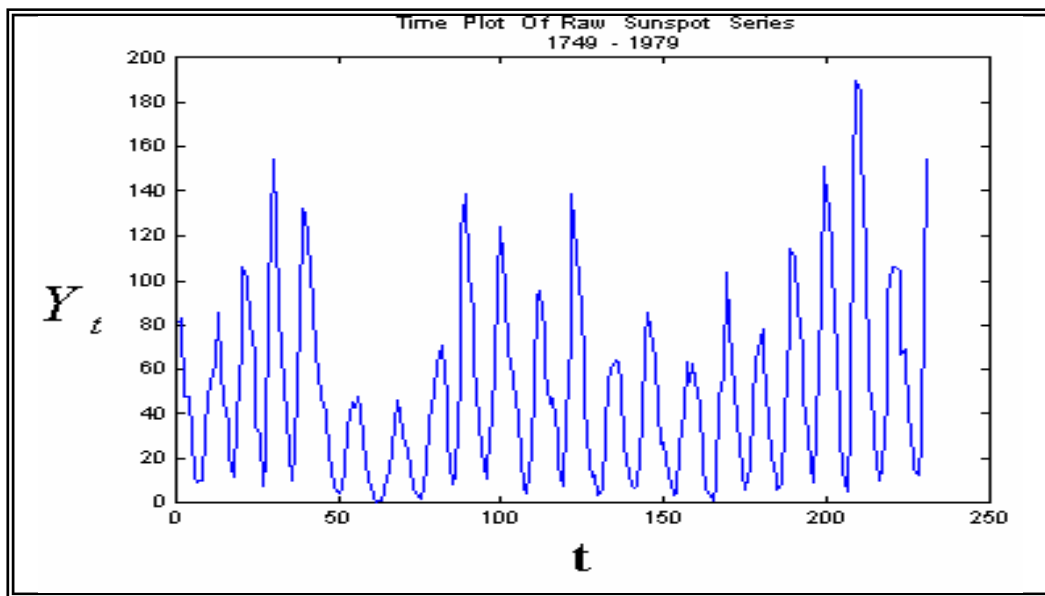


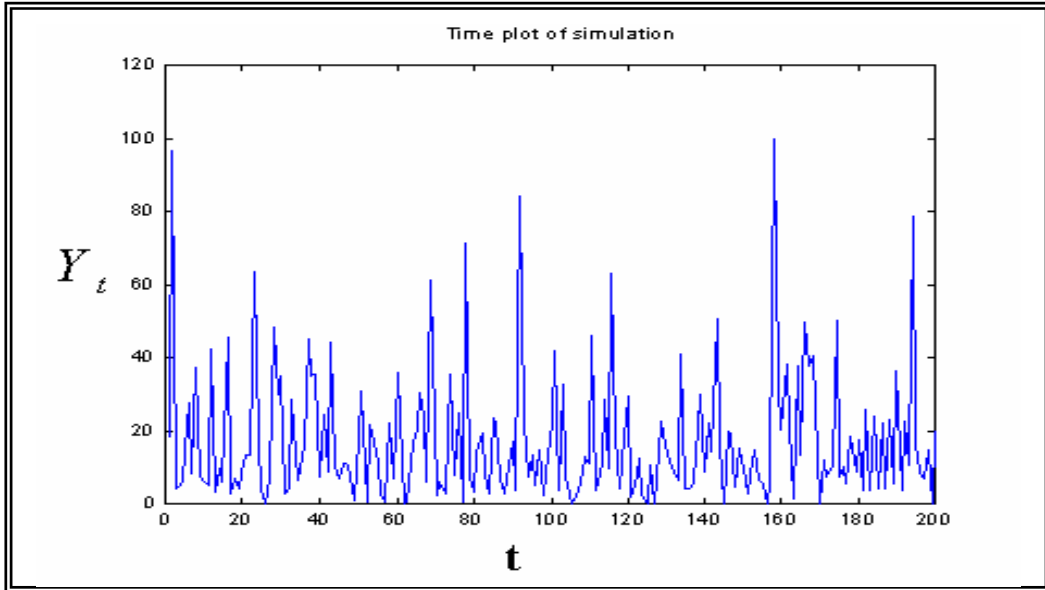
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: .1
Simulation of employment similarity of Exponential Function
(4)

Minitab .

(5)





(5)

3. :
 .1 r_k 's $k=1,2,\dots,10$
 (data) (radial basis)
 (2)
 . (iid)
 (2)

()

lag	Raw data Estimated ACF	Simulated data with noise Estimated ACF	Simulated data without noise Estimated ACF
1	0.796929	0.993310	0.984346
2	0.414749	0.987584	0.967992

3	0.015018	0.982365	0.951826
4	-0.278502	0.978034	0.938254
5	-0.411543	0.973967	0.928331
6	-0.361532	0.969977	0.918804
7	-0.151552	0.966077	0.910871
8	0.146619	0.962292	0.903342
9	0.441206	0.958584	0.897001
10	0.606310	0.954852	0.891029

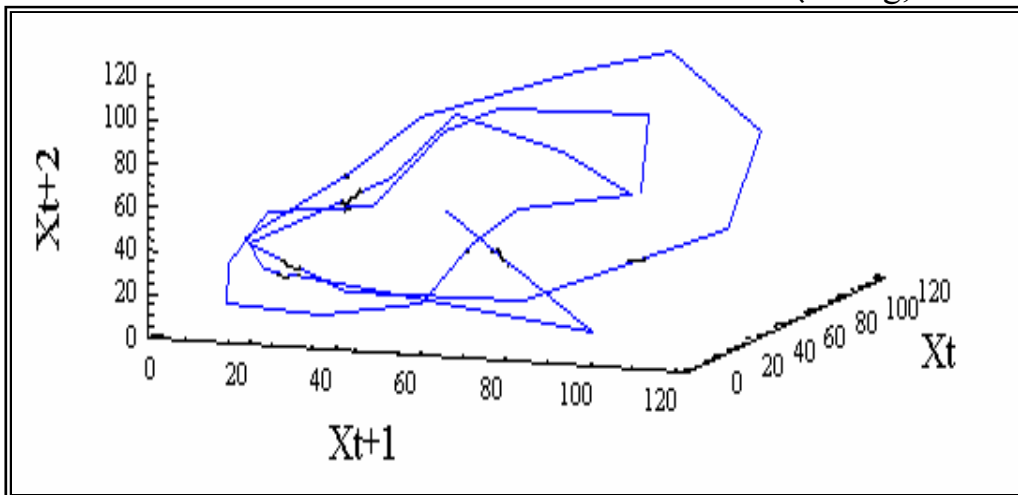
(6) .1

$d = 3$ $h = 1$ (5) Taken's

Taken's .2

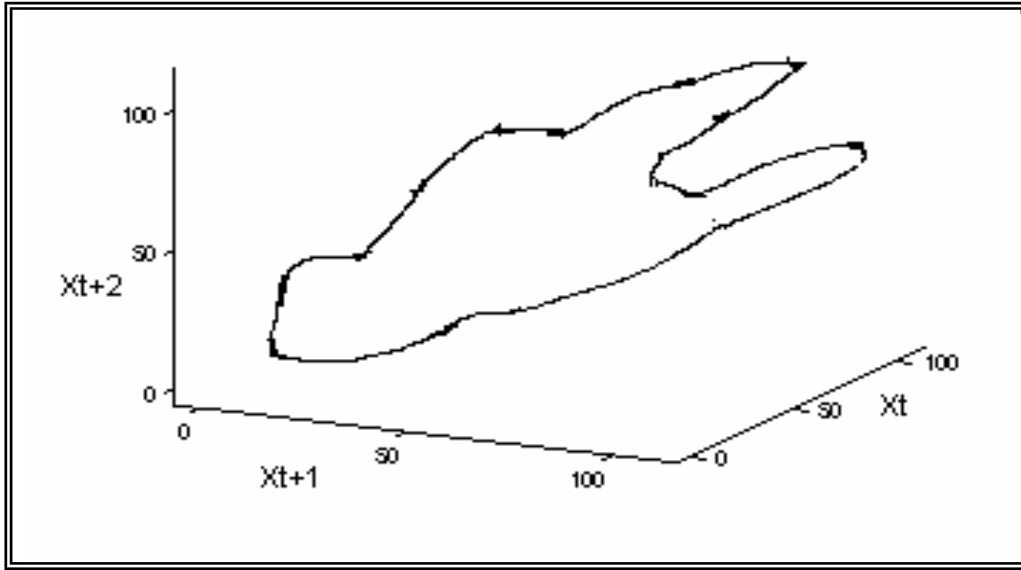
Tong, and B.) (7) $h=1$ $d=3$,

.(Cheng, 1992



(6)

(X_t, X_{t+1}, X_{t+2})



(7)

(X_t, X_{t+1}, X_{t+2})

4. حساب بعد الارتباط للسلسلة الزمنية المولدة بطريقة التشابه بالدالة الاسية :

()

(simulation)

(radial basis)

.(5)

(1980)

Minitab

(2)

BDS

(Radial basis) ()

.(3)

. (4)

(3)

(radial basis)

$$\varepsilon_j = (0.5)^j \quad j$$

d

(2)

$C(\varepsilon, d)$

J	(1980)									
	(d)									
	1	2	3	4	5	6	7	8	9	10
1	0.61014	0.61002	0.60995	0.6099	0.60987	0.60985	0.60983	0.60981	0.60979	0.60978
2	0.38085	0.38058	0.38041	0.38029	0.3802	0.38012	0.38005	0.37999	0.37995	0.3799
3	0.21627	0.21589	0.21563	0.21542	0.21524	0.21509	0.21495	0.21483	0.21471	0.21461
4	0.1152	0.11476	0.11442	0.11415	0.11393	0.11372	0.11354	0.11337	0.11322	0.11308
5	0.05898	0.05849	0.05813	0.05781	0.05756	0.057325	0.057106	0.056903	0.05672	0.056552
6	0.03001	0.02949	0.02911	0.02880	0.02852	0.028273	0.028045	0.027832	0.027635	0.027454
7	0.01516	0.01465	0.01426	0.01394	0.01367	0.013434	0.013219	0.013017	0.01283	0.012658
8	0.00761	0.00709	0.00671	0.00641	0.00615	0.0059211	0.0057167	0.0055362	0.005376	0.0052316
9	0.00384	0.00334	0.00299	0.00273	0.00250	0.0023184	0.0021581	0.0020211	0.001902	0.001796
10	0.00191	0.00145	0.00114	0.00094	0.00079	0.0006723	0.0005846	0.0005141	0.000458	0.0004102

(4)

(radial basis)

$$\varepsilon_j = (0.5)^j \quad j$$

d

(2)

C(ε, d)

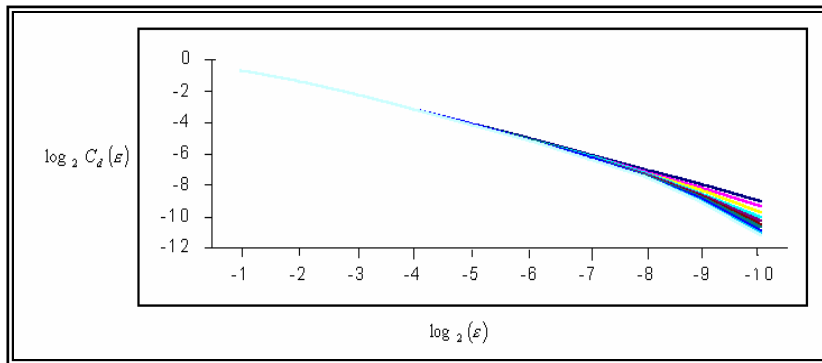
J	(1980)									
	(d)									
	1	2	3	4	5	6	7	8	9	10
1	0.75071	0.74979	0.74988	0.7500	0.75013	0.75027	0.75042	0.75057	0.75073	0.75088
2	0.43566	0.43491	0.43486	0.43487	0.4349	0.43495	0.43501	0.43507	0.43514	0.43522
3	0.22719	0.22652	0.22634	0.22622	0.22613	0.22607	0.22602	0.22599	0.22597	0.22596
4	0.11519	0.11461	0.11433	0.11411	0.11393	0.11377	0.11364	0.11352	0.11342	0.11333

5	0.05763	0.05709	0.05676	0.05649	0.05628	0.056096	0.055926	0.055774	0.055636	0.055515
6	0.02874	0.02819	0.02784	0.02755	0.02730	0.027086	0.026891	0.026709	0.026539	0.226379
7	0.01434	0.01382	0.01345	0.01315	0.01289	0.012672	0.012470	0.012285	0.012115	0.011956
8	0.00714	0.00661	0.00624	0.00594	0.00567	0.005435	0.0052313	0.005039	0.004866	0.0047073
9	0.00714	0.00661	0.00624	0.00594	0.00567	0.005439	0.0052313	0.005039	0.004866	0.0047073
10	0.00178	0.00129	0.00098	0.00076	0.00059	0.000477	0.0003835	0.000311	0.000259	0.0002158

\log_2 (4) (3)

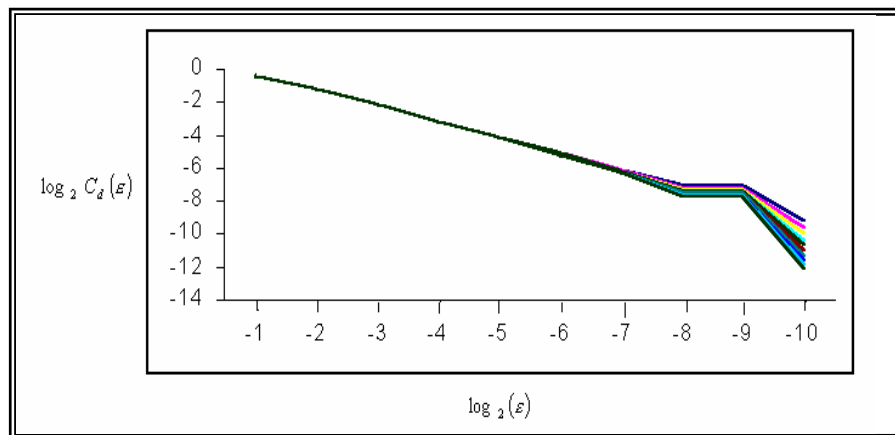
(9) (8)

Statgraph



(8)

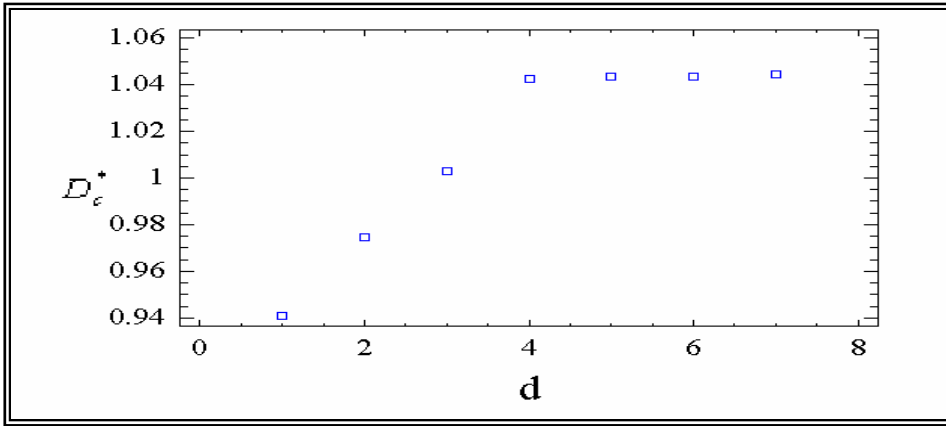
1980 d $\log_2 C_d(\epsilon)$
 $\log_2(\epsilon)$ $\log_2 C_d(\epsilon)$ ()



Statgraph

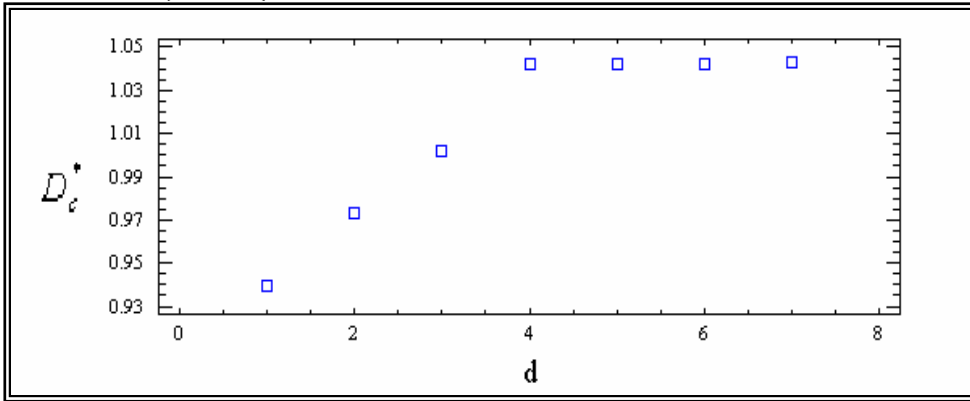
(6) (5)

. (11) (10)



(10)

()



(11)

()

(11) (10)

()

(1.0421) (1.0424) ()

. (4)

$$W_{d,n}(\varepsilon)$$

$$(8) \quad (7)$$

$$(7)$$

$$\alpha = 0.05 \quad \alpha = 0.01$$

$$\alpha =$$

$$(8)$$

$$\alpha = 0.05 \quad 0.01$$

$$(7)$$

d

$$W_{d,n}(\varepsilon)$$

$$\varepsilon_j = (0.5)^j \quad \mathbf{j}$$

$$(13)$$

J	(1980)								
	d								
	2	3	4	5	6	7	8	9	10
1	1.0724 E+2	1.2489 E+2	1.4831 E+2	1.8209 E+2	2.3033 E+2	2.9884 E+2	3.9619 E+2	5.3500 E+2	7.3387 E+2
2	1.665 E+2	2.6246 E+2	4.4359 E+2	8.0227 E+2	1.5322 E+3	3.0518 E+3	6.2784 E+3	1.3244 E+4	2.8493 E+4
3	2.7886 E+2	6.8034 E+2	1.8851 E+3	5.7815 E+3	1.9017 E+4	6.5649 E+4	2.3459 E+5	8.5996 E+5	3.2145 E+6
4	5.1029 E+2	2.1264 E+3	1.0688 E+4	6.0605 E+4	3.7026 E+5	2.3763 E+6	1.5784 E+7	1.0752 E+8	7.4652 E+8
5	9.8203 E+2	7.5183 E+3	7.2489 E+4	7.9427 E+5	9.3866 E+6	1.1653 E+8	1.4971 E+9	1.9724 E+10	2.6483 E+11
6	1.9144 E+3	2.7574 E+4	5.1356 E+5	1.09 E+7	2.4965 E+8	6.0086 E+9	1.4969 E+11	3.8245 E+12	9.9608 E+13
7	3.5145 E+3	9.5644 E+4	3.4198 E+6	1.396 E+8	6.1558 E+9	2.8541 E+11	1.3699 E+13	6.7447 E+14	3.386 E+16
8	6.1919 E+3	3.1318 E+5	2.1038 E+7	1.6174 E+9	1.3465 E+11	1.1792 E+13	1.0713 E+15	1.000 E+17	9.5291 E+18
9	9.7402 E+3	8.6513 E+5	1.0298 E+8	1.4097 E+10	2.0953 E+12	3.2879 E+14	5.3672 E+16	9.022 E+18	1.5488 E+21

(8)

j

d

$$W_{d,n}(\varepsilon)$$

$$\cdot \varepsilon_j = (0.5)^j$$

. (13)

J	(1980)								
	(d)								
	2	3	4	5	6	7	8	9	10
1	1.3511 E+2	1.4044 E+2	1.4696 E+2	1.5719 E+2	1.7145 E+2	1.9011 E+2	2.1378 E+2	2.4333 E+2	2.7991 E+2
2	2.2973 E+2	3.3445 E+2	5.119 E+2	8.3051 E+2	1.4147 E+3	2.5056 E+3	4.5772 E+3	8.57 E+3	1.6367 E+4
3	4.7631 E+2	1.1308 E+3	3.0382 E+3	9.0266 E+3	2.8771 E+4	9.6328 E+4	3.3418 E+5	1.1906 E+6	4.3298 E+6
4	1.0019 E+3	4.2418 E+3	2.172 E+4	1.2569 E+5	7.8493 E+5	5.1568 E+6	3.5117 E+7	2.4561 E+8	1.7536 E+9
5	2.0187 E+3	1.6065 E+4	1.6169 E+5	1.854 E+6	2.2969 E+7	2.9943 E+8	4.0465 E+9	5.6168 E+10	7.9598 E+11
6	4.0805 E+3	6.2321 E+4	1.2364 E+6	2.8022 E+7	6.867 E+8	1.7715 E+10	4.7383 E+11	1.3019 E+13	3.6525 E+14
7	7.5144 E+3	2.1949 E+5	8.4613 E+6	3.7322 E+8	1.7814 E+10	8.9587 E+11	4.6743 E+13	2.5067 E+15	1.373 E+17
8	1.300 E+4	7.0919 E+5	5.1542 E+7	4.2938 E+9	3.8785 E+11	3.6951 E+13	3.6542 E+15	3.7194 E+17	3.8699 E+19
9	1.3000 E+4	7.0919 E+5	5.1542 E+7	4.2938 E+9	3.8785 E+11	3.6951 E+13	3.6542 E+15	3.7194 E+17	3.8699 E+19
10	1.9735 E+4	2.8192 E+6	5.4872 E+8	.2331 E+11	3.0406 E+13	8.0004 E+15	2.2123 E+18	6.2926 E+20	1.842 E+23

Conclusions :

() .1

(radial basis)

(radial basis) () .2

(4) (1.0421) (1.0424)

BDS .3

: (4) (3)

$$C_d(\varepsilon) = (C_1(\varepsilon))^d \tag{15}$$

(Brock et al., 1991)

$$C_d(\varepsilon) / (C_1(\varepsilon))^d > 1 \tag{16}$$

: (16)

.a

(1) .b

.c

3,2,1

(radial basis) ()

التوصيات : Recommendations

.1

. ()

() .2

(Splin Function)

(Gaussian Function)

.3

.4

() .5

(Control) (Estimation)

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