Diyala Journal of Engineering Sciences

ISSN 1999-8716

Printed in Iraq

Vol. 04, No. 02 , pp. 1-11 , December 2011

ESTIMATION OF (N_γ) FOR STRIP AND CIRCULAR FOOTINGS USING THE METHOD OF CHARACTERISTICS

Lamyaa Najah Snodi
Associate Lecturer of Civil Engineering,
Tikrit University, Tikrit, Iraq.
*E-mail: Lamia_aa2001@yahoo.com
(Received:22-12-2009 ; Accepted:31-5-2011)

ABSTRACT:- By using the method of characteristics (commonly referred to as the slip line method), the values of the bearing capacity factor N_γ were computed for rigid surface strip and circular footings with smooth and rough bases. The analyses of bearing capacity for vertically loaded footings using the method of characteristics have been implemented in the publicly available computer program ABC, then N_γ –values were calculated theoretically. The supporting soil is modeled as a frictional Mohr – Coulomb material. For both footings, the value of the bearing capacity factor N_γ was found to increases significantly with an increase in the angle of internal frictionϕ. When friction angle of soil $\phi \leq 25°$, the computed values of N_γ for circular footing were found to smaller than those for strip footing and for larger values of ϕ, the magnitude of N_γ for circular footing were greater than those for strip footing for both smooth and rough base of footings. On the other hand, the magnitude of N_γ for rough footings was seen be higher than for a footings with smooth base. The obtained results were compared with those available in the literature, and reasonable agreements were observed.
Keywords: Bearing capacity, method of characteristics, circular footings, strip footing.

INTRODUCTION

Whereas, the magnitude of bearing capacity factors N_c and N_q given by Prandtl and Reissner (1924) based on the slip line method are known to be exact, a wide scatter between published values for the bearing capacity factors N_γ exists. The solutions for N_γ have been obtained by various researchers using (i) the method of stress characteristics (Sokolovski, 1960; Larkin, 1968; Bolton and Lau, 1993; Kumar, 2003), (ii) the limit equilibrium method (Terzaghi, 1943; Meyerhof, 1957; 1963), (iii) an upper bound limie analysis with the assumption of collapse mechanism (Chen and Liu, 1990; Michalowski, 1997; Zhu et al., 2001), and (iv) the lower and upper bound limit analysis with the use of finite elements (Sloan, 1988; Sloan and Kleeman, 1995; Ukritchon et al.,2003; Hjiaj et al., 2005). By using the slip line method, Martin (2005) computed N_γ with respect to variation in soil- footing interface

friction angle (δ) for different values of the angle of internal friction (ϕ). From the available analysis, it can been seen that the bearing capacity factor N_γ increases as the soil-footing interface changes from smooth to rough.

The present study aims at the estimation of the bearing capacity factor N_γ for strip and circular footings resting on (smooth and rough) surfaces using the method of characteristics have been implemented in the publicly available computer program ABC (Martin 2004). The user manual gives full details of the relevant theory, the numerical methods employed, and the extensive collections of test problems that have been used for validation. The obtained results from the present study were compared with solutions reported in the literature.

PROBLEM DEFINITION

The present study deals with the determination of the bearing capacity factor stemming from the self - weight of the soil. A rigid strip footing of 4m width, B, and circular footing of 4m diameter, D, are resting on a homogenous cohessionless soil of unit weight γ= 20kN/m^3 . The foundation is subjected to a vertical load at its centerline (Fig.1). Both rough and smooth interfaces between the footings and soil mass are considered. The soil is modeled as a rigid perfectly plastic Mohr-Coulomb yield criterion with friction angle ϕ ranging from 5° to 45°.

ANALYSIS

For determining the bearing capacity factor N_γ, the computer program ABC (Martin 2004) was used to analyze the bearing capacity of the footings. After obtaining the magnitude of the bearing capacity (q_u), the bearing capacity factor N_γ was defined with expression (Terzaghi, 1943)

$$N_\gamma = \frac{Q_u}{0.5\ \gamma B^2}$$

Analysis was repeated for different values of internal friction angle of sand (ϕ). For each analysis the magnitude of N_γ was obtained by using the expression above then the variation of the bearing capacity factor N_γ with values for angle of internal friction (ϕ) on a semi log plot is shown in Figs.(2-5) for strip and circular footings (smooth and rough) respectively. It can be seen that the magnitude of N_γ increases continuously with the increase in (ϕ).

The failure patterns generated from program ABC for ϕ = 20° for strip and circular footing are shown in (Fig.6) and (Fig.7), respectively.

RESULTS AND COMPARISONS

The computed values of N_γ for ϕ = 5° - 45° were compared with those reported in the literature.

Table(1) shows the values of N_γ for rough strip footings from different investigator using; (i)the method of characteristics; (ii) the limit equilibrium method; (iii) upper bound limit analysis. As can be seen, the values of N_γ obtained by Terzaghi (1943) and kumar (2003) are all higher than others. The results of present solution are agree with the result obtained by Hjiaj (2005) who using finite element upper bound method, with the corresponding values a little greater.

And the calculated values of N_γ for a rough circular footing were compared with those reported by (i) kumar (2005) using the method of characteristics; (ii) Erickson and Drescher (2002) using the FLAC with an associated flow rule. The comparison of the results is shown in Table(2). It can be seen that the present N_γ values are significantly smaller than those obtained by Kumar (2005). The N_γ values of Erickson and Drescher (2002) are found to be a little higher than the values obtained in the present analysis.

DISCUSSION AND CONCLUSIONS

As per the theorem of limit analysis, for an associated flow rule material, the method of characteristics provides a lower bound estimate with reference to the magnitude of the failure load (Lysmer, 1970). This method does not take into account the kinematics of the problem, and therefore, the solution obtained with this method cannot be said to be correct. At present much better numerical methods are available in literatures which consider the determination of lower and upper bound solution with help of finite element method (Sloan et al. 1982). However, these methods require much more computational efforts as compared to the method of characteristics.

The bearing capacity factor N_γ was computed for strip and circular footings (smooth and rough) using the method of stresses characteristics; the magnitude of the bearing capacity factor N_γ increases as the soil-footing interface changes from smooth to rough. The magnitude of the bearing capacity factor N_γ increases continuously with the increase in (ϕ). The computed values of N_γ for rough (strip and circular footings) were compared with those available in the literature and reasonable agreements were observed.

REFERENCES

1. **Bolton, M. D., and Lau, C.K. (1993). "Vertical bearing capacity factors for circular and strip footings on Mohr-Coulomb soil". Canadian Geotechnical Journal, 30(6).**

2. **Chen, W.F., and Liu, X.L. (1990). "Limit analysis in soil mechanics". Elsevier, Amsterdam, The Netherlands.**

3. **Erickson, H.L. and and A. Drescher.(2002). "Bearing capacity of circular footings". Journal of Geotechnical and Geoenviromental Engineering, ASCE, 128(1).**

4. **Hjiaj, m., Lyamin, A.V., and Sloan, S. W.(2005). "Numerical limit analysis solution for the bearing capacity factor N_γ ". International Journal of Solids and Structures, 42(5).**

5. **Kumar, J. (2003). "N_γ for rough strip footing using the method of characteristics". Canadian Geotechnical Journal, 40(3).**

6. **Kumar, J. and Ghosh, P.(2005). "Determination of N_γ for rough circular footing using the method of characteristics". EJGE.**

7. **Larkin, L.A.(1968). "Theoretical bearing capacity of very shallow footings". Journal of Soil Mech. And Foundation Engineering Division, ASCE, Vol. 94(6).**

8. **Lysmer, J. (1970). "Limit analysis of plane problems in soil mechanics". Journal of the Soil Mechanics and Foundation Division, ASCE, 96.**

9. **Meyerhof, G.G. (1957). "The ultimate bearing capacity of foundations on slopes". Proc., 4th.Int. Conf. on Soil Mech. and Foundation Engineering, London,1.**

10. Meyerhof, G.G. (1963). "Some recent research on the bearing capacity of foundations". Canadian Geotechnical Journal, 1(1).

11. Michalowski, R.L. (1997). "An estimate of the influence of soil weight on bearing capacity using limit analysis". Soils and Foundations, 37(4).

12. Martin, C.M. (2004). "ABC-Analysis of bearing capacity V1.0". Software and documentation available online from www- civil.eng.ox.ac.uk.

13. Martin, C.M. (2005). "Exact bearing capacity calculation using the method of characteristics". Proc.of 11th. IACMAG, Torino, Italy, 4.

14. Sokolovski, V.V. (1960). "Statics of soil media". Butterworths Publications.

15. Sloan, S.W. (1988). "Lower bound imit analysis using finite elements and linear programming". International Journal for Numerical and Analytical Methods in Geomehanics,12.

16. Sloan, S.W. and Kleeman, P.W. (1995). "Upper bound limit analysis using discontinuous velocity fields". Computer Methods in Applied Mechanics and Engineering, 127(1).

17. Terzaghi, K. (1943). "Theoretical soil mechanics". John Wiley and Sons, New York.

18. Ukritchon, B., Whittle A.W., and Klangvijit, C. (2003). " Calculation of bearing capacity factor N_γ using numerical limit analysis". Journal of Geotechnical and Geoenviromental Engineering, ASCE, 129(7).

19. Zhu, D.Y., Lee,C.F., and Jiang, H.D. (2001). "A numerical study of the bearing capacity factor N_γ". Canadian Geotechnical Journal, 38(5).

Table (1): Comparison of N_γ values for rough strip footing.

	N_γ for rough strip footing			
	Method of characteristics		Limit equilibrium	Upper bound analysis
ϕ	Present analysis	Kumar (2003)	Terzaghi (1943)	Hjiaj(2005)
5.0	0.11342	0.23	0.50	0.1196
10.0	0.43310	0.69	1.20	0.4552
15.0	1.181335	1.60	2.50	1.2378
20.0	2.83870	3.43	5.00	2.9612
25.0	6.49150	7.18	9.70	6.7379
30.0	14.7546	15.57	19.70	15.2372
35.0	34.4760	35.16	42.40	35.6491
40.0	85.5650	85.73	100.40	88.3901
45.0	234.2107	232.84	297.50	240.8801

Table (2): Comparison of N_γ values for rough circular footing.

	N_γ for rough circular footing		
	Method of characteristics with an associated flow rule		
ϕ	Present analysis	Kumar (2005)	Erickson and Drescher (2002)
5.0	0.08064	0.35	-
10.0	0.32232	0.99	-
15.0	0.93232	2.27	-
20.0	2.41546	4.87	2.80
25.0	6.0727	10.40	-
30.0	15.5251	23.23	-
35.0	41.878	56.01	45.00
40.0	123.6755	150.97	130.00
45.0	417.6747	471.70	456.00

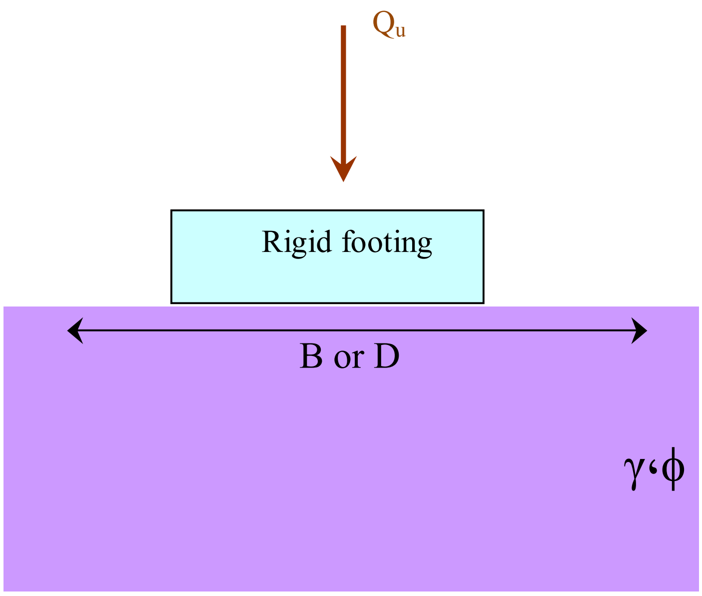


Fig.(1): Footing resting on a cohessionless soil.

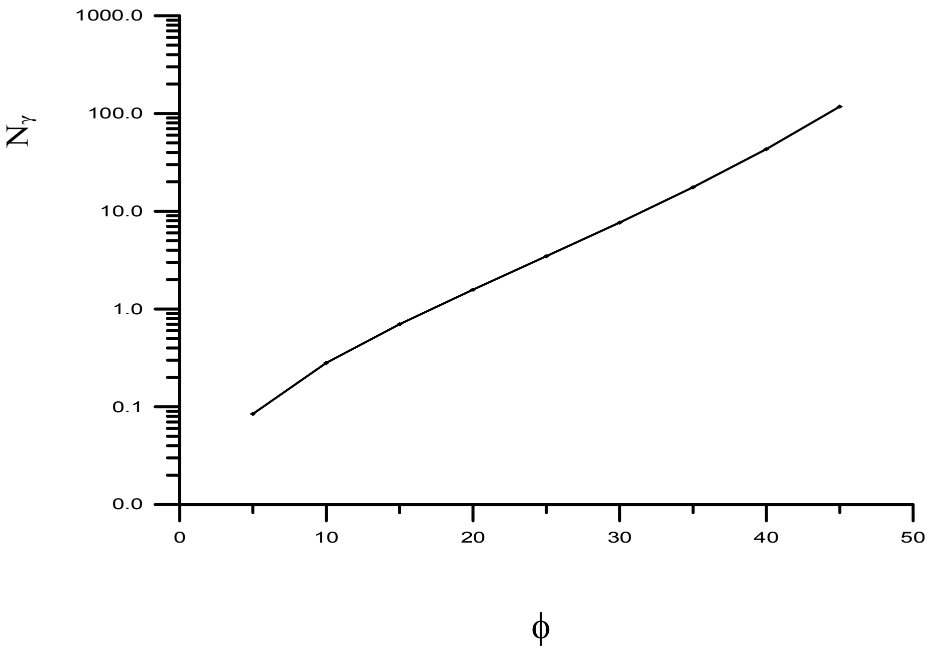


Fig. (2): The variation of N_γ with ϕ for a smooth strip footing.

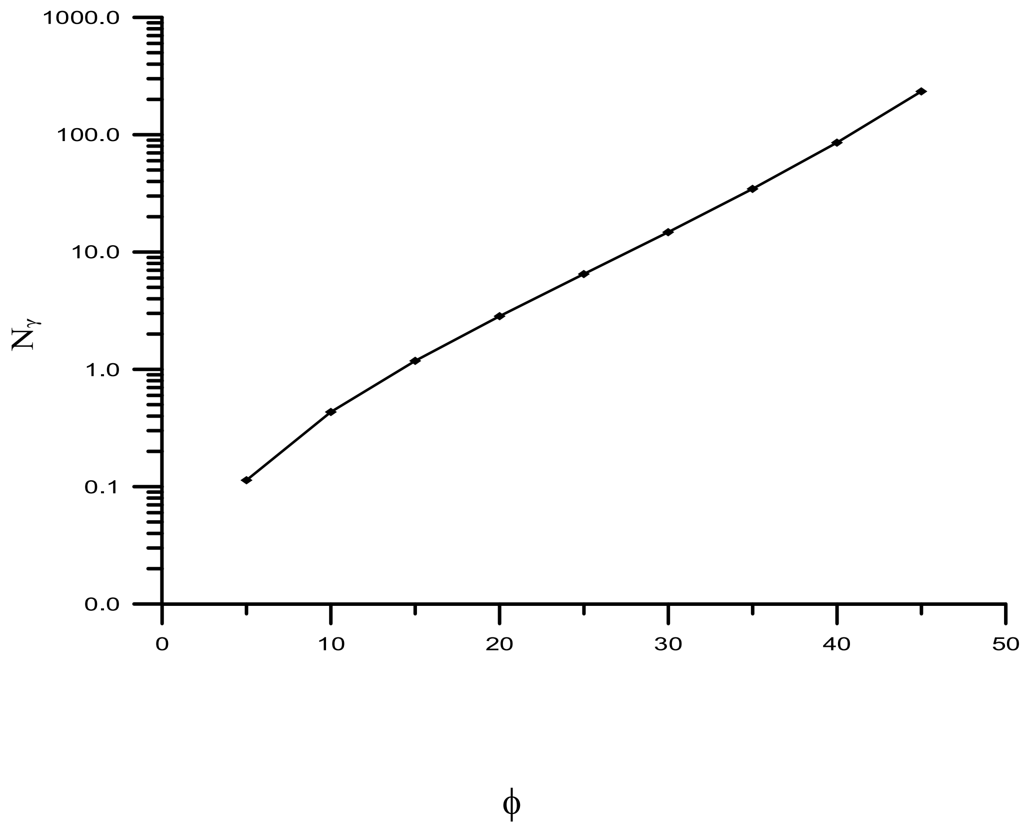


Fig. (3): The variation of N_γ with ϕ for a rough strip footing.

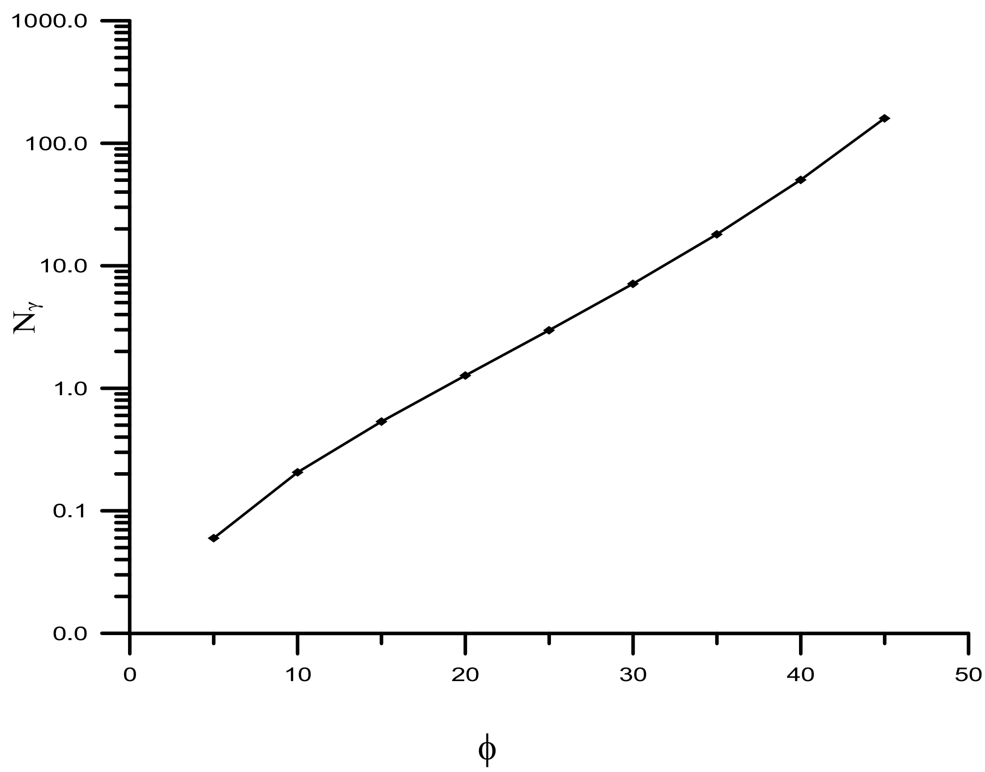
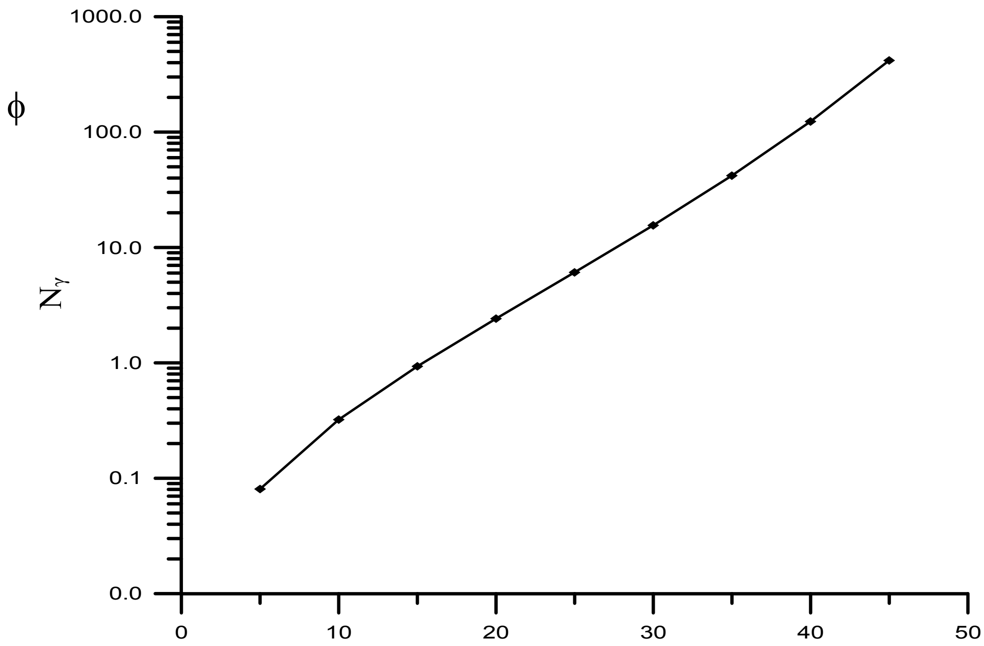
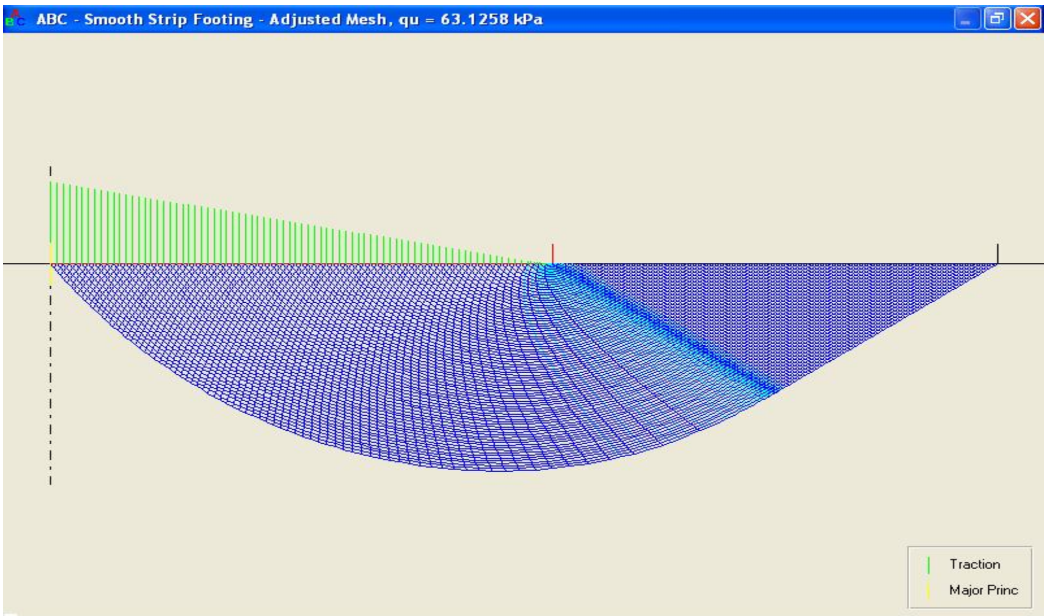


Fig. (4): The variation of N_γ with ϕ for a smooth circular footing.



(Fig.5): The variation of N_γ with ϕ for a rough circular footing.



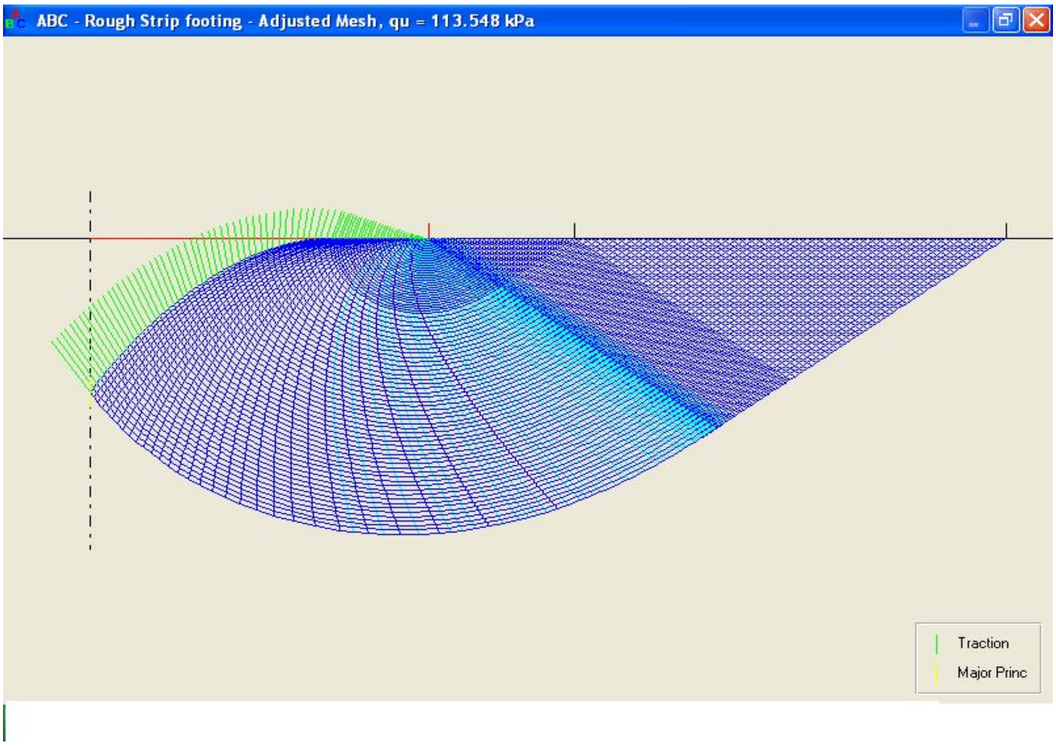
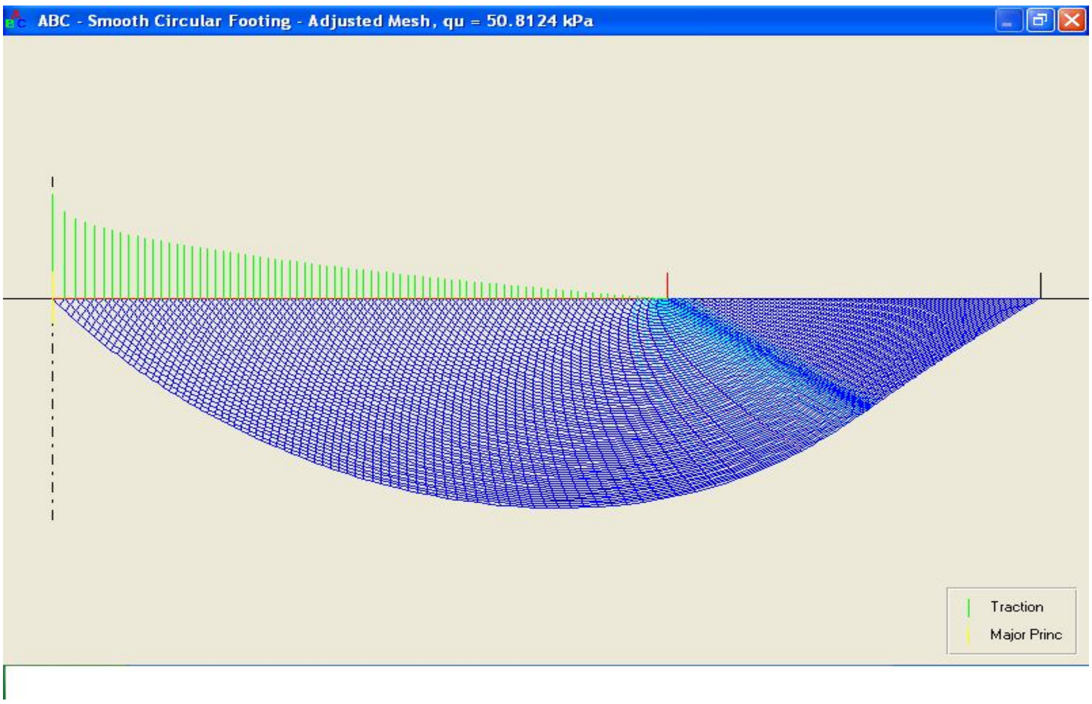


Fig.(6): The failure patterns for strip footing, $\phi = 20°$.



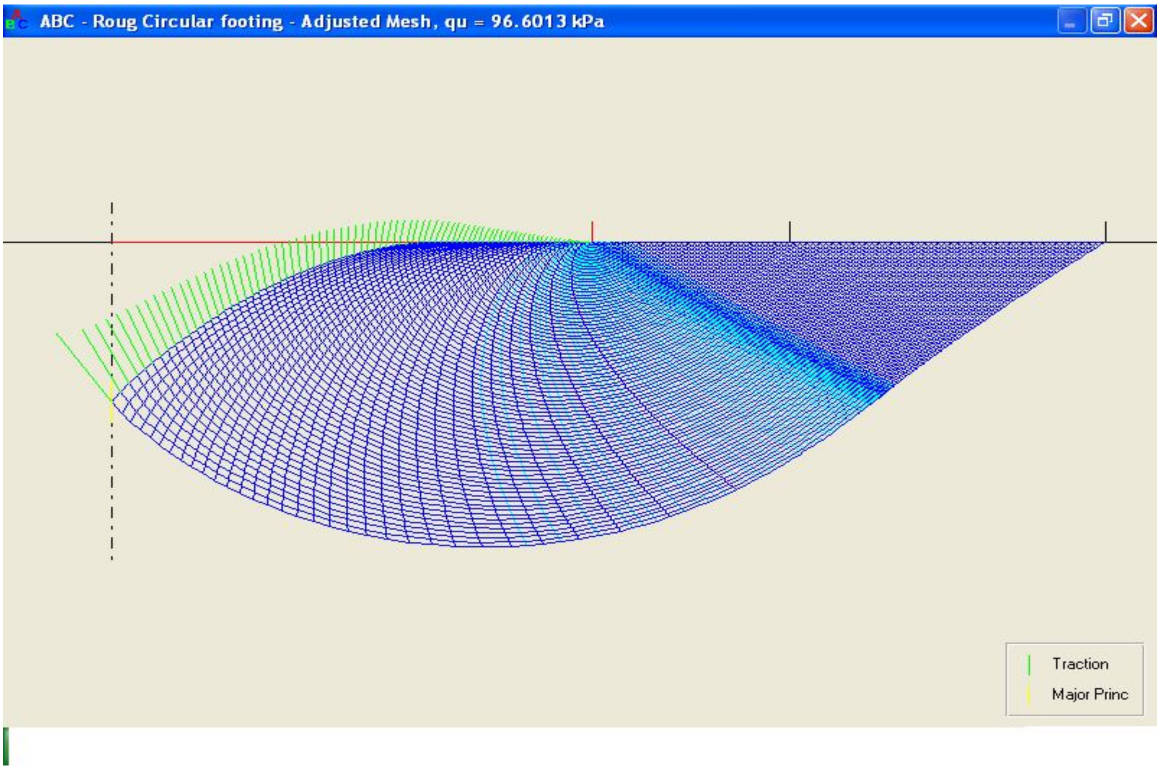


Fig.(7): The failure patterns for circular footing, $\phi = 20°$.

تخمين (N_γ) للأسس الشريطية والدائرية باستخدام طريقة الخصائص

لمياء نجاح سنودي

مساعد مدرس

كلية الهندسة ــ جامعة تكريت

الخلاصة

باستخدام طريقة الخصائص (عادة ما تسمى طريقة خط الانزلاق) تم إيجاد قيم معامل قابلية التحمل N_γ للأسس الشريطية والدائرية السطحية الصلدة (المستندة على قاعدة غير قابلة للانثناء) ذات قواعد ملساء وخشنة.

التحليل لقابلية التحمل لأسس محملة بشكل عمودي باستخدام طريقة الخصائص طبقت في برنامج الحاسوب المتوفر ABC بعدها تم حساب N_γ نظريا. التربة كانت احتكاكية غير متماسكة وحسب نظرية Mohr – Coulomb.

لكلا الأساسين ، وجد بأن قيمة معامل قابلية التحمل N_γ كانت تزداد بشكل ملحوظ مع زيادة زاوية الاحتكاك الداخلي ϕ. عندما كانت زاوية الاحتكاك للتربة $\phi \leq$ ٢٥° ، وجد بأن قيم N_γ المحسوبة للأسس الدائرية اصغر من قيم N_γ للأسس الشريطية وللقيم العليا من ϕ ، وان قيمة N_γ للأسس الدائرية كانت اكبر منها للأسس الشريطية لحالتي الأسس ذات القاعدة الملساء والخشنة من ناحية ومن ناحية أخرى، لوحظ بأن قيمة N_γ للأسس الخشنة كانت أعلى منها للأسس الملساء. قورنت النتائج المستخلصة من هذا البحث مع نتائج بحوث سابقة ، حيث لوحظ تقارب مقبول بينها.