EFFECT OF SINGLE OR MULTI ROTATING HORIZONTAL CYLINDERS ON THE MIXED CONVECTION HEAT TRANSFER INSIDE A TRIANGULAR ENCLOSURE

Dr. Ahmed K. M. Alshara
Mechanical Engineering Department - College of Engineering - Basra University

ABSTRACT

This study investigates the effect of rotating horizontal single or multi cylinders on mixed convection heat transfer in an equilateral triangular enclosure filled with air. The governing equations for the steady, laminar, two dimensional, incompressible flows with Boussinseq approximation and constant fluid properties are solved numerically using the finite element method with FlexPDE soft package. Three cases are performed: single rotating cylinder, three rotating cylinders at the same direction and three rotating cylinders at different directions. The main parameters are: Rayleigh number ($Ra = 10^2 - 10^5$), Prandtl number ($\Pr = 0.7$), the dimensionless angular velocity (Ω=0-1000) (for both directions clockwise CW and counter clockwise CCW) and dimensionless radius of rotating cylinder (R=0.1-0.25). It was found that the average Nusselt number for the single or multi rotating cylinder is increased with increasing Ra, R and Ω for all cases. Also the average Nusselt number of single rotating cylinder is greater than the multi rotating cylinders for the same ratio of the solid cylinder or cylinders volume to total enclosure volume. The results are compared with other authors in the literature and a good agreement was seen.

KEYWORDS: Mixed convection, Rotating cylinder, Triangular enclosures, Finite elements method

تأثير اسطوانات أفقية دوارة منفردة أو متعددة على انتقال حرارة الحمل المختلط في فجوة مثلثة

د. أحمد كاظم محمد الشرع
قسم الهندسة الميكانيكية- كلية الهندسة- جامعة البصرة

الموجز

هذه الدراسة تبحث تأثير اسطوانات أفقية دوارة مفردة أو متعددة على انتقال الحرارة بالحمل المختلط في حيز مغلق مثلث متساوي الإضلاع مملوء بالهواء. المعادلات الحاكمة لجريان مستقر، طباقي، ثنائي الأبعاد، غير انضغاطي مع تقاربBoussinseq و ثبوت خواص المائع ُحلت عدديا باستخدام طريقة العناصر المحددة مع الحقيبة البرمجية FlexPDE . ثلاث حالات أنجزت: اسطوانة دوارة مفردة، ثلاث اسطوانات دوارة بنفس الاتجاه و ثلاث اسطوانات دوارة باتجاهات مختلفة. العوامل الأساسية هي: رقم رايلي ($Ra = 10^2 - 10^5$)، رقم براندل

($Pr = 0.7$)، السرعة الزاوية ألللابعدية (Ω=0-1000) ولكلا الاتجاهين (باتجاه عقرب الساعة CW وعكس عقرب الساعة CCW) و نصف قطر الأسطوانة الدوارة ألللابعدي (R=0.1-0.25) . وجد بان معدل رقم نسلت للأسطوانة الدوارة المفردة أو المتعددة يزداد مع زيادة R, Ra و Ω و لكل الحالات. كذلك معدل رقم نسلت للاسطوانة الدوارة المفردة اكبر من الاسطوانات الدوارة الثلاثة لنفس نسبة حجم الأسطوانة أو الاسطوانات الصلبة إلى الحجم الكلي للحيز المغلق. النتائج قورنت مع نتائج باحثين آخرين في الأدبيات وبينت تطابق جيد.

NOMENCLTURE

B Base of triangular enclosure m
E Non-dimensional radius of the centers circle of three cylinders, e/H
g Acceleration due to gravity m/s^2
H Height of equilateral triangular enclosure m
k Thermal conductivity $W/m^2.K$
Nu_{av} Average Nusselt number
P Dimensionless pressure, $(p+\rho_c gy)H^2/\rho\alpha^2$
Pr Prandtl number, υ/α
R Dimensionless radius of the rotating cylinder, r/H
Ra Rayleigh number , $g\beta k(T_h-T_c)\,H^3/\upsilon\alpha$
T Temperature of the fluid in the enclosure K
U,V Non-dimensional velocity components , uH/α , vH/α
X, Y Non-dimensional coordinates, X=x/H, Y=y/H

GREEK SYMBOLS

α Thermal diffusivity of fluid m^2/s
β Thermal expansion coefficient K^{-1}
θ Dimensionless temperature, $\theta=(T-T_c)/(T_h-T_c)$
υ Kinematics viscosity m^2/s
ρ Density kg/m^3
ω Angular rotational velocity of solid cylinder rad/s
Ω dimensionless angular rotational velocity, $\omega H^2/\alpha$

SUBSCRIPTS

av Average
c Cold
h Hot
o center

INTRODUCTION

Mixed convection (forced and natural convection) in an enclosure is found in many industrial and environmental applications such as electronic systems, solar energy systems, heat sinks, saving of materials and cooling equipments, etc. To improve both conductive and convective heat transfer and heat exchange with fluids, the rotating solid bodies (e.g. rotating solid cylinder) are used which represent one of methods to produce mixed convection in the enclosures. Therefore a method of employing rotating circular cylinders in the enclosures is proposed to enhance the minute natural convection heat transfer of the enclosures. These configurations are important with several applications in engineering, science and environmental analysis. Heat exchangers, cylindrical cooling devices in plastics and glass industries, grain storage, food and chemical processing and furnaces and drying technologies are just some these applications. There are many geometric configurations of enclosures filled with convective fluids such as rectangular, triangular, elliptical and circular enclosures.

(**Shu et al., 2001**) studied numerically the natural convective heat transfer in a horizontal eccentric annulus between a square outer cylinder and a heated circular inner cylinder using the differential quadrature (DQ) method. They analyzed the natural convection systematically for $Ra = 3\times10^5$ and aspect ratio equal to 2.6 including the effects of outer cylinder location on average Nusselt number, flow and thermal fields. (**Kim et al., 2008**) carried out numerical calculations for natural convection induced by a temperature difference between a cold outer square enclosure and a hot inner circular cylinder. They investigated the effect of the inner cylinder location on the heat transfer and fluid flow for the range of Rayleigh numbers $10^3 < Ra < 10^6$. Also, they found that the presence of the secondary and tertiary vortices near the upper surfaces of the inner cylinder according to the variation of the inner cylinder position and Rayleigh number has a large influence on the distribution of the local and surface-averaged Nusselt numbers. (**Rahman et al., 2009**) analyzed the mixed convection in rectangular cavity with a heat conducting horizontal circular cylinder. They developed mathematical model to solve continuity, momentum and energy equations employing Galerkin weighted residual finite element method. The results show that the cavity orientation and the cylinder diameter have a great influence on the streamlines and isotherms distribution. (**Yu et al., 2010**) performed a numerical study of laminar convection from a horizontal triangular cylinder to its concentric cylindrical enclosure to investigate the Prandtl number effect on flow and heat transfer characteristics. The finite volume approach was used to solve the governing equations, in which buoyancy is modeled via the Boussinesq approximation. They found that the flow and heat transfer characteristics for a low Prandtl number fluid (Pr=0.03) are unique and they are almost independent of Prandtl number when Pr≥0.7.

A multi circular and square rods in cavities with laminar natural convection was studied by (**Braga and Lemos, 2005**). Governing equation (momentum and energy equations that resemble a conjugate heat transfer problem for in both the solid and the void space) were solved using finite volume method. They found that the average Nusselt number for cylindrical rods is slightly lower than those for square rods considering the same modified Rayleigh number.

(**Ashjaee et al., 2008**) studied the convective heat transfer from the inner cylinder of an annular gap of eccentric cylinders with air enclosed. The inner and the outer cylinder have low and high constant temperature, respectively. The rotational Reynolds number considered based on the inner and outer cylinders diameters is in the range 0-1200. They observed when the cylinders are rotated a decrease in the heat transfer from the inner cylinder in three different phases; a slight decrease, a dramatic decrease and becoming constant, while the rotation of the outer cylinder increases the heat transfer in comparison with the case with rotating inner cylinder when rotating with the same rotational Reynolds number, and decreases it when rotating with the same rotational speed. The mixed and forced convection heat transfer from an unsteady no-uniform stream shear flow past a rotating isothermal cylinder was studied by (**Abdella and Magpantay, 2007**).

(**Fu et al., 1994**) investigated numerically enhancement of natural heat transfer in enclosure by a rotating cylinder using a penalty finite element method with Newton-Raphson iteration algorithm. They found that the contribution of the rotating cylinder to natural heat transfer depends on the direction of rotation of the cylinder, also for counter clockwise rotation cylinder situation, the contribution was found to be substantial when the value of Gr/Re^2 is greater than 100, however the maximum enhancement of the heat transfer rate is approximately equal to 60%. (**Costa and Raimundo, 2010**) studied numerically mixed convection in a square enclosure with a rotating concentric cylinder. It is explored the influence of the cylinder through its radius, rotating velocity, thermal conductivity and thermal capacity on the resulting mixed convection. The overall thermal performance of the enclosure was analyzed through the overall Nusselt number. They observed that the size of the cylinder has strong influence on the resulting flow and heat transfer process, as it limits the space for fluid flow between the cylinder and the enclosure walls, giving rise to higher or lower local fluid velocities close to solid boundaries, also they concluded for high values of the cylinder radius, the overall Nusselt number is small if the rotating velocity is low, and it considerably increase in a nearly linear way, with the rotating velocity absolute value, for both the situations of possible combination of the natural and forced convection effects.

The aim of the present study is to investigate the effect of a single or multi rotating horizontal cylinders in an equilateral triangular enclosure on the mixed heat transfer. And to solve governing equations (continuity, momentum and energy) using penalty finite element method. And to show the influence of rotating cylinders velocity, the direction of rotating velocity, radius of cylinder and Rayleigh number on velocities and temperature distribution and then on the average Nusselt number. Also to illustrate the difference between single-cylinder and multi-cylinders having constant cylinder (cylinders) to total enclosure volume ratio.

THEORETICAL ANALYSIS

The schematic diagram of the problem of mixed convection heat transfer inside equilateral triangular enclosure with height H and base B for single and multi rotating solid horizontal cylinder at angular velocity ω, is shown in **Figure1 (a) and (b)** respectively. The ratio of solid rotating cylinder volume to total triangle enclosure volume for single or multi cylinder is constant and equal to 0.25. The viscous dissipation term in the energy equation and radiation are neglected. The governing equations for the steady, laminar, two dimensional, incompressible flow with Boussinseq approximation (**Kreith and Bohn, 1997**) and constant fluid properties can be written in non-dimensional form as follows:

-continuity equation

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

-the momentum equations

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

and

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\Pr\theta \tag{3}$$

-energy equation

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \tag{4}$$

Where the dimensionless variables are defined as:

$$X=\frac{x}{H},\;\; Y=\frac{y}{H},\;\;\; R=\frac{r}{H},\;\; U=\frac{uH}{\alpha},\;\; V=\frac{vH}{\alpha},\; \theta=\frac{T-T_c}{T_h-T_c}$$

$$P=\frac{(p+\rho_c\, gy)H^2}{\rho\alpha^2},\ \ \Pr=\frac{\nu}{\alpha},\ \ Ra=\frac{g\beta(T_h-T_c)H^3}{\nu\alpha}\ and\ \ \Omega=\frac{\omega H^2}{\alpha}$$

With boundary conditions

1- U=V=0 and θ=1 at the base of triangular enclosure
2- U=V=0 and θ=0 at the left wall of triangular enclosure
3- U=V=0 and $\frac{\partial\theta}{\partial n}=0$ at the right wall of triangular enclosure
4- $\mathrm{U}=\Omega(\mathrm{Y}-\mathrm{Y_o}),\ \mathrm{V}=-\Omega(\mathrm{X}-\mathrm{X_o})$ and $\frac{\partial\theta}{\partial n}=0$ at the wall of rotating solid cylinder
5- $\frac{\partial P}{\partial n}=0$ at all the walls

Where $(\mathrm{X_o},\mathrm{Y_o})$: the center of cylinder.

Hence: (1)The center of rotating solid cylinder is constant, at the origin point $\mathrm{X_o}=0, Y_o=0$ for single rotating cylinder, and $(0,E)$, $(E\sin°60,-E\cos°60)$ and $(-E\sin°60,-E\cos°60)$ for three rotating cylinders respectively, (2)The three cylinders are distributed on the centers circle which has the radius E and equally distributed on it by $(120°)$, (3)The sign of U and V at the wall of cylinder is varied with direction of rotating solid cylinder, and (4)Two directions of rotating are used: clockwise CW (positive direction) and counter-clockwise CCW (negative direction).

The average Nusselt number at the heated wall is calculated by

$$Nu_{av}=\int_{-1/2}^{1/2}-\left(\frac{\partial\theta}{\partial Y}\right)_{Y=-H/3}dX \tag{5}$$

And the bulk average temperature is defined as

$$\theta_{av}=\int\frac{\theta\, d\overline{V}}{\overline{V}} \tag{6}$$

Where $\overline{V}$: the volume of occupying fluid in triangular enclosure.

NUMERICAL SOLUTION

In the present study, a finite element software package Flexpde (**Backstrom, 2005**) is applied in the solution of the nonlinear system of equations (1) to (4). Hence, the continuity equation (1) is used to check the error of the solution throughout the grids of domain. The penalty finite element method are applied to overcome the linkage between velocity and pressure in the momentum equations using continuity equation

$$\nabla^2 P=\gamma\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right) \tag{7}$$

Where; γ is a setting parameter. The continuity equation is automatically satisfied for large value of penalty parameter γ. Typical values of γ that yield consistent solutions are 10^6. The relative error limit which is employed in this study is less than 10^{-3} .

VALIDATION AND COMPARISON OF THE STUDY

To check the validation of software, the continuity equation ($\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$) is used. **Figure 2(a)** shows the validity of continuity equation of single rotating cylinder with Ω=500CCW, **Figure 2(b)** illustrates the validity of continuity equation for three rotating cylinders with Ω=500CCW, it can be seen exactly validated of the velocity distribution of the values ($\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$) over the domain. Also, the present model is compared with results of (**Costa and Raimundo, 2010**) as shown in **Figure 3**, for R=0.2. The comparison represent the variation of the average Nusselt number with dimensionless angular velocity at constant the other parameters, $Ra=10^5, \Pr=0.7$ and the ratio of thermal conductivity of solid to fluid and the heat capacity of solid to fluid equal to one. The results show a good agreement and from this comparison it can be decided that the current code can be used to predict the flow field for the present problem.

RESULTS AND DISCUSSION

The results are represented for three cases: case (I) single rotating horizontal cylinder at constant position with two direction of rotating CW or CCW, case (II) three rotating horizontal cylinders at constant positions with the same direction of rotating CW or CCW, case (III) three rotating horizontal cylinders at constant positions with the different directions of rotating (the upper and right lower cylinders at the same direction while the left lower cylinder is inversed them). The fluid in enclosure is air, Pr=0.7. The values of Rayleigh number varying over the range of 10^2-10^5. The dimensionless rotating velocities of cylinder **Ω** are varied from 0 to 1000 with two directions CW and CCW. The ratio of rotating cylinder volume to total volume of triangular enclosure in single cylinder or three cylinders is constant and equal to 0.25, and the radius of centers circle of each cylinder from the center of triangular for the multi cylinders is E=0.25.

A-Flow and Isotherms Fields

I- Single Rotating Cylinder

Fig.4 shows the counters of U-velocity (left), V-velocity (middle) and isotherm lines (right) for $Ra=10^3, \Pr=0.7$ and $R=0.214$. It shows that when **Ω**=0, the both velocities U and V have the minimum values comparison with the **Ω**=500CW or **Ω**=500CCW, and for U-velocity distribution a cellular motion is formed near the upper and bottom surfaces of the cylinder as shown in **Figure 4a**(left). **Figure 4a** (middle) shows that the cellular motion of the V-velocity distribution. It is occurred near the upper surface of the cylinder, this is because of the effect of the secondary flow. For the rotating cylinder, the maximum values of V-velocity are occurred near the both left and right sides of the cylinder as shown in **Figure 4** (middle) b and c, this can be understood from the definition of U and V respect to dimensionless angular velocity **Ω**. **Figure 4a** (right) represents the isotherm lines for non-rotating cylinder. It can be seen a curvature lines are concentrated in the left corner of the enclosure, also the isotherm lines are symmetric between the cold and hot surfaces due to the dominant of the natural convection. The effect of the rotating flow by the cylinder increases the density of the isotherm lines which covers most of the enclosure spaces, also the effect of the rotation in both direction are predicted in **Figure 4b** (right) and **Figure 4c** (right). The rotating velocity tends to transfer the heat transfer toward the rotation direction.

Figure 5 shows the counters of U-velocity (left), V-velocity (middle) and isotherms (right) for high Rayleigh number $Ra=10^5$ and $R=0.214$. It can be seen the curves behavior of the U-velocity and V-velocity are the same as that in **Figure 4**, a deformation of velocity distributions is occurred in same regions near the cylinder surface due to the strong effect of secondary flow. For non-rotating cylinder shown in **Figure 5a** (right) the isotherms lines are covered most the cavity space while for the rotating cylinder as in **Figure 5** b and c the density of the lines is higher than that in **Figure 4b** and c respectively. This is because of the increasing of the vortices flow with increasing of Ra (effect of natural convection) and with the increasing the rotating velocity (effect of forced convection).

II-Multi Rotating Cylinders in the Same Direction

This case is for three rotating cylinders with constant ratio of the volume of cylinders to total volume of equilateral triangle which equal to 0.25. **Figure 6** illustrates the contours of U-velocity, V-velocity and isotherms for $Ra = 10^3$ and $R = 0.124$. **Figure 6a** indicates a multi cellular motion of U and V velocity distributions while the isotherms show a uniform curvature lines from the left corner of the enclosure toward the cold wall. **Figure 6** b and c, show the velocity fields and isotherms for rotating multi cylinder in both directions. The behavior of the curves is the same for single cylinder shown in **Figure 4** b and c. It shows that the maximum values of U and V are smaller than the values of single cylinder at constant angular velocity **Ω** because the radius of three cylinders is less than the radius of single cylinder. The isotherms lines become more interacted than the case of single cylinder, also the isotherms lines become more symmetric for non-rotating cylinders.

III-Multi Rotating Cylinders in the Deferent Directions

Figure 7 represents the behavior of three rotating cylinders in a different directions for $Ra = 10^3$, Pr=0.7 and $R = 0.124$. The volumetric ratio is 0.25 and the radius ratio of centers circle of cylinders is 0.25. Three configurations are studied, configuration (1): the three cylinders with **Ω**=0 as shown in **Figure 7a**, configuration (2): the upper cylinder and the right lower cylinder are rotated with **Ω**=500 CCW, while the left lower cylinder is rotated with **Ω**=500 CW as shown in **Figure 7b**. Configuration (3): the upper cylinder and the right lower cylinder are rotated with **Ω**=500 CW while the left lower cylinder is rotated with **Ω**=500 CCW as shown in **Figure 7c**. It can be show from this figure; the locations of maximum U-velocity and V-velocity near the rotating cylinders are dependent on the direction of the rotation. For isotherms, **Figure 7** (right) illustrates a curvature of isotherms lines are cover all the enclosure space in both directions. Also it can be seen the growth of the thermal boundary layer near the heated bottom wall of the cavity for configuration (3) is higher than that of configuration(2) due to the strongly effect of the rotation (forced convection).

B- Heat Transfers Field

I-Single Rotating Cylinder

Figure 8a illustrates the variation of the average Nusselt number with Rayleigh number for single rotating cylinder for $R = 0.214$ and **Ω**=500CCW, while **Figure 8b** shows the variation of the bulk mean temperature with Rayleigh number for the same above parameters. It indicates that the bulk air temperature is increased with increasing of Ra, it may be attributed to increase bouncy effecting so that the growth of the thermal boundary layer is increased near the heated surface leading to increase the average Nusselt number and enhancement the heat transfer process with increasing Rayleigh number as shown in **Figure 8a**. The effect of dimensionless angular velocity **Ω** of rotating single cylinder on the average Nusselt number and on the mean bulk temperature for $Ra = 10^3$ and $R = 0.214$ is shown in **Figure 9a** and **Figure 9b** respectively. Increment of **Ω** in both directions CW and CCW gives enhancement in the average Nusselt number due to increase the force convection effect and mixing of fluid i.e. increasing the heat transfer. For the isotherms the behavior is deferent with direction of rotating, which it inversed on the boundary conditions of θ, therefore the direction of rotating make the effect one surface more than the other. **Figure 10a** illustrates the variation of the average Nusselt number with dimensionless radius of cylinder and **Figure 10.b** shows the variation of mean bulk temperature with dimensionless radius of cylinder, both variations for $Ra = 10^3$ and Ω=500CCW. It can be seen Nu_{av} and θ_{av} are increased with increasing R, due to decrease the volume of fluid compared to the volume of cylinder and increasing the velocity to satisfy the continuity equation.

II- Multi Rotating Cylinders in the Same Direction

Figure 11 represents the effect of Rayleigh number on Nu_{av} and θ_{av} at R=0.124 and Ω=500CCW. Also, as the single rotating cylinder the Ra effect causes an increasing of both Nu_{av} and θ_{av}, it may be attributed to increase the temperature difference and bouncy effecting with increasing Ra. **Figure 12** shows variation of the average Nusselt number with dimensionless angular velocity and the variation of the average dimensionless temperature with dimensionless angular velocity for three rotating cylinders, at $\Pr = 0.7$, $R = 0.124$ and $Ra = 10^3$. Increment of Ω in both directions CW and CCW gives increasing in the heat transfer then average Nusselt number. The average dimensionless temperature is varied with varying the direction of rotating (increment or decrement), because the changing the direction of rotation makes the effect of cold or hot wall greater than the other.

III-Multi Rotating Cylinders in the Deferent Directions

Figure 13a illustrates the variation of the average Nusselt number with Rayleigh number for multi rotating cylinders when the upper cylinder and the right lower cylinder are rotated with Ω=500 CCW while the left lower cylinder is rotated with Ω=500 CW, at $R = 0.124$ while **Figure 13(b)** shows the variation of the bulk mean temperature with Rayleigh number for the same above configuration. It shows that the bulk air temperature is increased with increases of Ra; it may be attributed to increase bouncy effecting so that the average Nusselt number is increased.

Comparison among the Three Cases

The **Table 1** shows the comparison of Nu_{av} and θ_{av} among the three cases at $Ra = 10^3, \Pr = 0.7$ and $R = 0.214$ for single cylinder and $R = 0.124$ for multi cylinders. It illustrates that the varying the direction of rotation of any one of three rotating cylinder gives increment in the average Nusselt number due to increase the mixing of the fluid particles and then heat transfer between surfaces.

CONCLUSIONS

The governing equations (mass, momentum and energy) for the steady , laminar, two dimensional, incompressible flow with Boussinseq approximation and constant fluid properties for rotating single or multi cylinder in equilateral triangular enclosure are solved numerically using finite element method with FlexPDE soft package. The main conclusions:

1-The average Nusselt number is increased with increasing Ra, R and Ω for both direction of rotating.
2- The average Nusselt number of single rotating cylinder is greater than the multi rotating cylinders for the same the ratio of the volume of the solid cylinder or cylinders to total enclosure volume.
4-Muti rotating cylinders with deferent rotating direction give increment in the mixing effect and then the average Nusselt number becomes greater.
5-When the angular rotating velocity equal to zero the behavior becomes of natural convection only.

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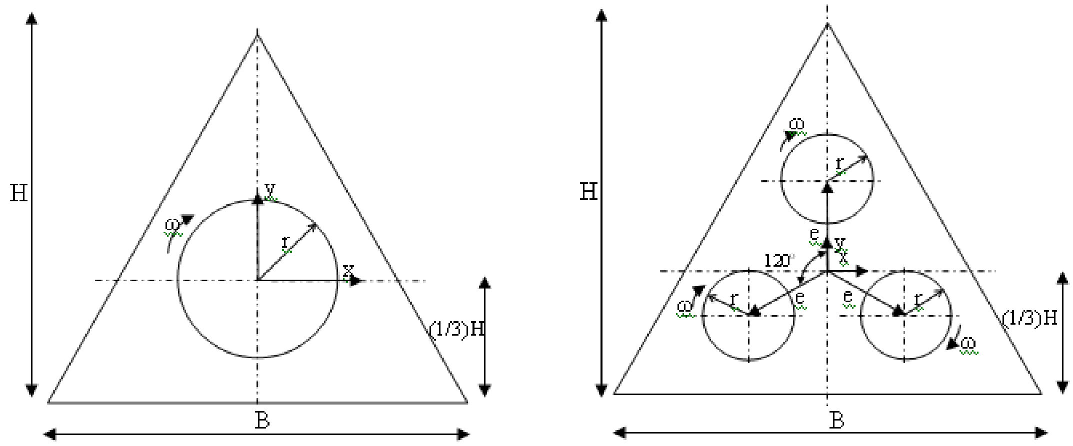
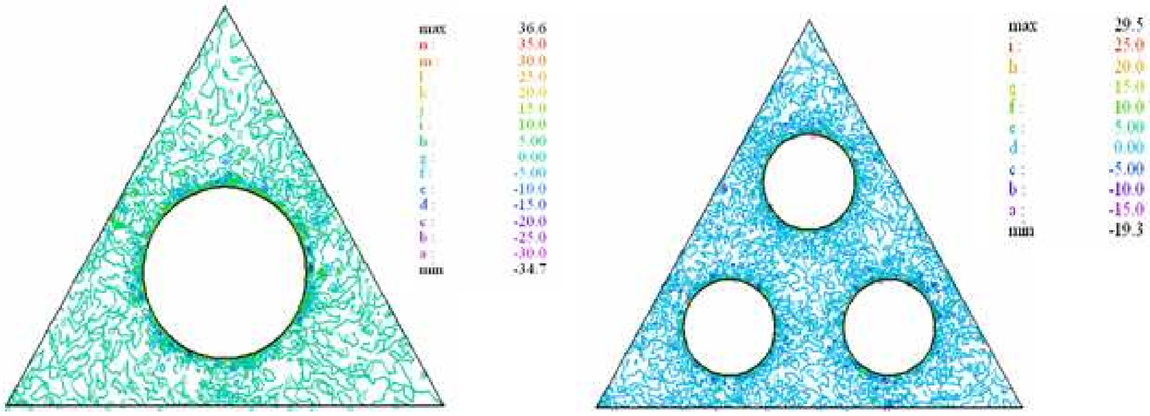
Table1 The comparison of Nu_{av} and θ_{av} among the three cases.

No. of case	Rotating velocity Ω	Nu_{av}	θ_{av}
Case I	500CCW 500CW	7.7612 7.7576	0.5584 0.4422
Case II	500CCW 500CW	7.0212 7.0113	0.5843 0.4166
Case III	i-500CCW the upper cylinder & right lower cylinder, 500CW left lower cylinder ii-500CW the upper cylinder & right lower cylinder, 500CCW left lower cylinder	7.5317 7.5304	0.4824 0.5184



(a) (b)

Figure 1 Sketch of the physical model (a) single rotating cylinder (b) three rotating cylinders.



(a) (b)

Figure 2 Verfication of numerical approach(countours of continuity equation) for $Ra = 10^3, \Pr = 0.7$ and Ω=500CCW. (a) single rotating cylinder, (b) multi rotating cylinders

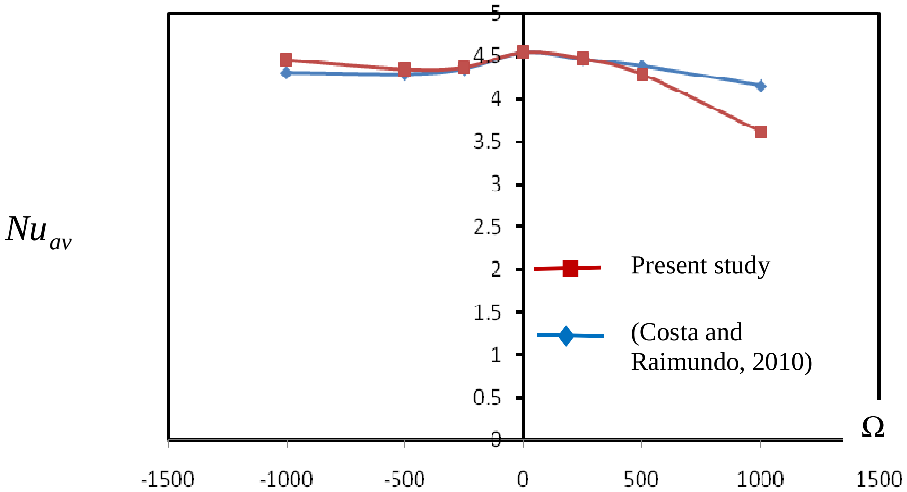
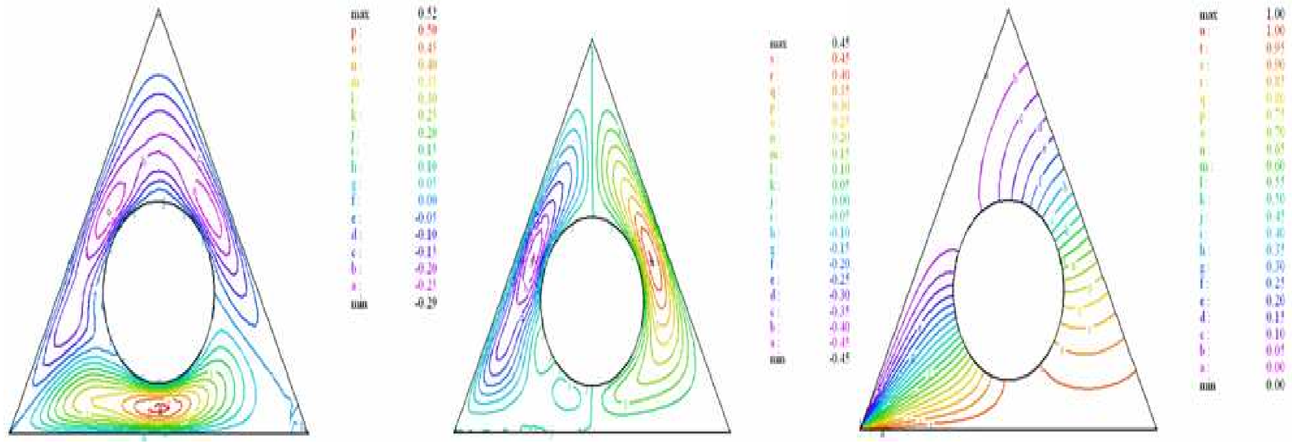
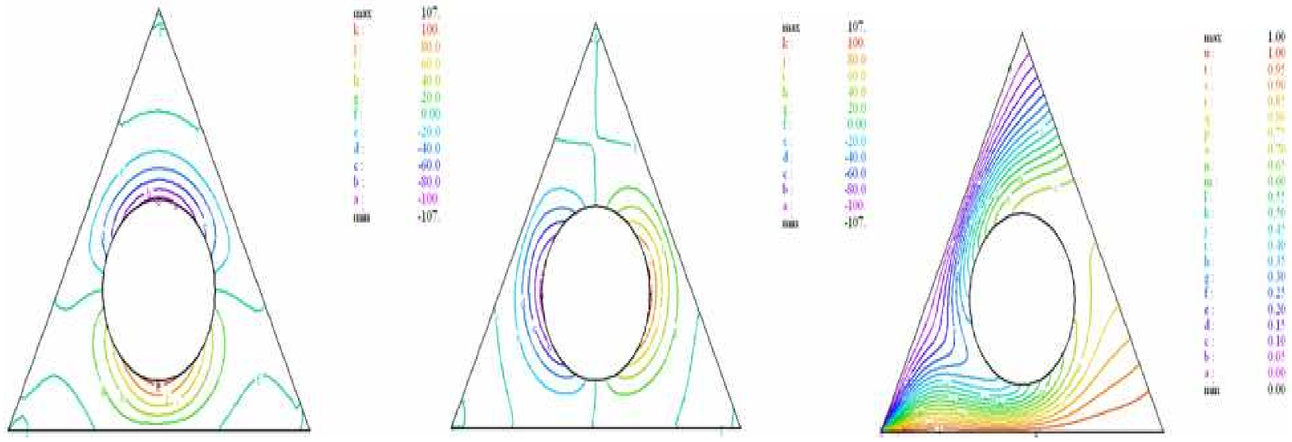


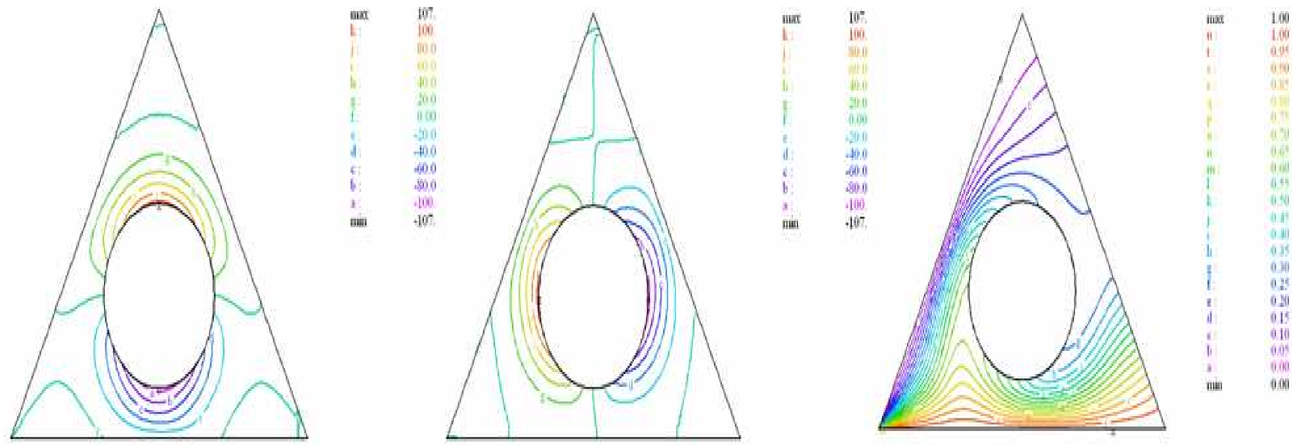
Figure 3 Comparison between the present study and results of (**Costa and Raimundo, 2010**) for $Ra = 10^5, \Pr = 0.7$ and R=0.2.



(a)

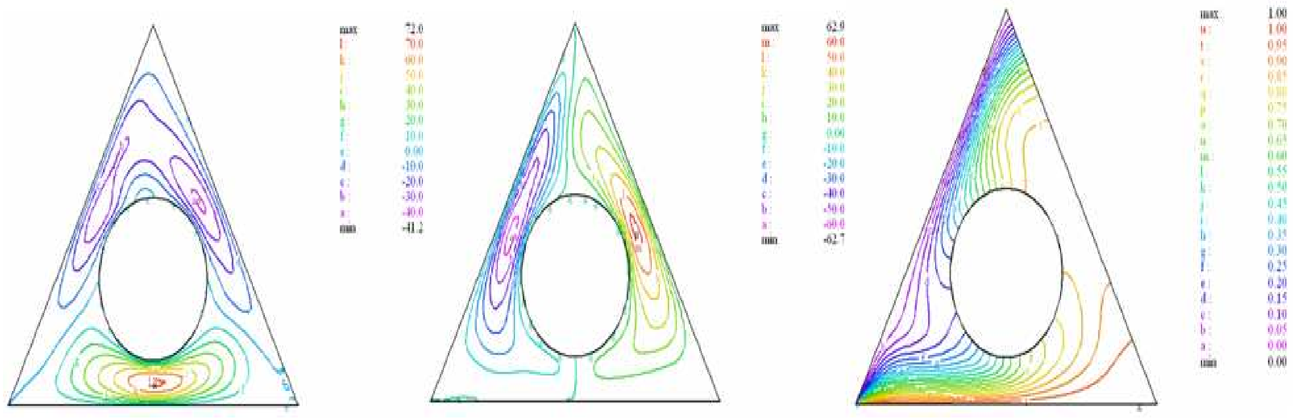


(b)

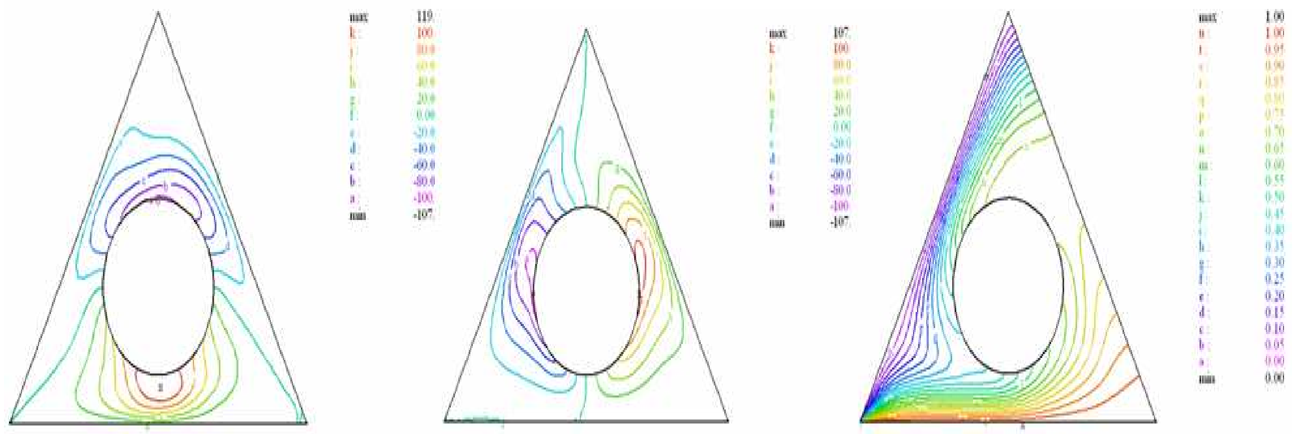


(c)

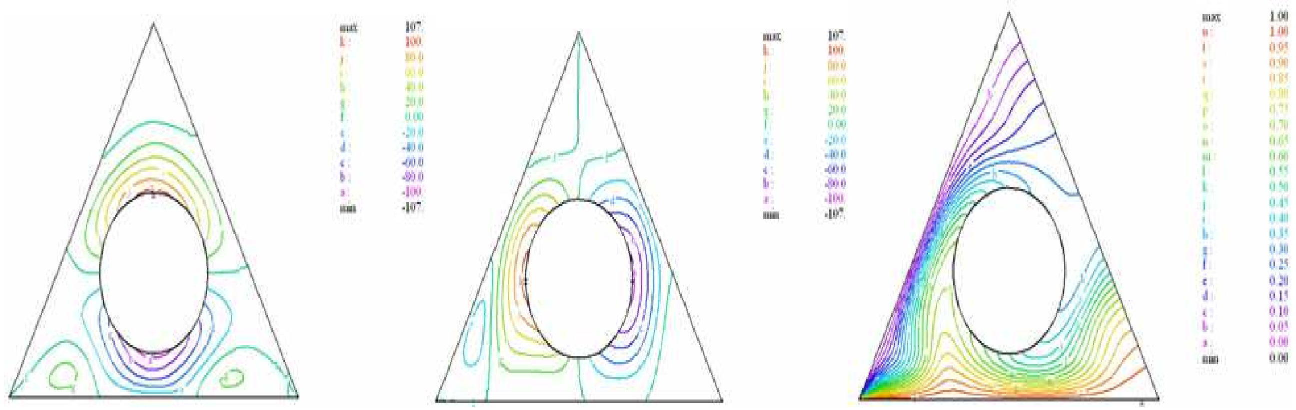
Figure 4 U-velocity (left), V-velocity (middle) and isotherms (right) for single cylinder, $Ra = 10^3$ and $R = 0.214$ and (a) **Ω**=0 (b) **Ω**=500CCW (c) **Ω**=500CW.



(a)

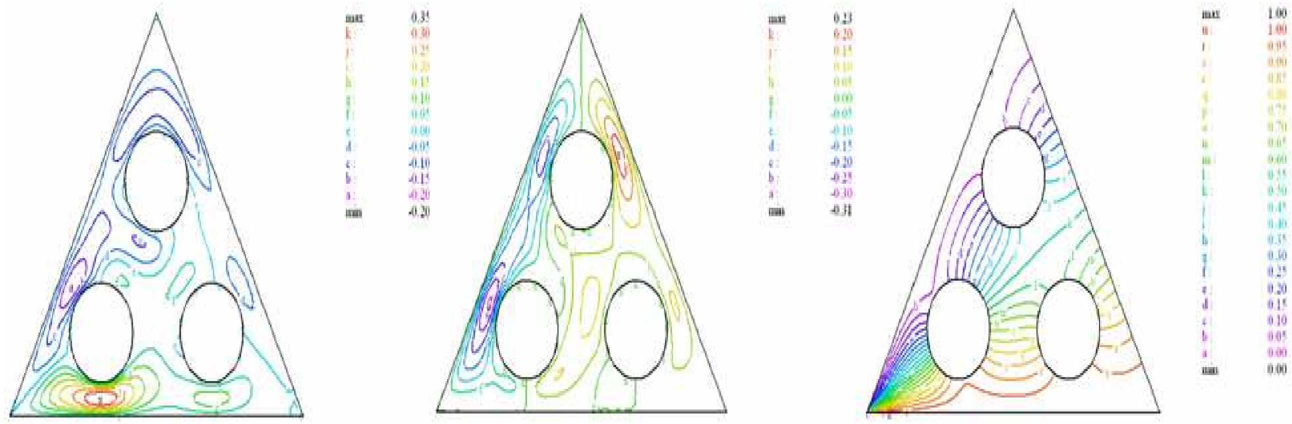


(b)

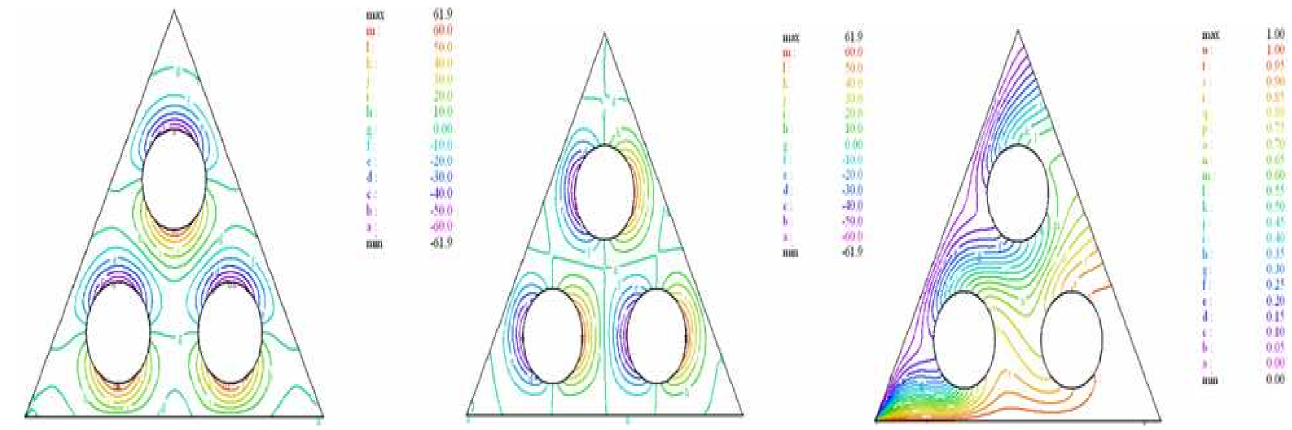


(c)

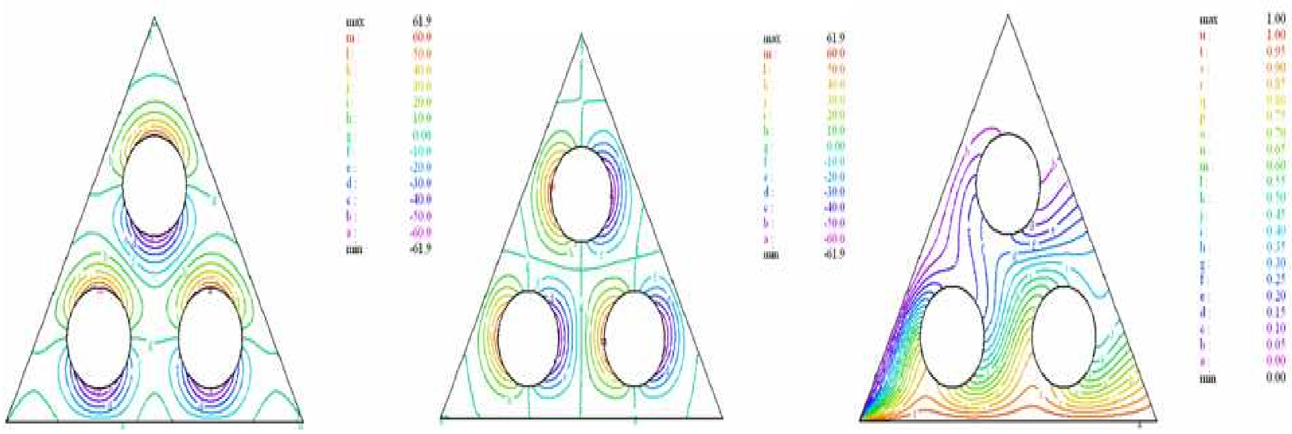
Figure 5 U-velocity (left), V-velocity (middle) and isotherms (right) for single cylinder, $Ra = 10^5$ and $R = 0.214$. (a) Ω=0 (b) Ω=500CCW (c) Ω=500CW.



(a)

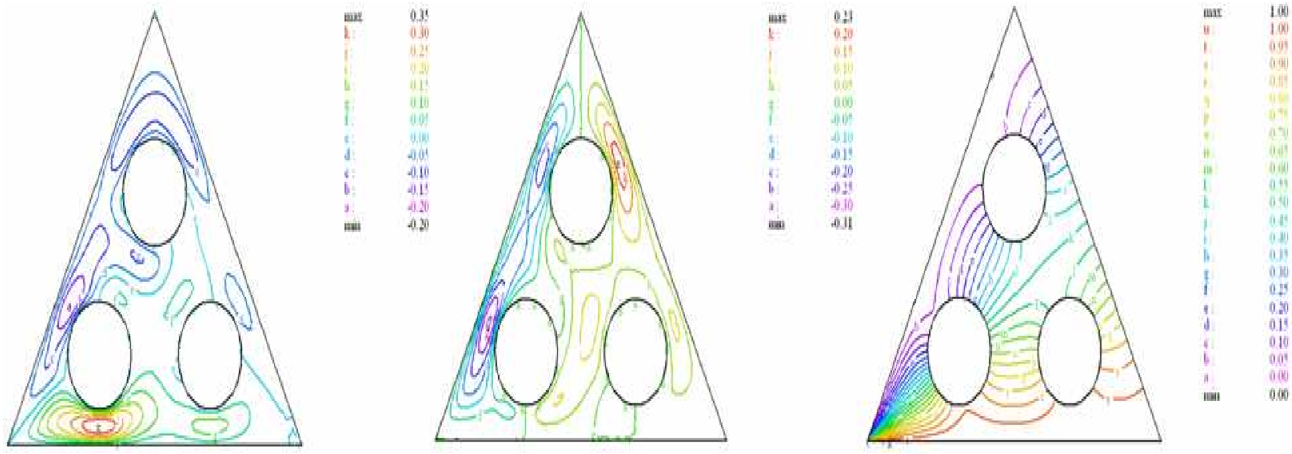


(b)

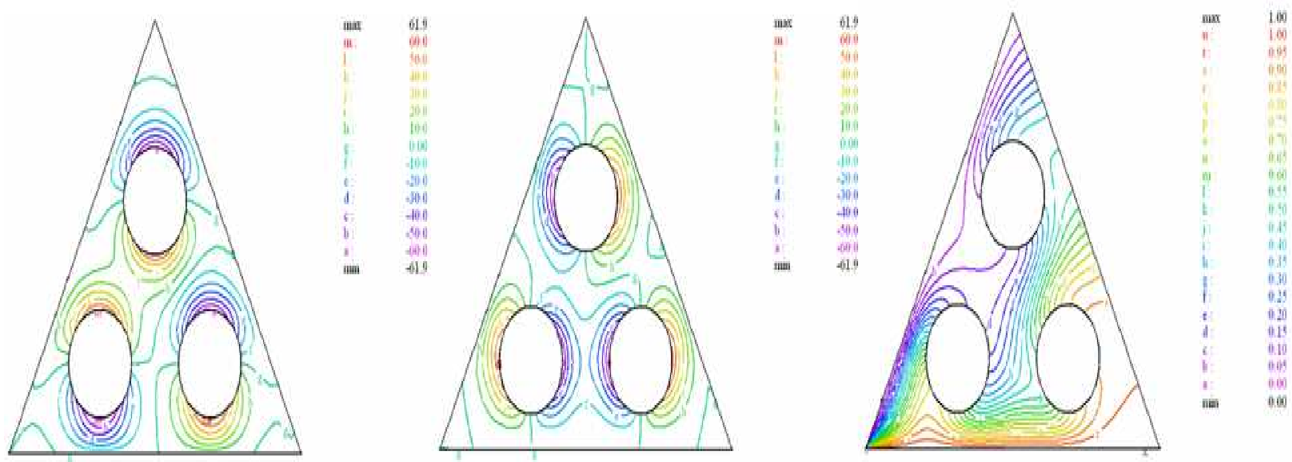


(c)

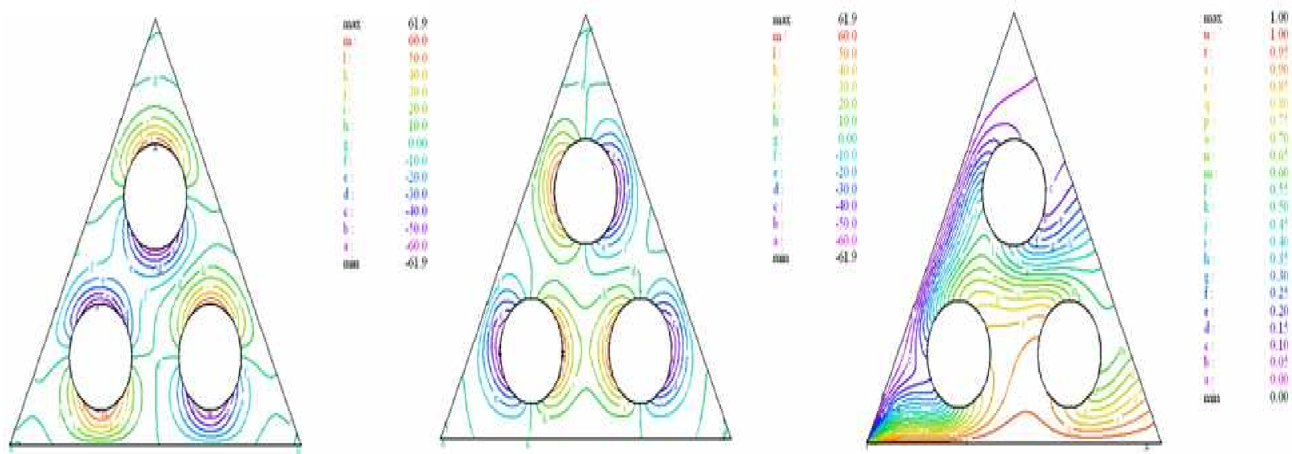
Figure 6 U-velocity (left), V-velocity (middle) and isotherms (right) for multi cylinders rotating at the same direction, $Ra = 10^3$ and $R = 0.124$. (a) Ω=0 (b) Ω=500CCW (c) Ω=500CW.



(a)

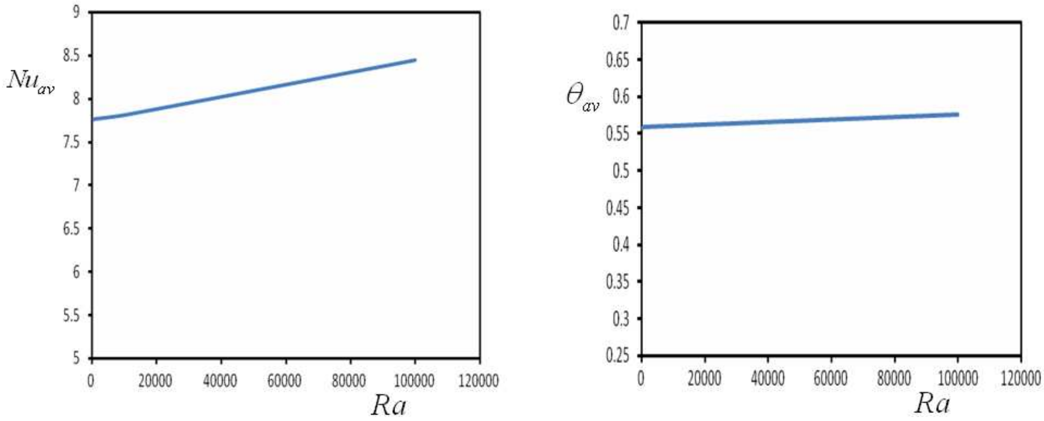


(b)



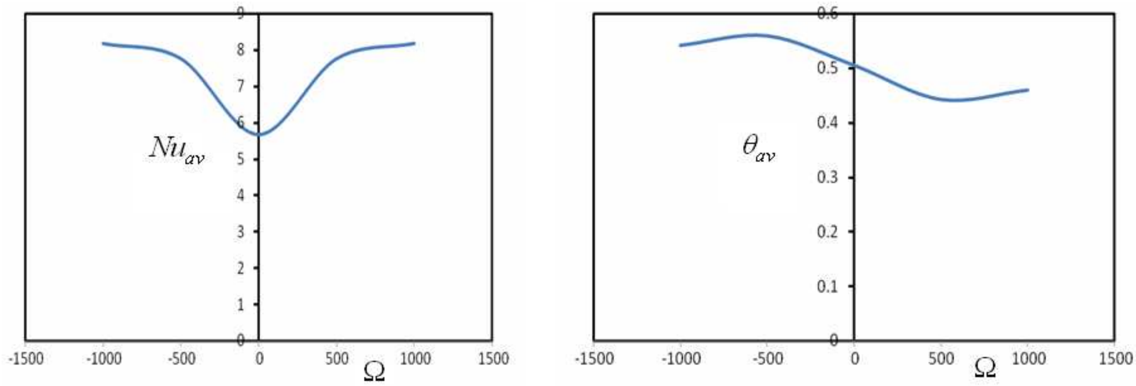
(c)

Figure 7 U-velocity (left), V-velocity (middle) and isotherms (right) for multi cylinders, $Ra = 10^3$ and $R = 0.124$. (a)the three cylinders have Ω=0 (b) the upper and the right lower cylinders are rotated with 500CCW and the left lower cylinder is rotated by 500CW, (c)the upper and the right lower cylinders are rotated 500CW and the left lower cylinder is rotated by 500CCW.



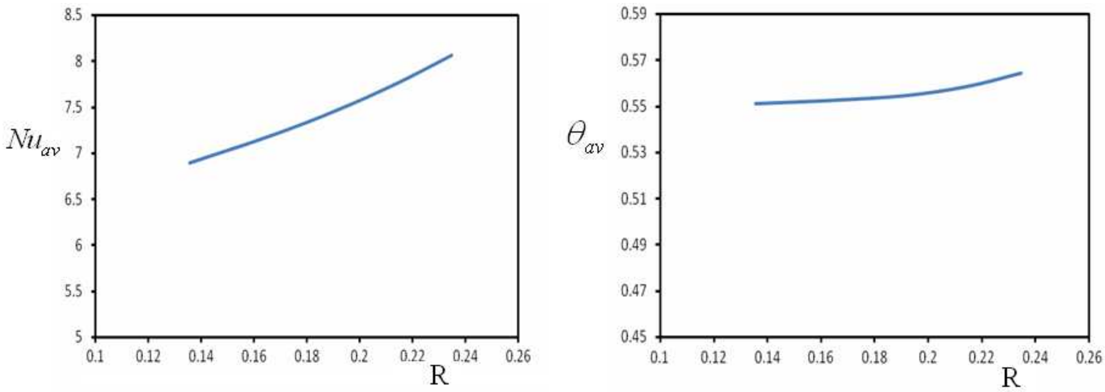
(a) (b)

Figure 8 (a) Variation of the average Nusselt number with Rayleigh number (b) Variation of the average dimensionless temperature with Rayleigh number, for single rotating cylinder, at $R = 0.214$ and **Ω**=500CCW.



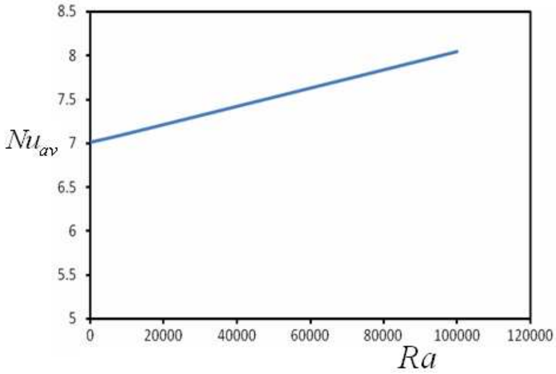
(a) (b)

Figure 9 (a) Variation of the average Nusselt number with dimensionless angular velocity (b) variation of the average dimensionless temperature with dimensionless angular velocity, for single cylinder at $Ra = 10^3$ and $R = 0.214$.



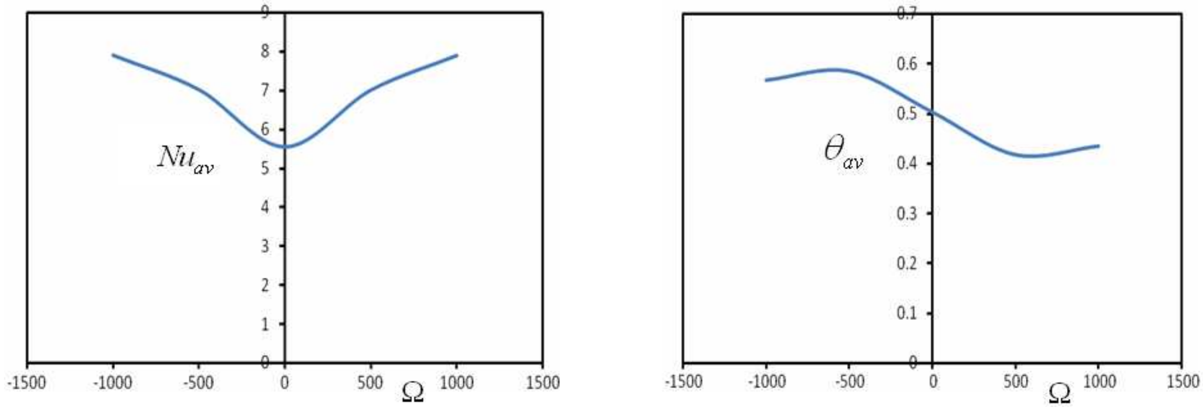
(a) (b)

Figure 10 (a) Variation of the average Nusselt number with dimensionless radius of cylinder (b) Variation of the average dimensionless temperature with dimensionless radius of cylinder, for single cylinder, at $Ra = 10^3$ and **Ω**=500CCW.



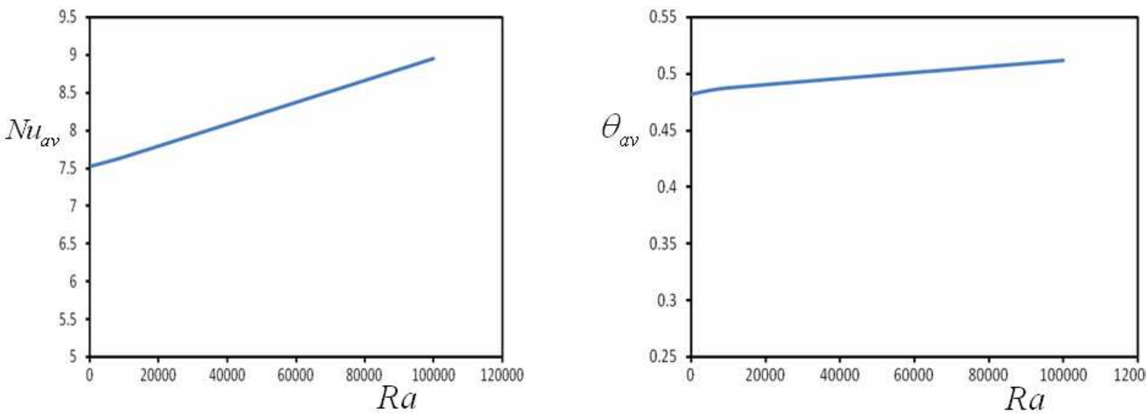
(a) (b)

Figure 11 (a) Variation of the average Nusselt number with Rayleigh number (b)Variation of the average dimensionless temperature with Rayleigh number for three cylinders rotating at the same direction, and $R = 0.124$ and Ω=500CCW.



(a) (b)

Figure 12 (a) Variation of the average Nusselt number with dimensionless angular velocity (b)Variation of the average dimensionless temperature with dimensionless angular velocity, for three cylinders rotating at the same direction, at $Ra = 10^3$ and $R = 0.124$.



(a) (b)

Figure 13 (a) Variation of the average Nusselt number with Rayleigh number (b)Variation of the average dimensionless temperature with Rayleigh number, for three cylinders rotating at deferent directions, and $R = 0.124$.