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Mathematical Technique for Controlling the Gaps in the Cubic Bezier Curves for Two Dimensional

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Abstract

Designs are generated in the Bezier curves at most contains gaps in form that is drawn up in curve. This state is causing misshaping final form that depended on curves generating.

In this research applies of suggested method is making Bezier curve generating don't contains upon gaps and can controlled to eliminate gaps by using added parameters to control it.

The research applies modified equations by using different shapes in different cases and brought shapes (curves) don't contain the gaps and smoothing curves' .

Keywords: Bezier Curves, Gallier modify equation, Computer graphics, Smoothing curve, Gaps elimination in curve, enhancement drawing algorithms

التقنية رياضية للسيطرةِ على الفجواتَ المنحنياتِ Bezier الثنائي الأبعادِ

الخلاصة

التصاميم المولدة بواسطة المنحنيات *Bezier* تحتوي في كثير الأحيان على فجوات في صورة المنحي المرسوم. وهذا يتسبب بتشوه التصاميم النهائية المعتمدة على هذا المنحنيات.

في هذا البحث تم اقتراح طريقة لتوليد منحنيات *Bezier* بحيث لا توجد هذا الفجوات ويتم التحكم بذلك من خلال التحكم بمعاملات مضافة لسيطرة على ذلك.

تم تطبيق المعادلات المعدلة الجديدة باستخدام التصاميم المختلفة الإشكال والمختلفة الحالات ودلت النتائج على أن المنحنيات مصقولة ولا تحتوي على الفجوات.

1. Introduction

Smoothing curves generated in designs are very important, because it overcome the drawn up defects (gaps) that offers in designs. See of the original curve Bezier equation:

$$P(t)=(1-t)^3P_1+3(1-t)^2tP_2+3(1-t)t^2P_3+t^3P_4 \qquad \dots (1)$$

Where: p(t) is X(t),Y(t) of point drawn up of curve and t is value 0<=t<=1 and P1, P2, P3, P4 are a four control points that are effecting in curve shape. [Faux, 83],[Gerald ,99], [Watt ,00].

In the generating of curve, needed to increase factor to value t in equation 1 to drawn up curve, in this case the increase factor is effective either a generating gaps or a smoothing curve, for example if increase factor =0.5 therefore curve points generating=3 , but if increase factor =0.2 therefore curve points generating=6 and finally if curve points are much more therefore

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smoothing curve, otherwise the misshaping of curve by gaps generating and can concluded curve points generating in curve drawn up by

$(\frac{1}{\text{increase factor}}+1)$.

In classic methods used modification of increase factor to improve curve shape if gaps are appeared therefore modify(and/or) change the increase factor but the classical method is weak for two causes that generating gaps after change (and/or) modify increase factor'

Case(1) scattering four control points look figure 1 and figure 2 show problem And figure 2 show that smoothing curve effective in scattered position control points

Case(2) if curve is enlarge therefore gaps appear look figure 3 and figure 4 to show another problem. And another problem if Bezier curve is big curve then lead too generate gaps between curve points generation figure 3 show that curve smoothing before scaling: Figure 4 show that gaps appear if scaling Bezier curve .The classic method is not efficient in this two crises.

In main idea of research, it process in this crises and smoothing of curve generating by using added parameters can be controlled to smooth curve and eliminate gaps in curve generating without modify or change of increase factor and control curve points generating and make increase factor is independent in original Bezier curve and this parameters is effective to eliminate gaps and improve to smoothing curve.

2- Gallier Modified Algorithmic.[Gallier,00].

The modified De-Castejalu algorithm [Faux, 83], [Gerald, 99].and Bezier curve is developed by Gallier as form below [A.M.J, 2010] , [Jaber, 2005]:

$$P(t) = (1-\Omega)^3 P_1 + (1-\Omega)^2 \Omega P_2 + (1-\Omega)\Omega^2 P_3 + \Omega^3 P_4 \quad \text{.....(2)}$$

Equation (2) called original Gallier modified Bezier curve is dependent on interval *[r, s]*.

P_1, P_2, P_3, P_4 are control points equation (2)

Where $\Omega=\frac{(t-r)}{(s-r)}$ and $1-\Omega=\frac{(s-t)}{(s-r)}$

And r, s are integer number and r < s

And t is value r<=t<=s

$$P(t)=(\frac{(s-t)}{(s-r)})^3P_1+(\frac{(s-t)}{(s-r)})^2\ \frac{(t-r)}{(s-r)}P_2+(\frac{(s-t)}{(s-r)})\ (\frac{(t-r)}{(s-r)})^2P_3+(\frac{(t-r)}{(s-r)})^3P_4 \quad \text{........(3)}$$

Equation (3) called Gallier modified Bezier curve is dependent on interval *[r, s]*. [Gallier,00]

But if *r =0 and s =1* in equation (3) then return to equation (1) [original Bezier curve is dependent on interval *[0, 1] in* De-Castejalu algorithm [Faux ,83], [Gerald ,99].

3. Origin of appearing gaps on Bezier Curves generating

See in section 1 the curve points generate by $(\frac{1}{\text{increase factor}}+1)$ where increase factor boundary interval (0..1] (0<increase factor<=1) if increase factor is a big value (reach in 1 'increase factor<1') the number of curve points generate a little number, therefore generate a much number of gaps and lose the smoothing of curve but if increase factor is small value (reach in 0 'increase factor >0') the number of curve points generate a much number, therefore generate of smoothing curve and eliminated gaps of between curve points

But another problem if control points of Bezier curve are scattered position or long distance, therefore it generates gaps and lose the smoothing curve.

Where in equation 3 enable to control for curve points are generated by using another factors not of the increase factor in classic method and advantage of the Gallier modification equation to solve it. [Gallier 00].

4. Gaps eliminating through Gallier modification equation

the proposal system is advance of Gallier equation "equation 3" because the equation found two parameters r, s where these parameters give controlled to generate for curve points that help to block gaps and smoothing curve and conclusion if difference r, s is big ,therefore the gaps eliminate and otherwise the gaps increasing between curve points .

Look at figure 5 and figure 6 the increase factor is unchanged in two figures and r, s are controlling for smoothing curve and curve points generating.

See Table 1.Properties of the difference value of r ,s if inc. factor =0.001. See Figure 7 show that relationship between smoothing with difference s-r ,and figure 8 show relationship between gaps curve points with difference s-r.

5. Optimal curve Bezier

Optimal curve Bezier is curve if it used equation (1) for original Bezier curve and addition the t value is between 0 to 1 '0<=t<=1' and can be used in equation (2) (3) to generate optimal curve Bezier if r=0 and s=1 and finally the curve points generated same of number and properties is similar. But another new fact if difference (s-r) =1 therefore generate optimal curve Bezier see table 2.

But difference (s-r) >1 then generate curve Bezier is not optimal curve Bezier that different by number of curve points generated and properties (ex. smoothing gaps see table 1)

See Table 2.Properties of the value of (s-r)=1 in optimal curve Bezier

6. The Proposed Method System

in the propose system advanced of Gailler modification equation (3) " see, the properties of table 1 and section 4" that can be eliminated gaps and smoothing curve by advantage of increasing subtraction (s-r) "see, the table 1". Finally, the algorithm in proposed method system as following :

<u>Input:</u> **r** value, **s** value: where r<s, 4 control points as **(X1,Y1), (X2,Y2), (X3,Y3), (X4,Y4)**

<u>Output:</u> Number of curve points and drawing final Bezier curve.

1. Start
2. t=r
3. Generate curve point by using Gailler modify equation "equation 3 "

a. $Xcp = (\frac{(s-t)}{(s-r)})^3 X_1 + (\frac{(s-t)}{(s-r)})^2 \frac{(t-r)}{(s-r)} X_{2+} (\frac{(s-t)}{(s-r)}) (\frac{(t-r)}{(s-r)})^2 X_{3+} (\frac{(t-r)}{(s-r)})^3 X_4$

b. Ycp = $(\frac{(s-t)}{(s-r)})^3 Y_1 + (\frac{(s-t)}{(s-r)})^2 \frac{(t-r)}{(s-r)} Y_{2+} (\frac{(s-t)}{(s-r)}) (\frac{(t-r)}{(s-r)})^2 Y_{3+} (\frac{(t-r)}{(s-r)})^3 Y_4$

4. Plot curve point (Xcp, Ycp) // draw point in Bezier curve
5. Cp =Cp+1 //account no. of Curve points
6. t=t+ increase factor // update t to generate
7. if t <= s then go to step 3
8. drawing final curve and show no. of curve points that generating.
9. end

Summary the Top level in DFD represents for proposal system in *figure 9.*

7. Implementation

In image process use for example to obtain pixels information in color image (true color). See figure 10 and figure 11

This pixels information's are very important & useful if it used on computer security's

8. Consolations

The research can conclude its:

1. Finally" the curve points generated in equation (1) is $(\frac{1}{increasefactor}+1)$ see section 1, but curve points generated in equation (2) (3) is $(\frac{(s-r)}{increasefactor}+1)$.

See table 1.

2. In modified method, the designer can control the smoothing curve and gaps eliminated of curve by using Gallier modified Bezier curve depending on interval [*r, s]* instead of increase factor.
3. That parameters r, s are effecting to time period in generations of Bezier curve because, it generated more curve points to cover this a gaps.
4. The method for controlling the smoothing and gaps of generating the curve which used in the design, effect by the type of the computer used in generating, and different (s-r) used its.
5. controlled points or the curve except smoothing and gaps.
6. if increase factor = 0 therefore generating infinity curve points but Increase factor =1 therefore generating only two points If difference (s-r)>1 that it leads to smooth curve and eliminated gaps generated than in (s-r) =1.
7. Parameters r, s that enable to control to generate curve points but if (s-r)=1 therefore generate curve points is same as equation(1) & finally can be controlled without modify or change for increase factor.
8. Can be used in image processing upon computers security domains.
9. This proposes can be done without changing any of the (first control point of Bezier curve, end control point of Bezier curve) in original Bezier curve and Gallier modified Bezier curve if the values r=0, s=1 OR (s-r)=1.
10. This work proposes technique for controlling the smoothing curve and gaps eliminated, depends on Gallier modified cubic Bezier curve.
11. this work is benefit to enhancements of Bezier curves in 3D dimensional.

9. References

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Table 1.Properties of the value of r ,s if inc. factor =0.001

S	r	(s-r)	No. curves points
1	0	<u>1</u>	<u>100</u>
2	0	2	200
1	-1	2	200
7	-1	8	801
12	4	8	801
101	100	<u>1</u>	<u>100</u>
120	100	**20**	**2000**
130	100	*30*	*3000*
0	-20	**20**	**2000**
-50	-80	*30*	*3000*

Table 2.Properties of the value of (s-r)=1 if inc. factor =0.001

S	r	**(s-r)**	No. curves points
1	0	**1**	*100*
2	1	**1**	*100*
0	-1	**1**	*100*
-7	-8	**1**	*100*
12	11	**1**	*100*
101	100	**1**	*100*
120	119	**1**	*100*
-130	-131	**1**	*100*
-1019	-1020	**1**	*100*
1050	1049	**1**	*100*

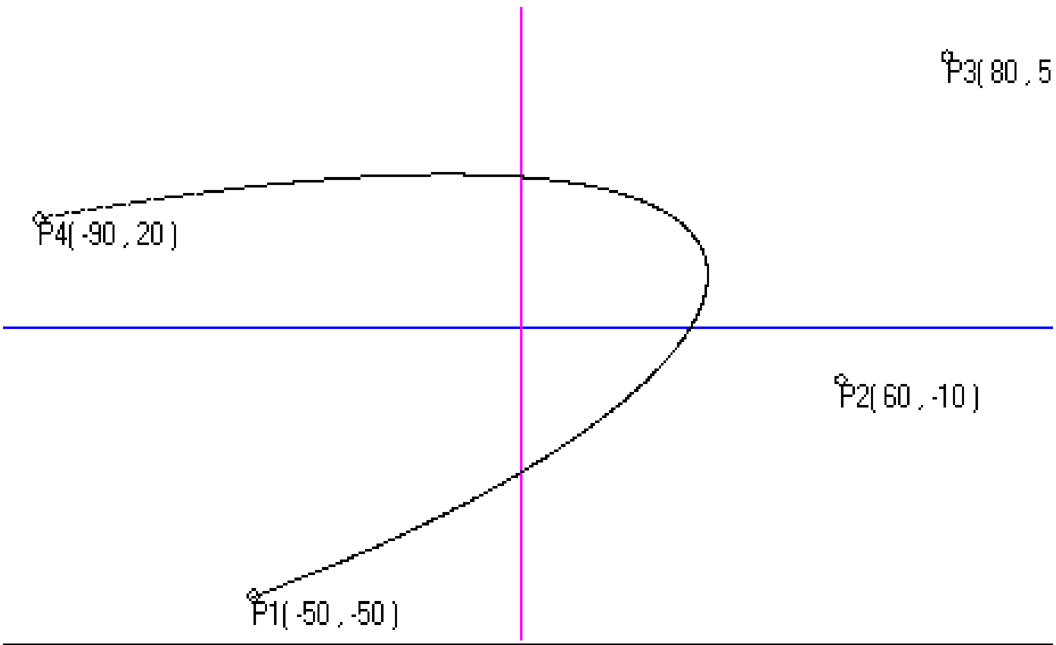

Figure1: Curve smoothing inc. factor =0.001

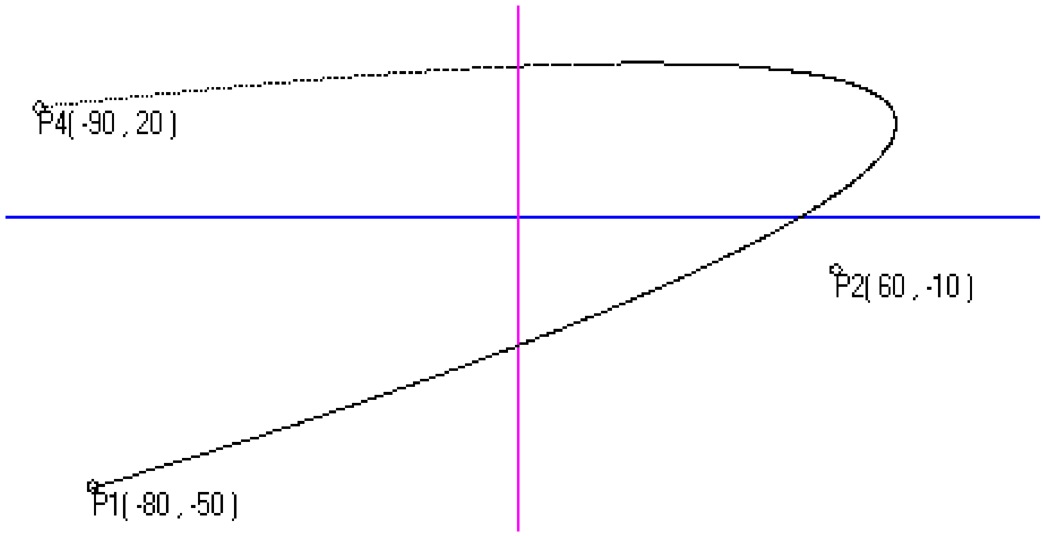

Figure2: Curve smoothing inc. factor =0.001in scattered control points

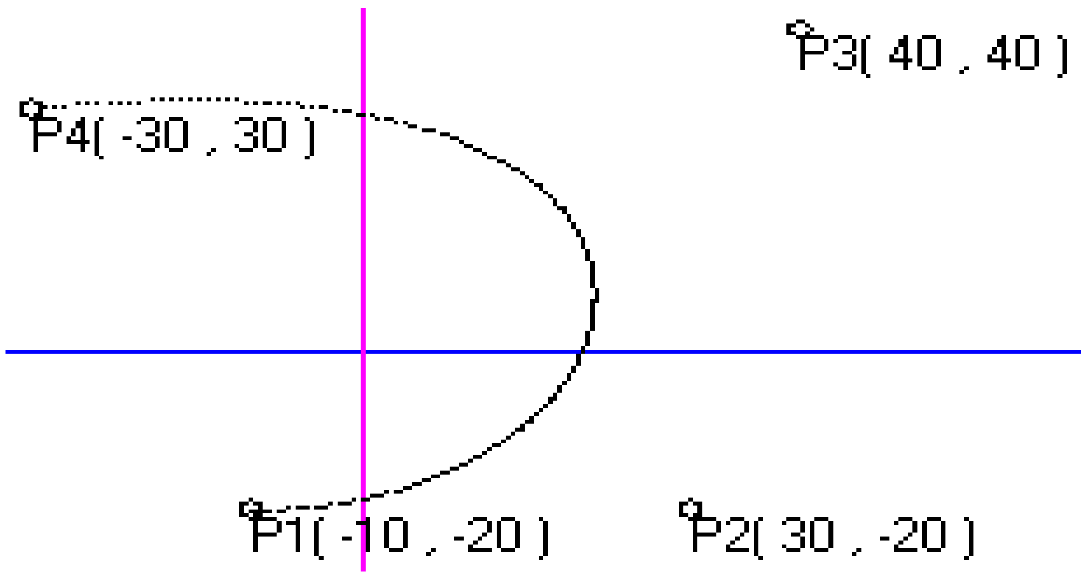

Figure 3: Curve smoothing inc. factor = 0.005 before scaling Bezier curve

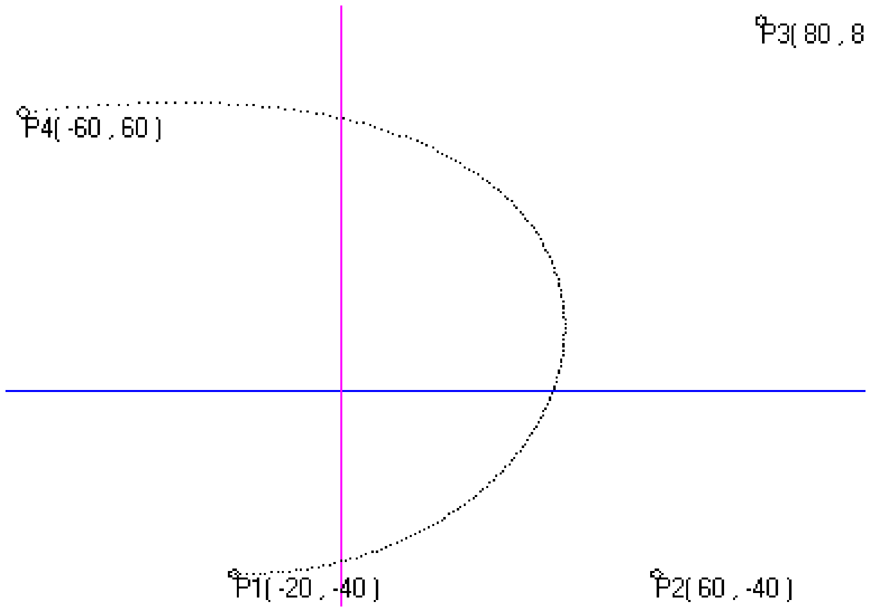

Figure 4: Curve smoothing inc. factor = 0.005 after scaling Bezier curve

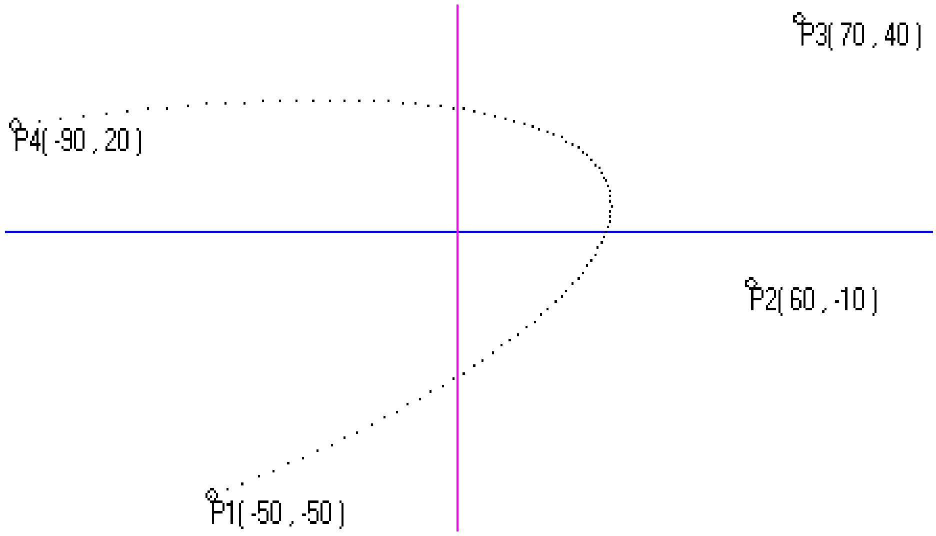

Figure 5: Increase factor =0.01, curves points=100, r=0, s=1,(s-r)=1

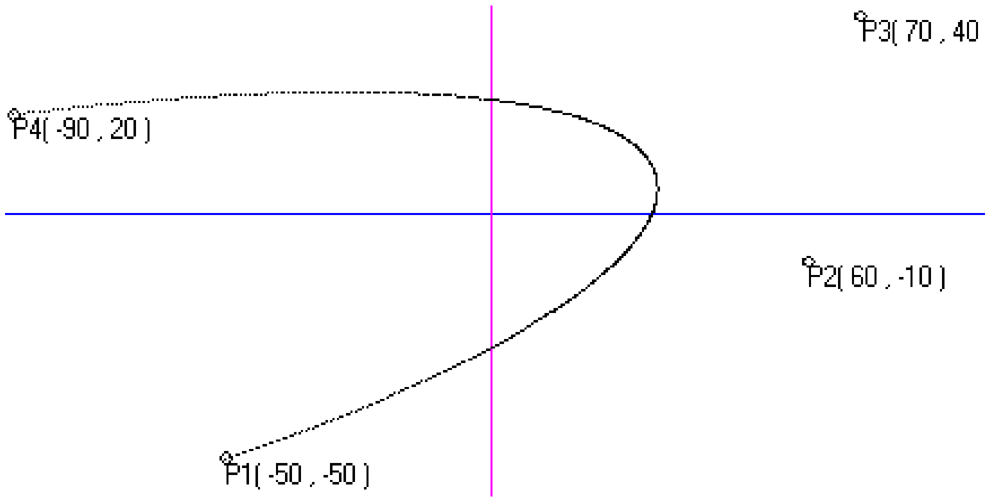

Figure 6: Increase factor =0.01, curves points=501, r=0, s=5, (s-r*) =5*

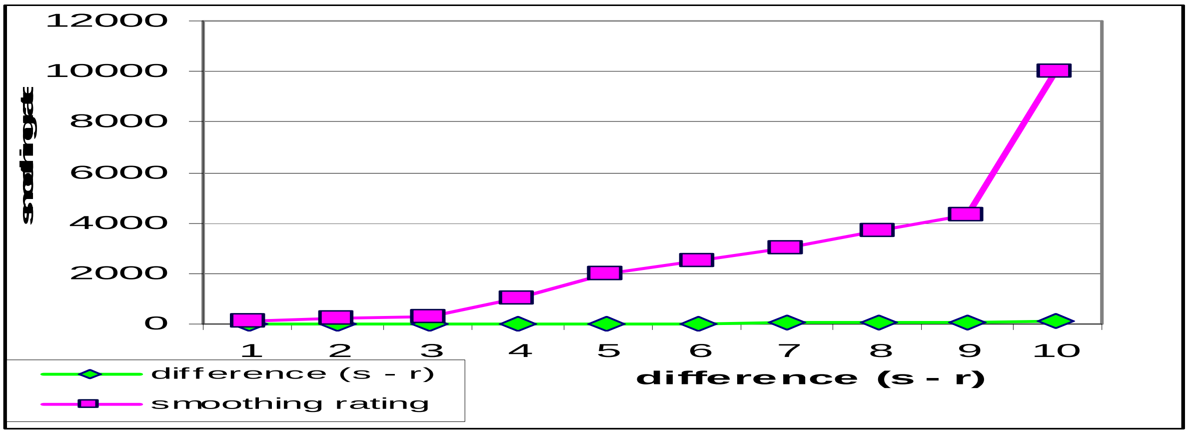

Figure 7: relationships between smoothing with difference s-r in increase factor=0.01

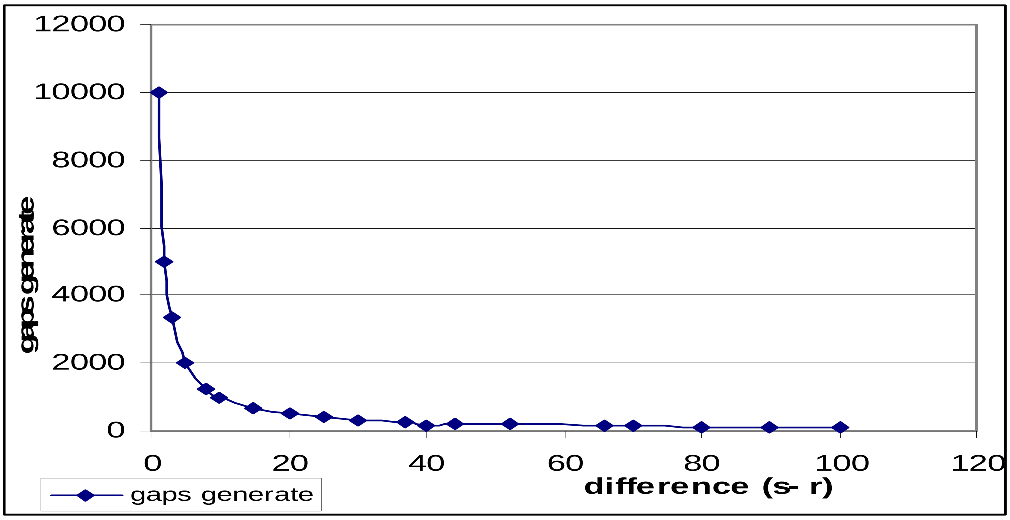

Figure 8: show relationship between gaps curve points with difference s-r in increase factor =0.01

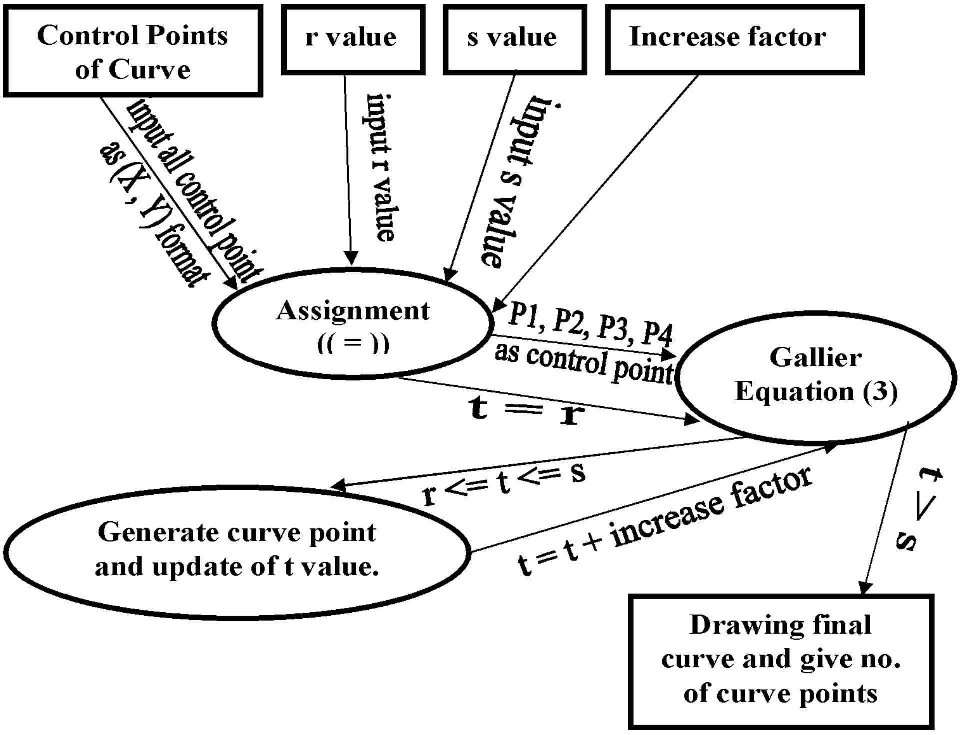

Figure 9: Show top DFD level of proposal work.

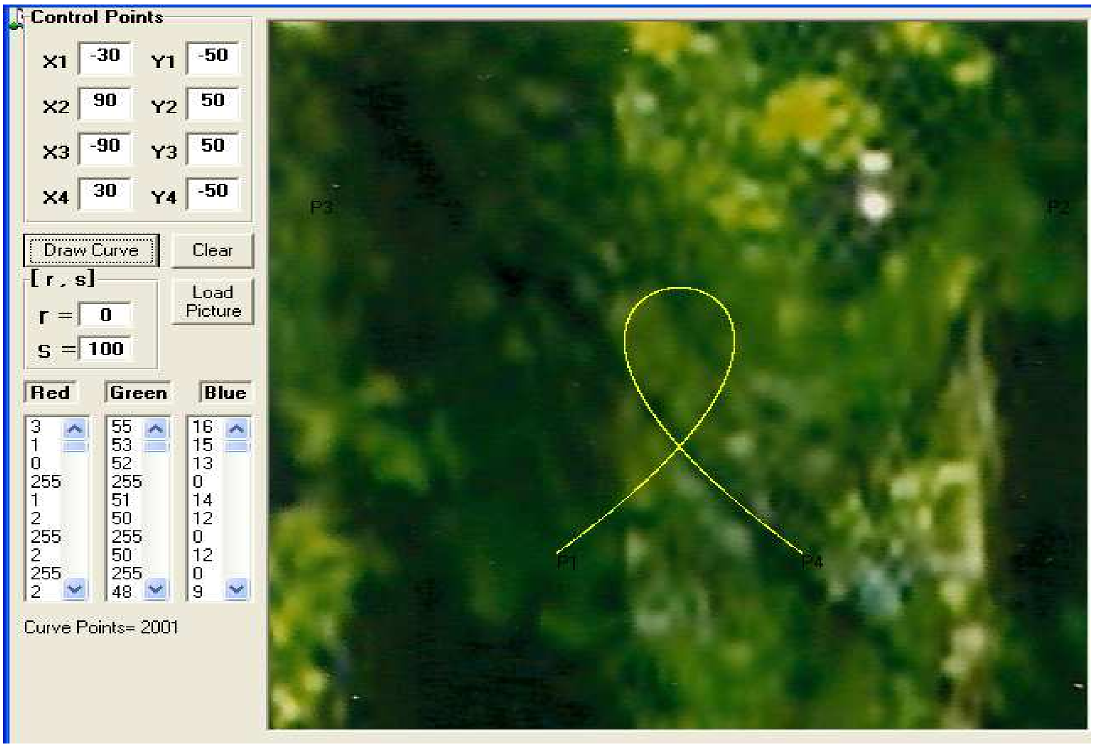

Figure 10 used in image process

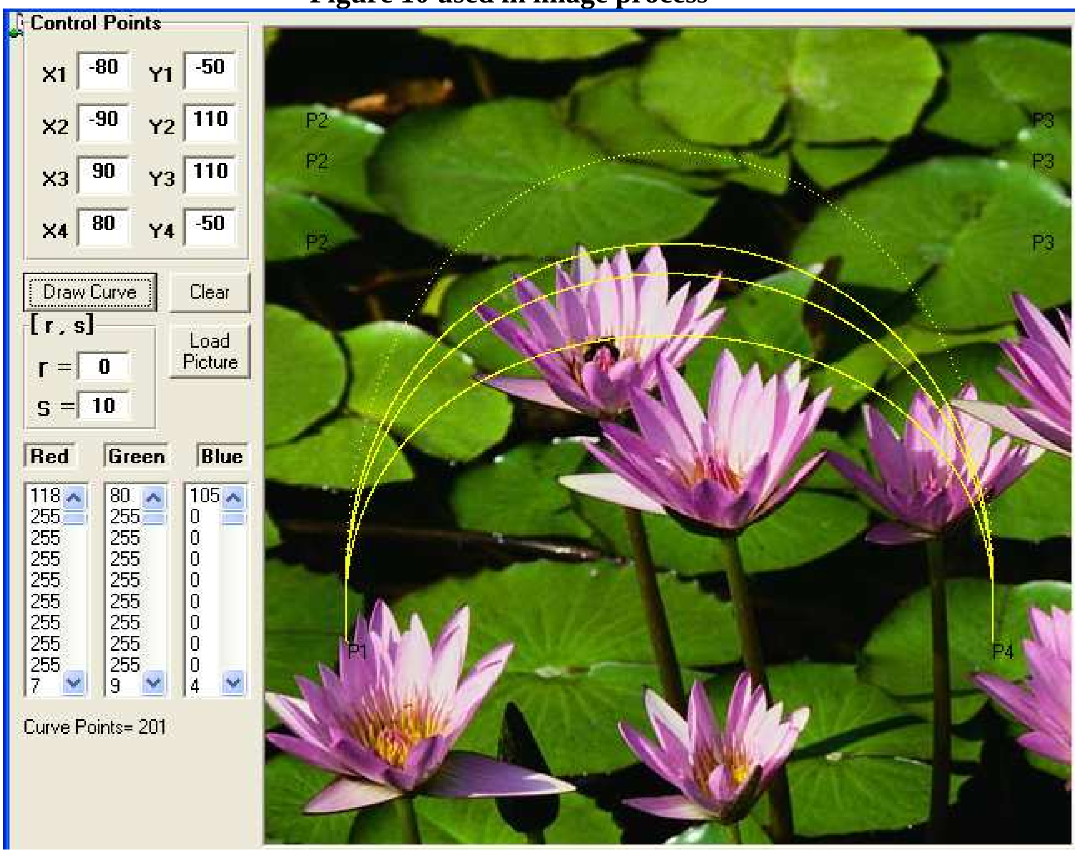

Figure 11 how can obtain pixels information