ESTIMATED EQUATIONS FOR WATER FLOW THROUGH PACKED BED OF MULTI –SIZE PARTICLES

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Abstract

Different parameters affecting the pressure drop of fluid flow through packed bed have been studied. These parameters are fluid velocity, bed porosity, bed diameter, sphericity, particle diameter, packing height and wall effect.
A semi-empirical equation for water flow through packed bed has been proposed, which can be used for several types and kinds of packing materials with different sizes, depending on B k h π. h o r
The results of calculations for the proposed equation have been compared with many documented experimental literatures. This comparison gave a very good agreement, and has been represented and curves. The results from Ergun equation using similar conditions have been represented in the curves for the sake of comparison. Ergun equation results were far away from the experimental data and the semi-empirical equations results. The main reason of this deviation was that Ergun's equation neglect wall effect on fluid flow, differences in bed dimensions, packing shapes and sizes.
The working range of the proposed equation is within the fixed region of the fluid flow diagram, i.e., the estimated equation can be used for fluid flow up to the fluidization point. A semi-empirical equation based on Leva equation had been modeled to evaluate the minimum fluidization velocity.
Key words: semi-empirical equation, pressure drop, Ergun equation, , minimum fluidization.

معادلات مقترحة لجريان الماء خلال العمود الحشوي لحشوات متعددة الأحجام

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الخلاصة

تم دراسة العوامل المختلفة التي تؤثر على هبوط الضغط عند جريان الموائع في عمود حشوي كل على حده، هذه العوامل هي سرعة جريان الموائع، مسامية الحشوة، طول الحشوة في العمود الحشوي، قطر العمود الحشوي، معامل كروية الحشوات، قطر الحشوة ودراسة تاثير جدار العمود الحشوي.
تم صياغة معادلة شبه عملية لجريان الماء خلال العمود الحشوي تصلح لعدة انواع واشكال الحشوات ولاحجام مختلفة بالاعتماد على نظرية باكنكهام. تم مقارنة النتائج المستحصلة من الحسابات لجريان الموائع خلال عمود حشوي مع عدد كبير من النتائج العملية المستحصلة من المصادر الموثقة،و هذه المقارنة اعطت تطابق جيد جدا، وتم عرض ذلك في جداول و رسومات. تم وضع النتائج المستحصلة باستخدام معادلة ايرجن في هذه الرسومات لنفس الضروف السابقة لغرض المقارنة. ان نتائج معادلة ايرجن بعيدة عن النتائج العملية ونتائج المعادلات شبه العملية المكتوبة. والسبب الرئيسي لذلك يعود الى اهماله تاثير جدار العمود الحشوي على جريان الموائع، وهناك عدة اسباب اخرى للاختلاف كاستخدامه ابعاد مختلفة للعمود الحشوي،واشكال وحجوم مختلفة للحشوات.تم دراسة فترة عمل معادلة جريان الموائع خلال المنطقة الخاصة بالجريان المنتظم أي انه المعادلات المكتوبة تصلح للعمل الى نقطة الجريان الغير المنتظم . تم كتابة معادلة شبه عملية بالاعتماد على معادلة ليفا لحساب السرعة الدنيا للجريان وذلك لحساب نقطة تغير الجريان الى غير المنتظم.

NOTATIONS

A	=	The bed cross sectional area (m).
D_R	=	Diameter of the bed (m).
d_p	=	Diameter of the particle (m).
dp_{eff}	=	Effective particles diameter (m).
d_{pi}	=	Diameter of particle i in mixture (m).
e	=	Porosity of the bed.
g	=	Acceleration due to gravity, 9.81 (m/s^2).
L	=	The height of packing in the bed (m).
l	=	Thickness of the bed (m).
ΔP	=	Pressure drop through packed bed, Pa ($kg/m.s^2$).
Re_{mf}	=	Reynold number at minimum fluidization velocity.
Re_p	=	Modified Reynolds number for packed bed.
Re	=	Reynolds number.
u	=	Superficial velocity (m/s).
u_{mf}	=	Minimum fluidization velocity (m/s).
u_t	=	Terminal velocity (m/s).
V	=	Volume of the fluid flowing through bed in time t.
x_i	=	The weight fraction of particle i.

Greek Symbols

ε	=	Porosity of the bed.

ε_{mf} = Minimum fluidization porosity of the bed.
= Fluid viscosity (kg/m.s).
Φ = Sphericity.
ρ = Density of fluid (kg/m^3).
ρ_p = Density of particle (kg/m^3).

Introduction:

The single phase flow through a packed bed extensively for many chemical engineering applications, particularly for the design of fixed catalytic beds and therefore expressions are needed to predict pressure drop across beds (Back et al., 2004),. There are several parameters affected on the pressure drop, some of them related to the physical properties of the fluid flowing through the bed such as viscosity, density, and rate of fluid flow, and others related to the nature of the bed such as shape and size of the particles, container walls effects, porosity of the bed, surface roughness of the particle and orientation of particles (Chung et al., 2002)

A packed bed is simply a vertical column that is partially filled with small media varying in shape, size, and density. A fluid (usually air or water) is passed thought this column from the bottom and the pressure is measured by two sensors above and below the packed bed. This packed bed becomes "fluidized" when the fluid flows at such a high velocity that the closely packed particles are freed and the space between the packing increases and the particles appear to float and oscillate slightly in the column so that the mixture behaves as though it is a fluid, and this velocity is defined as the **minimum fluidization velocity (u_{mf})** (Basu et al., 2003).

The most important factor in concerning the packed bed from a mechanical perspective is the pressure drop required for a liquid or a gas to flow through the column at a specified flow rate (Edison and Kim, 2006). A simple model for predicting pressure drop through packed columns was developed by **Ergun** in 1952. This model is now commonly referred as the Ergun equation (Ergun, 1949), and can be expressed as follows:

$$\frac{\Delta P}{L} = 150\,\frac{\mu u}{\phi^2 d_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75\,\frac{\rho u^2}{\phi d_p}\frac{(1-\varepsilon)}{\varepsilon^3} \quad (1)$$

For beds consisting of a mixture of different particle diameters, the effective particle diameter (dp_{eff}) can be used instead of d_p in eq. (1) as (Geankoplis, 2003):

$$dp_{eff} = \frac{1}{\sum_{i=1}^{n} \frac{x_i}{d_{pi}}} \quad (2)$$

Ergun equation applies to a broad spectrum of fluids and packing materials, but it does not predict pressure drop behavior after the point of fluidization because of bed expansion and changes in packing void fraction (Ergun, 1953). Ergun's equation does not take in consideration wall effects, which represents pipe like flow around the edges of the column (Coulson and Richardsen 1985, Niven 2002). The fluid flow through packed bed has attracted considerable attention from many investigators. **Leva in 1959** (Leva 1959), predicted the pressure drop of flow rate based on the study of single incompressible fluids through an incompressible bed of granular salts. **Dullien** and **MacDonald** addressed the problem of multi-sized particles present in a packed bed (Dullien et al., 1976). **Bey** and **Eigenberger in 1997 (Bey, 1997)** have represented the pressure drop in the packing by modifying the Ergun equation for a cylindrical coordinated system. **Shenoy et al**. **in 1996** developed a theoretical model for the prediction of velocity and pressure drop for the flow of a viscous power law fluid through a bed packed with uniform spherical particles. **Gibson and Ashby in 1988, Duplessis in 1994 and Richardson in 2000** (Moreira 2004) studied the influence of several structural parameters, such as porosity, tortuosity, surface area and pore diameter, in predicting the pressure drop through packed bed. **Basu in 2003** (Basu et al., 2003) studied the effect of various velocity ranges on the packed bed column and took their observations of the packing height and pressure drop in the column. **Hellström and Lundström in 2006** (Hellström and **Lundström** 2006) suggested a model for flow through porous media taking into consideration the inertia-effects. They compared their results with Ergun equation, and it fits well to Ergun equation. **Chung and Long in 2007** (Chung and Long 2007) studied how the pressure drop of a packed bed is related to the flow rate of the fluid coming into the column , they compared their results to the pressure drop predicted by the Ergun equation**.**

The aim of this case study is to:

i. Writing a general semi-empirical equation to predicate the pressure drop for water flow through packed bed that can be used for all types of packing systems.

ii. Studying the effect of different parameters on pressure drop of fluid flow through packed beds, like fluid velocity, height of packing, type of packing materials, particles size, bed porosity and bed diameter.

iii. Writing a semi-empirical equation to evaluate the minimum fluidization velocity, in order to determine the working range of the written equations, this is in the fixed region of the fluid flow diagram.

Theory of the model:

Semi-Empirical Equations Model for Water Flow through Packed Bed

Semi-empirical formulas for modeling water flow through packed bed was estimated for the parameters affecting the pressure drop using Buckingham π (Buckingham, 1914). This formula consists of multiplied dimensionless terms raised to certain powers; these powers were evaluated from experimental data taken from literatures with statistical fitting. The method of modeling used to derive the expression for the pressure drop was based on curve fitting of the available literatures experimental data, and by implementing dimensional analysis. This analysis can be summarized as follows:

The pressure drop was assumed to be dependent on fluid velocity (u),packing diameter (d_p), bed length (L), l s (ρ), l s o s (μ), o ro (sphericity ϕ), and can be written in the following expression:

$$\Delta P = f\left(u, d_p, L, \rho, \mu, \varepsilon, \phi\right) \quad (3)$$

Th B k h 's π h o r w s e semi-empirical formula of h s the fluid flow equation. In this theorem the dimensions of a physical quantity are associated with mass, length and time ,represented by symbols M, L and T respectively, each raised to rational powers.

Th B k h heorem forms the basis of the central tool of the dimensional analysis. This theorem describes how every physically meaningful equation involving n variables can be equivalently rewritten as an equation of *n-m* dimensionless parameters, whereas, the number of fundamental dimensions used. Furthermore, and the most important is that it proves a method for computing these dimensionless parameters from the given variables. According to this theorem n=8 and m=3,then this theorem gave us five dimensionless groups. The dimensions of parameters used in expression are shown in **table1**

Selecting the variables particle diameter, fluid velocity, and fluid density.

The particle diameter (d_p) has the dimension L therefore $L = d_p$

The fluid velocity (u) has dimensions $L\ T^{-1}$ therefore $T = d_p\ u^{-1}$

Th l s (ρ) h s^3 h r so ro $_p$M s = M$_p$L

Th rs $_1$) o = ΔP (M ft^{-1} L T^2)

$$\pi_1 = \frac{\Delta P}{\rho u^2} \quad (4)$$

Th s o $_2$) ro L (L^{-1}) (π

$$\pi_2 = \frac{L}{d_p} \quad (5)$$

Th h r $_3$) o = μ (M ft^{-1} L T)

$$\pi_3 = \frac{\mu}{\rho u d_p} \quad (6)$$

Th o r h $_4$) ro ε (π

$$\pi_4 = \varepsilon \quad (7)$$

Th h $_5$) ro ϕ (π

$$\pi_5 = \phi \quad (8)$$

Therefore the equation for the pressure drop dependence on fluid velocity (u), packing diameter (d_p), b l h (L), l s (ρ), l s o s (sphericity (ϕ)will be as follows:

$$\frac{\Delta P}{\rho u^2} = b_1 \left(\frac{L}{d_p} \right)^{b_2} \left(\frac{\mu}{\rho d_p u} \right)^{b_3} \varepsilon^{b_4} \phi^{b_5}$$

(9)

While Reynolds number is defined as:

$$\mathrm{Re} = \frac{\rho d_p u}{\mu}$$

(10)

Then eq. (9) can be written as follows:

$$\frac{\Delta P}{\rho u^2} = b_1 \left(\frac{L}{d_p} \right)^{b_2} \left(\frac{1}{\mathrm{Re}} \right)^{b_3} \varepsilon^{b_4} \phi^{b_5}$$

(11)

Where b_1, b_2, b_3, b_4 and b_5 are constants which can be evaluated from experimental data taken from literature by statistical fitting. The above equation can be used for different types of packing system.

S ($\Delta P/\rho^2$) prescribes fluid flow through packed bed, therefore; equation 11 can be considered as a semi-empirical equation of fluid flow through packed bed. Each term of this equation is a dimensionless group, because ($\Delta P/\rho^2$) is dimensionless number.

Equation Model for Minimum Fluidization Velocity

The semi-empirical estimated equation model can be used for fluid flow up to the fluidization point. The minimum fluidization velocity is an indication for the fluidization point, therefore; the minimum fluidization velocity must be evaluated to find the fluidization point.

The basic theory for prediction of the minimum fluidization velocity is that the pressure drop across the bed must be equal to the effective weight per unit area of the particles at the point of incipient fluidization, this expressed mathematically as follows (Thornhill, 1990):

$$\frac{\Delta P}{L} = (\rho_p - \rho)(1 - \varepsilon_{mf}) g$$

(12)

Eq. (1) can now be used for small extrapolation for packed beds to calculate the minimum fluidization velocity at which fluidization begins as follows (Thornhill, 1999):

$$\frac{1.75\, d_p^2 u_{mf}^2 \rho^2}{\phi \varepsilon_{mf}^3 \mu^2} + \frac{150\,(1-\varepsilon_{mf}) d_p u_{mf} \rho}{\phi^2 \varepsilon_{mf}^3 \mu} - \frac{d_p^3 \rho\,(\rho_p - \rho) g}{\mu^2} = 0$$

(13)

Defining a Reynolds number at the minimum fluidization as:

$$Re_{mf} = \frac{d_p\, u_{mf}\, \rho}{\mu}$$

(14)

So that eq. (13) will be as follows:

$$\frac{1.75\, \mathrm{Re}_{mf}^2}{\phi\, \varepsilon_{mf}^3} + \frac{150\,(1-\varepsilon_{mf})\mathrm{Re}_{mf}}{\phi^2 \varepsilon_{mf}^3} - \frac{d_p^3 \rho (\rho_p - \rho) g}{\mu^2} = 0$$

(15)

When $Re_{mf} < 10$ (small particles), the first term can be dropped as follows:

$$u_{mf} = \frac{(\rho_p - \rho) g\, d_p^2\, \varepsilon^3}{150\, \mu (1-\varepsilon)}$$

(16)

And when $Re_{mf} > 1000$ (large particles), the second term can be dropped out (Brandenet al., 2003).

Leva in 1959 made a semi-empirical equation for the prediction of minimum fluidization velocity ***for gas fluidization*** as shown below:

$$u_{mf} = \frac{0.0093\, d_p^{1.82} (\rho_p - \rho)^{0.94}}{\mu^{0.88} \rho^{0.06}} \quad (17)$$

Wen and Yu in 1966 produced an empirical correlation for u_{mf} ***for gas fluidization*** the Wen and Yu correlation is often taken as being most suitable for particles larger than 100 μm, whereas the correlation of **Baeyens and Geldart** in 1974, shown below in eq. (18), is best for particles less than 100 μm.

$$u_{mf} = \frac{d_P^{1.8} (\rho_P - \rho)^{0.934} g^{0.934}}{110\, \mu^{0.87} \rho^{0.066}}$$

(18)

In the present work **Leva** equation have been modified to be used for the fluidization in liquid phase, by using experimental data from literatures for liquid

phase, and making statistical fitting for this data. The modified **Leva** equation can be written as follows:

$$u_{mf} = 0.088 \frac{d_p^{0.005} \left(\rho_p - \rho\right)^{-0.05}}{\mu^{0.03} \rho^{0,17}}$$

(19)

Results and Discussion:

Semi-Empirical Estimated Equations for Water Flow through Packed Bed

The estimated semi-empirical equation (11) was fitted for water flow through packed beds of multi sized of packing system (it includes all different types of packing systems (mono size spherical particles system, mono sized non spherical particles system, binary sized spherical particles system, ternary sized spherical particles system, quaternary sized spherical particles system, and quinary sized spherical particles system). In this fitting 150 sets of data from literatures (Boss and Lim 2001, Brown and Carothers 2001, Dence and Leifeste 2001, Miller and Shah 2001, Zekia 2001, Pierce et al., 2002, Betler et al., 2002, Britton and Donegan 2003, Chopard and Welsh 2003, Basu et al., 2003, Back et al. 2004, Chung et al., 2004, Saw and Yang 2004, Dileo and Hung 2005, Arffa et al., 2005, Sandidge and Shin 2005Dhanani et al., 2005, Pierce and Williams 2005, Chung and Henry 2006, Hersman and Hunter 2006, Edison and Kim 2006, Chopard and Welsh 2007, Kovell and Jordan 2007, Osayawe and Patel 2007, Hana 2007, Wekar 2007) were used, which includes 1300 values of pressure drop versus velocity. Many types of packing were used in the present work such as Pea Gravel, Marbles, Glass Marbles, Black Marbles, Clear Marbles, Acrylic balls, Glass spheres, Rasching Rings and Glass Marbles. The diameters of the packing materials used in this model are from the range of (0.02-8.89) cm, the bed diameters used is from the range of (7.62-15.24) cm, the height of packing used is from the range of (15.15-67.3) cm, the porosity used is from the range of (0.3-0.5) and the Sphericity range of (0.3 -0.9). So the estimated model for water flow through packed beds was found to be as follow:

$$\frac{\Delta P}{\rho u^2} = 110.6 \left(\frac{L}{d_p}\right)^{-1.24} \left(\frac{1}{\mathrm{Re}}\right)^{0.33} \varepsilon^{0.64} \phi^{0.1}$$

(20)

The average percentage errors were found to be 5.7% between experimental work and the estimated equation model.

The results of the estimating equation (eq. (20)) for ***multi-sized*** packing system are presented in this section. This presentation takes into account a comparison between these results and the experimental results taken from literatures, as well as comparisons were made between all these results and similar results taken by using Ergun equation for air and water flow through packed bed.

It could be noticed from **figures 1 to 8** that the estimated model gave good fitting for the experimental results and better than Ergun equation. This is due to:

- Ergun equation assumes smooth geometric of the particles, but the irregular of the surface of the particles area would increase the drag force of the fluid moving past the particles (frictions) as well as the pressure drop (Boss 2001). So there is a cretin deviation between Ergun results and experimental results. This deviation was also found between the modified equation results and the Ergun equation.
- Ergun's equation is based on a large ratio of column diameter to particle diameter, neglecting wall effect. (In order to neglect wall effects a ratio of 10 or greater should be used (Kececigoglu 1994)). These neglections cause a great difference from experimental results. The wall effects was included in all experiments, also the present model includes this effect through the equation constants.
- The differences in beds dimensions, packing shapes and sizes used by Ergun.
- Ergun derived the values for the constants through experiments where the packing was small, non-spherical, and rough

Effect of variables on the Estimated Equation Model

This section shows the effect of different parameter on the estimated equation 11, a certain range for each parameter was taken in this study according to the available experimental data from literatures.Most of the experimental previous works were studying the effect of different parameters of fluid flow on the pressure drop. So to get good comparison for the estimated model form with the available experimental data, q o 11 h b ^{2}).The new form of the equation will be a pressure drop equation. The fluid physical properties used in estimated equation were taken from experiments held at temperature of (25°C). Fluid velocity used was taken within the fixed region.The important parameters affecting the pressure drop in the equation was found to be particles diameter, porosity and bed length. The effect of

these parameters on pressure drop have been studied and shown in the following subsections.

- **Effect Of Particle Diameter On Pressure Drop**

Figure 9 indicates that an increase in the particle diameter causes decrease in the pressure drop, this is due to the fact that when the particle diameter increase's the surface area decreases, the reason of this relation is that when the surface area decreases the resistance of fluid flow decreases which leads to a decrease in pressure drop.

- **Effect of porosity on pressure drop**

Figure 10 show that when the porosity increase the pressure drop decreases, where the void fraction between particles become larger this leads to less resistance to fluid flow through the bed. For example at velocity 0.3 m/s when the porosity is 0.5 the pressure drop is 16.797 Kpa, while for the same velocity with porosity of 0.3 the pressure drop is 23.373 Kpa .

- **Effect of bed length on pressure drop**

Figure 11 show that whenever the length of the packing height increases the fluid flow resistance increases this leads to an increase in pressure drop. For example at velocity 0.3 m/s when the packing height is 0.1m the pressure drop is 5.9941Kpa, while for the same velocity with packing height of 0.26 m the pressure drop increased to 19.6516 Kpa, further increase in the packing height to 0.5 m for the same velocity the pressure drop increased to 44.3828 Kpa.

The Minimum Fluidization Velocity Modified Equation Model

The results of the semi-empirical equation 19 are shown in (**Table 2**). This table show the parameters used in the equation from experiments. It also represents the experimental values for minimum fluidization velocity found in literatures. From (**Table 2**), it can be seen that the values of the minimum fluidization velocity of the modified Leva equation model used are comparable with the experimental values of the minimum fluidization velocity for water flow. So the range of calculations for water flow in the modified model was taken to be not exceeding this minimum value of velocity.

Conclusions:

In this work, a new semi-empirical equation model was introduced to overcome problems the problems of the pressure drop through packed bed of multi-sized of

packing systems. The estimated equation had successfully described the effects of different parameters on pressure drop of water flow through packed beds, like fluid velocity, height of packing, type of packing particles, particles size, bed porosity and bed diameter, compared with the experimental results

An increase in particle diameter causes a decrease in pressure drop, this is due to the fact that when the particle diameter increase's the specific surface area of it decreases, and this leads to a decrease in the resistance to fluid flow.

The particle size and size distribution highly affect the bed porosity. For mono size packing, the lower the particle size, the lower is the bed porosity. The porosity of multi- size systems are generally less than those of mono size systems, because the particles of smaller sizes tend to fill the void spaces between the larger sizes particles.

The bed porosity highly affects the pressure drop and inversely proportional to it, this is because that when the porosity increases the resistance to fluid flow through the bed decreases

The pressure drop through a packed bed is highly sensitive to the packing height and that as the packing height increases the pressure drop increases.

Comparing the results of the estimated equations of pressure drop versus velocity curves with those of experimental data from literature and Ergun equation results; it indicates that the estimated equations results coincide with experimental results, while the results from Ergun equation was far away from them.

The modified **Leva** equation of minimum fluidization velocity that has been obtained in the present work is comparable with the experimental values of the minimum fluidization velocity for water flow; therefore it can be used with confidence to find the working region of the fluid within the fixed region

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Table 1: The minimum fluidization velocity results

Particles Type	**u_{mf} (m/s) (Experiment)**	**u_{mf} (m/s) (Model)**	**dp (m)**	**Dr (m)**	**ρ_p (kg/m^3)**	**L (m)**	**References**
Pea Gravel	0.02025	0.02166	0.0011	0.089	2500	0.556	Dileo and Hung (2005)
Glass Marbles	0.01795	0.02191	0.0127	0.1524	2500	0.52	Britton and Donegan (2003)
Black Marbles	0.02701	0.02184	0.0127	0.1524	2600	0.445	Sandidge and Shin (2005)
Pea Gravel	0.01898	0.02285	0.0031	0.1524	1600	0.552	Sandidge and Shin (2005)
Pea Gravel	0.0222	0.02285	0.0031	0.1524	1600	0.58	Sandidge and Shin (2005)
Pea Gravel	0.0167	0.02191	0.0899	0.1524	2800	0.352	Branden et.al. (2003)

Pea Gravel	0.01753	0.02191	0.0899	0.1524	2800	0.346	Branden et.al. (2003)
Pea Gravel	0.0191	0.02285	0.0031	0.08	1600	0.56	Britton and Donegan (2003)
Glass Marbles	0.01716	0.02191	0.0127	0.08	2500	0.485	Britton and Donegan (2003)
Pea Gravel	0.01918	0.02191	0.0899	0.1524	2800	0.362	Branden et.al. (2003)
Pea Gravel	0.01687	0.02205	0.0899	0.1524	2600	0.372	Branden et.al. (2003)

Table2: The estimated equations technical sheet values

System Type	**Type Of Packing Material**	**d_p (cm)**	**Dr (cm)**	**L (cm)**	**ε**	**References**
Mono Size Spherical Particle	Pea Gravel, Marbles, Glass Marbles, Black Marbles, Clear Marbles, Acrylic balls and Glass spheres	0.2-8.89	7.62-15.24	26.03 - 55.88	0.3 - 0.47	Brown and Carothers 2001, Dence and Leifeste 2001, Miller and Shah 2001, Pierce et al., 2002, Betler et al., 2002, Britton and Donegan 2003, Chopard and Welsh 2003, Basu et al., 2003, Saw and Yang 2004, Dileo and Hung 2005, Arffa et al., 2005, Dhanani et al., 2005, Pierce and Williams 2005, Chung and Henry 2006, Hersman and Hunter 2006, Edison and Kim 2006, Kovell and Jordan 2007, Osayawe and Patel 2007
Mono Size Non Spherical Particles	Rasching Rings And Pea Gravel [Sphericity Range (0.3 -0.9)]	0.02-1.27	8.89-15.24	41.91 - 67.3	0.32 - 0.4	Miller and Shah 2001, Chopard and Welsh 2003, Back et al. 2004, Chung et al., 2004, , Dileo and Hung 2005, Sandidge and Shin 2005, Kovell and Jordan 2007, Chopard and Welsh 2007
Binary Sized Spherical Particles	Glass Marbles and Acrylic balls	0.42-1.2	7.62-8.89	15.15 - 50.8	0.3 - 0.43	Zekia 2001, Boss and Lim 2001, Pierce et al., 2002, Hana 2007, Wekar 2007
Ternary Sized Spherical Particles	Glass Marbles and Acrylic balls	0.42-1.01	7.62-7.64	15.15 – 20	0.35-0.43	Zekia 2001, Boss and Lim 2001, Pierce et al., 2002, Hana 2007, Wekar 2007
Quaternary Sized Spherical Particles	Glass Marbles	0.42-1.01	7.62-7.64	15.15 – 20	0.36-0.39	Wekar 2007
Quinary Sized Spherical	Glass Marbles	0.42-1.01	7.62	20	0.3624	Wekar 2007

Particles						

Note:

- The physical properties used in all estimated equations were taken from experiments held at temperature 25°C for water flow through packed bed.
- In the packing of binary size particles the mixture contains two sizes of sphere particles. The percentage of each size is equal 1/2 from the total packing.
- In the packing of ternary size particles the mixture contains three sizes of sphere particles. The percentage of each size is equal 1/3 from the total packing.
- In the packing of quaternary size particles the mixture contains four sizes of sphere particles. The percentage of each size is equal 1/4 from the total packing.
- In the packing of quinary size particles the mixture contains five sizes of sphere particles. The percentage of each size is equal 1/5 from the total packing.

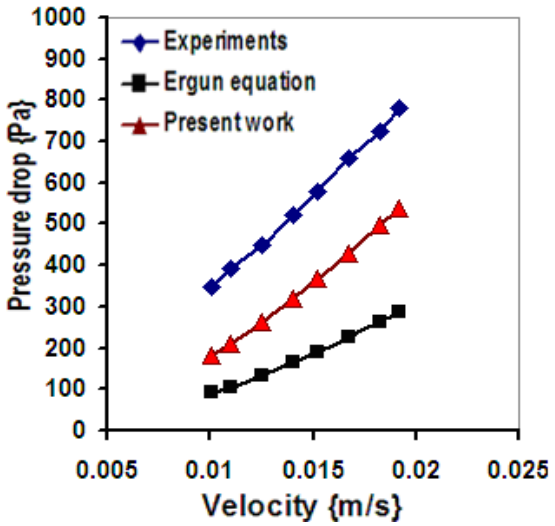


Figure 1 Pressure drop versus velocity for pea gravel of particles diameter 1.27 cm, bed porosity of 0.36, packing height of 41.3 cm, bed diameter of 8.89 cm (Basu 2003)

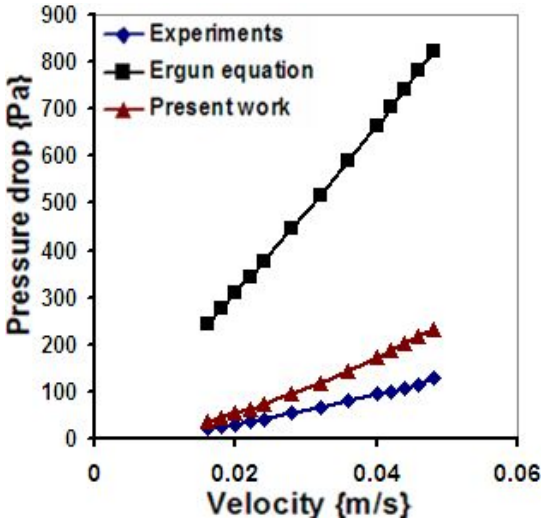


Figure 2 Pressure drop versus velocity for pea gravel of particle diameter 0.02cm, sphericity of 0.7, bed porosity of 0.3, packing height of 43cm, bed diameter of 8.89cm (Chung 2003)

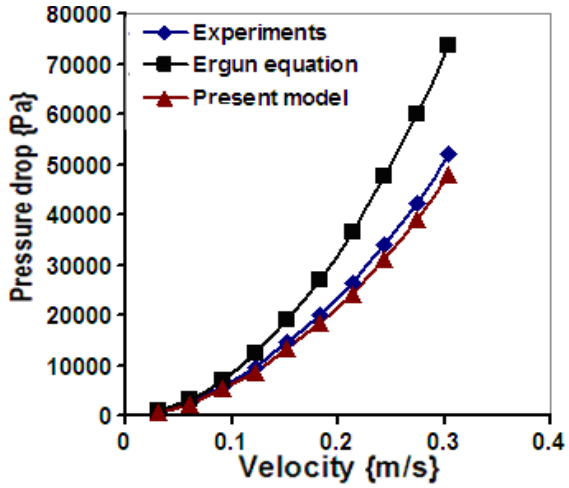


Figure 3 Pressure drops vs. velocity for spherical particles diameter of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.61 cm), bed porosity of 0.36, packing height of 15.15 cm, bed diameter of 7.62 cm (Wekar 2007)

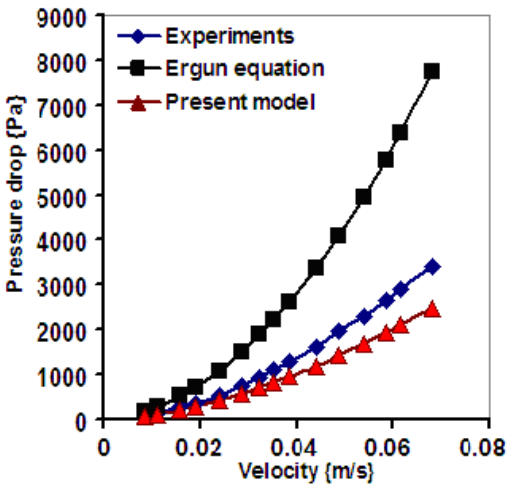


Figure 4 Pressure drop vs. velocity for Acrylic balls of diameter (dp_1=0.655cm, dp_2=1.27cm,with dp_{eff}=1.016 cm),fractions of(x_1=0.25,x_2=0.75),bed porosity of 0.37, packing height of 49.53cm, bed diameter of 8cm (Mahalec 2007)

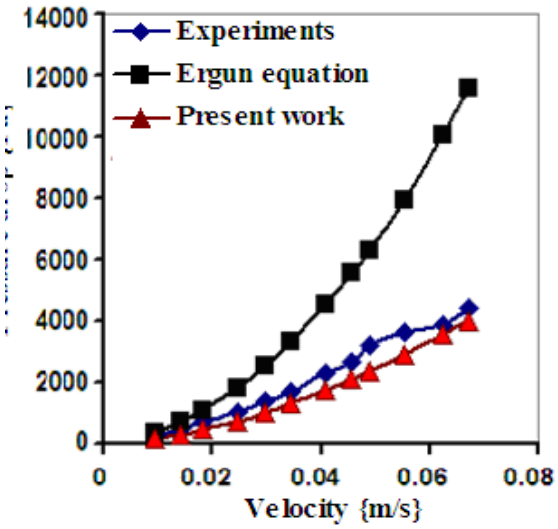


Figure 5 Pressure drops versus velocity for Acrylic balls of diameters (dp_1=0.655,dp_2=1.27, and dp_{eff}=0.73cm), fractions of (x_1=0.75, x_2=0.25),bed porosity of 0.37, packing height of 50.8 cm, bed diameter of 8 cm (Mahalec 2007)

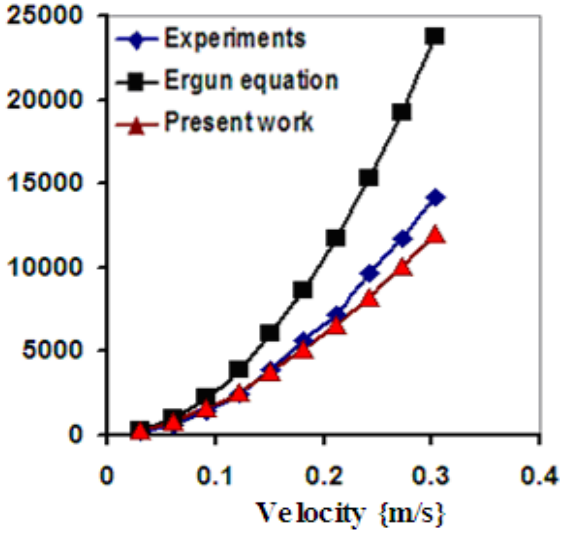


Figure 6 Pressure drop versus. velocity for glass sphere of diameters (0.9987,0.7955and0.6015 cm, with dp_{eff}=0.77 cm),bed porosity of 0.38,packing height of 15.15 cm, bed diameter of 7.62 cm (Hana 2007)

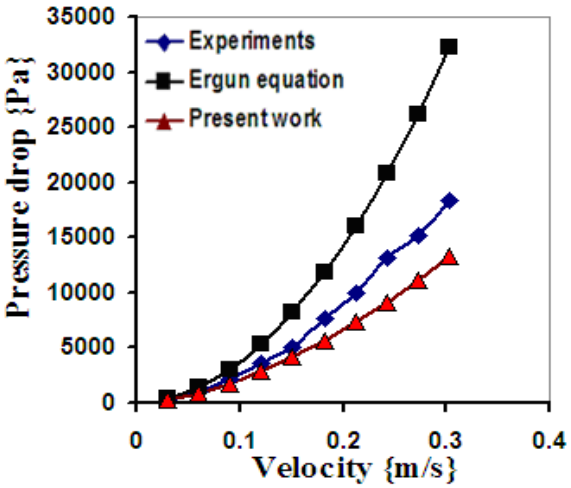


Figure 7 Pressure drop versus velocity for glass sphere of diameters (0.9987, 0.7955 and 0.509 cm, with dp_{eff}=0.71 cm), bed porosity of 0.38, packing height of 15.15 cm, bed diameter of 7.64 cm (Hana 07)

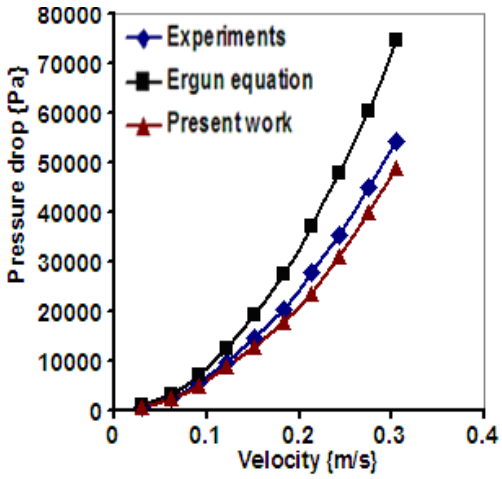


Figure 8 Pressure drop versus velocity for glass spherical particles diameter of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.55 cm), bed porosity of 0.37, packing height of 15.15 cm, bed diameter of 7.62 cm (Zekia 2001)

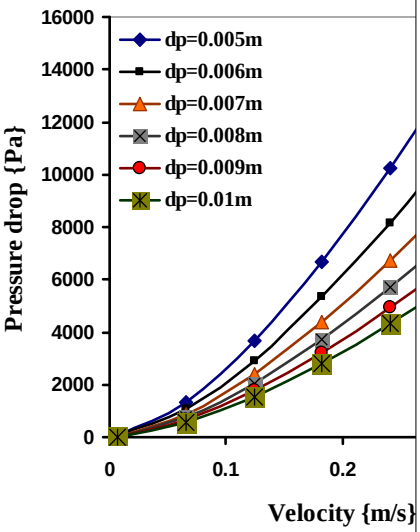


Figure 9 Pressure drop vs. veloc conditions bed diameter 0.08m, bed length 0.1m, at different particle diameters.

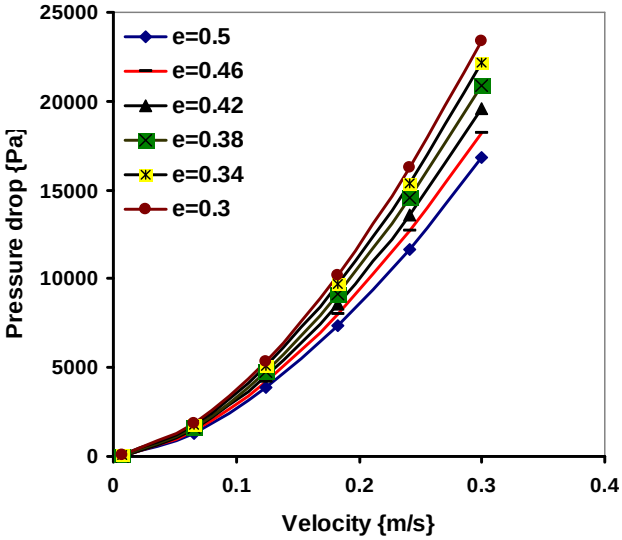
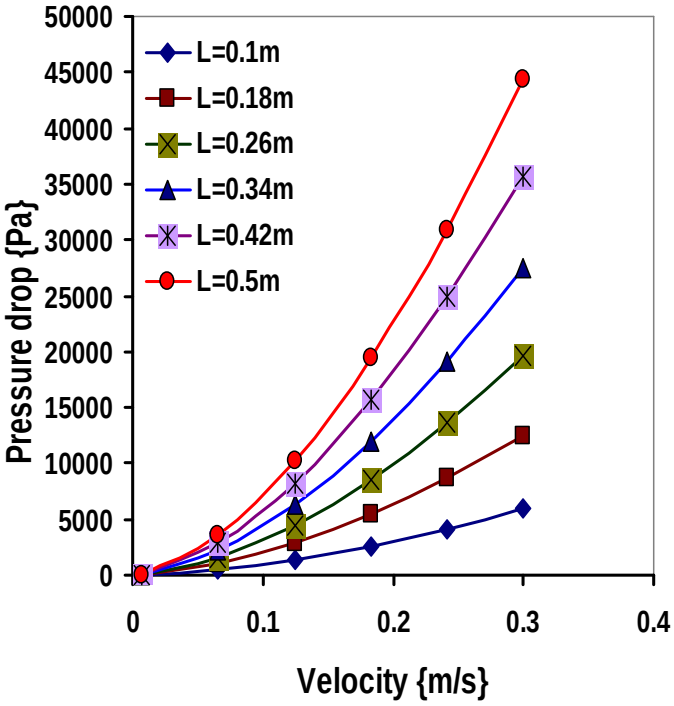


Figure 10 Pressure drop vs. velocity for the conditions bed diameter 0.08 m, particles diameter 0.005m, bed length 0.1m, at different porosities.



conditions bed diameter 0.08m, particles diameter 0.01m, porosity 0.33m, at different bed lengths.