NUMERICAL SIMULATION OF MELTING SOLIDIFICATION PROCESS IN AN ALLOY METAL WITH A SQUARE SECTION

Mr.Hameed K. Al Naffiey
Babylon University – College of Engineering
Department of Mechanical Engineering

Abstract

This study is used to construct a mathematical model to analyze melting solidification process considering condition phenomena to an alloy metal in a square section. The aim of the present study, know the time that the metal is solidification in the mold to Know the time that open the mold. After the fluid inters the mold as a liquid, the heat is transferred by conduction and convection, including de thermal phase change phenomena. The mathematical model consists of square section which has length L and [a*b] dimensions. The metal enters the mold from upper end and go to fill all the mold use explicit technique is used to calculate the temperature during the mold and use the thermal phase phenomena from liquid to solid. In this study used Finite difference method to solve the mathematical model also used computer program **Fortran 90** to solve this model. The result represented by **Golden Software Surfer 8**. Also this study may be used in refrigeration of water and studying solidification from the water to ice.

Keywords /Heat transfer, Numerical, conduction, convection, alloy, molds, and solidification.

التمثيل العددي لعملية التجمد الحاصل في السباكة المعدنية الحاصلة في قالب مربع المقطع

السيد حميد كاظم حمزة / مدرس مساعد

جامعة بابل/كلية الهندسة/قسم الميكانيك

الخلاصة:

في هذة الدراسة تم عمل نموذج رياضي لعملية التجمد الحاصلة في قالب ذي مقطع مربع حيث تم تحليل النموذج تحليلا عدديا وبتالي دراسة إلية التجمد الحاصلة فيه المصحوبة بانتقال الحرارة بنوعية الحمل والتوصيل. الهدف من هذه الدراسة هو معرفة الزمن ألازم لتجمد المائع داخل فراغ القالب وبالتالي تحديد

الزمن اللازم لفتح القالب ويتم ذلك بدراسة انتقال الحرارة كدالة لزمن التجمد.الموديل الرياضي يتضمن قالب أبعادة [a*b] وطولة L.المائع يدخل بشكل سائل إلى القالب من الأعلى ثم ينتقل تدريجيا إلى جميع أجزاء القالب وبمرور الزمن يتحول إلى الحالة الصلبة بسبب الفقدان الحراري الحاصل بنوعية الحمل والتوصيل .في هذةالدراسة تم استخدام طريقة الفروقات المحددة لحل الموديل رياضي المستخدم، وكذلك استخدم برنامج حاسوبي بلغة Fortran 90 لاستخراج النتائج التي تم تمثيلها لاحقا باستخدام برنامج Golden Software Surfer 8 .كذلك ممكن الاستفادة من هذه الدراسة بتغيير نوع المائع في معامل صناعة الثلج ودراسة آلية التجمد .

2-Nomenclature:

C_p: Specific heat capacity; J/Kg K

F_o: Fourier number;-

K: Thermal conductivity; W/m^2 K

L: Length; m

l : Latent heat capacity ; J/Kg K

M: Sub- region number; -

T: Temperature; k

$Q^{\bullet}$: Heat generation rate per volume; W/m

$q^{'}$: Heat flux; W/m^2

t: Time; sec

x: Distance ;m

Subscript

m: mold

c: casting

Superscript

n:Time denoted

Greek symbols

ρ: Density of material; Kg/m^3

γ : partition ratio

Δt: Time interval

Δx: Distance interval

Introduction

The solidification rate of alloys is important processing variable, and solidification rate relates directly to the coarseness of dendritic structures and hence controls the spacing and distribution of micro heterogeneities, such as dendritic ,second phases, inclusions and micro porosity microsegregation,**by D.r Poirier 1993** . This research presents a study about solidification of alloy of metals that is known in the literature as Solid – Liquid Phase Change problem. From this study, the interfacial heat transfer coefficient has been found to depend on many factors including the presence and thickness of surface. Mold material applied pressure, Liquid alloy surface tension **by Carslaw , H.S. 1959** .

The effect, of the direction of gravity in relation to the interface has been examined by investigation with the mold place on the bottom. An a temperature distribution of heat transfer during liquid alloy solidification in a casting mold depends on determination of the boundary conditions during the solidification**,** properties of the mold, properties of the casting alloy **by Sully LJD.1976**.The heat transfer during the mold caused phase change, and then transfer by conduction and convection to wall, the thickness of the wall may be design according to the alloys process and metals **by Welty(1997).**The mold that used in this study has square cross section see Fig.1.

Mathematical model

The mathematical analysis is based on the following assumptions:

1. One dimension and unsteady state heat transfer model.
2. All the physical properties are assumed to be constant.
3. The fluid is considered incompressible with constant properties.

Heat flow in the chill.

The amount of heat that escapes of the chill is very important to determine the temperature distribution in the ingot. The heat flow through the casting can be approximated as a one dimension heat transfer problem .Unsteady state (transient) condition heat transfer in a one dimension body is given **by Yunus, A.1998.**

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \qquad 0 \le x \le L, \quad t > 0 \tag{1}$$

Where T is the temperature, t is the time and x is the Cartesian coordinate. The term is the thermal diffusivity of the conduction material which is given by

$$\alpha = \frac{k}{\rho\, C_p} \tag{2}$$

Where **k** the thermal conductivity, **ρ** is the density and $\mathbf{C_p}$ **is** the specific heat capacity.

2-Heat flow in the casting

The governing equation that describes the casting heat flow for solidifying metals is given by

$$\mathbf{k}\frac{\partial^2 \mathbf{T(x,t)}}{\partial \mathbf{x}^2} + \mathbf{Q}^\bullet = \boldsymbol{\rho}\,\mathbf{C}_p \frac{\partial T\mathbf{(x,t)}}{\partial \mathbf{t}} \tag{3}$$

The term $Q^\bullet$ represents heat source term, To account for the change of phase from liquid to solid in a binary alloy, each computation cell contains a fraction of solid (f_s) and fraction of liquid(f_l) ,where the sum of fractions must equal unity. **by M.Rapaaz,(1988)**.

$$\mathbf{Q}^\bullet = \boldsymbol{\rho}\mathbf{l}\frac{\partial \mathbf{f_s}}{\partial \mathbf{t}} \tag{4}$$

Where **l** is the latent heat of fusion and $\mathbf{f_l}$ term is determined **by I.Imafoku (1983).**

$$\mathbf{f_l} = \frac{\mathbf{T} - \mathbf{T_s}}{\mathbf{T_l} - \mathbf{T_s}} \tag{5}$$

This equation is based on the assumption freezing.

$$\mathbf{f_l} = \frac{\mathbf{C_\circ m_1} - \gamma(\mathbf{T} - \mathbf{T_m})}{(\mathbf{1} - \gamma)(\mathbf{T} - \mathbf{T_m})} \tag{6}$$

The term $\frac{\partial \mathbf{f_s}}{\partial \mathbf{t}}$, can be related to temperature from

$$\frac{\partial \mathbf{f_s}}{\partial \mathbf{t}} = \frac{\partial \mathbf{f_s}}{\partial \mathbf{T}}\frac{\partial \mathbf{T}}{\partial \mathbf{t}} \tag{7}$$

Substitution of Eq. (7), in Eq. (4), gives

$$\mathbf{Q}^\bullet = \boldsymbol{\rho}\mathbf{l}\frac{\partial \mathbf{f_s}}{\partial \mathbf{T}}\frac{\partial \mathbf{T}}{\partial \mathbf{t}} \tag{8}$$

The latent heat is added to the energy by using an effective specific heat .It has been found that method tends to be inefficient.

$$\mathbf{C_p} = \mathbf{C}_p(T) + \mathbf{L}\frac{\partial \mathbf{f_l}}{\partial \mathbf{t}} \tag{9}$$

Substitution of Eq. (8), in Eq.(1), gives

$$\mathbf{k}\frac{\partial^2 \mathbf{T}}{\partial \mathbf{x}^2} + \boldsymbol{\rho}\mathbf{l}\frac{\partial \mathbf{f_s}}{\partial \mathbf{T}}\frac{\partial \mathbf{T}}{\partial \mathbf{t}} = \boldsymbol{\rho}\mathbf{C_p}\frac{\partial \mathbf{T}}{\partial \mathbf{t}} \tag{10}$$

This equation can be rearranged to give

$$\mathbf{k}\frac{\partial^2 \mathbf{T}}{\partial \mathbf{x}^2} = \boldsymbol{\rho}\left(\mathbf{C}_p - \mathbf{l}\frac{\partial \mathbf{f_s}}{\partial \mathbf{T}}\right)\frac{\partial \mathbf{T}}{\partial \mathbf{t}} \quad (11)$$

Then Eq.(11), can be written as

$$\frac{\partial^2 \mathbf{T(x,t)}}{\partial \mathbf{x}^2} = \frac{\boldsymbol{\rho}\,\mathbf{C_p}}{\mathbf{k}}\frac{\partial \mathbf{T(x,t)}}{\partial \mathbf{t}} \quad (12)$$

3-- Finite difference formulation

Finite difference methods are use to solve Eq.12, Equation12 represent temperature of distribution in the mold. Now construct mesh along the mold as shown in fig.(2) .In the finite difference analysis of one-dimension conduction of element ,The central finite difference are used for grid as shown , by Petrovetc, Z.(1996).

However, these solutions can be generated for an assortment of simple geometries and boundary condition, and they are well documented in the literature. On the other hand, analytical solutions to transient problems are restricted to simple geometries and boundary conditions.

In the present work, this problem has been approached in one dimensional geometry for a region with a finite dimension L shown in Fig. (2) as follows:

The region ($0 \le x \le L$) is divided into M equal size meshes.

$$\Delta \mathbf{x} = \frac{\mathbf{L}}{\mathbf{m}} \quad (13)$$

m subscripts are used to designate the x location of the discrete node points in Fig.2. Besides being discredited in space, the problem must also be discredited in time. The integer **n** is introduced for this purpose

$$\mathbf{t} = \mathbf{n}\,\Delta \mathbf{x} \quad (14)$$

The finite difference approximation to the time derivatives of T, and the time derivatives in left side of Eq.(12) is expressed as

$$\left.\frac{\partial \mathbf{T}}{\partial \mathbf{t}}\right|_{\mathbf{m}} = \frac{\mathbf{T}_{\mathbf{m}}^{\mathbf{n+1}} - \mathbf{T}_{\mathbf{m}}^{\mathbf{n}}}{\Delta \mathbf{t}} \quad (15$$

The superscript n is used to denote the dependence of T , and the time derivative is expressed in terms of the difference in temperature associated with the new (n+1) and the previous (n) time steps.

Eq.(12) solved using an explicit Finite deference methods for the chill and casting by

$$\frac{\mathbf{T}_{\mathbf{m+1}}^{\mathbf{n}} - 2\mathbf{T}_{\mathbf{m}}^{\mathbf{n}} + \mathbf{T}_{\mathbf{m-1}}^{\mathbf{n}}}{(\Delta x)^2} = \frac{\mathbf{1}}{\boldsymbol{\alpha}}\frac{\mathbf{T}_{\mathbf{m}}^{\mathbf{n+1}} - \mathbf{T}_{\mathbf{m}}^{\mathbf{n}}}{\Delta \mathbf{t}} \quad (16)$$

This can be rearranged to give,

$$\mathbf{T_m^{n+1} = T_m^n (1-2F\ ) + F\ (T_{m+1}^n + T_{m-1}^n)} \quad (17)$$

Where **F** is a finite difference form of the Fourier number, which is given by

$$\mathbf{F = \frac{\alpha \Delta t}{\Delta x^2}} \quad (18)$$

The term Δx and Δt in this study, refer to the space and time increments used in the calculation. In this work, the differential elements are select as Δx=2 mm and Δt = 0.5 sec for both casting and the chill, complying $\mathbf{F \leq 0.5}$.

4-The boundary conditions

From the symmetrical of the system, the boundary conditions are

$$\mathbf{At \quad x = 0 \quad -K.A.\left(\frac{dT}{dx}\right) = hP(T_c - T_\infty)} \quad (19)$$

$$\mathbf{At \quad x = a \quad -K.A.\left(\frac{dT}{dx}\right) = hP(T_c - T_\infty)} \quad (20)$$

$$\mathbf{At \quad t = 0, \quad T(x,0) = Tm} \quad (21)$$

Equation17 represent temperature distribution along casting after construct computer program to solve this equation and depend on boundary condition also we needed **Gauss elimination** method to solve this equation.

5. Result and Discussion.

Fig.(3) represented the relation between the heat flux and the time of remaining the metal in the mold. We conclude that when the time increases the heat flux decrease for steal chill or cupper chill because the heat is transfer to the surrounding increase. The higher curve for cupper chill and the lower represent the steal chill.

Fig 4 Fig.(5), and Fig6 represented temperature distribution along cross section of mold at different time .From these figures conclude that the temperature decreases with time and as is very high in the center of mold and decreases towards the surface of mold because the heat transfer to surrounding.

Fig.(4) represents the temperature distribution at time 200 sec .the line in this fig. represent the temperature distribution. Fig.(5) represents the temperature distribution at time 500 sec .the line in this fig. represent the temperature distribution. Fig.(6) represents the temperature distribution at time 700 sec .the line in this fig. represent the temperature distribution.

From these figures conclude when the time increase the metal convert from liquid to solid and solidification accurse and from these figure we needed time more than 700 sec to open the mold. Fig.(7) represent temperature distribution at the different location along the mold from this figure conclude the temperature decrease as the time increase from the center of the mold to the wall. And this time change as a function to fluid and the dimension of the mold also the metal of chill.

This result compact with the Theoretical result **by C.P Hong,(1990).** As shown in Fig. (8).

6. Conclusion.

1- The difference of the thermal properties between liquid and solid gives influence to change the whole solidification behavior.
2- The heat transfer from the corner of the mold is very high because the heat is transfer to surrounding through two walls.
3- The time that used to open the mold depend on the temperature of metal at inters and the properties of metal also the location of raiser.

7. References.

- Carslaw , H.S. and Jaeger , J,C. " Condution of Heat in Solids ",SecondEdition ,Oxford University Press , London , 1959 .
- C.P.Hong,T. Umeda,and Y.Kimura, "Numerical models for casting solidification "Metal Transactions B, Vol.15B, March 1990, p.91.
- D.R.poirier and E.j.Poirier ,"Heat transfer Fundamentals for metal casting" Second edition ,United states of America,1994
- I.Imafoku and Chijiiwa ,"A mathmatica model for shrinkage cavity prediction in steal casting",AFS Transaction,91,pp.527-540 (1983)
- M.Rappaz,D.M Stefanescu, "Modeling of microstructure Evolution". Metals Hand book," ASM,(1988)
- Sully LJD.The thermal interface between castings and chill molds .AFS Trans 1976; 84:735-44.
- Petrovetc, Z. Stupar, S .“CFD one, Computational fluid Dynamics one”, Mechanical Engineering Faculty, Belgrade 1996.
- Welty, Wicks, Wilson “Fundamentals of Momentum, Heat, and Mass Transfer, 3rd edition”, John Wiley & Sons, p.252-295. ,(1997)
- Yunus, A. Cengel, “Heat Transfer A Practical Approach”, Mc-Graw Hill, Inc., 1998.

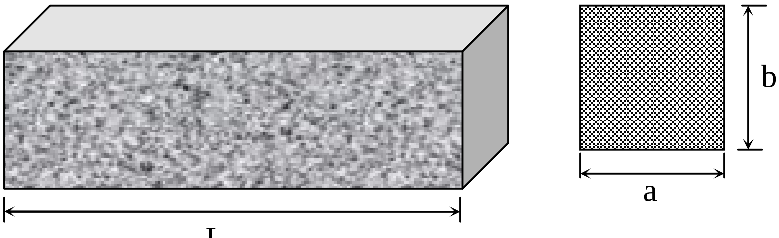


Fig.1: The dimension of the mold

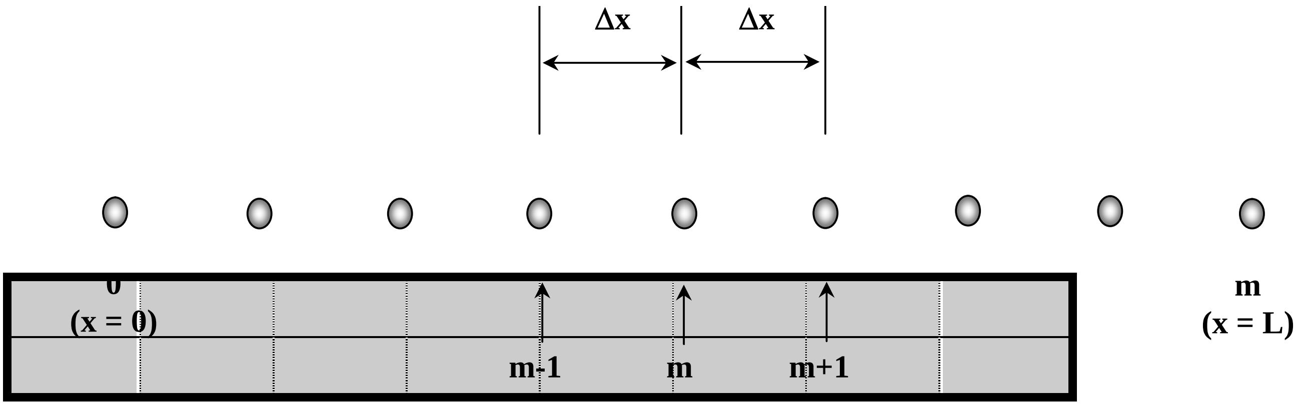


Fig.2. Finite difference node points in one- dimension conduction.

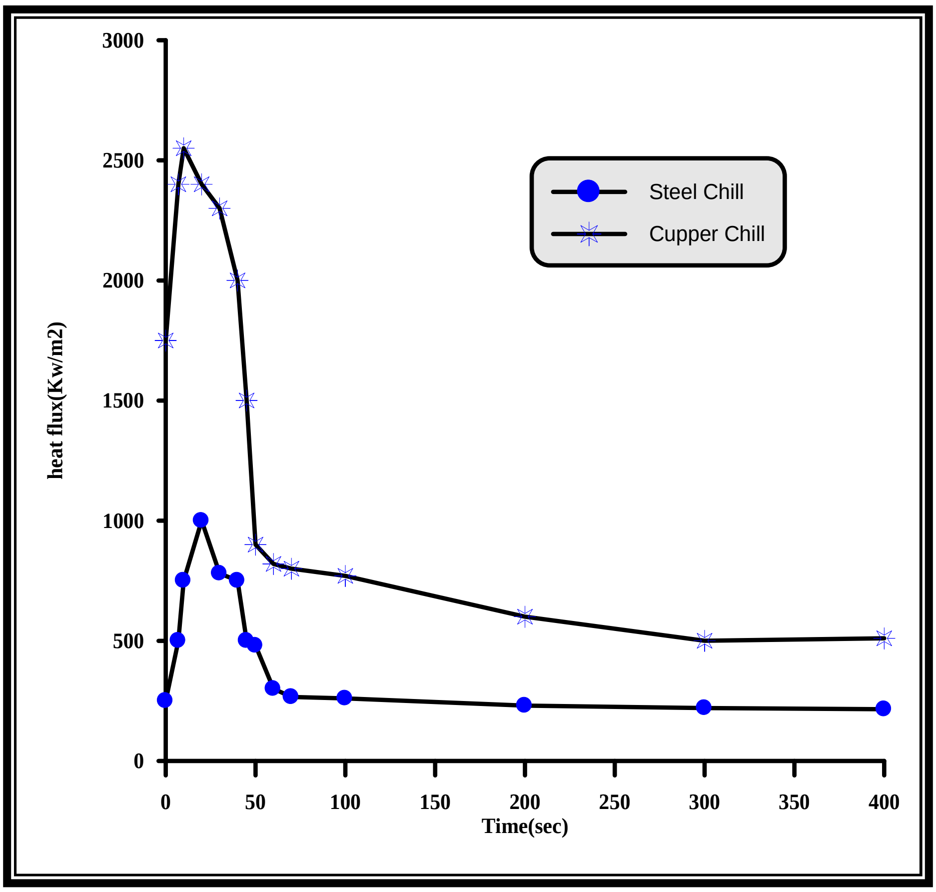


Fig.3: Heat flux between casting and chill as a function of time.

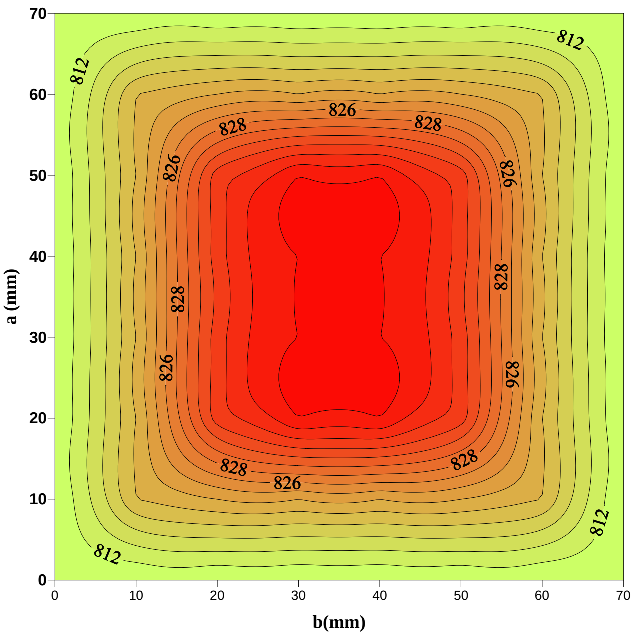


Fig.4a

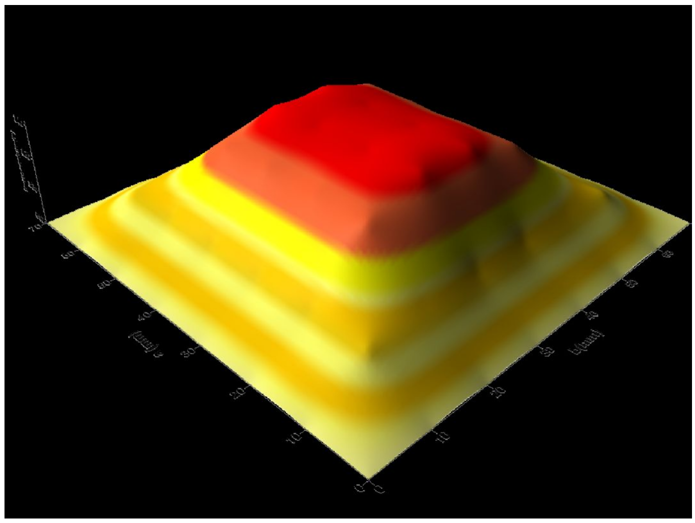
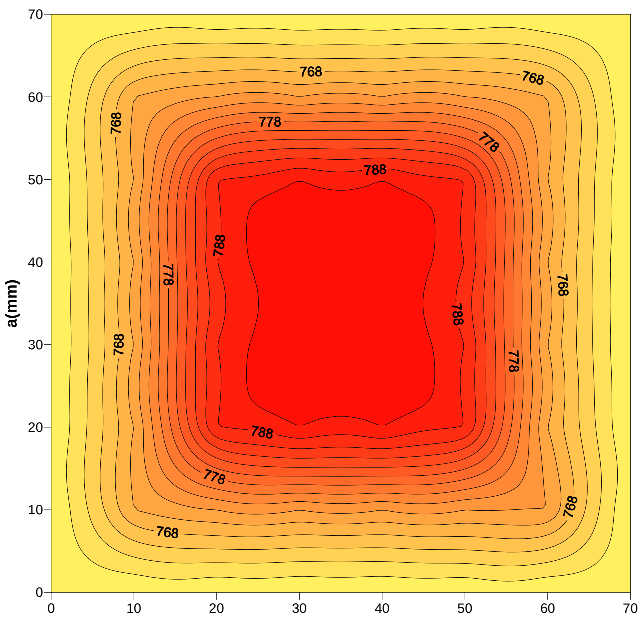
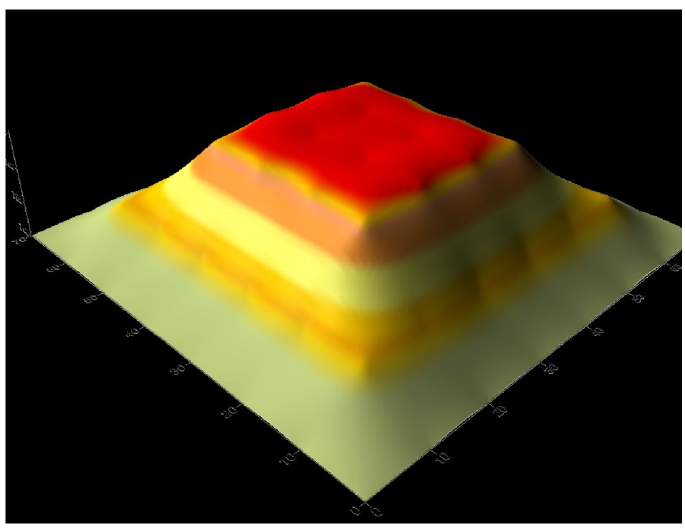


Fig.4b





b(mm)

a b

Fig.5: Temperature distribution at time 500 sec in mold

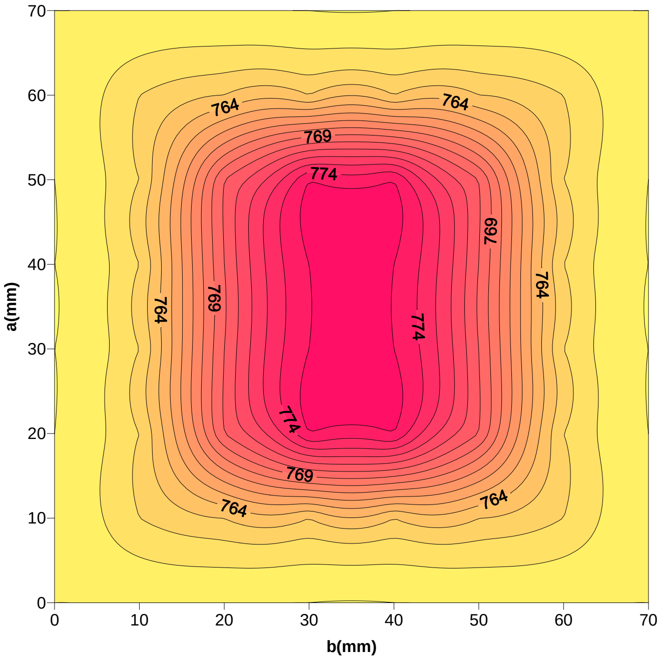


Fig.6a

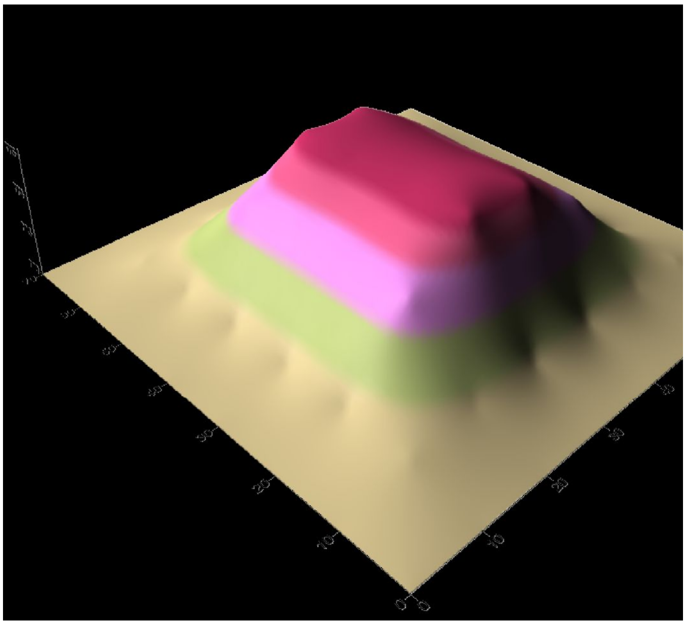


Fig.6b

Fig.6: Temperature distribution at time 700 sec in mold

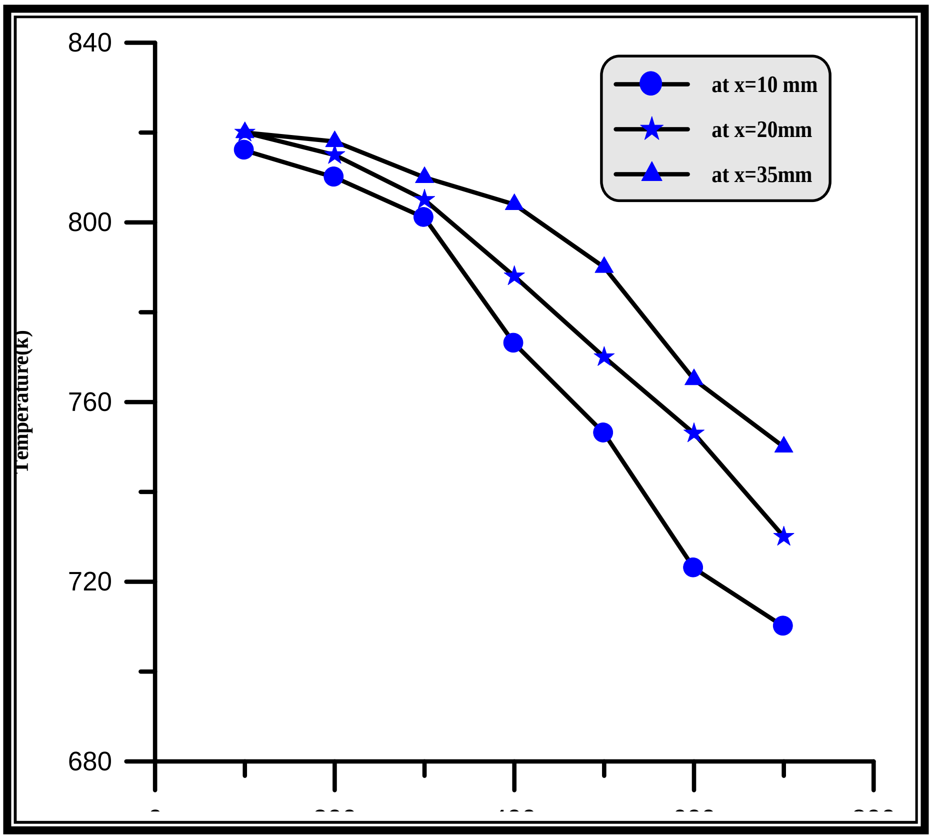


Fig.7: Temperature distribution at several distances in mold

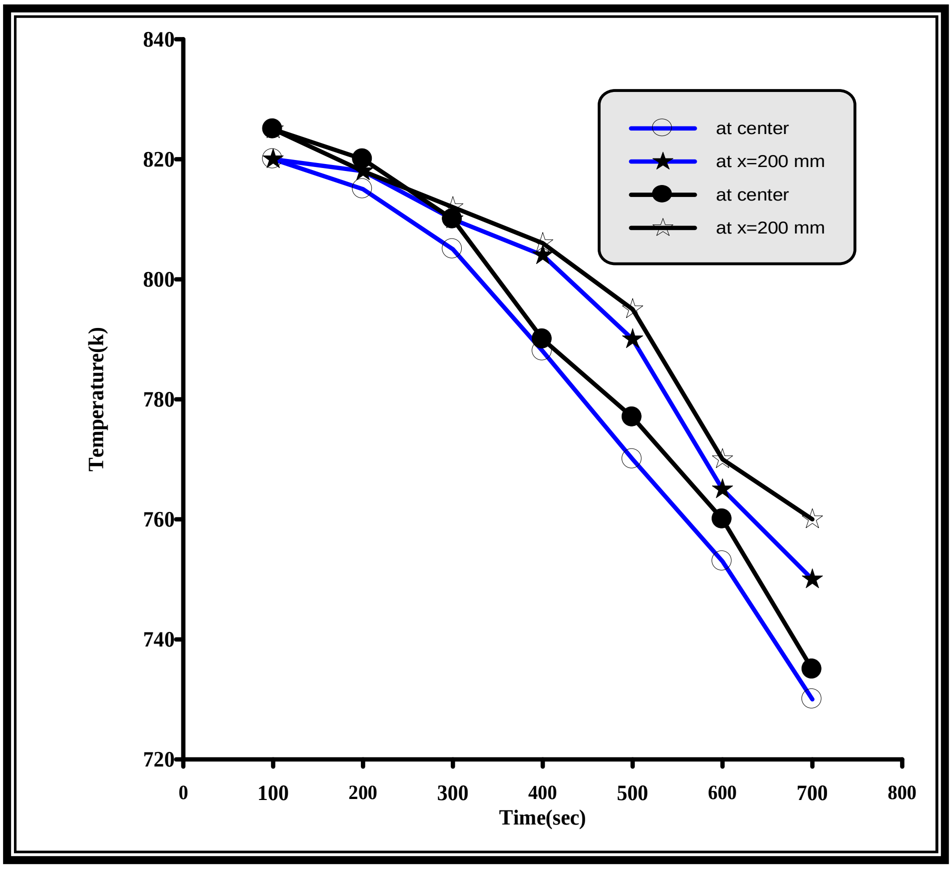


Fig.8: Temperature distribution Compare with result by C.P Hong,(1990).