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Optimum Design of Composite Laminated Plate Using Genetic Algorithm and RSM

Dr. Ammar Ali Hussein Al-filfily*

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Abstract

The paper is focused on the application of the response surface method (RSM) in structural optimization. Applications of the response surface method in the design of composite laminated plate have been discussed. The response surface method consists of two stages. In the first stage, the random variables is selected in order to perform a deterministic computer simulation (finite element solution) in the sample points. In the second stage, the approximation of the function (which represent the buckling load) is performed in order to obtain response surfaces using PDS module included in the ANSYS Program. This response surface is incorporated into a genetic algorithm (GAs) for optimization of random input variables to obtain maximum buckling load for composite laminated plate subjected to both mechanical and thermal loading. GAs are stochastic optimization algorithms based on natural selection and genetics. In contrast to traditional gradient-based methods, GAs work on populations of solutions which evolve typically over hundreds of generations. Four and five different variable formulations are examined. It was found that for SSSS boundary condition and two layer laminate the optimum values of buckling load for all thermal loading occur at $\theta_1=33^o$, $\theta_2=59^o$, $t_1=1.23$ mm and $t_2=$ 1.25 mm, also it can observe that the significant random variable are t_1 and t_3 (in the case of five independent variables) since the value of buckling load effected with t_1 and t_3 more than for t_2.

Keywords: composite plate; finite element method; genetic algorithm; RSM

التصميم الامثل لصفيحة مركبة باستخدام الخوارزمية الجينية وطريقة استجابة السطح

الخلاصة

تم في هذا البحث التركيز على طريقة استجابة السطح في ايجاد التصميم الامثل للهياكل، حيث تم تطبيق هذه الطريقة على صفيحة مركبة.تتكون طريقة سطح الاستجابة من مرحلتين اساسيتين حيث يتم في المرحلة الاولى اختيار المتغيرات العشوائية ومن ثم يتم حل المسالة بطريقة العناصر المحددة للنقاط العينية. وفي المرحلة الثانية يتم تكوين الدالة التقريبية لرسم سطح الاستجابة (في هذه الحالة حمل الانبعاج) باستخدام البرنامج PDSالمدعم مع برنامج ANSYS. وبعد ذلك يتم ادخال الدالة التقريبية (دالة استجابة السطح) الى الخوارزمية الجينية لايجاد القيم المثلى للمتغيرات العشوائية للحصول على اعظم حمل انبعاج للصفائح الطبقائية عند تعرضها الى احمال ميكانيكة وحرارية. تعتبر الخوارزمية الجينة احدى طرق ايجاد الامثلية والتي تعتمد على الاختيار الطبيعي والقوانين الجينيه: وعلى عكس الطرق التقليدية التي تعتمد على المشتقة فان الخوارزمية الجينية تعتمد على مبدأ التوزيع والذي يتضمن مئات التولدات. تم تطبيق حالتين للصفائح المركبة الاولى لاربعة متغيرات عشوائية والثانية لخمسة متغيرات عشوائية. اظهرت النتائج في حالة الظروف

*Technical College, Foundation of Technical Education /Baghdad

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University of Technology-Iraq, Baghdad, Iraq/2412-0758

الحديـة (SSSS) ووجـود طبقتـين ان القيمـة المثلـى لحمـل الانبعـاج تحـدث عنـد $\theta_1=33^o$, $\theta_2=59^o$,
(t_1=1.23 mm and t_2= 1.25 mm) كذلك لوحظ ان هنـاك تـاثير فعـال للمتغيـرين العشـوائيين (t_1
and t_3) (في حالة وجود خمسة متغيرات مستقلة) كـون قيمـة الانبعـاج تتـاثر بهـاتين القيمتين اكثر
من المتغير (t_2) .

List of Abbreviations

ANSYS	Analysis System
APDL	ANSYS Parametric Design Language
CCD	Central Composite Design
CCFF	Clamped-Clamped-Free-Free
DOF	Degree of Freedom
FE	Finite Element
GA	Genetic Algorithm
PDS	Probabilistic Design System
RSM	Response Surface Method
SSCC	Simply-Simply-Clamped-Clamped
SSFC	Dimply-Simply-Free-Clamped
SSFF	Simply-Simply-Free-Free
SSFS	Simply-Simply-Free-Simply
SSSS	Simply-Simply-Simply-Simply

Introduction

The present paper is focused on finding a minimum buckling load of composite plate subjected to both mechanical and thermal loading using Response surface method implemented in the ANSYS and Genetic algorithm. Response surface models have been utilized throughout scientific research to fit expressions to experimental data. However, they have also found use in optimization work to approximate more complex systems, for which the governing equation is unknown or expensive to solve many times. A few recent studies have examined the use of response surfaces in optimization work. Ragon*et al.* [1] incorporated a response surface model into a global-local optimization strategy. A response surface was used to model the weights of optimized composite panels, subject to a series of load and stiffness requirements, over a range of local design variables. Deng [2] presented a new practical and user-friendly FE model updating method. The new method utilizes the response surface method for the best experimental design of the parameters to be updated based on which numerical analysis can be performed in order to obtain explicit relationships between the structural responses and parameters from the simulation results. The parameters are then be updated using the genetic algorithm (GA) by minimizing an objective function.

Susuki [3] proposed a ranking method for strength optimizations of laminated composites for in-plane loadings. For general stacking sequence optimizations, genetic algorithms (GA) are becoming popular [4–13] because GAs are generally admitted to be effective for combinatorial optimizations.

Todoroki [14] proposed a new criterion to judge optimality of stacking sequences optimizations using a genetic algorithm with response surface in lamination parameters, In the case that 5% error is admitted, a global response surface is sufficient. However, the present method provides the real optimal laminate with 100 analyses to obtain the real optimal stacking sequence.

Genetic algorithms are ideally suited to optimization problems which involve discrete design variables, like composite material stacking sequence design. For this reason, a genetic algorithm optimization code was determined to be the most suitable, and was utilized to examine composite plate designs.

Genetic Algorithms

Genetic algorithms(GAs) are optimization algorithms which search the design space using the principles of evolutions. A design's success or failure is determined by its fitness. Survival of the fittest ultimately allows the algorithm to converge to an optimum design.

GAs are have significant advantages and disadvantages. Their ability to search an entire design space gives them the ability to handle optimization problems with highly non-linear objective functions with many local minima. In addition, since they are not gradient based algorithms, they are ideally suited to optimization problems which utilize discrete design variables, such as the optimization of composite laminate stacking sequence. Soremekun[15] utilized GAs in his study of composite laminate design. On the other hand, genetic algorithms can be extremely expensive. Each member of a population of designs requires an evaluation of the fitness function. Tens of thousands of fitness function evaluations may be required over the course of single GA run. GAs utilize a string of numeric values, which are analogous to the biological chromosome, to describe a potential design in terms of its design variables. These numerical values may be the values of the design variables themselves or the chromosome may require decoding to arrive at the design variables. Table 1 shows examples of GA chromosomes and their decoding.

Figure 1 shows a schematic of a typical GA. To begin a GA run, an initial population of designs is randomly determined. This initial population is then decoded, and the fitness of the design can be determined by evaluating the value of the fitness function for the design. Often, this fitness function incorporates the constraints required by the optimization problem in the form of penalty terms.

These penalty terms penalize the value of the fitness function for violation of constraints. After the fitness of each design is determined, the designs are ranked for mating according to this fitness value. A "roulette wheel" is used to select parent designs for mating. Designs which have the best values of the fitness function are more likely to be selected than designs with poor fitnesses.

A series of genetic operators is performed on two selected designs to arrive at two new (child) designs. The two most common genetic operators are crossover and mutation. Crossover works by splitting the two design chromosomes at a randomly determined point and switching portions of the chromosome, mutation randomly selects a gene in the chromosome and randomly changes it to another value. Tables 2 and 3 show examples of these genetic operators.

Whether or not a given genetic operator is performed on a chromosome is determined by user defined probabilities. Typically, crossover is performed all of the time (>90% probability). Selection and genetic operations are performed until a new generation has been filled.

After an entire child generation has been determined, the process of fitness function evaluation, ranking, selection, and mating repeats until convergence is reached. Often, an elitist scheme is used, which incorporates the top design, or a number of the top designs, from the parent generation into the child generation. This ensures that the information contained in the best design of one generation is not lost to the next. Convergence id reached when the fitness function of the best design no longer improves. A variety of convergence criteria can be used, but a typical criterion is to set the number of generations which must pass without improvement in the fitness function.

Response Surface Methods

Response surface methodology (RSM) is a statistical technique in which smooth functions, typically polynomials, are used to model an objective function. For example, a quadratic response surface model for p variables has the form [16]:

$$y = c_0 + \sum_{1 \le i \le p} c_i x_i + \sum_{1 \le i \le p} c_{ij} x_i x_j \quad (1)$$

Where the x_i are the variables, the c_i are the polynomial coefficients, and y is the measured response. For p variables, equation (1) has $n=(p+1)(p+2)/2$ terms. In such a mode the polynomial coefficients are estimated using the technique of forward-stepwise regression. Throughout this work, ANSYS program is used to generate response surfaces.

A response surface analysis consists of two stages:

1. performing the simulation loops to calculate the values of the random output parameters that correspond to the sample points in the space of random input variables.
2. Performing a regression analysis to derive the terms and the coefficients of the approximation function.

The fundamental idea of response surface methods is that once the coefficients of a suitable approximation function are found, then we can directly use the approximation function instead of looping through the finite element model. To perform a finite element analysis might require minutes to hours of computation time: in contrast, evaluating a quadratic function requires only a fraction of a second. Hence, if using the approximation function, we can afford to evaluate the approximated response parameter thousands of times.

For response surface analysis, you can choose from three sampling methods: Central composite design, Box-Behnken matrix and user-defined. In this work the central composite design method is used.

Central Composite Design Sampling

A central composite design consists of a central point, the N axis point plus 2N-f factorial points located at the corners of an N-dimensional hypercube [16]. Here, N is the number of random input variables and f is the fraction of the factorial part of the central composite design. A fraction f=0 is a called a full factorial design, f=1 gives a half-factorial design, and so on. The PDS gradually increases the fraction f as you increasing the number of random input variables. This keeps the number of simulation loops reasonable. The fraction f is automatically evaluated such that a resolution V design is always maintained. A resolution V design is a design where none of the second order terms of the approximation function are confined with each other. This ensures a reasonable accuracy for the evaluation of the coefficients of the second order terms. The locations of the sampling points for a problem with three random input variables are illustrated in Fig. (2).

Model Description

Physical Model

The physical structure that used in this work is a fiber reinforced composite plate, shown in Fig.3. The length of the plate is 100mm, with a width of 100mm. The thickness of the three layers and the ply orientation of each layer are treated as a design variable.

Finite Element Model

Analyses performed in this design study utilized a finite element model of the plate. The model was developed in ANSYS 12.1, using the 100 elements. The global x coordinate is directed along the length of the plate, while the global y coordinate is directed along the width and the global z direction is taken to be the outward normal of the plate surface. There are 10 elements in the axial direction and 10 along the width. Reasons for choosing the particular mesh used in this study will be described later in the discussion on convergence study. A linear buckling analysis was performed on the model to calculate the minimum buckling load of the structure.

The plates were analyzed under six different boundary conditions: SSSS, SSFS, SSFF, SSFC, SSCC and CCFF. Figs.(4 to 9) visually show the boundary conditions and the applied load as they were entered into ANSYS.

Program Verification Case Study

In order to confirm the reliability of the ANSYS program a verification case is considered.

The geometry and properties of cross - ply square laminated composite plate with various boundary conditions [17] were chosen as verification case for the ANSYS computer program.

Table (4) presents a comparison of results of critical buckling load obtained from Reddy [17] and the ANSYS computer program.

Design Variables

The finite element model is described via an input file using APDL ANSYS language of various parameters. A group of five design variables was used in this study. They are outlined in Table 5.

Material Properties

The material properties used throughout this study are shown in Table 6. These properties are obtained by using mechanics of material approach [18] except the property G_{23} at which the semiempirical stress partitioning parameter (SSP) technique by Barbero [19] is used to calculate its value. It is assumed that $E_2=E_3$, $G_{12}=G_{13}$, $\nu_{12}=\nu_{13}$.

Mesh Convergence

A convergence study was performed to determine the appropriate finite element mesh to be used in the linear buckling analysis of the plate model. Three meshes were developed, with increasing numbers of elements in the x and y directions. These are shown in Figure 10.

The buckling loads for each of these models is shown in Table 7. There is only a 0.2%difference between the load calculated for mesh 1 (4x4) and mesh 2 (5x5). A smaller difference (0.1%) is observed between mesh 2 and mesh 3(10x10). This indicates that the coarsest mesh is capable of performing the analysis within a reasonable degree of accuracy.

Optimization Results

The developed finite element code is used to provide critical buckling load. A response surface, spanning the design space, is generated from a set of design points. This response surface is incorporated into a genetic algorithm for optimization of composite laminated plate. Four and five different variable formulations are examined.

An optimization analysis was performed to the laminated composite plate for two different cases. Figures (11) to (17) represent the sample response surface fit for two layer composite plates with four input random variables. Second, figures (18) to (21) represent the response surface fit for three layer composite plate with five input random variables.

The detailed fitting equation and the optimum value for each response surface mentioned above was arranged in the tables (8) to (23).

The table (8) and figures from (11) to (14) represent the study of two layers composite plate when there is no thermal loading ($T=20^oC$) while a set of tables (9) to (11) and figures (14) to (17), represent the study of the two layers composite plate when it is subjected to both mechanical and thermal loading to obtain optimum value of thickness and ply orientation with respect to minimum buckling load with various boundary conditions.

It can be shown that in the case of no thermal loading (room temperature $T=20^oC$) and SSCC boundary condition the best value of critical buckling load occurs at $\theta_1=67^o$ and $\theta_2=71^o$ while the thickness of two layers are 1.47 mm and 1.19 mm respectively. This value is reduced to 48 % when the boundary condition changes to SSSS. Also it can be noted that ply orientation of layer 1 has significant effect than ply orientation of layer 2 on the critical buckling load as shown in figures (11) to (14).

When the plate is subjected to both thermal and mechanical loading the situation is different in which at all boundary condition the optimum value of all input variables are nearly constant as shown in tables from (8) to (11). For SSSS boundary condition and two layer laminate the optimum values for all thermal loading occur at $\theta_1=33^o$, $\theta_2=59^o$, $t_1=1.23$ mm and $t_2=$ 1.25 mm as shown in Figs from (15) to (17).

The same thing can be reported for three layer composite plates when there are five input variables namely (θ_1, θ_2, t_1, t_2 and t_3) at temperature ($T=60^oC$, $T=80^oC$) where this values becomes($\theta_1= 58.3^o$, $\theta_2=13.46^o$, $t_1=1.17$ mm, $t_2=1.2$ and $t_3=1.46$ as shown in tables from (12) to (15). Another important note can be reported from these tables where the maximum buckling load occurs at SSSS boundary condition and T=20 ^{o}C and this value is reduced to about (89%) for SSSS boundary condition and T=60 ^{o}C .

Figures (18) to (21) show the response surface for buckling load when there are five independent variables namely (θ_1, θ_2, t_1, t_2 and t_3). One can observe that the significant random variable are t_1 and t_3(the thickness of outer layers of composite plate) since the value of buckling load differ with $t_1$and t_3 more than for t_2, while the other random variables are not significant. It is also noted from these figures that some points not lay on the response surface and this problem can be treated by taking more samples points in the range of random variables.

Conclusions

In this study, RSM imposed in the ANSYS is used with the GA to obtain optimum value of a critical buckling load for plate subjected to thermal and mechanical loading under various boundary conditions. It was found that when the plate subjected to thermal loading the values of optimum input variables are different from that of plates subjected to only mechanical loading.

Also, it was found that the effect of t_1 and t_3 have a significant effect on critical buckling loading than other random input variables.

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Table (1) Chromosome Encoding

Chromosome	Alphabet Encoding Type	Decoded Chromosome
{1,0,1,1,0,1,0}	Binary encoding	180
{1,1,2,1,3}	1 = $[0^o]_2$, 2 = $[+45^o/-45^o]$, 3 = $[90^o]_2$	$[0^o/0^o/0^o/0^o/+45^o/-45^o/0^o/0^o/90^o/90^o]_s$
{10,24,17}	Direct	{10,24,17}

Table (2) Example of Crossover

Genetic Operator	Parent Chromosome	Child Chromosome
Crossover	{1,0,0,1,1,0,1,0} {1,0,1,**0,1,1,0,1**}	{1,0,0,**0,1,1,0,1**} {1,0,1,**1,1,0,1,0**}

Table (3) Example of Mutation

Genetic Operator	Initial Chromosome	Final Chromosome
Mutation	{1,0,1,0,1,**1**,0,1}	180

Table (4) Comparison of Results of Critical Buckling Loud obtained from ANSYS and Ready [17].

No. of Layer	Theory	SSSS	SSFC	SSFS	SSFF	SSSC
2-Layer	**Ready**	11.3530	6.1660	5.3510	4.8510	16.437
	ANSYS	11.4708	6.0504	5.3385	4.6281	16.1740
	Diff %	1.0372	-1.8742	- 0.2327	- 4.5954	- 1.6260
10-Layer	**Ready**	25.4500	14.3580	12.5240	12.0920	32.614
	ANSYS	24.9763	14.1693	12.3451	11.9682	32.9151
	Diff %	-1.8611	-1.3145	- 1.4281	- 1.0235	0.91477

Table (5) Design Variable Descriptions

Design Variables	Description
θ_1	Play Angle of Top and Bottom Layer
θ_2	Play Angle of Middle Layer
t_1	Ply Thickness of Top Layer
t_2	Ply Thickness of Middle Layer
t_3	Ply Thickness of Bottom Layer

Table (6) Material Properties (Glass – Epoxy)

E_1 (MPa)	E_2 (MPa)	E_3 (MPa)	G_{12} (MPa)	G_{23} (MPa)	G_{13} (MPa)	ν_{12}	ν_{23}	ν_{13}
28522	5235	5235	2312	2345	2312	0.26	0.34	0.26

Table (7) Critical Buckling Load Convergence Study for Orthotropic Plate. (Simply-Simply-Simply-Simply)

No. of Element	No. of Nodes	No. of DOF	Critical Buckling Load (Nx)cr. N/mm
			7
			7
			7

Table (8) Optimum Buckling Load For the Selected Parameter of the Two Layer Laminated Composite Plates for Various Boundary Conditions at T=20°C

	θ_1(°)	θ_2(°)	t_1(mm)	t_2(mm)	P_{cr}(N/mm)
CCFF	9.75	9.39	1.31	1.29	194.41
SSCC	67.0	71.0	1.474	1.197	254.49
SSFC	18.87	15.98	1.39	1.22	72.82
SSFF	1.05	3.86	1.49	7.61	36.31
SSFS	7.19	6.67	1.02	1.49	59
SSSS	43.9	54.6	1.41	1.27	132

Table (9) Optimum Buckling Load For the Selected Parameter of the Two Layer Laminated Composite Plates for Various Boundary Conditions at T=40°C

	θ_1(°)	θ_2(°)	t_1(mm)	t_2(mm)	P_{cr}(N/mm)
CCFF	43.57	23.59	1.34	1.10	26.65
SSCC	33.53	59.69	1.23	1.25	7.41
SSFC	33.53	59.69	1.23	1.25	35.42
SSFF	43.57	23.59	1.34	1.10	22.24
SSFS	33.53	59.69	1.23	1.25	23.81
SSSS	33.53	59.69	1.23	1.25	27.72

Table (10) Optimum Buckling Load For the Selected Parameter of the Two Layer Laminated Composite Plates for Various Boundary Conditions at T=60°C

	θ_1(°)	θ_2(°)	t_1(mm)	t_2(mm)	P_{cr}(N/mm)
CCFF	33.53	59.69	1.23	1.25	38.52
SSCC	33.53	59.69	1.23	1.25	3.73
SSFC	33.53	59.69	1.23	1.25	20.73
SSFF	43.57	23.59	1.34	1.10	16.65
SSFS	43.57	23.59	1.34	1.10	14.00
SSSS	33.53	59.69	1.23	1.25	14.99

Table (11) Optimum Buckling Load For the Selected Parameter of the Two Layer Laminated Composite Plates for Various Boundary Conditions at T=80°C

	θ_1(°)	θ_2(°)	t_1(mm)	t_2(mm)	P_{cr}(N/mm)
CCFF	33.53	59.69	1.23	1.25	16.28
SSCC	33.53	59.69	1.23	1.25	2.49
SSFC	33.53	59.69	1.23	1.25	14.50
SSFF	33.53	59.69	1.23	1.25	13.02
SSFS	33.53	59.69	1.23	1.25	10.12
SSSS	33.53	59.69	1.23	1.25	10.24

Table (12) Optimum Buckling Load For the Selected Parameter of the Three Layers Laminated Composite Plates for Various Boundary Conditions at T=20°C

	θ_1(°)	θ_2(°)	θ_3(°)	t_1(mm)	t_2(mm)	t_3(mm)	P_{cr}(N/mm)
CCFF	1.09	63.95	1.09	0.87	1.22	0.94	266.63
SSCC	58.33	13.46	58.33	1.17	1.20	1.46	744.43
SSFC	20.21	55.63	20.21	1.46	0.50	1.44	15.79
SSFF	9.22	63.95	9.22	0.87	1.09	0.94	72.23
SSFS	58.33	13.46	58.33	1.17	1.20	1.46	101.81
SSSS	58.33	13.46	58.33	1.17	1.20	1.46	363.30

Table (13) Optimum Buckling Load For the Selected Parameter of the Three Layers Laminated Composite Plates for Various Boundary Conditions at T=40°C

	θ_1(°)	θ_2(°)	θ_3(°)	t_1(mm)	t_2(mm)	t_3(mm)	P_{cr}(N/mm)
CCFF	80.70	71.54	80.70	1.08	0.99	0.24	24.69
SSCC	58.33	13.46	58.33	1.17	1.20	1.46	14.62
SSFC	58.33	13.46	58.33	1.17	1.20	1.46	91.87
SSFF	20.21	55.63	20.21	1.46	0.50	1.44	60.18
SSFS	58.33	13.46	58.33	1.17	1.20	1.46	57.83
SSSS	58.33	13.46	58.33	1.17	1.20	1.46	72.16

Table (14) Optimum Buckling Load for the Selected Parameter of the Three Layers Laminated Composite Plates for Various Boundary Conditions at T=60°C

	$\theta_1(^o)$	$\theta_2(^o)$	$\theta_3(^o)$	t_1(mm)	t_2(mm)	t_3(mm)	P_{cr}(N/mm)
CCFF	32.28	83.76	32.28	1.13	1.01	0.96	92.27
SSCC	58.33	13.46	58.33	1.17	1.46	1.20	10.59
SSFC	58.33	13.46	58.33	1.17	1.46	1.20	50.17
SSFF	58.33	13.46	58.33	1.17	1.46	1.20	39.73
SSFS	58.33	13.46	58.33	1.17	1.46	1.20	31.93
SSSS	58.33	13.46	58.33	1.17	1.46	1.20	38.10

Table (15) Optimum Buckling Load For the Selected Parameter of the Three Layers Laminated Composite Plates for Various Boundary Conditions at T=80°C

	$\theta_1(^o)$	$\theta_2(^o)$	$\theta_3(^o)$	t_1(mm)	t_2(mm)	t_3(mm)	P_{cr}(N/mm)
CCFF	58.33	13.46	58.33	1.17	1.20	1.46	35.28
SSCC	58.33	13.46	58.33	1.17	1.20	1.46	5.85
SSFC	58.33	13.46	58.33	1.17	1.20	1.46	34.44
SSFF	58.33	13.46	58.33	1.17	1.20	1.46	30.65
SSFS	58.33	13.46	58.33	1.17	1.20	1.46	21.97
SSSS	58.33	13.46	58.33	1.17	1.20	1.46	25.86

Table (16) Regression Equations For the Buckling Load For the Selected Parameter of the Two Layer Laminated Composite Plates for Various Boundary Conditions at T=20 °C

	Cons.	θ_1	θ_2	θ_1^2	θ_2^2	t_1	t_2	t_1^2	t_2^2	θ_1*t_1	θ_1*t_2	θ_2*t_1	θ_2*t_2	t_1*t_2
CCFF	15.5		-6.4	7.1		17.4	19.5		5.8	-11.4	-8.1		-8.0	12.4
SSCC	48.2				-3.8	41.4	41.1	10.5	8.3				4.9	21.3
SSFC	8.8	-2.3	-2.2			8.9	8.8	3.98	2.4	-1.5	-2.4	-1.9	-2.5	5.5
SSFF	3.98	-2.3	-2.1			4.78	4.02	2.06	1.49	-2.1	-0.8	-1.4	-1.6	2.0
SSFS	8.3	-2.0	-3.0			8.3	7.3	1.6	2.1	-2.3	-1.4		-2.2	3.2
SSSS	7.4	-0.6	-0.7	0.45	0.74	5.9	5.7							

Table (17) Regression Equations For the Buckling Load For the Selected Parameter of the Two Layer Laminated Composite Plates for Various Boundary Conditions at T=40 °C

	Cons.	θ_1	θ_2	$\theta_1*\theta_2$	θ_1^2	t_1	t_2	t_1^2	t_2^2	θ_1*t_1	θ_1*t_2	θ_2*t_1	θ_2*t_2	t_1*t_2
CCFF				0.7				1.5	0.6					1.9
SSCC	2.7	0.14	0.12	0.18		1.6	1.5	0.26	0.19	0.12	0.13			0.38
SSFC	8.9					7.6	7.3	1.6	1.6					2.9
SSFF	5.5	-0.9	-1.2		-0.5	4.1	4.0	1.3		-0.6		-0.8	-0.4	1.8
SSFS	6.0					5.2	4.8	1.2	1.0					2.0
SSSS	8.0	-0.6	-0.8			5.5	5.7	1.4	0.98					1.8

Table (18) Regression Equations For the Buckling Load For the Selected Parameter of the Two Layer Laminated Composite Plates for Various Boundary Conditions at T=60 °C

	Cons.	θ_1	θ_2	$\theta_1*\theta_2$	θ_1^2	t_1	t_2	t_1^2	t_2^2	θ_1*t_1	θ_1*t_2	θ_2*t_2	t_1*t_2
CCFF	12.1	-0.8	-1.0			8.7	8.5	1.9	1.1				3.2
SSCC	1.4	0.007	0.005	0.009		0.8	0.7	0.1	0.009	0.005	0.007		0.01
SSFC	6.2	0.6	0.5			4.4	4.4	0.9	0.8	0.5		0.5	1.37
SSFF	4.8	-0.5	-0.7		-0.5	3.1	3.4	1.1					1.4
SSFS	3.9					3.0	2.9	0.7	0.7				0.95
SSSS	4.6	-0.4	-0.5			2.9	3.0	0.7	0.4				0.9

Table (19) Regression Equations For the Buckling Load For the Selected Parameter of the Two Layer Laminated Composite Plates for Various Boundary Conditions at T=80 °C

	Cons.	θ_1	θ_2	$\theta_1*\theta_2$	t_1	t_2	t_1^2	t_2^2	θ_1*t_1	θ_1*t_2	t_1*t_2
CCFF	6.4		0.58		3.8	3.5		0.73			0.75
SSCC	0.95	0.004	0.003	0.006	0.5	0.5	0.008	0.006	0.003	0.004	0.01
SSFC	4.5	0.5	0.4		3.0	3.0	0.6	0.5	0.36	0.38	0.89
SSFF	3.3	-0.3	-0.3		2.79	2.7	0.7	0.5			1.15
SSFS	2.9				2.1	2.1	0.51	0.51			0.61
SSSS	3.2	-0.2	-0.4		1.9	2.1	0.5	0.3			0.6

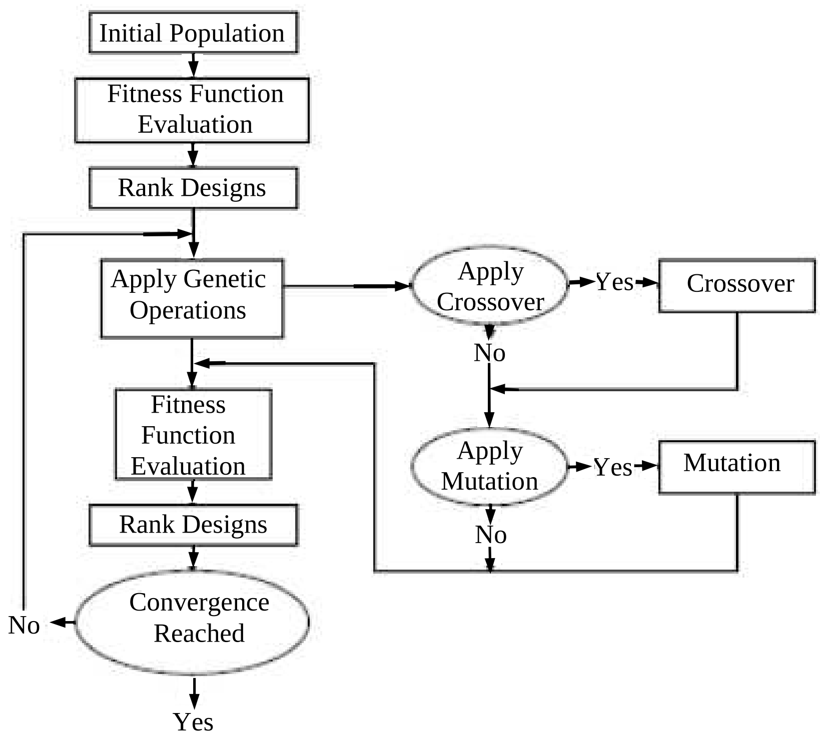


Figure (1) Genetic Algorithm Schematic

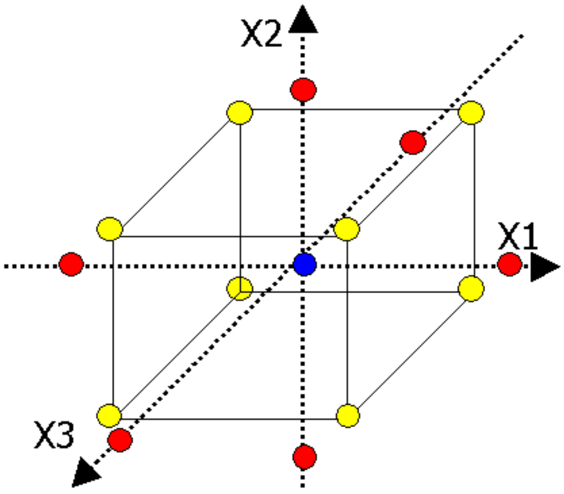


Figure (2) Locations of Sampling Points for Problem with Three Input Variables for CCD

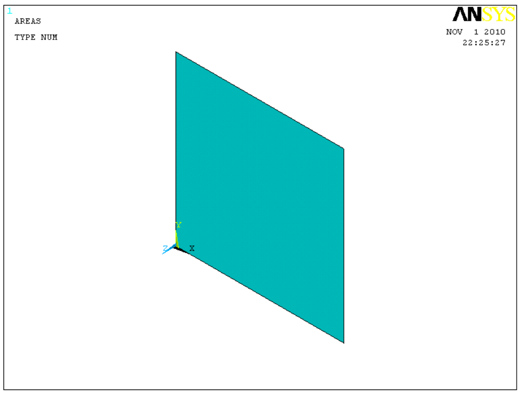


Figure (3) Plate Model

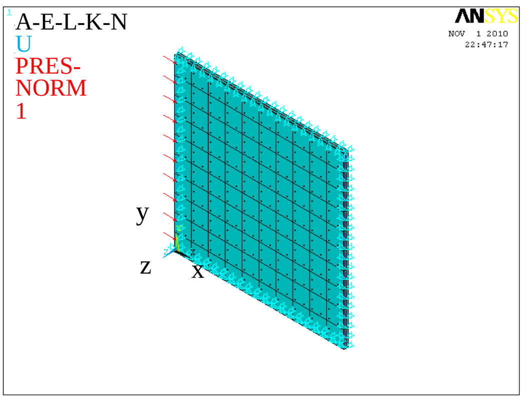


Figure (4) SSSS Boundary Condition

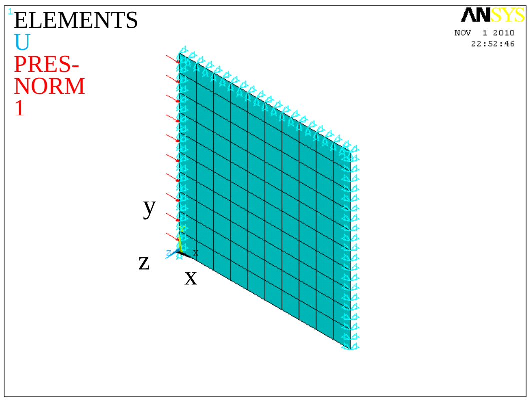


Figure (5) SSFS Boundary Condition

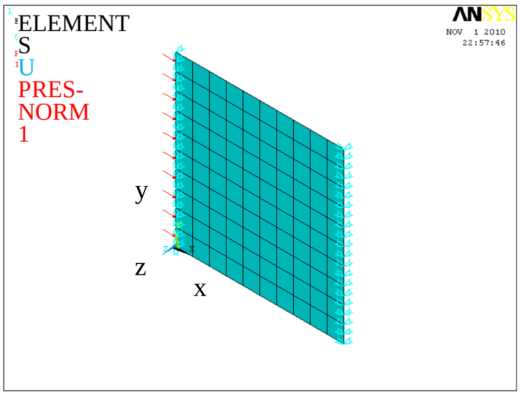


Figure (6) SSFF Boundary Condition

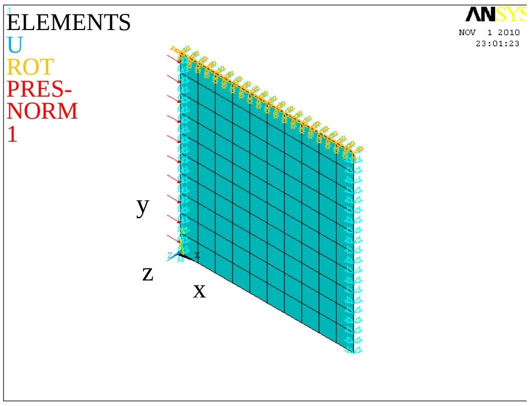


Figure (7) SSFC Boundary Condition

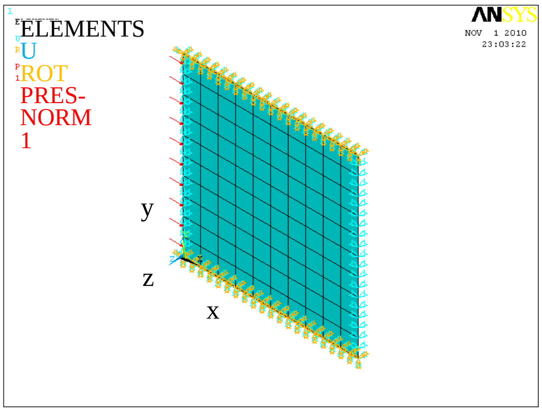


Figure (8) SSCC Boundary Condition

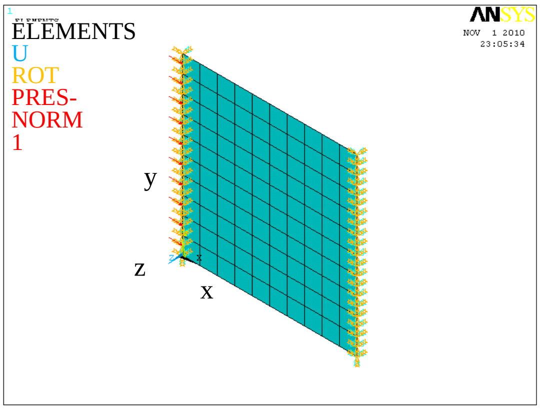
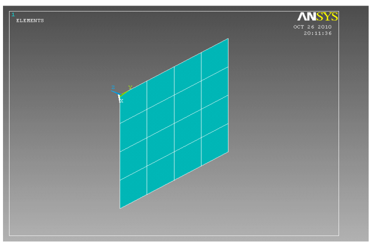
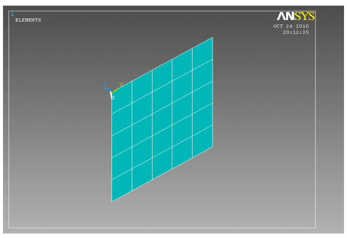


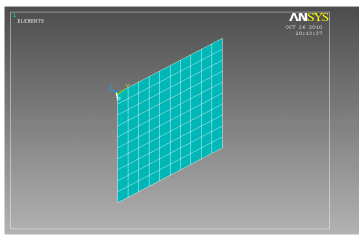
Figure (9) CCFF Boundary Condition



(a) 16 Element



(b) 25 Element



(c) 100 Element

Figure (10) Convergence Study

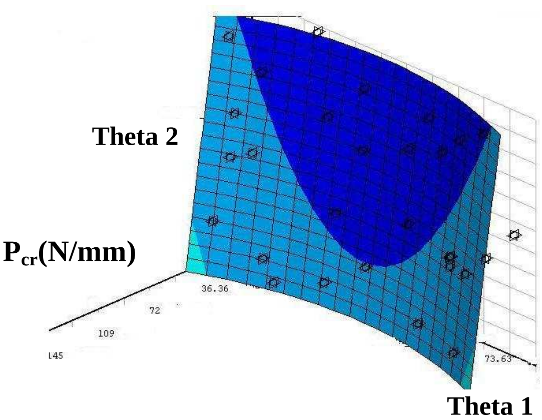


Figure (11) Response Surface Approximation (P_{cr}) For Two Layer CCFF Laminated Plate at T=20^{o}C (No Thermal Loading)

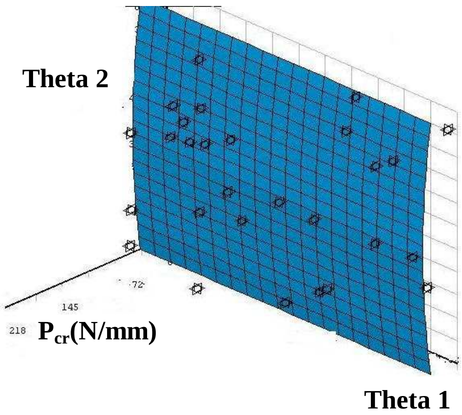


Figure (12) Response Surface Approximation (P_{cr}) For Two Layer SSCC Laminated Plate at T=20^{o}C (No Thermal Loading)

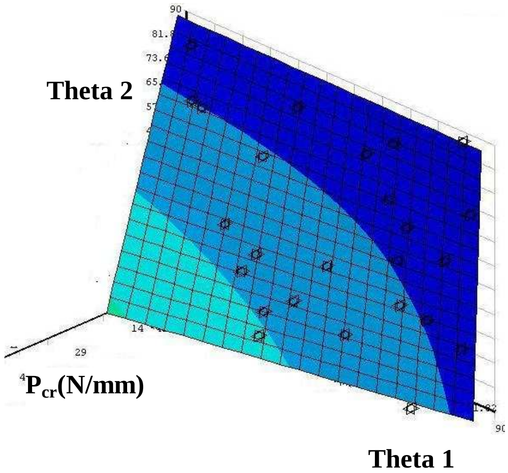


Figure (13) Response Surface Approximation (P_{cr}) For Two Layer SSFS Laminated Plate at T=20^{o}C (No Thermal Loading)

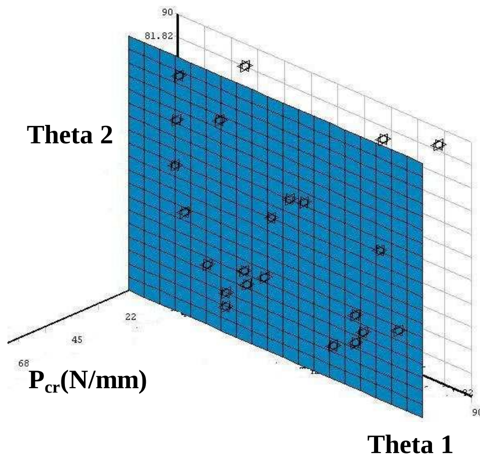
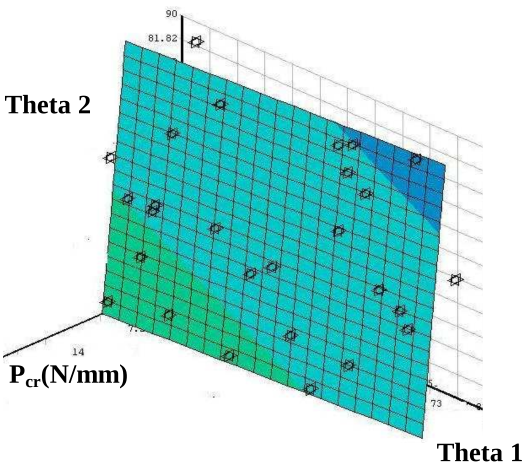
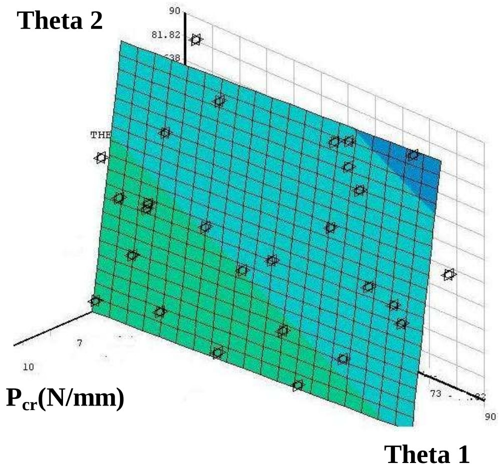


Figure (14) Response Surface Approximation (P_{cr}) For Two Layer SSSS Laminated Plate at T=20^{o}C (No Thermal Loading)



Figure(15) Response Surface Approximation (P_{cr}) For Two Layer SSSS Laminated Plate Subjected to Thermal Loading (T=40^{o}C)



Figure(16) Response Surface Approximation (P_{cr}) For Two Layer SSSS Laminated Plate Subjected to Thermal Loading (T=60^{o}C)

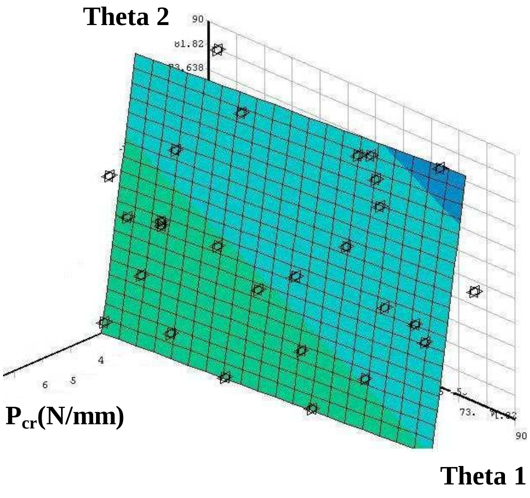


Figure (17) Response Surface Approximation (P_{cr}) For Two Layer SSSS Laminated Plate Subjected to Thermal Loading (T=80^{o}C)

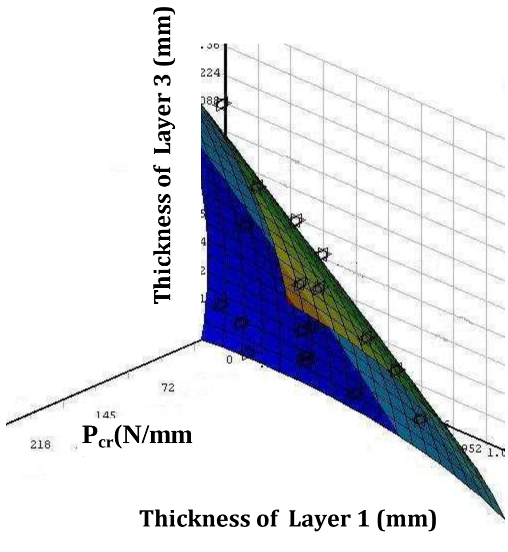


Figure (18) Response Surface Approximation (P_{cr}) For Three Layer SSSS Laminated Plate without Thermal Loading

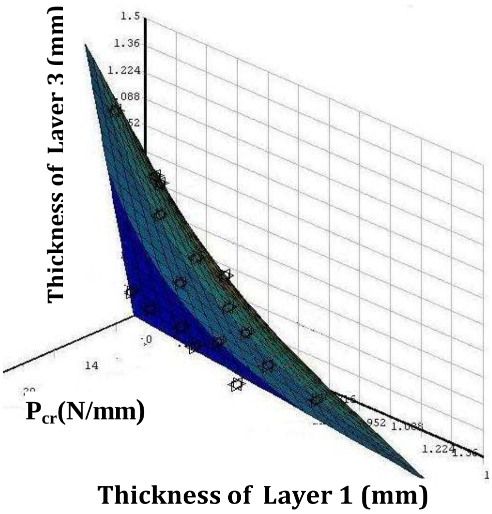


Figure (19) Response Surface Approximation (P_{cr}) For Three Layer SSSS Laminated Plate Subjected to Thermal Loading (T=40^{o}C)

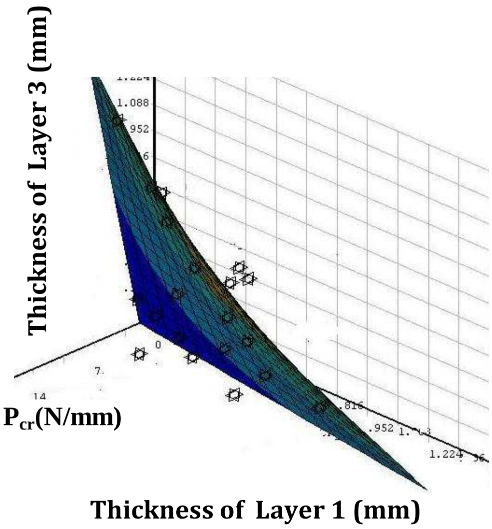


Figure (20) Response Surface Approximation (P_{cr}) For Three Layer SSSS Laminated Plate Subjected to Thermal Loading (T=60^{o}C)

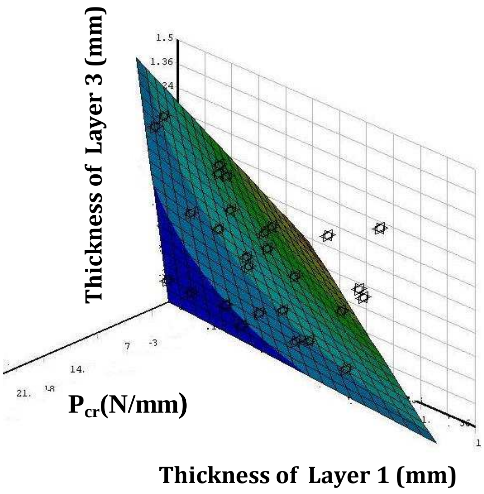


Figure (21) Response Surface Approximation (P_{cr}) For Three Layer SSSS Laminated Plate Subjected to Thermal Loading (T=80^{o}C)