DEGREES OF FREEDOM ANALYSIS IN PRODUCT ASSEMBLING

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ABSTRACT:

Assembly sequence generation is one of the major difficulties in assembly planning process. In this paper, disassembly sequence generation has been used as approach to generate products assembly sequence. Assembly sequence is generated here by tracing disassembly steps starting with completed product and working backward down to its basic components. Practically, in this work degrees of freedom DOFs analysis has been used to generate disassembly sequence. The approach has involve six steps proposed generate assembly sequence. This approach has been implemented on a virtual product and shows its ability to generate assembly sequence for the product in question.

KEYWORDS: Degrees of Freedom Analysis, Assembly, Sequence, Planning, AND/OR Graph.

تحليل درجات الطلاقة في تجميع منتوج

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الخلاصة:

تمثل عملية توليد تسلسل التجميع احد الصعوبات الكبرى في عملية التخطيط للتجميع. في هذا البحث، اتخذ تسلسل التفكيك طريقا لتوليد تسلسل التجميع حيث تم تتبع عملية التفكيك بالعودة إلى الخلف بدءا من المنتوج النهائي ولحين الوصول إلى مكوناته الأساسية. تم هنا الاستفادة من تحليل درجات الحرية لمكونات المنتوج لغرض الوصول إلى التسلسل الممكن لعمليات التفكيك. وعليه تم في هذا البحث تقديم منهجية مقترحه متكونة من ٦خطوات لتوليد تسلسل التجميع. تم تطبيق هذه المنهجية على منتوج افتراضي وقد أظهرت قابلية على توليد تسلسل لتجميع ذلك المنتوج.

NOMENCLATURE

DOFs	*Degrees Of Freedom*
$\mathscr{R}^3$	*Space around an object*
Ma	*All possible motion parameters*
Md	*Disallowed motion parameters*
Mp	*Permissible motion parameters*

INTRODUCTION

Survey

1- (**De Mello and Sanderson, 1991**) suggested to divide assembly sequence representations into explicit and implicit ones, where formal definitions of different representations, correctness and completeness proofs, and mutual relationships between them are given.
2- (**Jones and Wilson 1996**) found a systematic and comprehensive listing of constraints to be considered in assembly planning.
3- (**Lai and Huang 2004**) derived a liaison matrix from the connection: each one of the n files or columns corresponds to a part, and the elements of the matrix are equal to 1 if a connection between these two parts exists, 0 otherwise (elements of the diagonal are obviously null).
4- (**Wang et al 2005**) applied a disassembly Completed Graph represents the search space of all possible disassembly sequences (where each node corresponds to a part and a disassembly direction, i.e., to an elemental disassembly operation DO), planning consists in finding a path that joins nodes with different part identification numbers, respecting the geometric precedence constraints.
5- (**Le et al 2009**) performs simultaneous path planning and (dis-)assembly sequencing and relies only on collision detection, seems to provide evidence on the contrary.

Assembly sequence planning plays an important role in the product design and manufacturing. Assembly planning includes generation, representation and selection of part assembly sequences (**Gao, Qian, Li, & Wang, 2010**). Assembly sequence generation means finding the order and direction of part or sub assembly insertion which will assemble the product in a collision-free manner (**Chakrabarty & Wolter, 1994**). AND/OR graph can be used to represent all the possible assembly sequences (**Wol, 2006**). In this graph, each node represents a subassembly, and each AND'ed group of children for that node represents a set of smaller subassemblies in which the given subassembly can be decomposed. When there are multiple ways to break an assembly or a subassembly, there are multiple sets of children OR'd together. For the usual case of two-handed assembly sequencing, each node in the AND/OR disassembly graph will have its AND'ed children in pairs. **Figure 1** shows an example of such disassembly graph. The AND/OR structure captures the necessity of recursively disassembling all of the subassemblies resulting from a partitioning in order to completely disassemble the parent assembly (**Romney, 1997**).

DEGREES OF FREEDOM CONCEPT (DOFs)

The motions of the subassemblies can be classified in several important ways. (**Romney *et al***) (**Romney, Godard, Goldwwasser, &Ramkumar, 1995**) classified these motions into local and global. Where:

a. Local motion: is motion over an infinitesimal distance. Although such motions are never used in a final assembly sequence, the concept of local motion is important because it gives rise to that of local freedom. A subassembly is said to be locally free if it can move an infinitesimal distance without colliding with other parts; this is a necessary condition for the subassembly to be removed.
b. Global or extended motion: is motion over an arbitrary long distance. A subassembly is said to be globally free if it can removed to infinity. A single-step translation is a type of extended motion in which the subassembly moves to or from infinity in a single, straight line without rotating.

As a fundamental concept, degrees of freedom (DOFs) are the set of independent displacements and/or rotations that specify completely the displaced or deformed position and orientation of the body or system. Therefore, any free object can have at most three translational degrees of freedom and three rotational degrees of freedom. **Figure 2** shows these degrees of freedom. Translation is the ability of an object to move along a path without rotating while rotation is angular motion about some axis.

Basically, translational DOFs exist only along the local coordinate directions; thus only the coordinate directions were tested for translational motion. Similarly, rotations would occur only about axes that coincide with the local coordinate axes.

In geometric realization, (*Ma*) is assumed as the space of all possible motion parameters. *(Mp)* is a subspace in *(Ma)* that represents all permissible motion parameters.
Therefore

$$Mp \subseteq Ma \tag{1}$$

Since free object has no mating surfaces. Thus, for such object (*Mp = Ma)*. A mating surface element prevents the object from moving along a certain well defined set of directions which is expressed in terms of motion spaces as (*Ma)*. The subspace over which motion is disallowed is referred to by (*Md).* Therefore, each additional mating surface element reduces the space (*Mp)* and expands the space (*Md)* such that the following relations are satisfied:

$$Ma = Mp\ U\ Md \tag{2}$$

Therefore

$$Mp \cap Md = \phi \tag{3}$$

Depending on the above bases, *(Mp)* is considered as the solution space. However, motion spaces can be illustrated on a Venn diagram as in **Figure 3**.

TRANSLATIONAL DEGREES OF FREEDOM

When the geometry of contact between an assembly (A) and another assembly (B) is given, the directions along which (A) possesses instantaneous translational motion relative to (B) can be found. Hence, a representation of the space of all possible directions for translational motion can be constructed. Furthermore, the effect of each mating element on these directions can be considered next.

Figure 4 shows a single mating (contact) face between object (A) and object (B). The effect of this mating face on the space of allowed directions for translation is shown in **Figure 5**. The sphere in this figure is generated by constructing a unit vector ($\vec{d}$) with its tail at the center of the sphere and its head at a point (*p*) on its surface. It can be referred to this vector by referring to the point *(p)* on the sphere. In the same way, any arbitrary direction vector can be mapped onto a point on this unit sphere. The sphere generated is called the direction sphere. The hemisphere that is shaded (*Md*) is an open set of points. All directions that lie in this hemisphere are translation directions that are disallowed by contact face. The complement of this set of directions is the set of allowed directions (*Mp*). For multiple contact faces, an upper position of the restraint hemispheres produced by each face of contact produces a set of disallowed directions (**Mattikalli, Khosla, and Xu, 1990**). For example, the object (A) in **Figure 6** has four mating faces (right, left, down, and back) so its motion is only possible at the arc (ab) in **Figure 7** due to the constraints exist.

Rotational Degrees of Freedom

When the geometry of contact between an object (A) and another object (B) is given, axes about which (A) possesses instantaneous rotational motion relative to (B) can be found. Doubtless, the space of rotational motion is divided to allow and disallow portions depending on the contact surfaces exist.

Obviously, the previous argument is applicable for polygonal planar objects, general planar objects and three dimensional polyhedral objects. But, only DOFs for polygonal planner object will be explained in this work in order to reduce the scope of the study.

ROTATIONAL DOFs FOR POLYGONAL PLANER OBJECTS

The best explanation of rotational DOFs for polygonal planer objects is given by Mattikalli and Khosla (**Mattikalli & Khosla, 1992&1991**). Their explanation is as follows; Consider a polygonal object (A) in the (XY) plane which has mating constraints as shown in **Figure 8 (a)**. Consider an axis of rotation ($\vec{r}$, $\vec{p}$) where ($\vec{r}$) is the direction vector and *(* $\vec{p}$ *)* the position vector of a point on the axis. For planar objects, *(* $\vec{r}$ *)* can be parallel to either the (+Z) or (-Z) coordinate directions. Although the range of *(* $\vec{p}$ *)* could be the space ($\mathscr{R}^3$), for a given ($\vec{r}$) it would suffice if ($\vec{p}$) is restricted to a plane that is perpendicular to ($\vec{r}$). Thus we observe that two planes [one for each direction of *(* $\vec{r}$ *)*] are a sufficient (geometric) representation of the space of all possible rotational axes for planar objects. The plane corresponding to the +Z coordinate direction is referred to as (M^+_a). *(*M^-_a*)* is the corresponding space for rotational axes in the (-Z) coordinate direction with

$$M_a = M^+_a \cup M^-_a \tag{4}$$

Consider the mating element on (A) represented by edge *(*E_a*)* with inward normal *(* $\vec{f}$ *)* as shown in **Figure 8 (b)**. Thus, the influence of *(*E_a*)* on allowed axes of rotation can be obtained by identifying region *(*M_d*)* within *(*M_a*)*.

To find *(*M_d*)* due to *(*E_a*)* refer to **Figure 8 (b)**. Let *(t)* be an arbitrary point on *(Ea)*. A rotation about an arbitrary point $\wp$ along direction $\vec{r}$ (the +Z direction in this case) is disallowed if

$$(\bar{r} \times \overline{\wp t}) \,.\, \bar{f} < 0 \qquad\qquad \forall t \in E_a \tag{5}$$

Which is the scalar triple product of the vectors *(*$\bar{r}$ *,* $\overline{\wp t}$ and $\bar{f}$ *)*. Points in the right half space of line (ℓ) [shown shaded in **Figure 8 (c)**] represent axes disallowed by the mating element E_a. Other mating edges impose similar restrictions on permissible axes of rotation. A superposition of all these subspaces produces the subspace of disallowed axes (M^+_d). The complement of this subspace with respect to (M^+_a) gives (M^+_p*)*. The solution to simultaneous linear inequalities can be considered to be one of intersection of half planes. An important property of the intersection of half planes is that the result is always a convex polygon (**Mattikalli & Khosla, 1992**).

Figure 8 (d) shows the result of superposing the constraints due to individual mating edges. It shows a polygon whose edges $E_1; \ldots ; E_4$ are mating edges. Lines $\ell_1, \ldots, \ell_4$ define the constraints on rotation that is imposed by each of the edges. For each ℓ the half plane shown hatched is the disallowed region. The advantage of using a geometric realization over an algebraic one is to be able to predict as to which constraints are redundant. The regions constrained by edges E_2 and E_3 are contained within those imposed by E_1 and E_4 rendering them redundant. In what follows, it will make use of the fact that some constraints are redundant and need not be accounted for to begin with. It will be identifying to those constraints that define the boundary of the permissible subspace as (M^+_p); the remaining constraints are ignored (**Mattikalli & Khosla, 1991**).

THE PROPOSED APPROACH

Assembly sequence generation is the major difficulty in assembly planning process. In this paper, degrees of freedom analysis has been adopted and implemented to create the best disassembly sequence. This sequence will be reversed to generate assembly sequence. Basically, degrees of freedom analysis is a technique used to solve the problem of determining the positions and orientations of a set of rigid objects that satisfy a set of geometric constraints. Reasoning about the geometric bodies is performed symbolically by generating a sequence of actions to satisfy each constraint incrementally, resulting in the reduction of the system's available DOFs.

The proposed technique consists of the following six steps:

1- assembly representation
2- generating AND/OR graph
3- specifying contact surfaces between parts
4- applying degrees of freedom DOFs analysis (translational and rotational degree of freedom analysis)
5- generating disassembly sequence and
6- generating assembly sequence

Figure 9 shows the proposed approach steps that

In order to test and explain the proposed approach, a product is taken as example for implementation. A virtual product has been considered and consists of three parts as shown in **Figure 10**.

The steps of the proposed approach will be applied as follow:-

Step-1 Assembly Representation

a- parts geometry representation

The geometry of the assembled product and its three parts will be represented using one of the geometric modeling techniques (CSG, B rep., wire frame, surface rep.). In this example, wire frame technique has been used to represent the parts as in **Figure 11**.

b- Relations between parts identification:

In assembly planning process, information about relations between parts is very important beside the information about part geometry. Liaison graph can be used to represent relations between parts. Liaison is defined as the physical contact (mating surfaces) between parts. The relation between the three parts of the product in our case is shown in **Figure 12**.

Step-2 Create AND/OR graph

AND/OR graph is used to represent the feasible assembly/disassembly sequences. The AND/OR graph for disassembling the product is shown in **Figure 13**.

Step-3 Specifying contact surfaces between parts

Depending on parts geometry and parts relations, the contact surfaces between parts can be represented as motions parameters *(Ma)* all possible motion parameters, *(Mp)* as permissible motion parameters, and *(Md)* as disallowed motion parameters) for each part. **Figure 14** illustrate contact surfaces between product parts.

Step-4 Applying DOFs analysis

The most crucial step in the proposed approach is DOFs analysis. However, as it has been discussed previously, degrees of freedom DOFs analysis should involves two types (translational and rotational degree of freedom analysis) as follows;

a- translational degree of freedom analysis

I. Create direction sphere for each part and net direction sphere

In order to generate direction sphere, a sphere of unit radius, as shown in **Figure 15(a)** should be considered. Thus, a unit vector ($\vec{d}$) with its tail at the center of the sphere and its head at a point (*p*) on the surface of the sphere should be constructed. It can refer to this vector by referring to the point (*p*)*on* the sphere. In the same way, any arbitrary direction vector can be mapped onto a point on the unit sphere. This mapping of direction vectors to points on a sphere can be used to represent all directions in the three dimensional space. The points on the surface of the sphere correspond to (*Ma)*. This sphere is the direction sphere.

Depending on contact surfaces between parts, translational degree of freedom for the parts A, B, & C has been obtained. **Figure 15 (a, b and c)** illustrates translational degree of freedom for these parts respectively. Taking the union of disallowed regions due to mating surfaces the net translational direction sphere shown in **Fig 15 (d)** is obtained.

II. Conclude part with free translational motion.

Now can notice that contact constraints permit translational motion between part C & A toward (Z^+) direction. That means part C should disassemble first. Then calculate the translational constraints to the parts A& B and find the net translational constraints between them as shown in **Figure 15 (a, b, c and d)**. The contact constraints permit translational motion between parts B & A toward (Z^+) direction. That means the part B can be disassembled second and the part A become free (float).

b- Rotational degrees of freedom analysis

I. Calculation of rotational constraints due to each mating face.

Figure 16 shows the direction sphere for the contact surfaces. Since the net DOFs will exist on a tangent plane whose normal is the (+Z) direction, therefore will focus on that plan referred to as (T_P) (where *P* is contact surfaces). The solution method consists of determining the constraints on (T_P) due to segment (ab, bc, cd, de) so as to cover the entire surface. Tangent planes on all points on the great circle ABCD will be taken. The effects of segment abcde along all points on the great circle ABCD are analyzed. If these solutions are superimposed, then the solution will be identical to analyzing the effect of the entire surfaces.

At the single point A on the surface of direction sphere, the boundary of constraints due to *f1* at A can be easily verified by projecting the normal at *f1* on to the tangent plan at A. Similar projections on the tangent plans at B, C, &D can be used to verify the other constraint boundaries. Note that we traverse the great circle from A to B and so on back to A, the boundary of the constraints region rotates about the point O from one maximum position at A to the other at C and then back. Similar analysis is carried out to find the constraints due to *f2*, *f3*, *f4,* as well as *f5*. The union of the disallowed regions at the four points gives the net constraints at any point on the great circle ABCD due to surface *P*. This is shown in figure 17 (a, b, c, & d).

II. Conclude part with free motion direction.

The analysis for rotational constraints between parts shown in **Figure 17** shows permit for motion between parts C &A (since part B is geometrically constraint). This mean part C should disassemble first. Then examine the constraints again to find that part B is free to motion now and can be disassembled. Then part A becomes free part.

Step-5 Generate disassembly sequence

Evidently, the translational degree of freedom and rotational degree of freedom generate the same disassembly sequence. This sequence is as follows;

Disassembly sequence	C – B – A

Step-6 Generate assembly sequence

Assembly sequence can be obtained by reversing disassembly sequence. Therefore it will be as follows;

Assembly sequence	A – B –C

CONCLUSION AND RECOMMENDATION

Assembly sequence generation is one of the major difficulties in assembly planning process. Assembly sequence generation means finding the order and direction of part or subassembly which will assemble the product in a collision-free manner. In this paper an approach of six steps has been proposed and implemented. This approach depends mainly on generating assembly sequence depending on reversing disassembly sequence. Disassembly sequence is created by means of degrees of freedom analysis. Hence this is the main contribution of this work. Implementing the proposed approach on a virtual product shows its ability to generate assembly sequence for this product. Two future works are recommended the first is to expand the application to be includes general planar objects and three dimensional polyhedral objects while the second is computerizing this approach.

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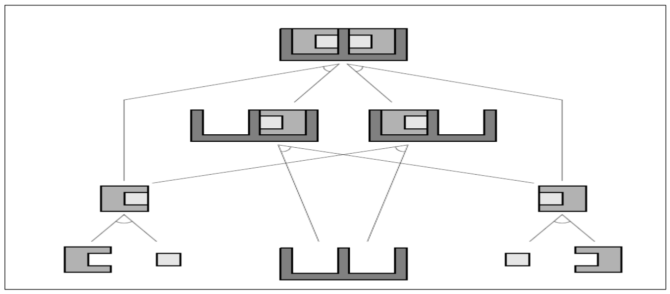

Figure 1 Example of an AND/OR disassembly graph [**Romney, 1997**].

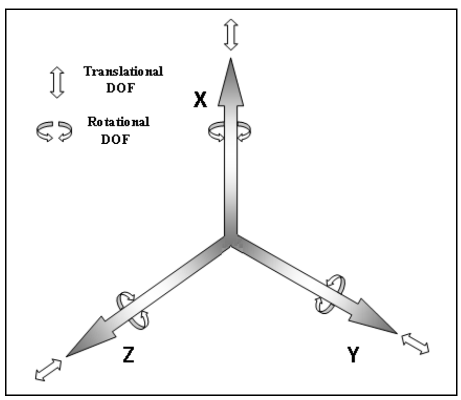

Figure 2 Degrees of freedom [the researcher]

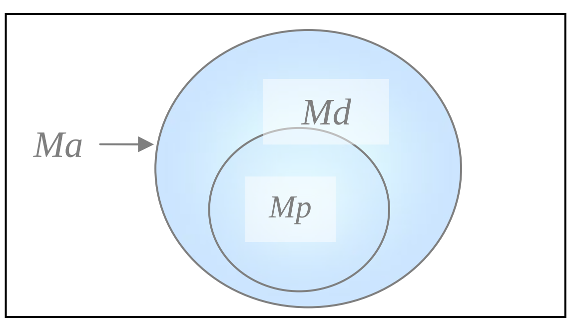

Figure 3 Venn diagram illustrates motion spaces

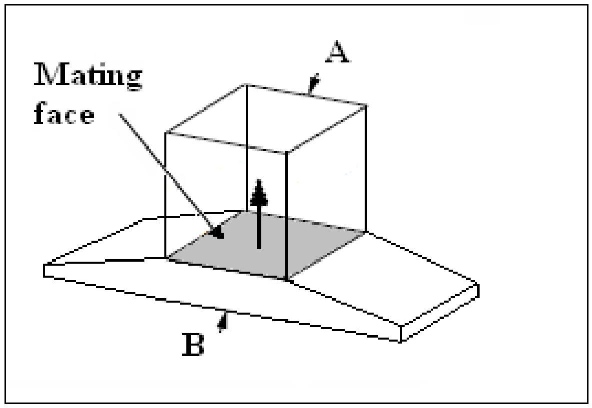

Figure 4 An object with single mating face

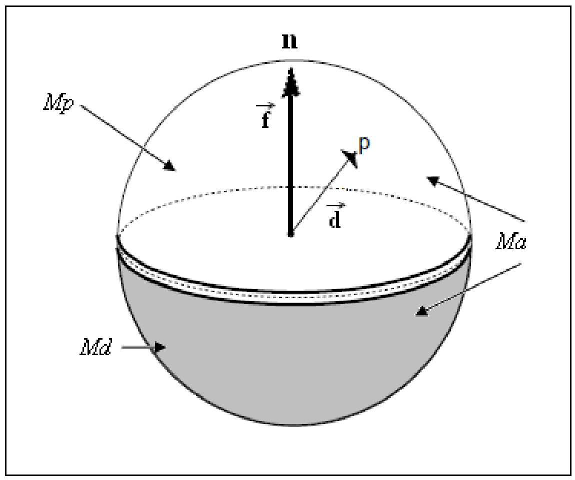

Figure 5 The direction sphere

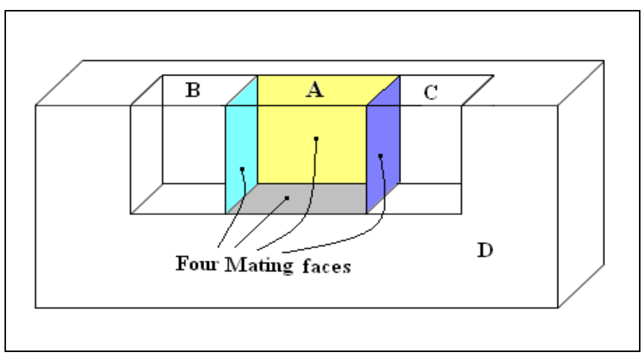

Figure 6 Object A has four mating faces

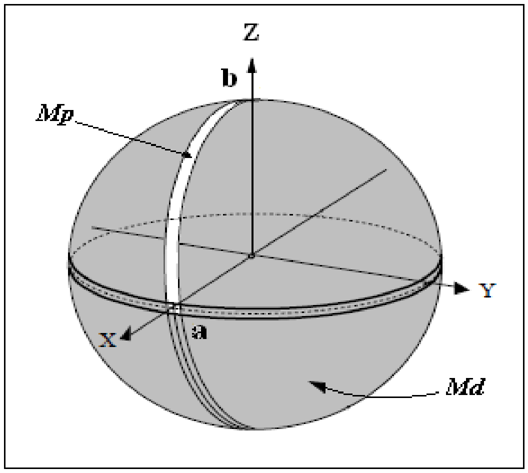

Figure 7 Net effects of the four mating faces. (Mp) portion shows DOFs [the arc (ab) only]

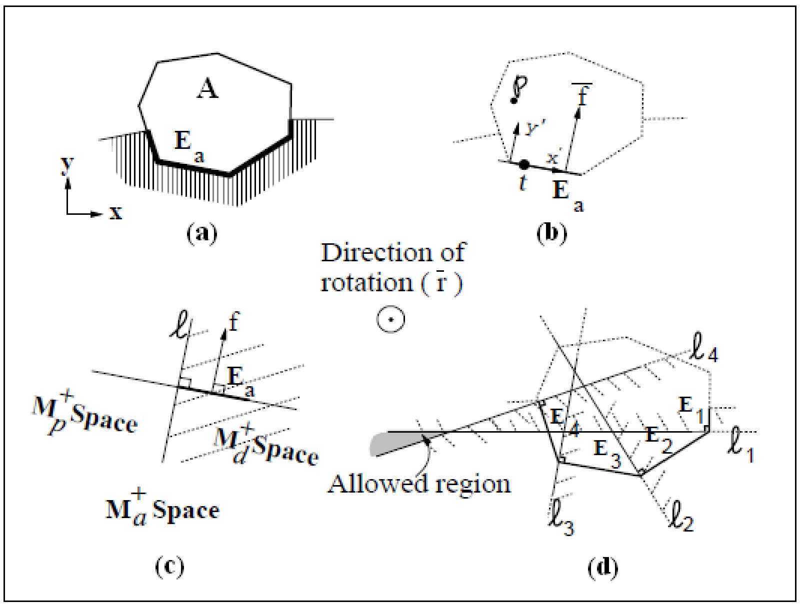

Figure 8 Rotational restraints on planar objects (a) A polygon with its mating conditions, (b) A single mating element Ea, (c) Anticlockwise rotation is disallowed about axes that lie in the shaded region, (d) Superposition of individual constraints due to contact over edges [**Mattikalli & Khosla, 1992**]

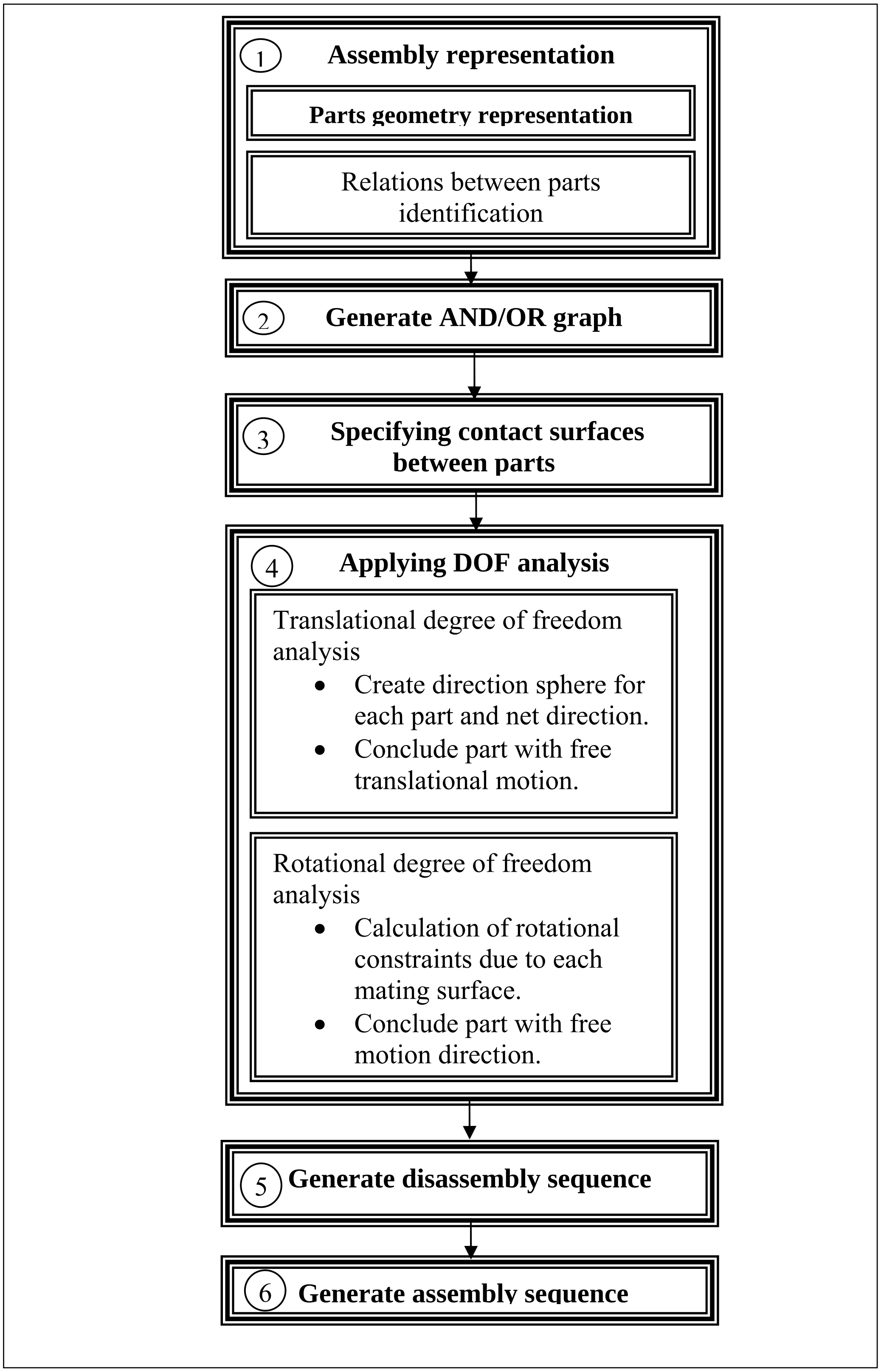

Figure 9 The proposed approach Block diagram

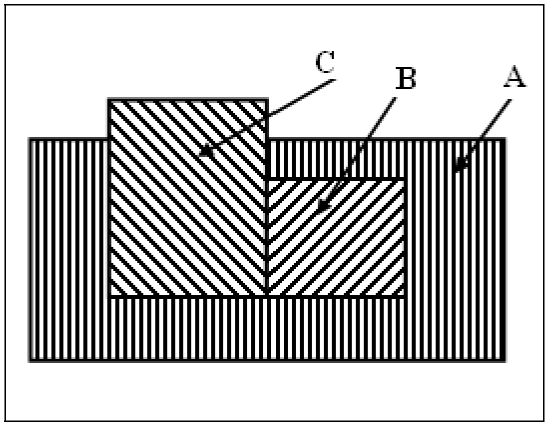

Figure 10 Cross section in the virtual product shows its parts

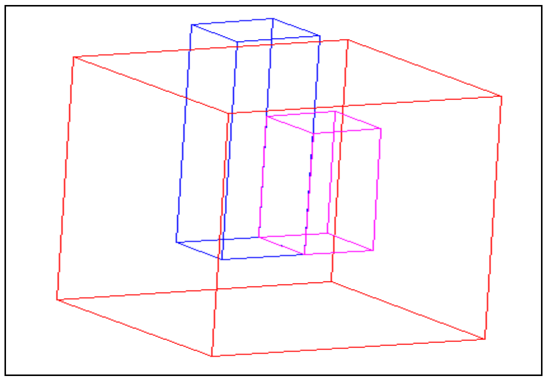

Figure 11 Product representations in wire frame representation

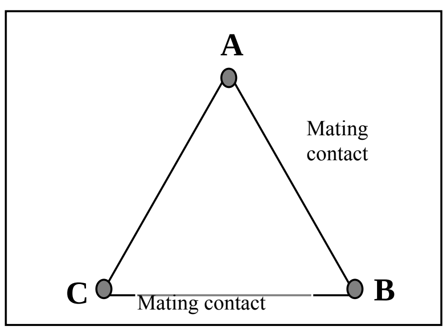

Figure 12 Mating relations between the three parts of the product

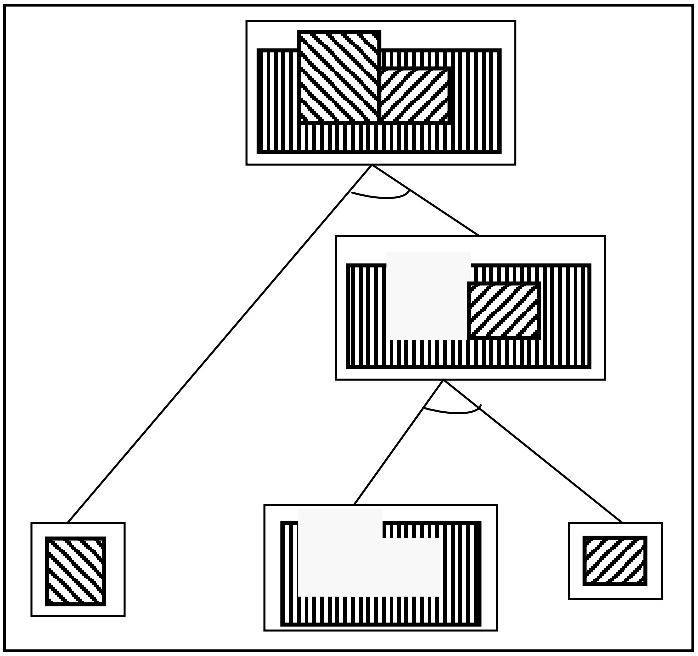

Figure 13 AND/OR graph representation

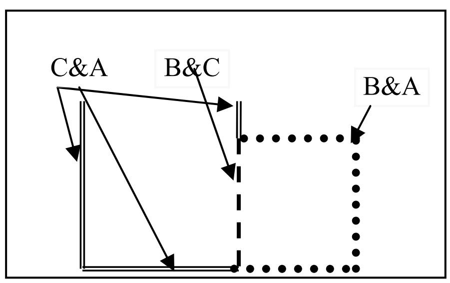

Figure 14 Contact surfaces between parts

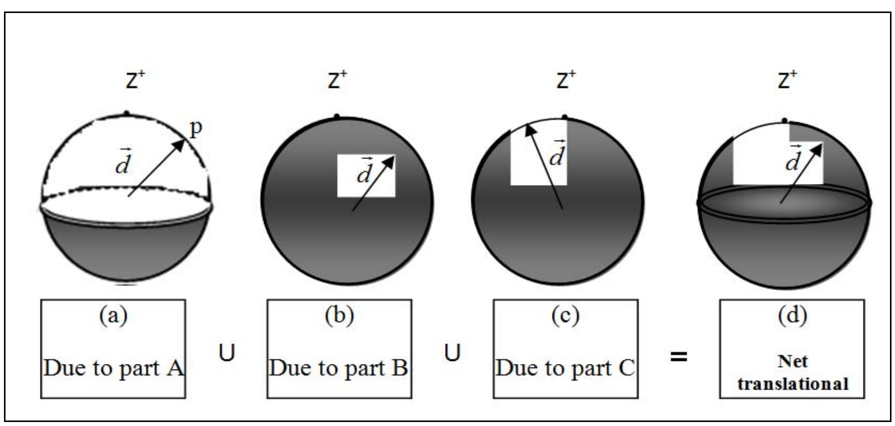

Figure 15 Translation constraints for the parts

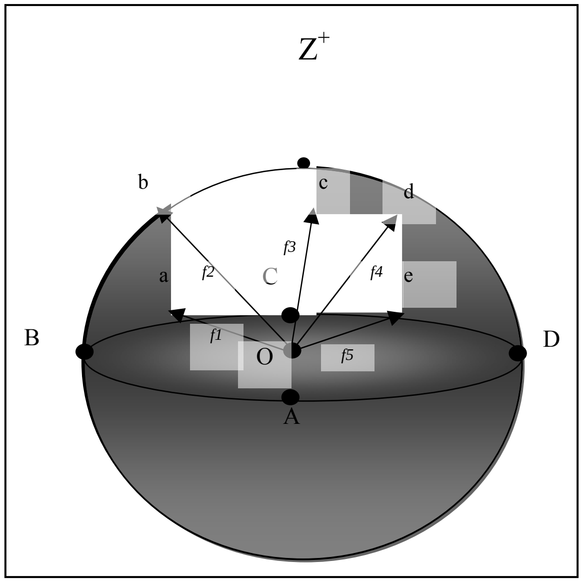

Figure 16 Direction sphere for contact surfaces

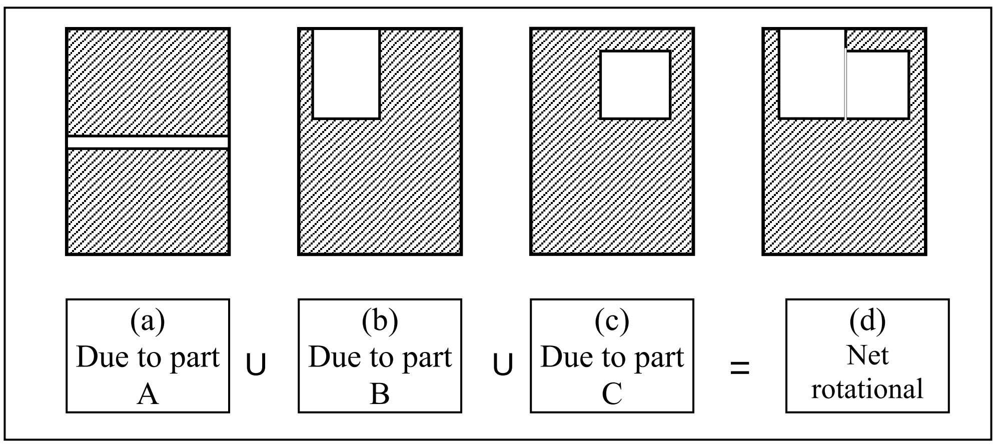

Figure 17 Rotational constraints for the parts