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Using maximum distances from unit line to the Embedding vectors to estimate the delay time with an application

Abstract

In this research a new method was proposed to estimate the delay time. Which can be used to determine the relationship between the input and output of the series, this means that after each time interval, the input is affected by the output.

The advantages of this method are the simplicity and the basic calculations in linear and nonlinear transformations. The proposed technique was found by examining the greatest expansion from the unit line in the impacted space.

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2008/ 2/19 :

2007/ 12/ 28:

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Ljung (1999)

(Astrom & Wittenmark,1997)

Tong(1996)

Delay Time

Effect Lag Cause
 ()
(Pollard,1981) ()

Input Forcing Function

(2002)

==:

Cross-correlation Function

.1

(u_t)

(y_t)

Box)

:-

(and Jenkins ,1976

$$\hat{\rho}_{uy}(k) = \frac{\sum_{t=1}^{n-k} (y_{t+k} - \bar{y})(u_t - \bar{u})}{\sqrt{\sum_{t=1}^n (y_t - \bar{y})^2 \sum_{t=1}^n (u_t - \bar{u})^2}} \quad , k=0,1,2,\dots,n-1 \quad \dots (1)$$

(\bar{u}), (\bar{y}) :

$k=0,1,2,\dots,L$

$\hat{\rho}_{uy}(k)$

$(\frac{n}{2})$

L

$$\hat{d} = \max_{k=0,1,2,\dots} \left| \hat{\rho}_{uy}(k) \right| \quad \dots \quad (2)$$

Impulse Response Function .2

Linear Combination

$$y_t = \sum_{i=0}^n h_i u_{t-i} \quad \dots \quad (3)$$

$$u_{t-i} \quad : \quad h_i \quad :$$

$$h_d \quad h_0 = h_1 = h_2 = \dots = h_{d-1} = 0$$

h_i Stable

$$H_0 : h_0 = h_1 = h_2 = \dots = h_{d-1} = 0 \quad , \quad h_d \neq 0$$

$$\hat{h}_i$$

i

(Makridakis et.al., 1983)

$$\hat{d}$$

Thanoon & Ibrahim (1993) .3

.4

Autoregressive with Exogenous Variables (ARX)

\hat{d} (Ljung,1995,CH.3-P.74)

ARX

:

ARX

$$) \quad B(q) \quad A(q) \quad (na = nb = 2) \quad (2)$$

10 1

$nb \quad na$

Loss Function

$$\hat{d}$$

: (Ljung,2004)

$$V = \frac{1}{2} \sum_{t=1}^n e_t^2 \quad \dots \quad (4)$$

e_t : n : V :

$$\hat{d} = \min_{k=0,1,2,\dots,L} V_k \quad : \quad \hat{d}$$

State Space Models .5

$$\begin{array}{l} X_t \quad X_{t+1} \quad (t+1) \\ \text{State Equation} \quad U_t \\ U_t \quad X_t \quad Y_t \\ \text{Observation Equation} \end{array}$$

Single)

-: SISO (Input Single Output

$$X_{t+1} = AX_t + BU_t \quad \dots \quad (5)$$

$$Y_t = CX_t + DU_t \quad \dots \quad (6)$$

-:

$(n \times n)$: A

$(n \times n)$: B

$(1 \times n)$: C

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(Nelles,2001)

State Space (2002)

(Pollard,1981)

\hat{d}
:
==

Time delay Coordinates method

State Space

Delay Component

(Takens, 1981) (Packard et al., 1980)

$\{x_1, x_2, x_3, \dots, x_N\}$ Time Series

:

$$Y_t = \{X_t, X_{t+d}, X_{t+2d}, \dots, X_{t+(m-1)d}\} \dots (7)$$

$$t = 1, 2, 3, \dots, N - (m - 1)d :$$

$$: d, m :$$

d

m, d

:

Autocorrelation Function Method

.1

:

$$\rho(d) = \frac{\sum_{n=1}^N (X_{n+d} - \bar{X})(X_n - \bar{X})}{\sum_{n=1}^N (X_n - \bar{X})^2} \quad \dots (8)$$

:

$$\bar{X} = \frac{1}{N} \sum_{n=1}^N X_n$$

d

.(Albano et al., 1988) $\rho(d) \leq 0$

Rosenstein et al.,)

.(1994

Mutual Information Method

.2

: (Fraser and Swinney,1986)

$$I(d) = \sum_{n=1}^N P(X_n, X_{n+d}) \log_2 \frac{P(X_n, X_{n+d})}{P(X_n)P(X_{n+d})} \quad \dots (9)$$

$\cdot X_{n+d} \quad X_n \quad : P(X_n, X_{n+d}) :$

$\cdot X_n \quad : P(X_n)$

$X_{n+d} \quad X_n \quad : I(d)$

d

$I(d)$

Histogram

Average Displacement Method .3

(Rosenstein et al.,1993)

Quantifies reconstruction expansion from identity line
embedding space

$$\langle S_m(d) \rangle = \frac{1}{M} \sum_{i=1}^M \sqrt{\sum_{j=1}^{m-1} [X_{i+jd} - X_i]^2} \quad \dots (10)$$

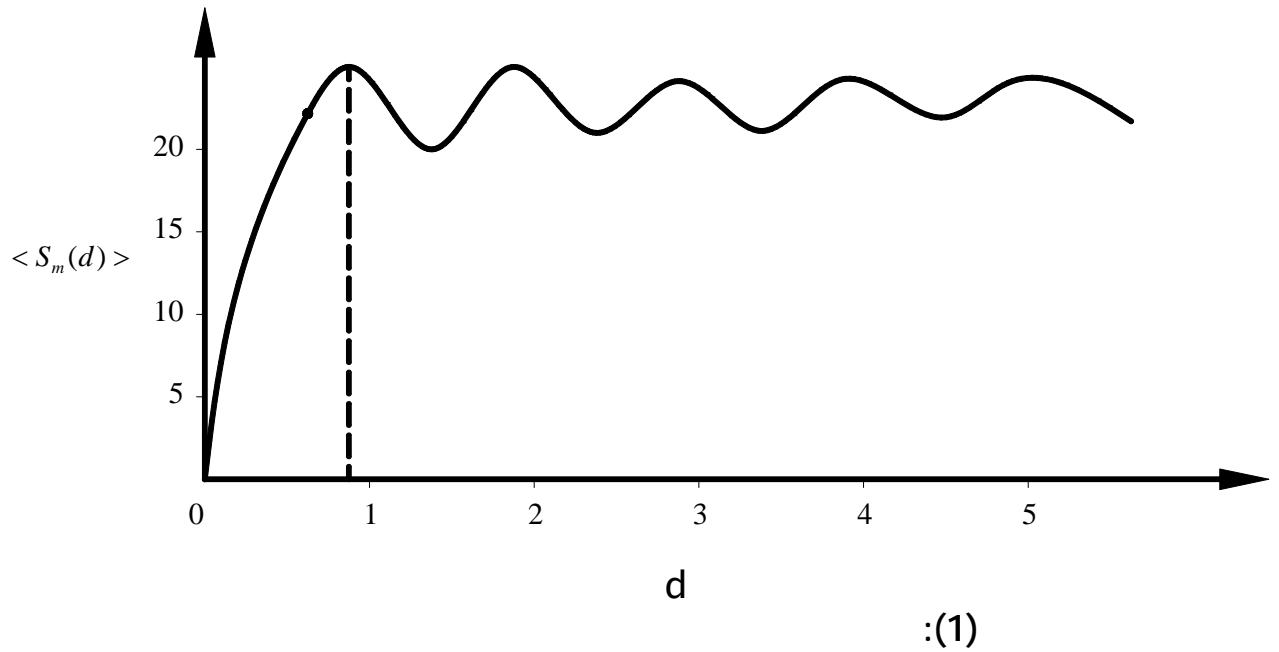
: M $\langle S_m(d) \rangle$:: m

%40

(Rosenstein et al.,1993)

:

 $\langle S_m(d) \rangle$



(

:

:

.1

$$Y_t^d = \{x_t, x_{t+d}, x_{t+2d}, \dots, x_{t+(m-1)d}\}$$

$$\{x_1, x_2, x_3, \dots, x_N\} :$$

$$.t = 1, 2, 3, \dots, N - (m - 1)d :$$

:

.2

$$L(d) = \frac{1}{M} \sum_{i=1}^M \max_{j=1, 2, 3, \dots, M} \{ \|Y_i^d - Y_j^0\| \}$$

$$\begin{matrix}
 Y_i^d & Y_j^0 & Y_j^0 & Y_i^d & & : & \| \| \\
 m=2 & & & & & : & M & d = 0 \\
 & & & & & & & :
 \end{matrix}$$

$$L(d) = \frac{1}{M} \sum_{i=1}^M \max \left\{ \sqrt{(x_i - x_1)^2 + (x_{i+d} - x_1)^2}, \sqrt{(x_i - x_2)^2 + (x_{i+d} - x_2)^2}, \dots \right\}$$

.3

$$\begin{matrix}
 d & L(d) \\
 d & L(d) \\
 & \vdots \\
 & \min \{L(d)\}_{d=1}
 \end{matrix}$$

Linear Models

(n=2000)

(u_t)

random gaussian signals "rgs"

randn

(e_t ~ N(0,25))

MATLAB

: (5)

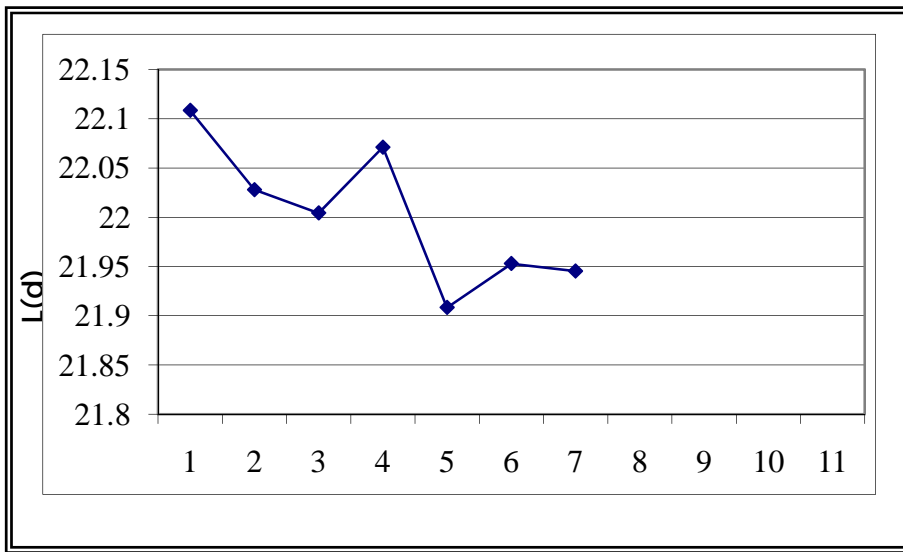
Nonlinear Models

randn

$$y(t) = 0.8 u(t - 5) - 0.6 u(t - 6) + 0.4 u(t - 7) + e(t);$$

(5)

:



d

(5)

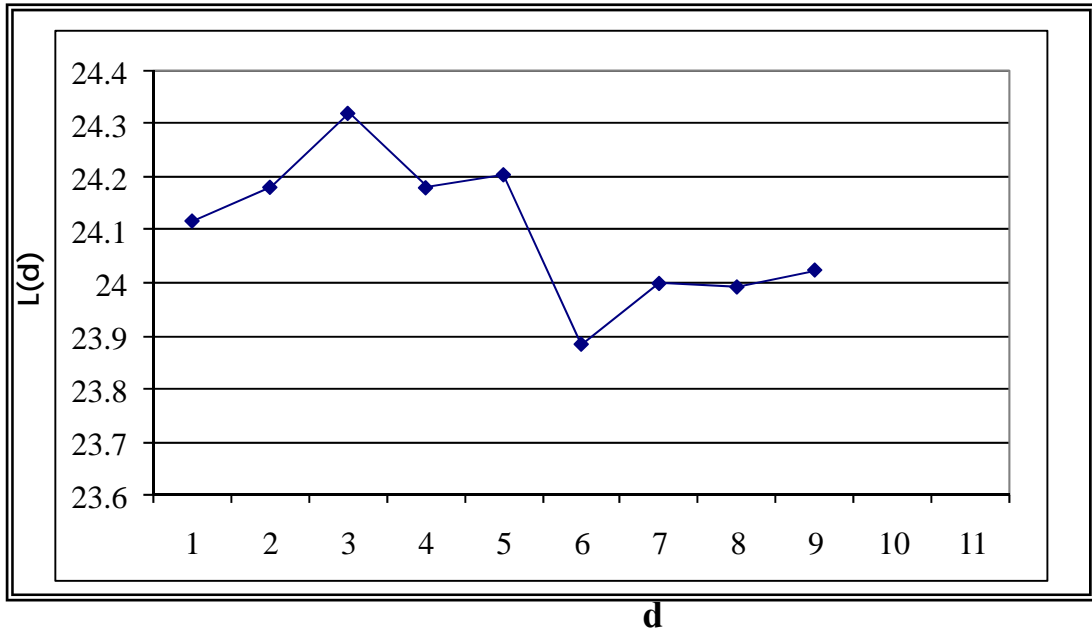
L(d) : (2)

: (6)

$$y(t) = 0.5 u(t - 6) + (1 + 0.3 \exp(-0.1 u(t - 7))) u(t - 7) + e(t);$$

(6)

:



(6)

L(d) : (3)

Thanoon & Ibrahim (1993)

:

1. $W_t = 1.5U_{t-2} + 0.8U_{t-3} + N_t$
2. $W_t = 0.4U_{t-3} + 0.2U_{t-4} - 0.6U_{t-3}U_{t-4} + N_t$
3. $W_t = 0.1U_{t-1}U_{t-2} + N_t$
4. $U_t = 0.2W_{t-3} - 0.1W_{t-2}W_{t-3} + Z_t$

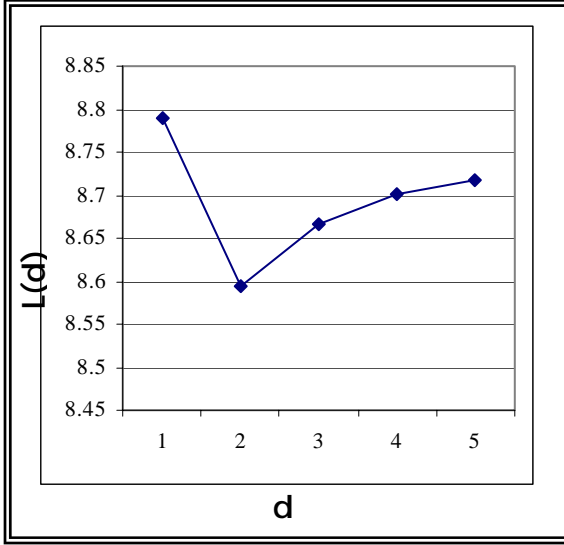
(2)

(Thanoon & Ibrahim (1993)P.P.92)

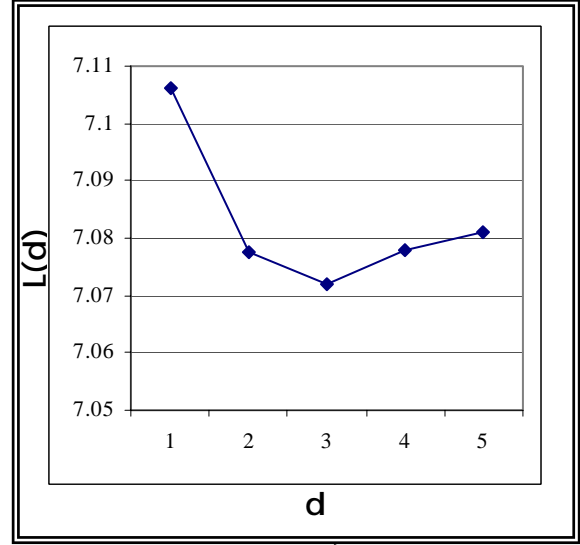
:

2	2	.1
3	3	.2
1	1	.3
2	2	.4

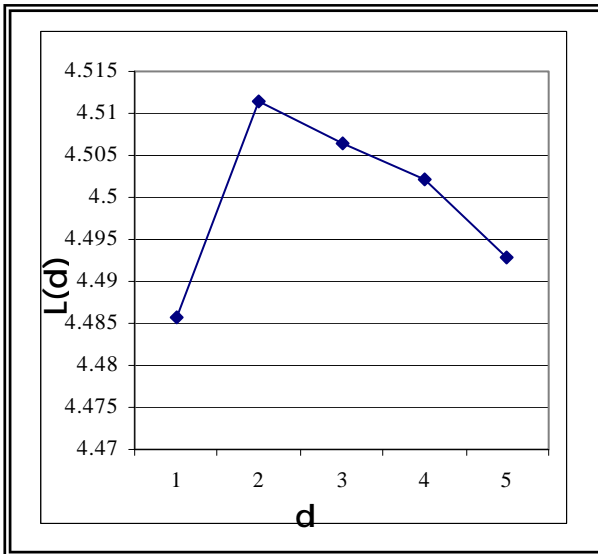
: (4)



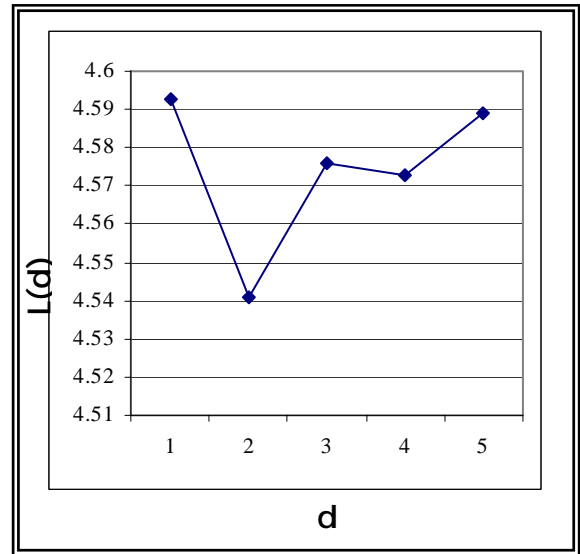
النموذج الأول



النموذج الثاني



النموذج الثالث



النموذج الرابع

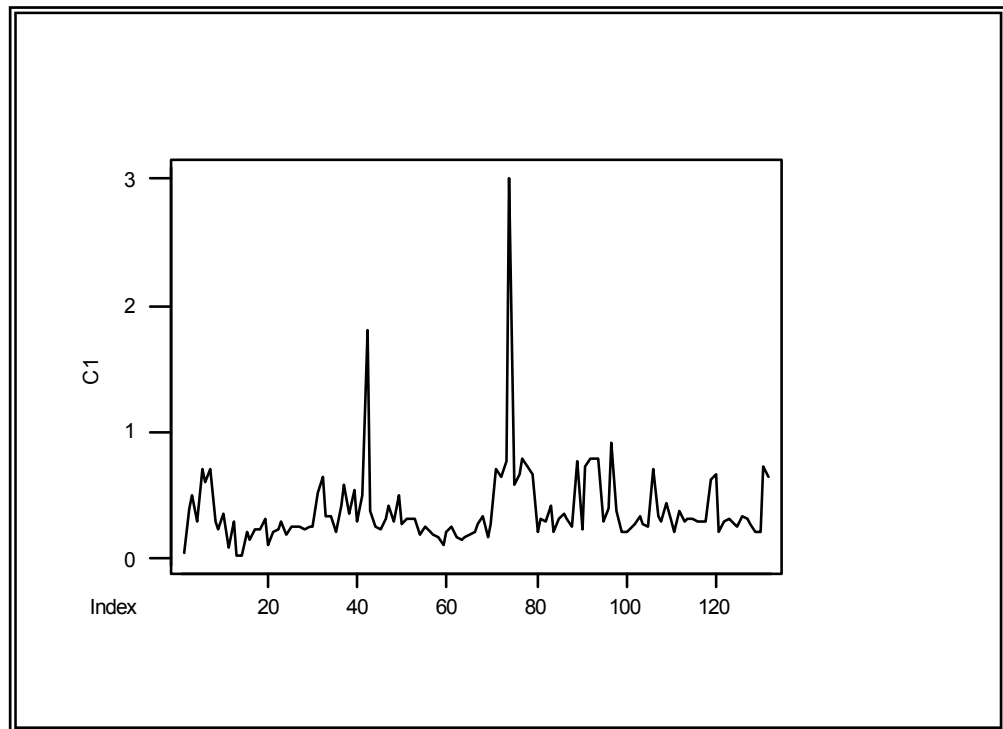
Thanoon & Ibrahim

L(d) : (4)

: (

(2004) (9)

: 132

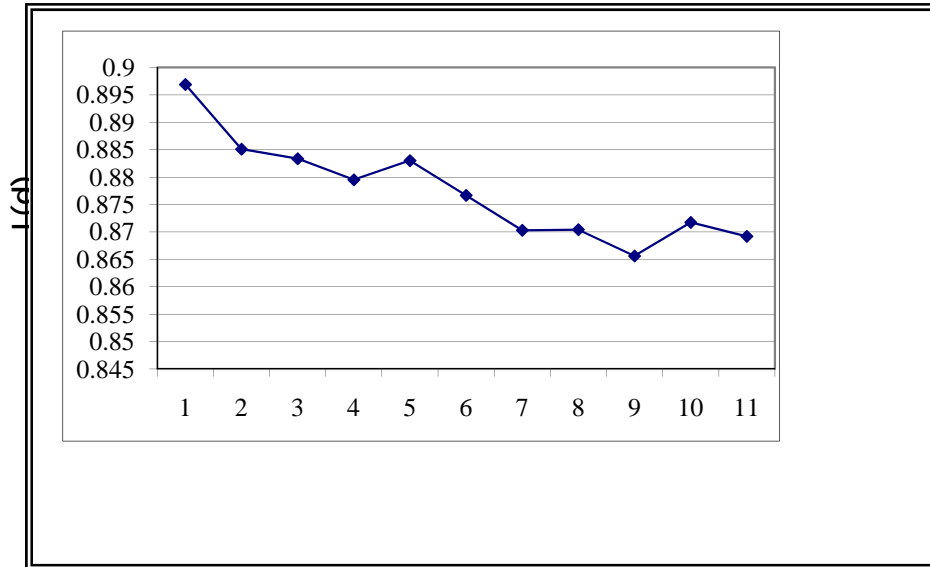


الشكل (5):

(6):

L(d)

(9)



L(d) : (6)

:

-1

L(d)

-2

(5)

(6)

...

(Chaotic Data)

(3)

(4)

-3

(6)

$(m \geq 2)$

m

-4

d

...

$m = 3 \quad m = 2$

$d = 1, 2, \dots, 10$

":(2002)

.1

"

":(2004)

.2

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