**Eng.&Tech.Journal, Vol.27,No.3,2009**

# Simuation Study of the Factors Effect on the Directional Controllability of Vehicles

**Dr. Ahmed N. Al-khazraji***

Received on:4/5/2005
Accepted on:6/6/2006

**Abstract**

This paper present the result of the work carried out the mathematical model of two degree of freedom movement study the influence of cornering stiffness of tires, the distance from the position of the center of gravity to front and rear axel and mass of vehicles.

Theoretical calculations have been carried out the vehicle have known mass and the position of the center of gravity.

Results show that for the state which is the front tires are bias ply tires and the rear tires are radial ply tires is the best because the state gives under steer behavior. Also the state will be the center of gravity nearest from the front axel gives the same behavior and also owing to the existence of center of gravity being nearest to the front axle.

**Keywords:** mechanical engineering, model analysis, simulation, controllability, vehicles

**الخلاصة**

في هذا البحث تم دراسة النموذج الرياضي لدرجتين من حرية الحركة لحركة سيارة تحت تـأثير الجساءة الجانبية للاطارات، موقع مركز ثقل السيارة عن المحاور الامامية والخلفية، كتلة السيارة وتأثير هذه العوامل على الاستقرارية الاتجاهية للمركبة.

اثبتت نتائج الاختبار بأن حالة المركبة التي تكون اطاراتها الامامية شـبكية والخلفيـة شـعاعية، هـي الافضل من حيث السلوكية لانها أعطت سلوكية تحتية under steer وكذلك نتائج كـون مركـز ثقـل السيارة اقرب الى الاطارات الامامية منه الى الخلفية يعطي نفس السلوكية التحتية.

## Introduction

Steady state handling performance is concerned with the directional behavior of a vehicle during a turn under non-time varying conditions. The expression for the steer angle ( $\alpha_s$ ) is:

[$\alpha_s$=L/R +K $V^2$/gR]; where (K) is under steer coefficient & expressed in radians. Depending on the values of the under steer coefficient (K), handling characteristics may be classified into three categories; neutral steer, under steer and over steer. When K=0, a vehicle having this handling property is said to be neutral steer, when a neutral steer, the vehicle follows straight line path at an angle to the original. When K>0 the handling property is said to be under steer a characteristic speed $V_{ch}$ may be identified, it is the speed at which the steer angle required to negotiate a turn is equal to (2L/R)

When K<0, vehicle with this handling property is said to be over steer. For an over steers vehicle, a critical speed Vcr. Can be identified. It is the speed at which the steer angle required to negotiate any turn is zero. For an over

***Mechanical Engineering Department, University of Technology/Baghdad**

https://doi.org/10.30684/etj.27.3.14
2412-0758/University of Technology-Iraq, Baghdad, Iraq

steer vehicle the under steer coefficient (K) has a negative sign. The critical speed also represents the speed above which an over steer vehicle exhibits directional instability.

**$G_a$**=statical gain of directional deviation of CG

**$G_{MS}$**=statical gain of moment on steering wheel (N.m/rad)

**C** αp'=directional stiffness of front, axle of vehicle [kN/rad].

**e** = distance between aerodynamic Pressure center and the center of Gravity [m].

**g** = gravitational acceleration [$m/s^2$]

**$H_P$**, = tractive force on front, rear

**$H_z$** = axle of vehicle respectively [kN]

**$I_s$** reduction ratio

**$J_z$** = polar moment of inertia of center of gravity [$kg.m^2$]

**$K_{us}$** = under steer coefficient [$S^2/m$]

**$L_P$**, = distance between front rear axle

**$L_z$** And center of gravity [m]

**m** = mass of vehicle [kg]

**$M_v$** = moment on the steering wheel [kN.m]

**$M_s$** = return moment of tire [kN.m]

**N** = lateral aerodynamic force[KN]

**$n_k$** = design distance [ mm ]

**$n_s$** = lateral force arm [ mm ]

**$S_p$ , $S_z$** = lateral force on front, rear axle of Vehicle respectively[kN]

**$t_p$ , $t_z$** = distance between front, rear wheels of vehicle respectively [m]

**v** = vehicle velocity [ m/s ]

**$V_{ch}$** = characteristic velocity [ m/s ]

**$V_{cr}$** = critical velocity [ m/s ]

**$Y_{stat}$** = statically centrifugal acceleration [ $m/s^2$ ]

a = deviation angle with longitudinal axis of center of gravity [deg]

***bv**** = steering angle [rad]

***bp*** = front wheels steering angle [rad]

*e* = coiling angle (between longitudinal) axis of the system and stationary axis. $X_o$)

*e*'= coiling velocity [ rad/sec ]

**1. Theoretical part:**

In order to study the control and stability of a vehicle. it is appropriate to use the dynamic model of a vehicle as shown in Fig (1.1) and Fig (1.2), which as shown that the lateral forces on the front axle is ($S_p = S_1 + S_2$) and on the rear axle is ($S_z = S_3 + S_4$). The radial tractive forces are equal to ( $H_p = H_1 + H_2$ ). ( $H_z$= H3 + H4) At this point the rolling resistance, together with wheel return moment is neglected.

$-m\acute{v}\cos\alpha + mv(\acute{\varepsilon}+\acute{\alpha})\sin\alpha - S_p\sin\beta_p + H_p\cos\beta_p + H_z - R\alpha=0$ ……(1)

$-m\acute{v}\sin\alpha + mv(\acute{\varepsilon}+\acute{\alpha})\cos\alpha + S_p\cos\beta_p + S_z H_p\sin\beta_p + N = 0$ ………(2)

$J_z\varepsilon + S_pL_p\cos\beta_p - S_zL_z + H_pL_p\sin\beta_p + N.e = 0$ ……(3)

Making these equations linear i.e, by substitution yield.

The lateral force on the axles can be computed as follows:

$Sp = \acute{C}\ \alpha p.\alpha_p$ , $S_z = C_{az},\ ,\ \alpha$ …(4)

Where ($\acute{C}\ \alpha p$ ) is directional stiffness of the tires of the front axles that is the sum of directional stiffness of the right and left tires

($C_{\alpha z}$ ) is the directional stiffness of the tires of the rear axle. The angles of directional deviation of the front axle ( $\alpha_p$).

And rear axle ($\alpha_z$) determine from Fig(2).

The speed of front wheel ( $V_p$ ) which is calculated by obtaining the directional sum of the speed of gravity ( v ) and the rotation speed of front axle center relative to center of gravity { $L_p\ \acute{\varepsilon}$ }

$\sin\ \alpha \sim 0$

$\cos\ \alpha \sim 1$

The speed of rear wheels ($V_z$) is the sum of speed { $L_z\ \acute{\varepsilon}$ and v }. The angle of the wheel directional deviation as defined as being the angle between the wheel longitudinal level and direction of wheel movement therefore the following equation can be applied to the small angles:

$\alpha_p = -\alpha - L_p/V\ \acute{\varepsilon} + \beta_p$ ………(5)

$\alpha_z = -\alpha + L_2/V - \acute{\varepsilon}$ …………(6)

The vehicle static control is determined by the response of its system to the rotation of the steering wheel where the system moves at constant speed in a circular path with constant diameter (R = constant).

in the other words : ($e = a = 0$ ).
the equation of motion will take following form after air effects are neglected.

$1/V[mv^2 - (C_{\alpha z} L_z - \acute{C}_{\alpha p} L_p)]\ \acute{\varepsilon}_{stat} + (\acute{C}_{\alpha p} - C_{\alpha z})\ \alpha_{stat} = \acute{C}\ ap\ \beta v^*_{stat}$……(6)

$1/V[(\acute{C}_{\alpha p} L^2_{\ p} - C_{\alpha z}\ L^2_{\ p})]\ \varepsilon_{stat} - (C_{\alpha z}\ L_z + \acute{C}_{\alpha p}\ L_p)\ \alpha_{stat} = \acute{C}\ \beta v^*_{stat}$…(7)

The centrifugal acceleration when moving at constant speed in circle

$\ddot{Y}_{stat} = V^2/R = V.\ \acute{\varepsilon}_{stat}$ ……..… (8)

The wheel stastical response to the rotation of the steering wheel ($B_v^{\alpha}$ ) has three responses { $\ddot{Y}_{stat}$, $\alpha_{stat}$, $\acute{\varepsilon}_{stat}$ } called the ratio of statical response and steering wheel rotation angle which produces at certain speed relative to static capacity .

Using equations (6), (7) we can easily calculate the statical gain of coiling velocity

$[\acute{\varepsilon}/\beta v^*]_{stat} = V/(L+K_{us}\ V2)$……(9)

Where ($K_{us}$) under-steer coefficient of control
$K_{us}=m[C_{\alpha z}L_z - \acute{C}_{\alpha p}L_p/\ \acute{C}_{\alpha p}C_{az}\ L]$…(10)

The vehicle behavior will have understeer if ($k_{us}$>0) and in a oversteer if ($k_{us}$<O).
The other response which results from solving the above equations the statical gain of directional deviation of gravity

$[a/\beta_v^*]_{stat} = C_{\alpha z}L_zL-$
$L_pV^2\ /\ C_{\alpha z}L\ (L+KuV2)$
$[\acute{\varepsilon}/\beta_v^*]_{stat}\ [L_z\ mL_p/V\ C_{\alpha z}L]$ ……(11)

The statical gain of lateral acceleration is the magnitude obtained from statical gain of coiling velocity.

$[\ddot{y}/\beta_v^*] = v[\acute{\varepsilon}\ /\ \beta_v^*]$ ………(12)

The last statical control response derived for linear model of the vehicle is the statical gain of moment on steering wheel.

$[M_v/\beta_v^*] = (mL_z(n_k+n_s)/L_sL^2).$
$(v^2/1 + [v/vch])$ ………(13)

Where ($V_{ch}$) is the characteristic velocity defined from the following equation:

$V_{ch} = \sqrt{L/K_{us}}$ ………(14)

The critical velocity ( $V_{ct}$ ) can be expressed as following:

$V_{cr} = \sqrt{L/-K_{us}}$ ……(15)

**2. Experimental Part:**

One of the most important parameters to be covered in the experimental work covers the vehicle behavior and its response through studying most of the factors affecting this behavior the most important factors to be taken into consideration the directional stiffness of the tires and the position of center of gravity of vehicle as well as the mass of the vehicle.

All these factors are studied to find the vehicles statical response that is the effect these factors on the statical gain of coiling velocity and the statical gain of the directional deviation of the center of gravity and statical gain of lateral acceleration and steering wheel moment, the study covers the following cases .

**3. Results and Discussions:**

The effect of tire directional stiffness on vehicle stability:

Parameters of vehicle used:

To fully understand this effect, two types of tires radial and bias ply tires were used. Measurements were made of different tires in terms of directional stiffness. In the first case radial tire with directional stiffness of (30000 N/rad) and another bias with stiffness (23075 N/rad) were used.

| Symbol | Unit | value |
|---|---|---|
| m | kg | 1500 |
| L | m | 2.5 |
| $L_p$ | m | 1.25 |
| $I_s$ | / | 17 |
| $n_k+n_s$ | m | 0.4 |

Four experiments were made. In the first cases the front tires were of bias ply type and the rear tires were radial type, this experiment shows the vehicle was in understeer which moving in a curve with constant radius. The vehicle had characteristic velocity of (25.819 m/s).

In the second case was in constant the first case the front tires were radial rear of bias ply, the vehicle behavior changed from under steer to over steer. This affected the vehicle and made it unstable especially at speed higher than the critical which was

( $V_{ch}$=25.819 m/s ).

In the third & fourth case were same construction, in the third case were all radial and in the fourth case they were bias ply.

The result of these two cases shows that the vehicle was in neutral steer. Fig (3),(4),(5),(6) show the response of the vehicle,

Fig (3) shows die relation between statical gain of coiling velocity and velocity. In the first case the statical gain of coiling velocity was increasing with velocity and when the vehicle reaches the maximum value and then decrease with increasing in velocity of the vehicle.

In the second case to which the vehicle has front radial and rear bias ply tires the response increase with the increase in vehicle speed and reach to infinity at the critical velocity. This applies to the third and fourth cases in which the vehicle had one type of tires and the response increasing linearly with increasing of velocity.

Fig (4) gives the relation ship between the statical gain of directional of center of gravity and vehicle. In the first case the statical gain increases toward the negative value with increasing in velocity, this applies also to the third and fourth cases.

In the second case the statical cam increasing with increasing in velocity until it reaches infinity at the vehicle velocity. In general with regard to response its found that the ( 0 m/s ) the statical gain of directional deviation is ($L_z$/L ).

Fig (5) shows the relation between lateral acceleration and velocity for the first, third, fourth cases the response increasing with increasing in velocity in the second case the response increasing with increasing in velocity unit it reaches infinity at vehicle critical velocity.

Also this applies to Fig (6), which expresses the relation between statical gain of the moment on steering wheel and velocity.

This relation is similar to the relation between statical gain of lateral

acceleration and velocity. After the required values of the response were obtained the same procedure was used on two different type of tires. One type was radial having directional stiffness (33167.5 N/rad), the second was bias ply with directional stiffness of (30524 N/rad). The above mentioned parameters were studied. In the first case the vehicle was understeer and it characteristic velocity was
( $V_{ch}$ = 50.529 m/s ) .
In the second case these was a changed from understeer to oversteer the critical velocity was ( $V_{cr}$ = 50.522 m/s ) .
In the third and fourth cases the response of the vehicle was neutral steer. After obtaining the values from experiment the same collections were made but on two different tires - one radial tire with directional stiffness ( 37008 N/rad ) and the other bias ply tire with the stiffness (35507.5 N/rad ).

| Type of case | The meaning |
|---|---|
| First case | Front tires (bias ply)-rear tires (radical) |
| Second case | Front tires (radial)-rear tire (bias ply) |
| Third case | Front tires and rear tires (radial) |
| Fourth case | Front tires and rear tires (bias ply) |

From studying the four cases it is found that m the first case the vehicle was in understeer the characteristic velocity was( $V_{ch}$ = 76.44 m/s )which in second case in the vehicle was in oversteer with critical velocity
($V_{cr}$ = 16.41 m/s ).
In the third and fourth cases, the vehicle was in neutralsteer.

1- **The effect of the position of center of gravity on vehicle stability.**

The previous parameters were studied a gain but on a different vehicle the center of gravity was now ($L_p$ = 1m ), ( $L_2$ = 1.5 m ).
In the first case vehicle had bias ply front tires and radial rear tires, the vehicle was in understeer which give the vehicle stability at all speeds and characteristic velocity of
($V_{ch}$ = 16.23 m/s) in the second case the vehicle had radial tires and bias ply tires the vehicle was in under steer and characteristic velocity of
($V_{ch}$=35.37 m/s).
In the third & fourth cases the vehicle was in under steer and had characteristic velocity of ( $V_{ch}$ = 22.36 m/s ) ,
($V_{ch}$ = 19.62 m/s) in the third and fourth cases respectively Fig(7) shows the relation between statical gain of coding velocity and vehicle velocity, the statical gain increases for the four cases and increases in velocity reaching the maximum at the characteristic velocity for each case and then decreasing while the speed increases.

Fig. (8) represent the relation between the statical gain of directional deviation of center of gravity and velocity.
The statical gain decreases for the four cases with increases in vehicle velocity.
This applies also to Fig (9) & (10), which show the relation among the statical gain of lateral acceleration the statical gain of moment on the steering wheel and velocity in both cases the statical gain increases with increases in vehicle velocity the same parameter were studied but on different vehicle taking into consideration the position of center of gravity which was now (Lp =1.5m) , ( $L_z$ = 1m).
In the four cases the vehicle was in over steer the critical velocity on the first cases
($V_{cr}$= 35.3 7 m/s) for the second case
($V_{cr}$= 16.23 m/s) for the third case
($V_{cr}$= 19.62 m/s) and for the fourth case
($V_{cr}$= 22.36 m/s). After obtaining these values the experiment was repeated on different vehicle having the following values of the center of the gravity
($L_z$=1.019 m) , ( $L_p$=1.196 m) in the first case the vehicle was in under steer and had characteristic velocity of ($V_{ch}$=39.03 m/s). the other cases showed

different behavior opposite to the first case the vehicle was in over steer and the critical speed was ($V_{ch}$=19.136 m/s), for the second case ($V_{ch}$=29.2 m/s), for the third case and the fourth case ($V_{ch}$= 33.29 m/s).

**2- The effect of mass in vehicle stability**

Four tests were made on different vehicles with mass of (in-1000 kg). In the first case the vehicle was in under steer and had characteristic velocity ($V_{ch}$=31.62 m/s) , in the second case the vehicle was in over steer and had critical speed of ($V_{ch}$=31.62 m/s). In the third and fourth case it was in neutralsteer.

## 4. Conclusions

A vehicle in under steer and stabile at all speeds, when it is center of gravity is in middle and has bias ply front tires and radial rear tires on the other hand a vehicle which has center of gravity in the middle and the radial front tires on bias ply rear tires will be in over steer and instable a speed higher than the critical velocity of vehicle.

The vehicle which has the center of gravity in the middle and the same type of tires on the front and rear wheel (all radial or all bias ply) with the same directional stiffness will be in neutral steer and at all vehicle speed. The closer the center of gravity to the front well the closer the vehicle will be to under steer and stability.

## References

[1]. AN. Al-khazraji, "Theory of Military Wheeled Vehicle", First Edition. M.C.E. Baghdad (1992).

[2]. LY.wong, "Theory of Ground Vehicles "John Wiley & Sons, Canada (1978).

[3]. U.K. "Automobile Tires" by Chapman and hall Ltd. (1972).

[4]. Limport, R. "Vehicle system component Design and safety" U. S.A. (1982).

[5]. French. Tom "Tire Technology" Adam Hilger, Brstol, (1989).

[6]. Williaim, H. Crouse "Automotive Mechanics" MC. Hill, book company. Forth Edition (1960).

[7]. J.K. Ulelwi, "A study of the performance of Military Wheeled Vehicles in the center of Iraq" ph.D. Thesis, Military Collage of Engineering (2002).

[8]. J.L. Synge, "Steering Problem" Quarterly of Applied Mathematics, (1973).

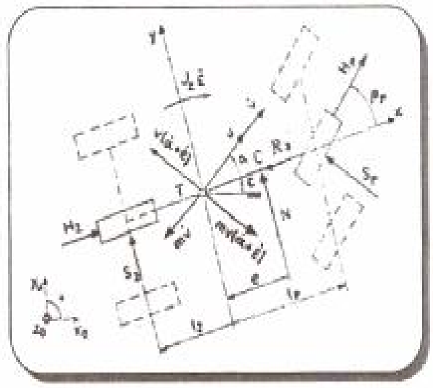


**Figure (1.1)**

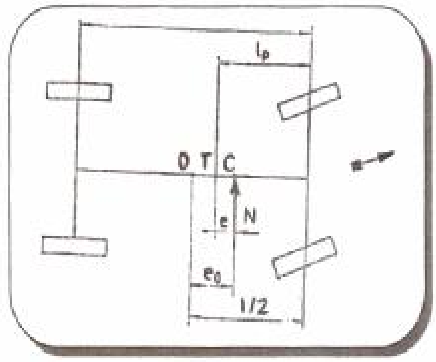


**Figure (1.2)**

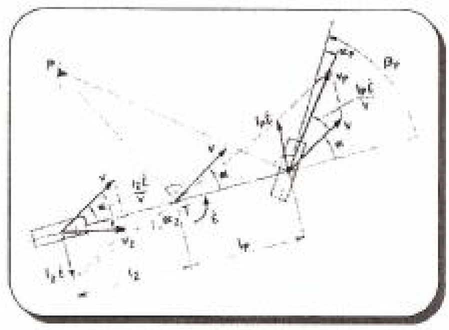


**Figure (2)**

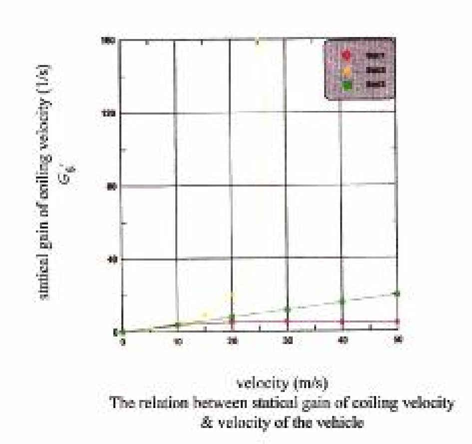



**Figure (3)**

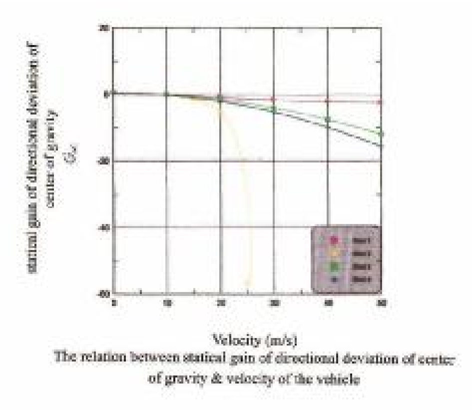



**Figure (4)**

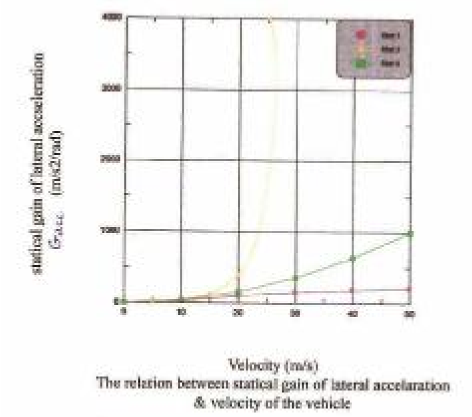



**Figure (5)**

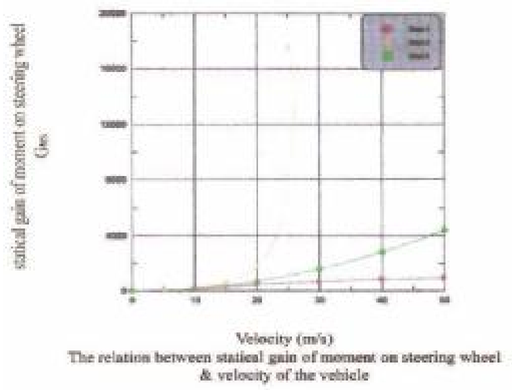



**Figure (6)**

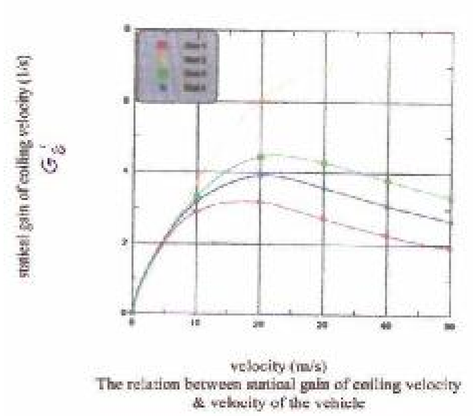



**Figure (7)**

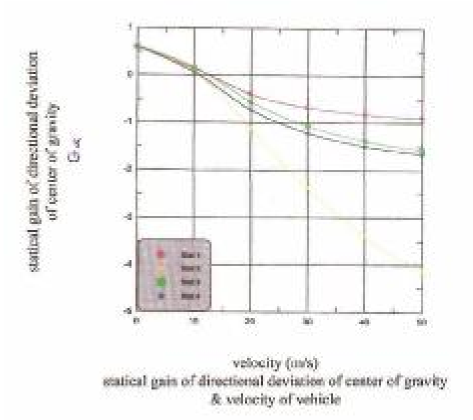
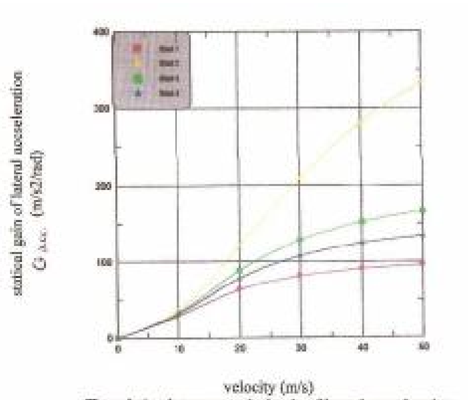
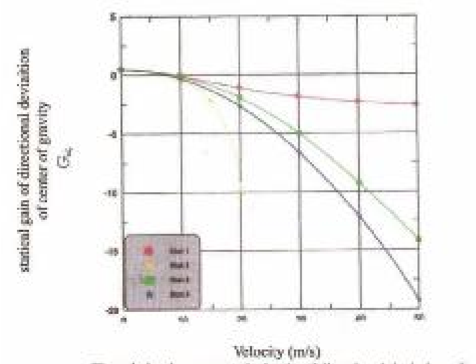



**Figure (8)**



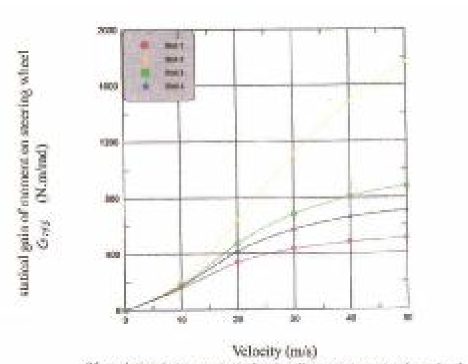
The relation between statical gain of lateral acceselaration & velocity of the velocity

**Figure (9)**



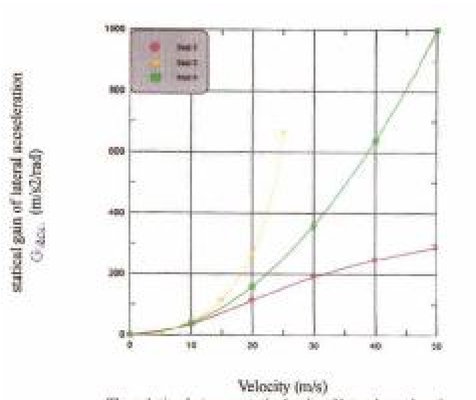
The relation between statical gain of directional deviation of center of gravity & velocity of the vehicle

**Figure (12)**



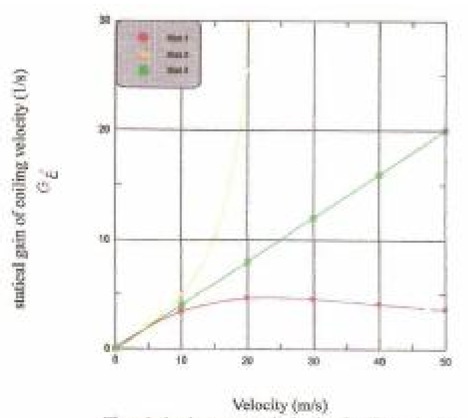
The relation between statical gain of moment on steering wheel & velocity of the vehicle

**Figure (10)**



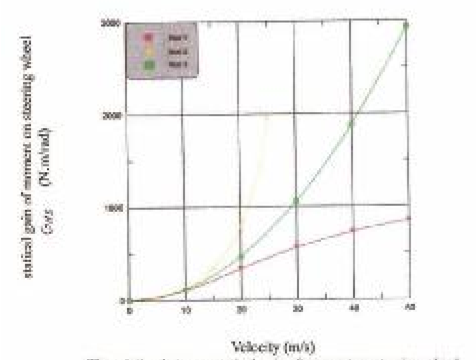
The relation between statical gain of lateral acceleration & velocity of the vehicle

**Figure (13)**



The relation between statical gain of coiling velocity & velocity of the vehicle

**Figure (11)**



The relation between statical gain of moment on steering wheel & velocity of the vehicle

**Figure (14)**