Iraqi Journal of Statistical Science (20) 2011
The Fourth Scientific Conference of the College of Computer Science & Mathematics
pp [271-290]

Speaker age detection using eigen value

Dr. Khalia I. Alsaif* **Hiba A. Abd alnaby****

abstract

In this research an algorithm was suggested for classifying a speaker age to two classes: (young and old classes) based on his speech signal. The suggested algorithm depend on a speech signal feature extraction in order to get a compact representation for this signal and to adopte these features in the classification process.

In this algorithm, the eigen values of the covariance matrix was adopted as a principle parameter in the recognition between the two classes. It was constructed from the data of the speech signal (usually one dimension)after rearranghng it into a number (2,4,8,16,32,64) of a two dimension square matrix array . The suggested algorithm include two main stages:

- 1st stage: includes data file preparation that contains the eigen values for a number of persons belong to both classes young and old (with different gender), in this stage the average of these values for each class to be calculated separately and the threshold curve(which represents the boundary seperating between two classes) were also computed.

Second stage: in this stage the classification process was done by comparing the curve that represents the eigen values of the speech signal, with the threshold curve, a different number of performance parameters are adopted in the evaluation the accuracy of the classification process. The measured correlation value was in range of (0.9610, 0.9994) when m=2 and m=64, respectively (this means whenever the number of arrays that the speech signal constructed from it increases, the correlation coefficient also increases) , while a clear difference can be seen with mmse parameter. After applying the suggested algorithm on 50 persons from both genders, the algorithm passed in applying 80% and failed in percentage 20% of them.

- Assit.Prof. \ College of Computers Sciences and Math.\ University of Mosul
- College of Computers Sciences and Math.\ University of Mosul

Received:1/10 /2011 ____________________Accepted: 21 /12 / 2011

الخلاصة

في هذا البحث تم اقتراح خوارزمية تعمل على تصنيف عمر المتكلم الى أحد الصنفين: صغار السن وكبار السن بالاعتماد على اشارة كلامه. تعتمد الخوارزمية المقترحة على استخلاص خواص اشارة الكلام من أجل الحصول على تمثيل ملخص لتلك الإشارة ومن ثم الاستفادة من تلك الخواص في عملية التصنيف.

تم في هذه الخوارزمية اعتماد القيم المميزة لمصفوفة التباين التي تم تكوينها من بيانات اشارة الكلام(تكون عادة احادية البعد) بعد اعادة ترتيبها على هيئة عدد(2,4,8,16,32,64) من المصفوفات الثنائية المربعة كعامل أساسي في عملية التمييز بين الصنفين.

تتكون الخوارزمية المقترحة من مرحلتين رئيستين:

- المرحلة الأولى: تضمنت إعداد ملف البيانات والذي ضم القيم المميزة لعدد من الأشخاص (80) من كلا الصنفين صغار وكبار السن (ولكلا الجنسين). إذ تم في هذه المرحلة ايضا ايجاد المعدل العام لتلك القيم لكل صنف بشكل مستقل ومن ثم ايجاد منحني حد العتبة الذي يمثل الحد الفاصل بين الصنفين.

- المرحلة الثانية: ويتم فيها عملية التصنيف من خلال مقارنة المنحني الذي يمثل القيم المميزة لاشارة الكلام مع منحني حد العتبة.

إذ تم اعتماد عدد من المقاييس في عملية تقييم دقة التصنيف. وإذ تراوحت قيم معامل الأرتباط (correlation coefficient) بين(0.9610، 0.9994) عندما m=2 وm=64 على التوالي(وهذا يعني أنه كلما زادت عدد المصفوفات المكونة منها اشارة الكلام زاد مقدار الترابط) ، مع وجود تباين كبير عند قياس عامل mmse.

فبعد تطبيق الخوارزمية المقترحة على 50 شخص من كلا الجنسين وجد أن الخوارزمية نجحت في تصنيف عمر الأشخاص بنسبة 80% وأخفقت بنسبة 20% منهم.

Introduction

Every human being goes through the process of ageing. This is a very complex process, which affects us in numerous ways, including the way we speak. Our voices and speech patterns change from early childhood to old age. Although most changes occur in childhood and puberty, age-related variation can be observed throughout our adult lives into old age. Consequently, our age is reflected in our speech, and speaker age can be – and has been – studied using several methodological approaches, mainly acoustic analysis and perception experiments. From young adulthood to old age, the speech production mechanism undergoes numerous anatomical and physiological changes. The Changes in the respiratory system affect speech breathing as well as the voice. The respiratory system reaches its full size after puberty but continues to change throughout adulthood to old age. Changes include decreased lung capacity (mainly due to loss of elasticity in lung tissue), stiffening of the thorax and weakening of respiratory muscles[1].

Speech signal

The speech signal produced by humans contains much more information than purely the message to be passed: it also gives the listener an idea of the gender and emotional state of the speaker, the language of communication, as well as the speaker's identity in some cases. Each of these components of information is processed by the listener, and then put together in the brain to generate a complete picture of the particular conversation.

Automatic methods have been developed for a long time to capture these various levels of information, and researchers have contributed to different areas of speech processing by machines, like Automatic Speaker Recognition(ASR), Language Identification, Natural Language Processing (NLP), Speaker Identification (Speaker ID), Emotion Recognition etc. In addition, research has also focused on giving machines the ability to speak, or Speech Synthesis[2].

Information classes in speech signal

The information in the speech signal can be classified into three levels[3]:

1. **linguistic information:**this include:
   - words
   - syllables
   - Phrases

   that the speech signal comprises from it.

2. **Paralinguistic information**

   paralinguistic information has two important features:

First: paralinguistic information is not inferable from the written counterpart and is deliberately added by the speaker.

Second: the strength of a given paralinguistic information type can vary continuously within one and the same category[4][5].

3. **non linguistic information**

   These communicate such information as the speaker's age, sex, state of health, etc., but cannot be used intentionally by the speaker for linguistic communication of any sort.

   They can be classified into **two** kinds:

   ✓ **individual variation**: includes those effects of phonation and resonance due to the speaker's physiology and the histology of the vocal tract.

- ✓ **Reflexes:** they are often an involuntary indication of genuine emotional stress. Extreme emotional states produce altered patterns in respiration, the endocrine system, and the metabolism in general, which may result in audible changes to speech[3].

Feature extraction

The goal from feature extraction process is to find a few number of characteristic features or rich with useful information for classification process (these small set of features reduce the complexity of classification algorithm, time, memory requirement for algorithm implementation), which can be represented as feature vectors that are constant with transformation that are independent of data[6][7].

The feature extraction usually can be got from arithmetic transformation of data. One of the most used transformation is linear transformation such as Principle Component Analysis(PCA)[6].

Literature survey

- ❖ **In 2003** Br¨uckl and Sendlmeier analysed speech samples of read speech, sustained vowels and spontaneous speech from 56 female speakers aged 20–87, and then carried out a direct age estimation test using 15 adult listeners (6 females and 9 males, aged 22–35). They found the highest correlation between PA and CA for the more naturally produced spontaneous stimuli (0.864), perhaps because they contained the most age information (e.g. in the semantic content, choice of words and sociolect), followed by read speech (0.862) and vowels (0.330–0.738)[8].
- ❖ **In 2005** Susanne Sch¨otz estimate in her research:
  "Effects of Stimulus Duration and Type on Perception of Female and Male Speaker Age" Speaker age from speech cues with respect to stimulus duration, stimulus type, and speaker gender. Four separate listening tests were carried out with four different sets of stimuli(10 and 3 seconds of spontaneous speech, one isolated word, 6 concatenated isolated words) all produced by the same 24 speakers. It was also found that stimulus duration influenced the listeners judgements of male speakers while stimulus type influenced the listeners judgements of female speakers[9].
- ❖ **In 2006** Susanne Sch¨otz concluded in her research: " CART estimation of direct age, age group and gender"
  Important features for automatic age estimation using the CART technique are mainly:
  - ✓ spectral features, including formant frequencies
  - ✓ HNR(Harmonic to Noise Ratio)
  - ✓ intensity

  though different features were important in different phoneme segments. A CART estimator based on a single segment of an solated word (mean error ± 14.45 years) does not reach the performance of human listeners (± 8.89 years). And the information about the gender does not influence the age estimation process using CART technique[10].

- **In 2007** Matthew Blackshaw, Eric Steinlauf calculated in their research "Gender and Age Determination of a Speaker " speaker age using a statistical model based on formant frequencies. The hypothesis was that the mean and standard deviation of formant frequencies for a speaker for a particular vowel would correlate with age. Isolating and identifying vowels from other characters in the sample were done[11].

Covariance matrix[12]

If the entries in the column vector $X=[X_1\ X_2\ \ldots X_n]$ are random variables, each with finite variance, then the covariance matrix Σ is the matrix explain in the equation:

$$\Sigma_{ij} = \mathrm{cov}(X_i, X_j) = \mathrm{E}\left[(X_i - \mu_i)(X_j - \mu_j)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{(1)}$$

Where:

$\mu_i = E(Xi)$

is the expected value of the ith entry in the vector X. the covariance matrix is :[12]

$$\Sigma = \begin{bmatrix} \mathrm{E}[(X_1-\mu_1)(X_1-\mu_1)] & \mathrm{E}[(X_1-\mu_1)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_1-\mu_1)(X_n-\mu_n)] \\ \mathrm{E}[(X_2-\mu_2)(X_1-\mu_1)] & \mathrm{E}[(X_2-\mu_2)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_2-\mu_2)(X_n-\mu_n)] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{E}[(X_n-\mu_n)(X_1-\mu_1)] & \mathrm{E}[(X_n-\mu_n)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_n-\mu_n)(X_n-\mu_n)] \end{bmatrix}.$$

properties of covariance matrix [13]

The covariance matrix is not just a convenient way of displaying numbers.As a matrix, it has several important properties which derive from the fact that a covariance matrix is always positive semidefinite. That fulfills the following equation:

$$[V][X][V']>=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where:

V: any non zero vector

X: covariance matrix

The converse is also true, any positive semidefinite matrix Σ is the covariance matrix of a random vector[13].

If X, Y, W, and V are real-valued random variables and a, b, c, d are constant, then the covariance matrix has the following properties:[14]

1- Cov(X,a)=0

2- Cov(X,X)=Var(X)

3- Cov(X,Y)= Cov(Y, X)

4- Cov(aX,bY)=ab Cov(X,Y)

5-Cov(X+a,Y+b)= Cov(X,Y)

6-Cov(aX+bY,cW+dV)=ac Cov(X,W)+ad Cov(X,V)+ bc Cov(Y,W)+bd Cov(Y,V)

7- for sequences x1,…,xn and y1,…,ym of random variables, we have:

$$\operatorname{Cov}\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} \operatorname{Cov}(X_i, Y_j).$$

8- for a sequence x1,…,xn of random variables, and constants a1,…,an, we have :

$$\operatorname{Var}\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 \operatorname{Var}(X_i) + 2\sum_{i,j:i<j} a_i a_j \operatorname{Cov}(X_i, X_j).$$

Eigen values definition of covariance matrix

Suppose A is a square complex matrix , and X is the value of a vertical non zero complex vector and is called the eigen vector.then λ is a complex number and represents the eigen value of A , this can be illustrated in the following equation:[15]

$$A X = \lambda X \qquad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

The eigen values have the following properties:

1) Eigenvalues and eigenvectors are defined only for square matrices.
2) Every eigenvalue has an infinite number of eigenvectors associated with it, as any nonzero scalar multiple of an eigenvector is also an eigenvector[16].
3) If A is an invertible matrix, then 1/ λ is the eigen value of A^{-1}.
4) If d is any number and In is an identity matrix, then λ+d is the eigen value of A+d In.
5) The sum of eigenvalues of a n*n matrix **A** is equal to the trace of **A.**

$$\sum_{i=1}^{n} \lambda_i = \mathrm{Tr}(\mathbf{A}) = \sum_{i=1}^{n} A_{ii}$$

6) The determinant of a n*n matrix **A** is equal to the product of its eigenvalues (counting multiplicities)[17].
   Det (A) = λ1 λ2 λ3 … λn

The proposed algorithm

The proposed algorithm deals with a new style, by treating a speech signal which is one dimension on two dimension matrix form in order to evaluate the eigen values of covariance matrix for the speech signal(which has a characteristic property of a square two dimension matrix).

proposed algorithm stages

the proposed algorithm included two main stages:

a- first stage :data file preparation

This stage consists of speech signals collection steps of large number of different samples for both genders in order to classify them into two groups and to start the preparation process of the threshold curves for each group(gender).

The recording process of speech signals was done using mp3 player device rather than using another media to acquire a speech signal, the mic on the sound set can also be used for recording.

The mp3 player device has the following setting:

- Samplingrate(FS):8000 sample/second
- Data type: double
- Record type: mono
- Data format transformation: AD pcm

In this stage of algorithm the signal with length 110000 samples was adopted for the sake of partitioning it into many equal size partitions (2 partition, 4 partition, 8 partition, 16 partition, 32 partition and 64 partition) and to convert each partition into a two-dimension square matrix then evaluate the eigen values of covariance matrix for each partition.

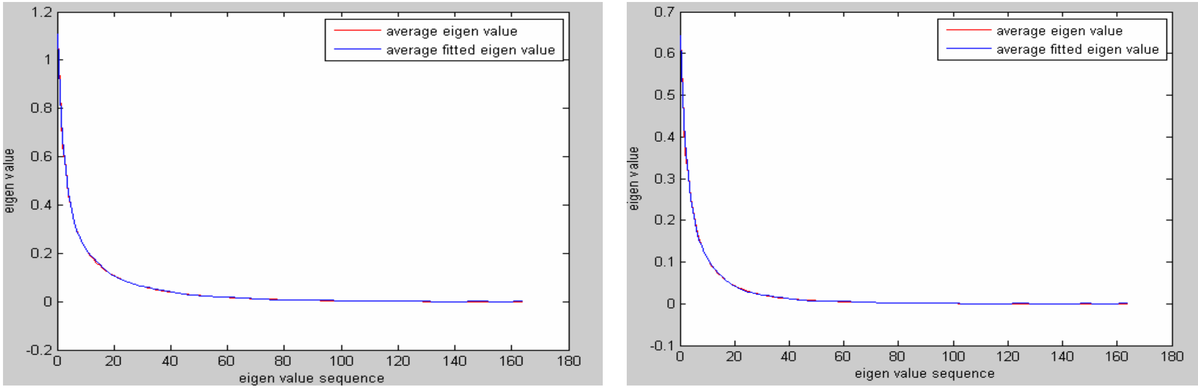
The above process was repeated for all speech signals which were collected in order to evaluate the average eigen values for all signals for each group and to find the threshold curve (which was adopted as a recognition parameter in the second stage of algorithm) for each dimension in the matrix.

The capability of transforming real eigen values of a speech signal to a fitted value has been studied using polynomial curve fitting of degree 15, then finding threshold for speech signals after fitting.

In this case the system consists of two thresholds: first threshold for real eigen values without fitting, second threshold for eigen values with fitting.

The figures from (1) to (6) explain average eigen values curves for speech signals for age group below 30 year and above 40 for males in six different casas. We can see the extent of convergence between average eigen values curves with and without fitting in both age groups.

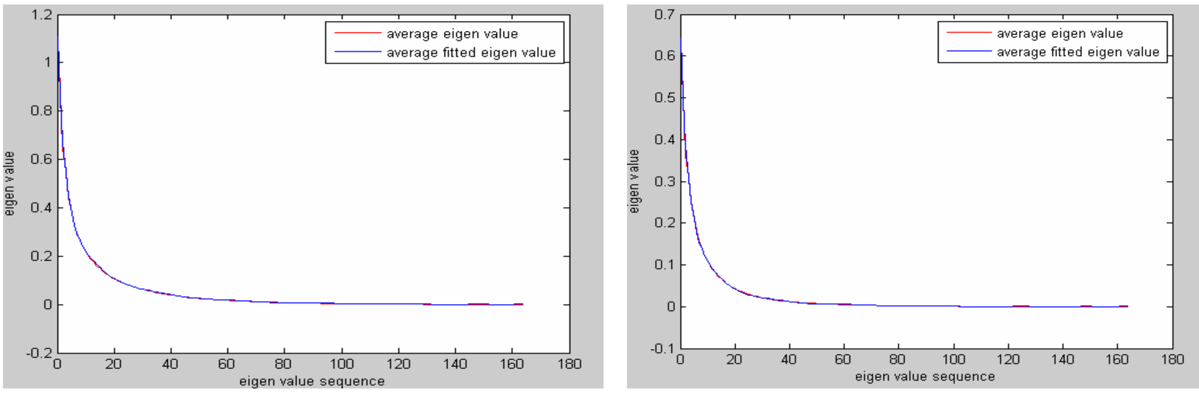
In the case of number of square matrix equal to 2, the first was ignored and the second was adopted only for evaluating the average eigen values of covariance matrix as it is explained in the figure(1).



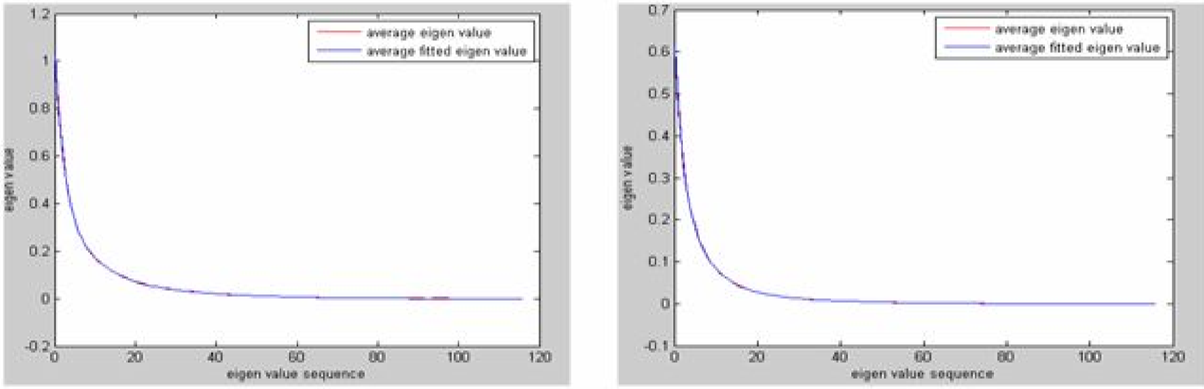
b- old a

Figure(1): average eigen values of speech signals when m=2

While in the case of number of matrices (4,8,16,32,64), the first one was ignored and adopted the rest for evaluating the eigen values of covariance matrix . The figures from (2) to (6) explain these cases respectively.

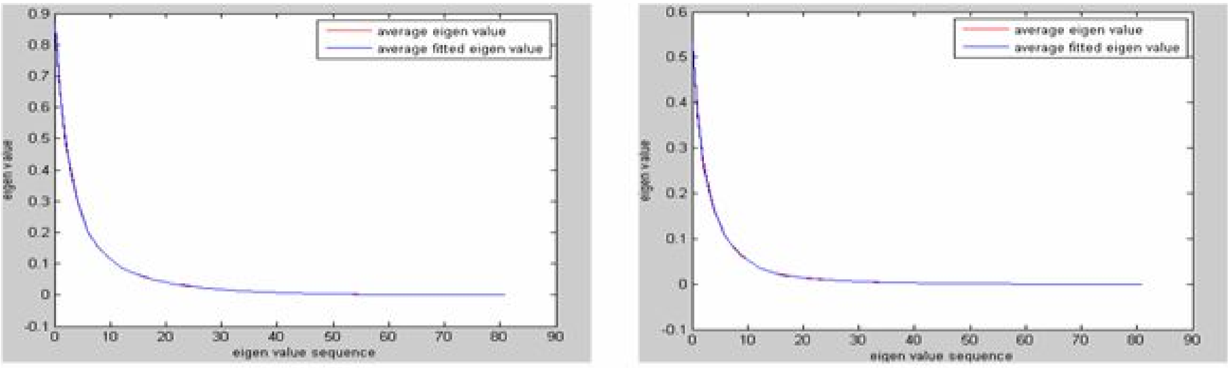


Figure(2): average eigen values of speech signals when m=4 before and after fitting



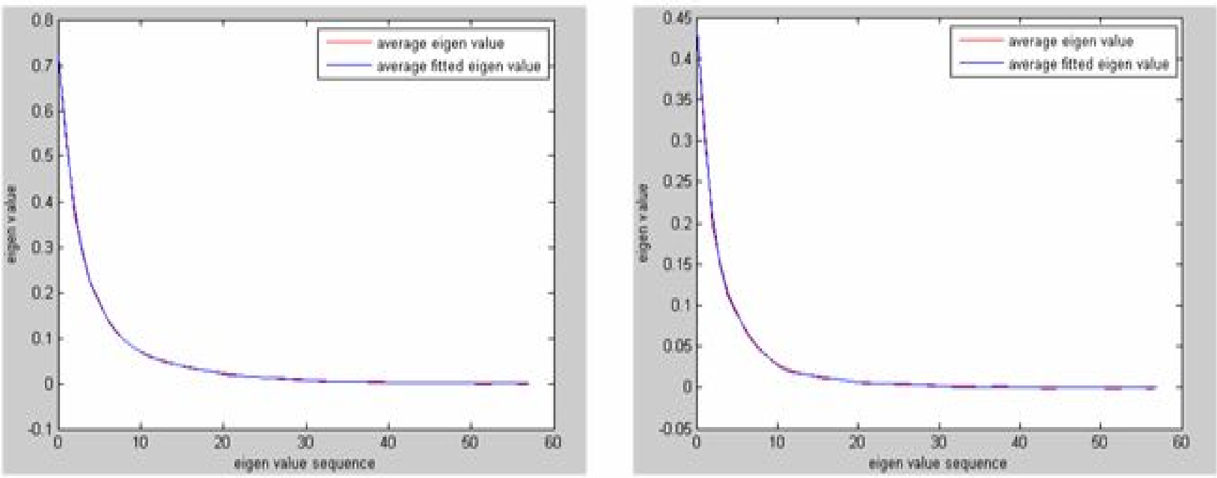
b- old a- young

Figure(3): average eigen values of speech signals when m=8 before and after fitting



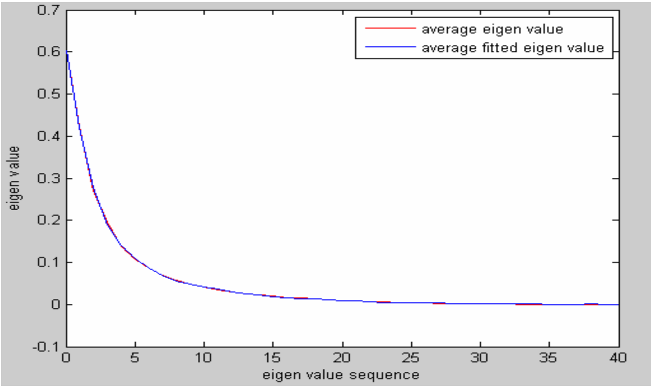
b- old a- young

Figure(4): average eigen values of speech signals when m=16 before and after fitting

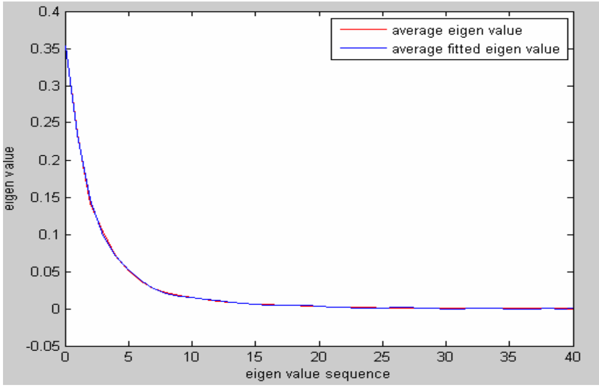


b- old a- young

Figure(5): average eigen values of speech signals when m=32 before and after fitting



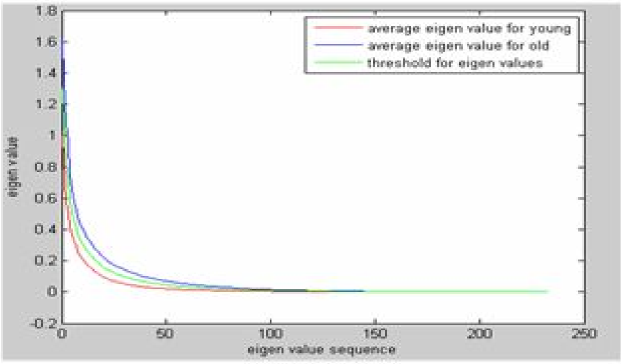
b- old



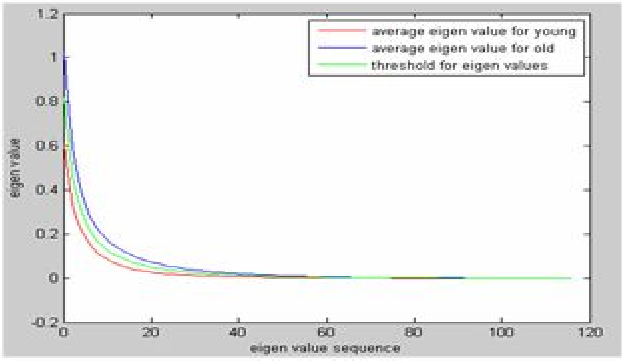
a- young

Figure(6): average eigen values of speech signals when m=64 before and after fitting

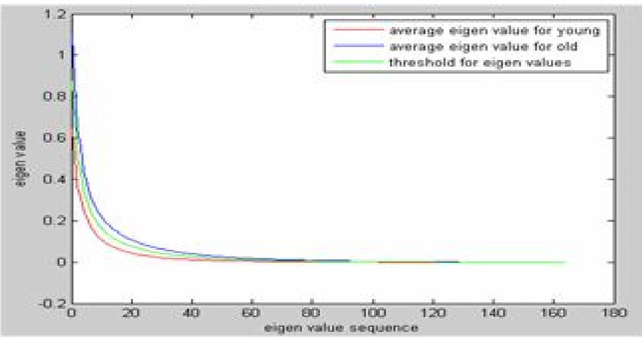
Based on average eigen values curves for both groups the threshold curve was evaluated for each case of previous cases as explained in figures(7)-(12):



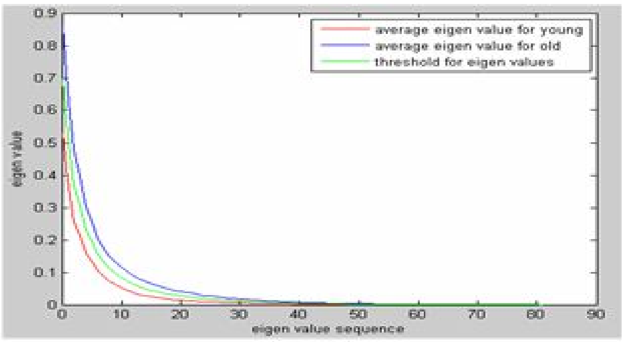
Figure(7): threshold curve when m=2 for young/old groups



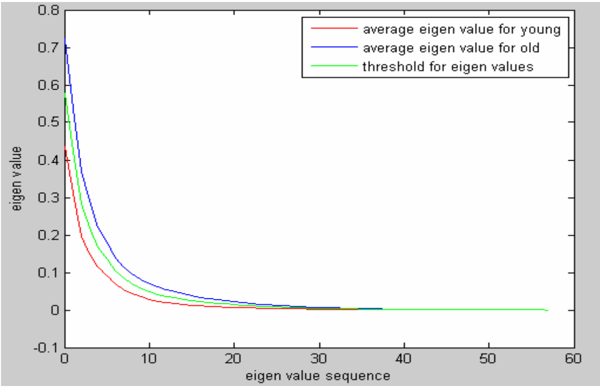
Figure(9): threshold curve when m=8 for young/old groups

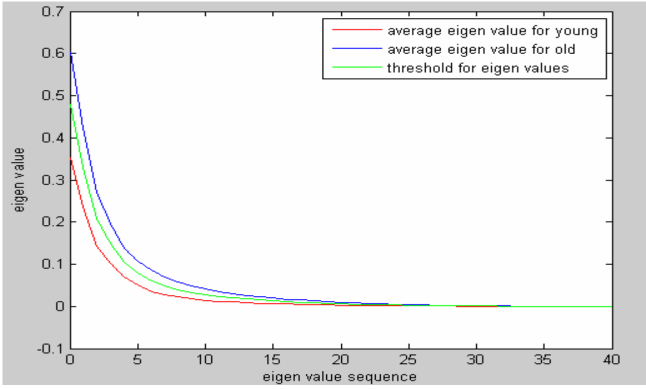


Figure(8): threshold curve when m=4 for young/old groups



Figure(10): threshold curve when m=16 for young/old groups





Figure(11): threshold curve when m=32 for young/old groups

Figure(12): threshold curve when m=64 for young/old groups

b- Second stage : classifying speaker age

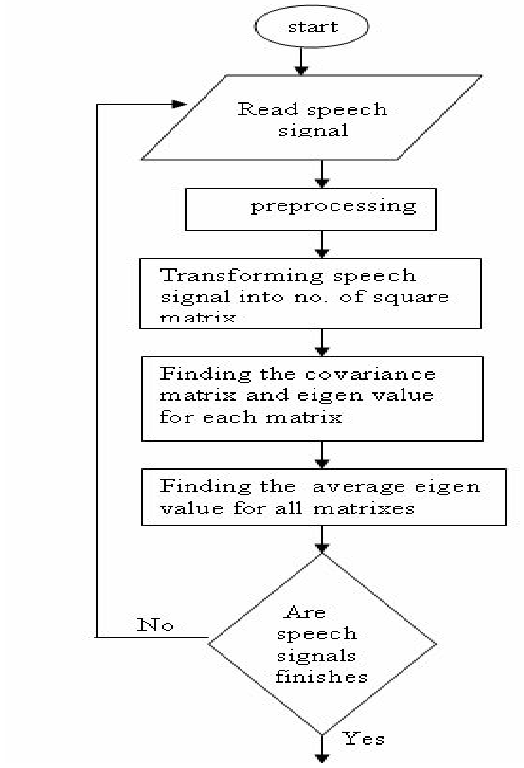
First step: read the speech signal .

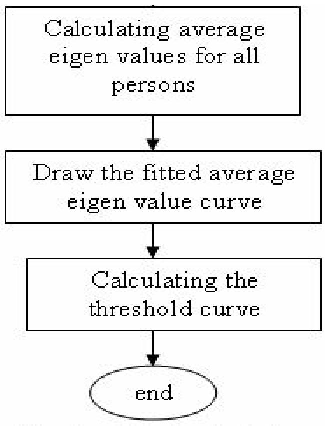
Second step: rearrange the speech signal.

Third step: find the covariance matrix and eigen values.

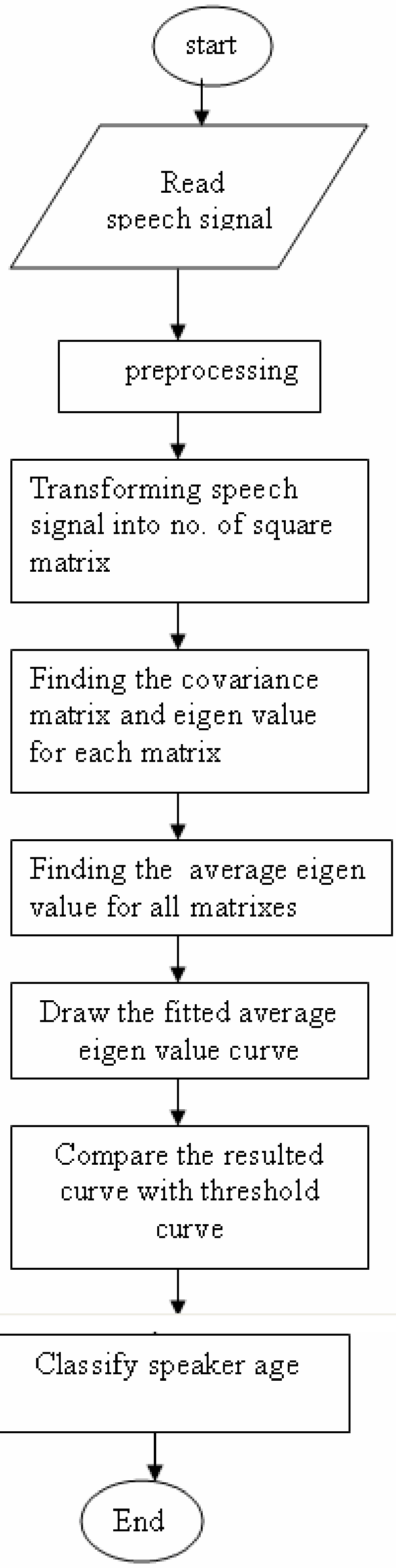
Fourth step: measure the extent of convergence from the threshold curve.

Figure(13),(14) explain the stages of the proposed algorithm.





Figure(13): flowchart for the first stage of algorithm

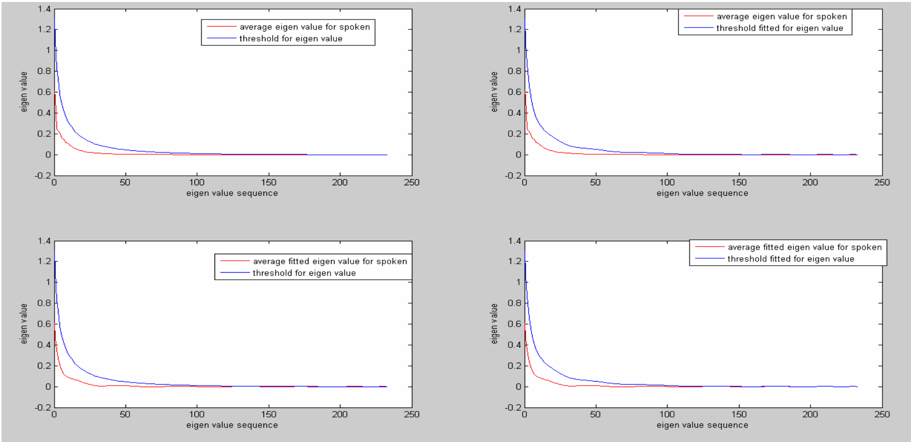


Figure(14): flowchart for the second stage of algorithm

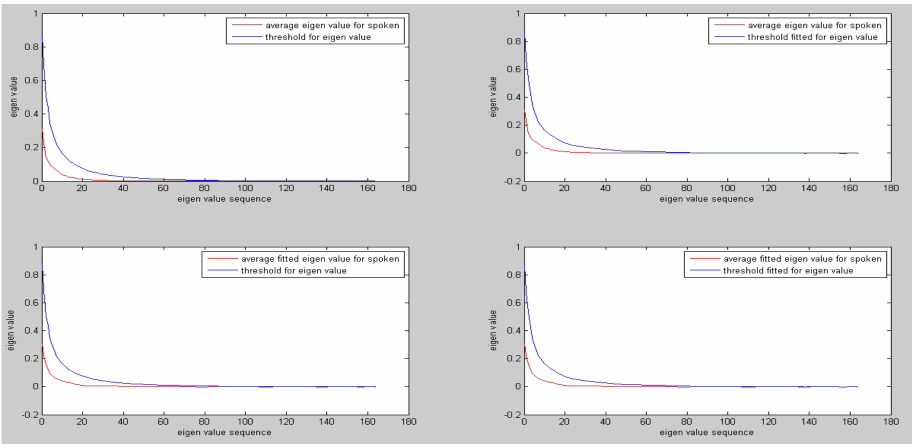
Results

- **Young case**

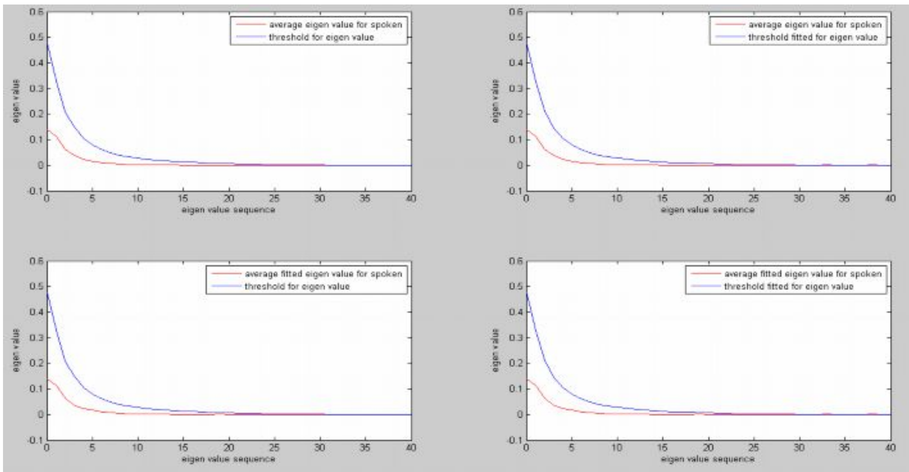
After applying the proposed algorithm on a male person belongs to a young group (22 years old), and the speech material is alfatiha sura, the duration of recording is about 20 second, the result is shown in the following figures:



Figure(15): average eigen values of speech signals when m=2(young case)



Figure(16): average eigen values of speech signals when m=4 (young case)



In all these previous cases it was observed that the fitted average eigen value curve falls under threshold curve. And this was for the most persons who belong to young group.

Convergence extent calculation

Many measures were used to evaluate the goodness or badness of the suggested algorithm. Some of them are:

1. Correlation coefficient

When calculated the Correlation coefficient between average eigen value of a person and both of average eigen value of young and old group, we found that Correlation coefficient with young group was greater than old group. For all cases and this means that the person belongs to a young group, as shown in tables (1), (2):

table(1): correlation with young group

No. of partition (n)	Correlation coefficient with young group	
	With out fit	With fit
2	0.9846	0.9898
4	0.9931	0.9944
8	0.9963	0.9985
16	0.9982	0.9990
32	0.9974	0.9977
64	0.9937	0.9937

table(2): correlation with old group

No. of partition (n)	Correlation coefficient with old group	
	With out fit	With fit
2	0.9724	0.9610
4	0.9855	0.9838
8	0.9926	0.9899
16	0.9946	0.9933
32	0.9955	0.9953
64	0.9898	0.9896

2. minimum mean square error(mmse)

When calculated the mmse between average eigen value of a person and both of average eigen value of young and old group, we found that mmse with young group was greater than old group. For all cases this means that the person belongs to a young group, as shown in tables(3),(4)

Table(3): mmse with young

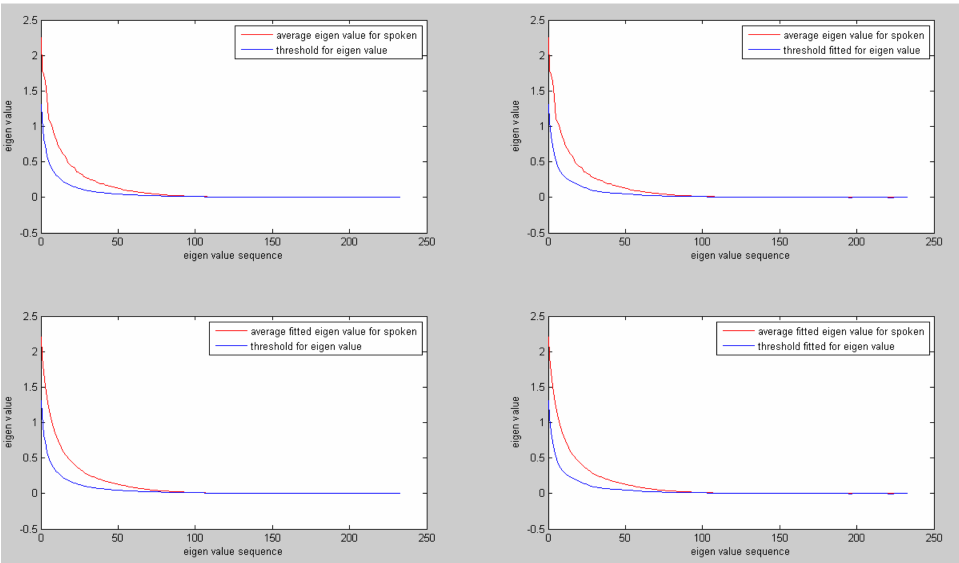
No. of partition (n)	mmse with young group	
	With out fit	With fit
2	0.0031	0.0031
4	0.0023	0.0023
8	0.0029	0.0029
16	0.0026	0.0026
32	0.0025	0.0025
64	0.0019	0.0019

Table(4):mmse with old group

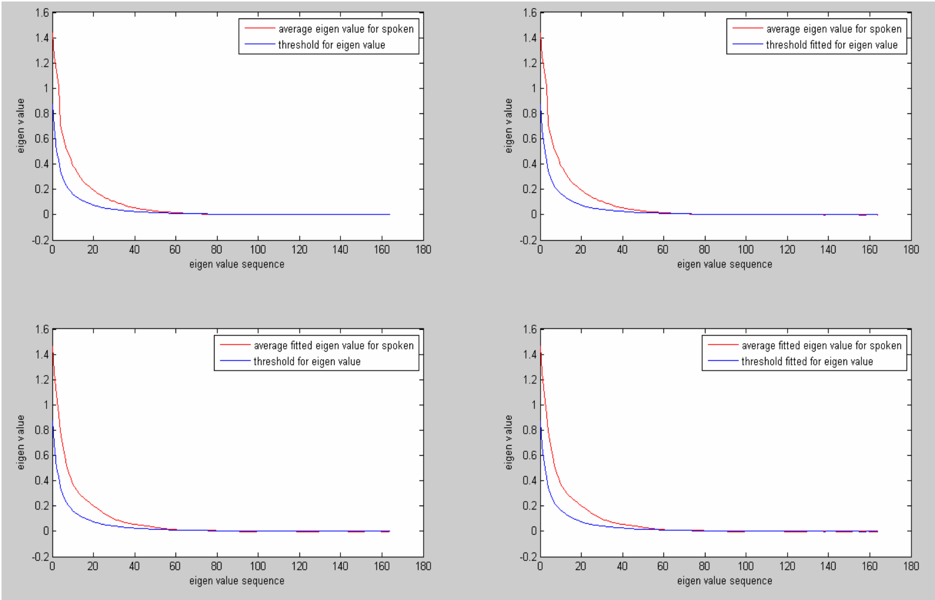
No. of partition (n)	mmse with old group	
	With out fit	With fit
2	0.0220	0.0219
4	0.0133	0.0133
8	0.0152	0.0152
16	0.0137	0.0137
32	0.0124	0.0123
64	0.0103	0.0103

☒ **Old case**

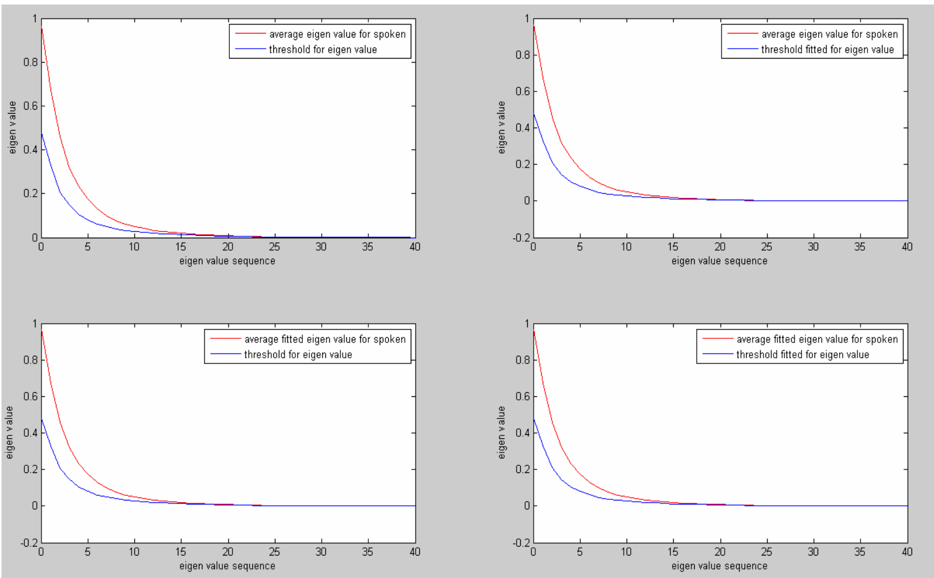
After applying the proposed algorithm on a male person belongs to an old group (52 years old), and the speech material is alfatiha sura, the duration of the recording is about 20 second, the result is shown in the following figures:



Figure(18): average eigen values of speech signals when m=2 (old case)



Figure(19): average eigen values of speech signals when m=4 (old case)



Figure(20): average eigen values of speech signals when m=64 (old case)

1. Correlation coefficient

The Correlation coefficient between average eigen value of a person and both of average eigen value of young and old group was calculated, it was found that Correlation coefficient with old group is greater than old group. For all cases and this means that the person belongs to old group as shown in tables(5), (6):

table(5): correlation with old group

No. of partition (n)	Correlation coefficient with old group	
	With out fit	With fit
2	0.9879	0.9862
4	0.9910	0.9889
8	0.9979	0.9973
16	0.9994	0.9993
32	0.9983	0.9982
64	0.9994	0.9993

table(6): correlation with young group

No. of partition (n)	Correlation coefficient with young group	
	With out fit	With fit
2	0.9710	0.9662
4	0.9832	0.9803
8	0.9929	0.9921
16	0.9958	0.9957
32	0.9954	0.9952
64	0.9976	0.9973

2. minimum mean square error (mmse)

The mmse between average eigen value of a person and both of average eigen value of young and old group was calculated, it is found that mmse with old group was lower than old group. For all cases And this mean the person belong to old group as shown in tables(7), (8):

Table(7): mmse with old group

No. of partition (n)	mmse with old group	
	With out fit	With fit
2	0.0187	0.0190
4	0.0074	0.0076
8	0.0084	0.0085
16	0.0088	0.0088
32	0.0084	0.0084
64	0.0064	0.0064

Table(8): mmse with young group

No. of partition (n)	mmse with young group	
	With out fit	With fit
2	0.0517	0.0521
4	0.0230	0.0232
8	0.0261	0.0261
16	0.0254	0.0254
32	0.0235	0.0235
64	0.0190	0.0190

Conclusions

- ✓ The idea of representing the speech signal(one-dimension) as two dimensions gives the capabilities of using the ideas and methods which are used in image processing (when image in spatial domain).
- ✓ The adoption of quad tree in square matrix decomposition results in reduction in the quantity of resulted error through speech signal acquision process, because the reflex of error(or acquired noise) on part of speech signal without other parts.
- ✓ The adoption of eigen value computation of covariance matrix as characteristic feature resulted in positive result. Since the elements of covariance matrix of a speech signal leads to compact the energy that contained in this matrix on vectors called eigen vectors. This indicates that eigen values are good representatives for speech signal elements.
- ✓ The eigen values were, after rearrange them descending had exponential layout, this meana that it were very high in the beginning and decreased step by step to be low and very convergence with others which could be eliminated because of their low influence.
- ✓ The dependence of a fewer number of coefficients by using polynomial equation of degree 15 leads to make the data file size(which was prepared for calculating the fitted average for both young and old group) not large.

Future works

- ✓ The capability of using neural network and genetic algorithm in speaker age classification
- ✓ Develop the idea of the research for classifying a speaker age in a number of age groups(5,7,10).
- ✓ Adopting other features for other matrices rather than covariance matrix such as co occurrence matrix in the recognition between young/ old groups.
- ✓ Adoption of other features of speech signal in the frequency domain such as fundamental frequency, formant frequency, and other features related to age.
- ✓ Adoption of different types of speech material rather than. changing in some influence such as the extent of near or far from recording device and study its influence on automatic classification of age.

References

[1] Sch¨otz, S., 2007, ”Acoustic Analysis of Adult Speaker Age”, In C. Müller (Ed.) Speaker Classification I, Lecture Notes in Computer Science, Springer, pp 88-107.
http://person2.sol.lu.se/SusanneSchotz/downloads/Spr_schoetz_07_1.pdf

[2] Vishnubhotla, S., 2007, “detection of irregular phonation in speech”, master thesis, College Park, University of Maryland.
[3] Roach, P. , et. al. , 1998,“Transcription of Prosodic and Paralinguistic Features of Emotional Speech”, Journal of the International Phonetic Association (1998) 28 (pp 83-94).
[4] Fujisaki, H. , 1994, “Aspects of Prosody Research and Topics for Further Investigation”, Proc. 1994 Autumn Meeting of ASJ, 1: 287-290.
[5] Fujisaki, H., 1997 “Prosody, Models, and Spontaneous Speech”, In Sagisaka et al ed. Computing Prosody. Springer, 1997.
[6] polikar, R., 2006, ”pattern recognition”, Wiley Encyclopedia of Biomedical Engineering.
[7] Liu, R., Gu, L., 2001," the application of optimization in feature extraction of speech recognition" ,IEEE Xplore, vol 1, pp. 745- 748.
[8] Br¨uckl, M., Sendlmeier, W., 2003, “Aging Female Voices: an Acoustic and Perceptive Analysis”, Proc of ISCA workshop Voqual ’03, pp. 163–168.
[9] Sch¨otz, S. , 2005,“Stimulus Duration and Type in Perception of Female and Male Speaker Age,” in Proc. Interspeech 2005, Lisbon.
[10] Schötz, S., 2006,”Perception, Analysis and Synthesis of Speaker Age”, Ph.D. thesis, Department of Linguistics and Phonetics, Centre for Languages and Literature, Lund University.
[11] Blackshaw, M., Steinlauf, E., 2007, “Gender and Age Determination of a Speaker”.http://mattblackshaw.com/project/agegender/paper.pdf
[12] Covariance matrix, from Wikipedia, the free encyclopedia http://en.wikipedia.org/wiki/Covariance_matrix
[13] Covariance matrix http://www.aiaccess.net/English/Glossaries/GlosMod/e_gm_covariance_matrix.htm
[14] Covariance, from Wikipedia, the free encyclopedia http://en.wikipedia.org/wiki/Covariance
[15] Katta G. Murty, 2001, “Computational and Algorithmic Linear Algebra and n-Dimensional Geometry”, Sophomore Level Self-Teaching Webbook. http://ioe.engin.umich.edu/people/fac/books/murty/algorithmic_linear_algebra/soph1-0.pdf
[16] Parasuram, 2001,“handout m.3 - eigen values and eigen vectors”, meen 364. http://www1.mengr.tamu.edu/aparlos/MEEN651/EigenvaluesEigenvectors.pdf
[17] Brian G. Higgins, 2003, “Primer on Eigenvalues and Eigenvectors”, Note book. http://www.ekayasolutions.com/ech259/ECH259ClassNotes/EigenValuesEigenvectors.pdf