Iraqi Journal of Statistical Science (20) 2011
The Fourth Scientific Conference of the College of Computer Science & Mathematics
pp [93-110]

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Numerical Solution for Non-linear Korteweg-de Vries-Burger's Equation Using the Haar Wavelet Method

Abstract

In this paper, an operational matrix of integrations based on the Haar wavelet method is applied for finding numerical solution of non-linear third-order korteweg-de Vries-Burger's equation, we compared this numerical results with the exact solution. The accuracy of the obtained solutions is quite high even if the number of calculation points is small, by increasing the number of collocation points the error of the solution rapidly decreases as shown by solving an example. We have been reduced the boundary conditions in the solution by using the finite differences method with respect to time. Also we have reduced the order boundary conditions used in the numerical solution by using the boundary condition at x=L instead of the derivatives of order two with respect to space.

الحل العددي لمعادلة korteweg-de Vries-Burger's غير الخطية باستخدام طريقة مويجة Haar

الملخص

في هذا البحث، تم تطبيق مصفوفة العوامل للتكاملات التي تعتمد على مويجة Haar لإيجاد الحل العددي لمعادلة korteweg–de Vries–Burger's غير الخطية من الرتبة الثالثة وقد قورنت النتائج مع الحل المضبوط. إن دقة الحلول التي حصلنا عليها عالية حتى إذا كان عدد نقاط الشبكة المحسوبة قليلاً وكلما زادت عدد نقاط الشبكة المحسوبة فان الدقة تزداد والخطأ يتناقص وقد تم توضيح ذلك من خلال حل مثال. لقد تم أيضا تخفيض رتبة الشروط الحدودية المطلوبة في الحل

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Received:1/10 /2011 ____________________Accepted: 21 /12 / 2011

العددي وذلك باستخدام طريقة الفروقات المنتهية بالنسبة للزمن وكذلك تم تخفيض رتبة الشروط الحدودية بالنسبة للبعد وذلك باستخدام الشروط الحدودية عند نهاية الفترة x=L بدلا من استخدام الشروط الحدودية للمشتقة الثانية.

1.Introduction:

As a powerful mathematical tool, Wavelet analysis has been widely used in image digital processing, quantum field theory, numerical analysis and many other field in recent years.

Haar wavelets have been applied extensively for signal processing in communications and physics research, and more mathematically focused on differential equations and even nonlinear problems. After discrediting the differential equation in a convential way like the finite difference approximation, wavelets can be used for algebraic manipulations in the system of equations obtained which may lead to better condition number of the resulting system [11].

Using the operational matrix of an orthogonal function to perform integration for solving, identifying and optimizing a linear dynamic system has several advantages: (1) the method is computer oriented, thus solving higher order differential equation becomes a matter of dimension increasing; (2) the solution is a multi-resolution type and (3) the answer is convergent, even the size of increment is very large [10].

The main characteristic of the operational method is to convert a differential equation into an algebraic one, and the core is the operational matrix for integration. We start with the integral property of the basic orthonormal matrix, $\phi(t)$ by write the following approximation:

$$\underbrace{\int_0^t\int_0^t\int_0^t\cdots\cdots\int_0^t}_{k} \phi(t)(dt)^k \cong Q_\phi^k \phi(t) \qquad \ldots(1)$$

Where $\phi(t)=[\vec{\varphi}_0(t)\ \ \vec{\varphi}_1(t)\ \ \ldots\ \ \vec{\varphi}_{m-1}(t)]^T$ in which the elements $\vec{\varphi}_0(t),\vec{\varphi}_1(t),\ldots,\vec{\varphi}_{m-1}(t)$ are the discrete representation of the basis functions which are orthogonal on the interval [0,1) and Q_ϕ is the operational matrix for integration of $\phi(t)$ [10].

Many authors have studied the solution for nonlinear third-order korteweg-de vries-burger's (KdVB) equation.

EL-Danaf T. (2002) is discuss the solution of the modified (KDVB) equation by using the collocation method with quintic splines and comparison between the numerical and exact solution, also he discuss the stability analysis of this method.

Darvishia M. T. , Khanib F. and Kheybari S. (2007) are using the spectral collocation method to solve the KDVB equation numerically, and to reduce round off error, they are use central left and right darvishi's preconditioning.

Lepik and Tamme (2007) derived the solution of nonlinear Fredholm integral equations via the Haar wavelet method, they are find that the main benefits of the Haar wavelet method are sparse representation, fast transformation, and possibility of implementation of fast algorithms especially if matrix representation is used.

Lepik Uio (2007) studied the application of the Haar wavelet transform to solve integral and differential equations, he demonstrated that the Haar wavelet method is a powerful tool for solving different types of integral equations and partial differential equations. The method with far less degrees of freedom and with smaller CPU time provides better solutions then classical ones.

Zhi S. LI-Y. and Qing-J. C. (2007) are establishes a clear procedure for finite-length beam problem and convection-diffusion equation solution via Haar wavelet technique, The main advantages of this method is its simplicity and small computation costs.

Bhatta D. (2008) is studied the modified Bernstein polynomials for solve korteweg-de veries-burger's equation over the spatial domain. B-polynomials are used to expand the desired solution requiring discreitization with only the time variable.

AL-Rawi Ekhlass S. and Qasem A. F. (2010) found the numerical solution for nonlinear Murray equation by the operational matrices of Haar wavelet method and compared the results of this method with the exact solution, they transformed the nonlinear Murray equation into a linear algebraic equations that can be solved by Gauss-Jordan method.

G. Hariharan · K. Kannan (2010) are develop an accurate and efficient Haar transform or Haar wavelet method for some of the well-known nonlinear parabolic partial differential equations. The equations include the Nowell-whitehead equation, Cahn-Allen equation, FitzHugh-Nagumo equation, and other equations.

In this paper, we study the numerical solution for nonlinear third-order korteweg-de vries-burger's equation by the operational matrices of Haar wavelet method and we compare the results of this method with the exact solution.

We organized our paper as follows. In section 2, the Haar wavelet is introduced and an operational matrix is established. Section 3 function approximation is presented. Section 4 we use Haar wavelets to solve nonlinear KdVB equation. Section 5 Reducing of the order boundary conditions used in the numerical solution is presented .Section 6 numerical results are presented. Concluding remarks are given in section 7.

2. Haar wavelet

The Haar functions are an orthogonal family of switched rectangular waveforms where amplitudes can differ from one function to another. They are defined in the interval [0,1] by [6]:

$$h_i(x)=\begin{cases}1 & \frac{k}{m}\le x<\frac{k+1/2}{m}\\ -1 & \frac{k+1/2}{m}\le x<\frac{k+1}{m}\\ 0 & otherwise \quad in \quad [0,1)\end{cases} \qquad \ldots(2)$$

Integer $m=2^j$ $(j=0,1,2,....,J)$ indicates the level of the wavelet; k=0,1,2,…,m-1 is the translation parameter. Maximal level of resolution is J. The indix i is calculated according the formula i=m+k+1; in the case of minimal values. m=1,k=0 we have i=2, the maximal value of i is $i=2M=2^{j+1}$. It is assumed that the value i=1 corresponds to the scaling function for which $h_1=1 \quad in\,[0,1]$. Let us define the collocation points $x_l=(l-0.5)/2M$, $(l=1,2,...,2M)$ and discredits the Haar function $h_i(x)$; in this way we get the coefficient matrix $H(i,l)=(h_i(x_l))$, which has the dimension 2M*2M.

The operational matrix of integration P, which is a 2M square matrix, is defined by the equation: [8]

$$P_{i,1}(x)=\int_0^{x_l} h_i(x)\,dx \qquad \ldots(3)$$

$$P_{i,v+1}(x)=\int_0^{x} P_{i,v}(x)\,dx \quad , \qquad v=1,2,... \qquad \ldots(4)$$

These integrals can be evaluated using equation (2) and first four of them are given:

$$P_{i,1}(x)=\begin{cases}x-\alpha & for\ x\in[\alpha,\beta)\\ \gamma-x & for\ x\in[\beta,\gamma)\\ 0 & elsewhere\end{cases} \qquad \ldots(5)$$

$$P_{i,2}(x)=\begin{cases}\frac{1}{2}(x-\alpha)^2 & for\ x\in[\alpha,\beta)\\ \frac{1}{4m^2}-\frac{1}{2}(\gamma-x)^2 & for\ x\in[\beta,\gamma)\\ \frac{1}{4m^2} & for\ x\in[\gamma,1)\\ 0 & elsewhere\end{cases} \qquad \ldots(6)$$

$$P_{i,3}(x)=\begin{cases}\frac{1}{6}(x-\alpha)^3 & for\ x\in[\alpha,\beta)\\ \frac{1}{4m^2}(x-\beta)-\frac{1}{6}(\gamma-x)^3 & for\ x\in[\beta,\gamma)\\ \frac{1}{4m^2}(x-\beta) & for\ x\in[\gamma,1)\\ 0 & elsewhere\end{cases} \qquad \ldots(7)$$

We also introduce the following notation: [8]

$$D_{i,v}=\int_0^1 P_{i,v}(x)\,dx \qquad \ldots(8)$$

3. Function approximation

Any square integrable function u(x) in the interval [0,1] can be expanded by a Haar series of infinite terms :

$$u(x)=\sum_{i=0}^{\infty} c_i\,h_i(x) \qquad i\in\{0\}\cup N \qquad \ldots(9)$$

Where the Haar coefficients c_i are determined as:

$$c_0=\int_0^1 u(x)h_0(x)dx \quad , \quad c_n=2^j\int_0^1 u(x)h_i(x)dx$$

$$i=2^j+k, \quad j\geq 0, \quad 0\leq k<2^j, \quad x\in[0,1)$$

Such that the following integral square error ε is minimized:

$$\varepsilon=\int_0^1\left[u(x)-\sum_{i=0}^{m-1}c_ih_i(x)\right]^2 dx, \quad m=2^j, \quad j\in\{0\}\cup N$$

Usually the series expansion of (10) contains infinite terms for smooth u(x). If u(x) is piecewise constant by itself, or may be approximation as piecewise constant during each subinterval, then u(x) will be terminated at finite m terms, that is:

$$u(x)=\sum_{i=0}^{m-1} c_i\,h_i(x)=c_{(m)}^T\,h_{(m)}(x) \qquad (10)$$

Where the coefficients $c_{(m)}^T$ and the Haar function vector $h_{(m)}(x)$ are defined as:

$c_{(m)}^T=[c_0,\ c_1,\ ....,\ c_{m-1}]$ And $h_{(m)}(x)=[h_0(x),h_1(x),......,h_{m-1}(x)]^T$

Where T means transpose. [6]

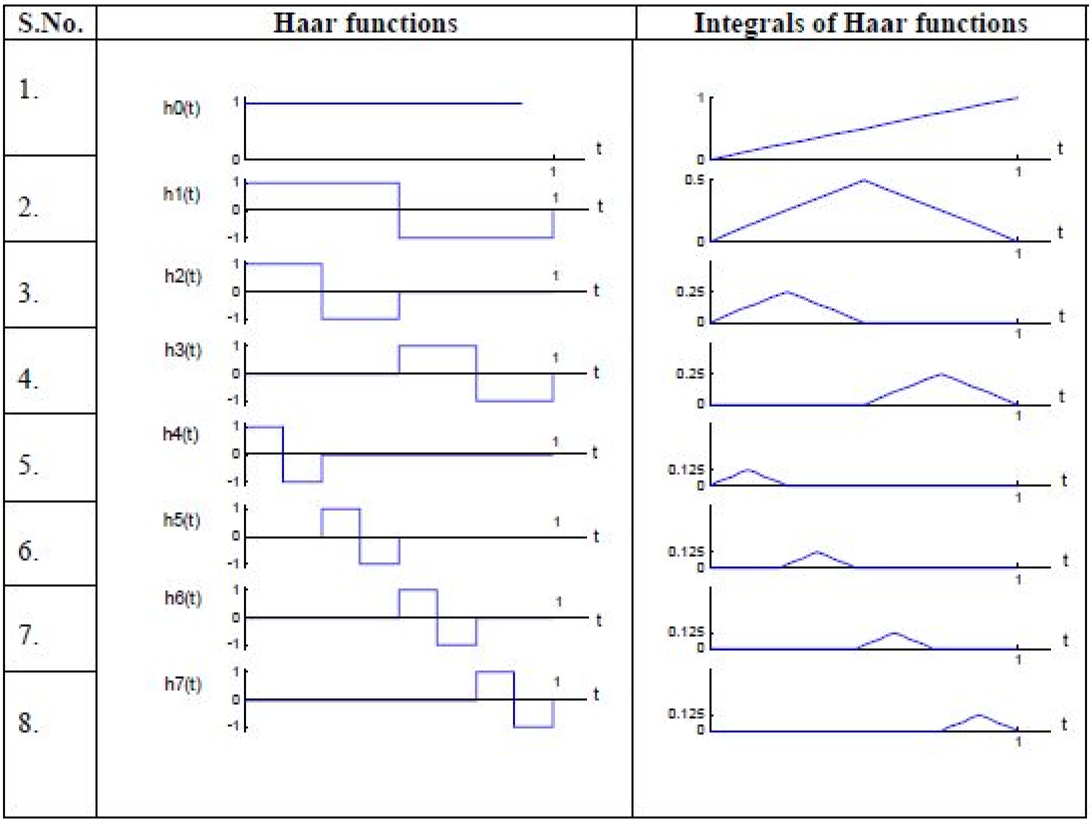


Fig. 1. First eight Haar functions [6]

4. Mathematical Model

Let us consider the nonlinear third-order korteweg-de Vries-Burger's (KdVB) equation which has the form [5]:

$$\frac{\partial u}{\partial t_*} + \varepsilon u \frac{\partial u}{\partial x_*} - \nu \frac{\partial^2 u}{\partial x_*^2} + \mu \frac{\partial^3 u}{\partial x_*^3} = 0 \qquad \ldots(11)$$

With the initial and boundary conditions:

$$\left.\begin{array}{ll} u(a,t_*) = \beta_1 & u(b,t_*) = \beta_2 \\ \frac{\partial u}{\partial x_*}(a,t_*) = \frac{\partial u}{\partial x_*}(b,t_*) = 0 & \\ \frac{\partial^2 u}{\partial x_*^2}(a,t_*) = \frac{\partial^2 u}{\partial x_*^2}(b,t_*) = 0 & \end{array}\right\} \quad t_* \geq 0 \qquad \ldots(12)$$

$$u(x_*,0) = f(x_*) \qquad a \leq x_* \leq b$$

where ε, ν and μ are positive parameters. ε is the coefficient of nonlinear terms, ν is the viscosity coefficient and μ is the coefficient of the dispersive term.

Since the Haar wavelets are defined for $x \in [0,1]$, we must first normalize equation (11) and initial-boundary conditions (12) in regard to x.
We changing the variables [9]:

$$x = \frac{1}{L}(x_* - a), \qquad t = t_* - 0, \qquad L = b - a$$

Then equation (11) and (12) becomes:

$$\frac{\partial u}{\partial t} + \frac{\varepsilon}{L} u \frac{\partial u}{\partial x} - \frac{v}{L^2}\frac{\partial^2 u}{\partial x^2} + \frac{\mu}{L^3}\frac{\partial^3 u}{\partial x^3} = 0 \qquad \ldots(13)$$

With the initial and boundary conditions:

$$\left.\begin{array}{l} u(0,t) = \beta_1 \qquad u(1,t) = \beta_2 \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 \\ \frac{\partial^2 u}{\partial x^2}(0,t) = \frac{\partial^2 u}{\partial x^2}(1,t) = 0 \end{array}\right\} \quad t \geq 0 \qquad \ldots(14)$$

$$u(x,0) = f(Lx + a) \qquad 0 \leq x \leq 1$$

In the paper, $\cdot \to \frac{\partial u}{\partial t}$ *and* $' \to \frac{\partial u}{\partial x}$, means differentiation with respect to (t) and (x) respectively.
Let us divide the interval (0,T] into N equal parts of length $\Delta t = T / N$ and denote to $t_s = (s-1)\Delta t$ s=1,2,…,N.

We assume that $u^{\bullet'''}(x,t)$ can be expanded in terms of Haar wavelets as follows:

$$u^{\bullet'''}(x,t) = \sum_{i=0}^{m-1} c_s(i) h_i(x) = c_m^T h_m(x) \qquad t \in (t_s, t_{s+1}] \qquad \ldots(15)$$

Where the row vector $c_{(m)}^T$ is constant in the subinterval $t \in (t_s, t_{s+1}]$.
Integrating (15) with respect to (t) from (t_s) to (t) and third with respect to (x) from (0) to (x) , we obtain:

$$u'''(x,t) = (t - t_s) c_m^T h_m(x) + u'''(x, t_s) \qquad \ldots(16)$$

$$u''(x,t) = (t - t_s) c_m^T P_{i,1}(x) + [u''(x,t_s) - u''(0,t_s)] + u''(0,t) \qquad \ldots(17)$$

$$u'(x,t) = (t - t_s) c_m^T P_{i,2}(x) + [u'(x,t_s) - u'(0,t_s)] + x[u''(0,t) - u''(0,t_s)] + u'(0,t) \qquad \ldots(18)$$

$$u\ (x,t) = (t - t_s) c_m^T P_{i,3}(x) + [u\ (x,t_s) - u\ (0,t_s) - x u'(0,t_s)] + \frac{x^2}{2}[u''(0,t\ ) - u''(0,t_s)] + x u'(0,t\ ) + u(0,t) \quad .(19)$$

Now the differential of equation (19) with respect (t), we get:

$$u^{\bullet}(x,t) = c_m^T P_{i,3}(x) + \frac{x^2}{2} u^{\bullet''}(0,t\ ) + x u^{\bullet'}(0,t\ ) + u^{\bullet}(0,t) \quad \ldots(20)$$

We can be reduce the boundary condition $u^{\bullet''}(0,t\ )$, $u^{\bullet'}(0,t\ )$ and $u^{\bullet}(0,t)$ in equation (20) by using the finite difference method , we get:

$$u^{\bullet}(0,t) = \frac{u(0,t) - u(0,t_s)}{(t - t_s)}$$

Then equation (20) becomes:

$$u^{\bullet}(x,t) = c_m^T P_{i,3}(x) + \frac{x^2}{2}\left[\frac{u''(0,t\ ) - u''(0,t_s)}{(t - t_s)}\right] + x\left[\frac{u'(0,t\ ) - u'(0,t_s)}{(t - t_s)}\right] + \left[\frac{u(0,t) - u(0,t_s)}{(t - t_s)}\right] \quad \ldots(21)$$

Now, by substitute equations (16)-(21) in equation (13), we get:

$$c_m^T P_{i,3}(x) - \frac{v\Delta t}{L^2} c_m^T P_{i,1}(x) + \frac{\mu\Delta t}{L^3} c_m^T h_m(x) = -\frac{\varepsilon}{L} u(x,t_s) u'(x,t_s) - \frac{x^2}{2\Delta t}\left[u''(0,t\ ) - u''(0,t_s)\right] - \frac{x}{\Delta t}\left[u'(0,t\ ) - u'(0,t_s)\right] - \frac{1}{\Delta t}\left[u(0,t) - u(0,t_s)\right] + \frac{v}{L^2} u''(x,t_s) + \frac{v}{L^2}\left[u''(0,t\ ) - u''(0,t_s)\right] - \frac{\mu}{L^3} u'''(x,t_s) \quad ...(22)$$

The Haar coefficients vector $c_{(m)}$ is calculated from the system of linear equations (22). The solution of the problem is found according to (19).

5. Reducing of the order boundary conditions:

We can be reducing of order boundary conditions used in equations (16)-(21) by using the boundary condition at x=1 and notation (8) instead of the derivatives $u''(0,t)$ and $u''(0,t_s)$.

The values of unknown term $u''(0,t)$ and $u''(0,t_s)$ can be calculated by integrating equation (17) from 0 to 1 and is given by:

$$\int_0^1 u''(x,t) = \int_0^1 (t-t_s)\, c_m^T\, P_{i,1}(x)\, dx + \int_0^1 u''(x,t_s)\, dx + \int_0^1 [u''(0,t) - u''(0,t_s)]\, dx$$

$$\Rightarrow u'(1,t) - u'(0,t) = (t-t_s)\, c_m^T\, D_{i,1}(x) + [u'(1,t_s) - u'(0,t_s)] + [u''(0,t) - u''(0,t_s)]$$

$$\Rightarrow [u''(0,t) - u''(0,t_s)] = -(t-t_s)\, c_m^T\, D_{i,1}(x) + [u'(1,t) - u'(1,t_s)] - [u'(0,t) - u'(0,t_s)] \qquad \ldots(23)$$

Such that

$$D_{i,1} = P_{i,2}(1) = \begin{cases} \frac{1}{2}(1-\alpha)^2 & \text{for } x \in [\alpha,\beta) \\ \frac{1}{4m^2} - \frac{1}{2}(\gamma-1)^2 & \text{for } x \in [\beta,\gamma) \\ \frac{1}{4m^2} & \text{for } x \in [\gamma,1) \\ 0 & \text{elsewhere} \end{cases} \qquad \ldots(24)$$

By substitute equation (23) in equations (16)-(21), we get:

$$u'''(x,t) = (t-t_s)\, c_m^T\, h_m(x) + u'''(x,t_s) \qquad \ldots(25)$$

$$u''(x,t) = (t-t_s)\, c_m^T\, P_{i,1}(x) + u''(x,t_s) + \left[-(t-t_s)\, c_m^T\, D_{i,1}(x) + [u'(1,t) - u'(1,t_s)] - [u'(0,t) - u'(0,t_s)]\right] \qquad ..(26)$$

$$u'(x,t) = (t-t_s)\, c_m^T\, P_{i,2}(x) + u'(x,t_s) + [u'(0,t) - u'(0,t_s)] + x\left[-(t-t_s)\, c_m^T\, D_{i,1}(x) + [u'(1,t) - u'(1,t_s)] - [u'(0,t) - u'(0,t_s)]\right] \qquad (27)$$

$$u(x,t) = (t-t_s)\, c_m^T\, P_{i,3}(x) + u(x,t_s) + [u(0,t) - u(0,t_s)] + x[u'(0,t) - u'(0,t_s)] + \frac{x^2}{2}\left[-(t-t_s)\, c_m^T\, D_{i,1}(x) + [u'(1,t) - u'(1,t_s)] - [u'(0,t) - u'(0,t_s)]\right] \qquad .(28)$$

Now the differential of equation (28) with respect (t), we get:

$$u^{\bullet}(x,t)=c_m^T P_{i,3}(x)+\frac{x}{(t-t_s)}[u'(0,t)-u'(0,t_s)]+\frac{1}{(t-t_s)}[u\ (0,t)-u\ (0,t_s)]$$
$$+\frac{x^2}{2(t-t_s)}\left[-(t-t_s)c_m^T D_{i,1}(x)+[u'(1,t)-u'(1,t_s)]-[u'(0,t)-u'(0,t_s)]\right] \quad .(29)$$

Now , by substitute equations (25)-(29) in equation (13), we get:

$$c_m^T P_{i,3}(x)+\frac{x^2}{2\Delta t}\left[-\Delta t\, c_m^T D_{i,1}(x)+[u'(1,t)-u'(1,t_s)]-[u'(0,t)-u'(0,t_s)]\right]$$
$$+\frac{x}{\Delta t}\left[u'(0,t\ )-u'(0,t_s)\right]+\frac{1}{\Delta t}\left[u(0,t)-u(0,t_s)\right]-\frac{v\Delta t}{L^2}c_m^T P_{i,1}(x)-\frac{v}{L^2}u''(x,t_s)$$
$$-\frac{v}{L^2}\left[-\Delta t c_m^T D_{i,1}(x)+[u'(1,t)-u'(1,t_s)]-[u'(0,t)-u'(0,t_s)]\right]$$
$$+\frac{\mu\Delta t}{L^3}c_m^T h_m(x)+\frac{\mu}{L^3}u'''(x,t_s)=-\frac{\varepsilon}{L}u(x,t_s)u'(x,t_s)$$

Then

$$c_m^T P_{i,3}(x)-\frac{x^2}{2}c_m^T D_{i,1}(x)-\frac{v\Delta t}{L^2}c_m^T P_{i,1}(x)+\frac{v\Delta t}{L^2}c_m^T D_{i,1}(x)+\frac{\mu\Delta t}{L^3}c_m^T h_m(x)$$
$$=-\frac{\varepsilon}{L}u(x,t_s)u'(x,t_s)-\frac{x^2}{2\Delta t}\left[[u'(1,t)-u'(1,t_s)]-[u'(0,t)-u'(0,t_s)]\right]$$
$$-\frac{x}{\Delta t}\left[u'(0,t\ )-u'(0,t_s)\right]-\frac{1}{\Delta t}\left[u(0,t)-u(0,t_s)\right]+\frac{v}{L^2}u''(x,t_s)$$
$$+\frac{v}{L^2}\left[[u'(1,t)-u'(1,t_s)]-[u'(0,t)-u'(0,t_s)]\right]-\frac{\mu}{L^3}u'''(x,t_s) \quad ...(30)$$

The Haar coefficients vector $c_{(m)}$ is calculated from the system of linear equations (30). The solution of the problem is found according to (28).

6. Numerical results

In this section, we have solved Kdv-Burger's equation (13) with the initial-boundary conditions (14) by using two formula:

a-) we have solved equation (13) with the initial-boundary conditions (14) by using the equation (22) such that [5]:

$$\left.\begin{array}{ll} u(0,t\ )=1 & u(1,t\ )=0 \\ \frac{\partial u}{\partial x}(0,t\ )=\frac{\partial u}{\partial x}(1,t\ )=0 & \\ \frac{\partial^2 u}{\partial x^2}(0,t\ )=\frac{\partial^2 u}{\partial x^2}(1,t\ )=0 & \end{array}\right\} \quad t\ \geq 0 \quad \ldots(31)$$

$$u(x\ ,0)=\frac{12\,v^2}{\varepsilon\,\mu}\left[1-\frac{e^{\frac{2v(xL+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v(xL+a)}{\varepsilon\,\mu}}+E\right)^2}\right]\qquad 0\le x\ \le 1\qquad \ldots(32)$$

Now, by substitute the boundary condition (31) in equations (16)-(22) we get:

$$c_m^T\,P_{i,3}(x)-\frac{v\,\Delta t}{L^2}c_m^T\,P_{i,1}(x)+\frac{\mu\,\Delta t}{L^3}c_m^T\,h_m(x)=-\frac{\varepsilon}{L}u(x,t_s)\,u'(x,t_s)$$

$$+\frac{v}{L^2}u''(x,t_s)-\frac{\mu}{L^3}u'''(x,t_s)$$

Where

$$u\ \ (x,t_{s+1})=\Delta t\ c_m^T\,P_{i,3}(x)+u\ \ (x,t_s)$$

$$u'(x,t_{s+1})=\Delta t\ c_m^T\,P_{i,2}(x)+u'(x,t_s)$$

$$u^{\bullet}(x,t_{s+1})=c_m^T\,P_{i,3}(x)$$

$$u''(x,t_{s+1})=\Delta t\ c_m^T\,P_{i,1}(x)+u''(x,t_s)$$

$$u'''(x,t_{s+1})=\Delta t\ c_m^T\,h_m(x)+u'''(x,t_s)$$

This process is started with:

$$u(x\ ,t_s)=u(x\ ,0)=\frac{12\,v^2}{\varepsilon\,\mu}\left[1-\frac{e^{\frac{2v\,(xL+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v\,(xL+a)}{\varepsilon\,\mu}}+E\right)^2}\right]\qquad 0\le x\ \le 1$$

$$u'(x\ ,t_s)=u'(x\ ,0)=\frac{\partial}{\partial x}\left[\frac{12\,v^2}{\varepsilon\,\mu}\left[1-\frac{e^{\frac{2v\,(xL+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v\,(xL+a)}{\varepsilon\,\mu}}+E\right)^2}\right]\right]\qquad 0\le x\ \le 1$$

$$u''(x\ ,t_s)=u''(x\ ,0)=\frac{\partial^2}{\partial x^2}\left[\frac{12\,v^2}{\varepsilon\,\mu}\left[1-\frac{e^{\frac{2v\,(xL+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v\,(xL+a)}{\varepsilon\,\mu}}+E\right)^2}\right]\right]\qquad 0\le x\ \le 1$$

$$u'''(x\ ,t_s)=u'''(x\ ,0)=\frac{\partial^3}{\partial x^3}\left[\frac{12\,v^2}{\varepsilon\,\mu}\left[1-\frac{e^{\frac{2v\,(x\,L+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v\,(x\,L+a)}{\varepsilon\,\mu}}+E\right)^2}\right]\right]\qquad 0\le x\ \le 1$$

The exact solution of KdVB equation (13) in a closed form is given by [5]:

$$u(x\ ,t\ )=\frac{12\,v^2}{\varepsilon\,\mu}\left[1-\frac{e^{\frac{2v}{\varepsilon\,\mu}((x\,L+a)-w\,t)}}{\left(e^{\frac{v}{\varepsilon\,\mu}((x\,L+a)-w\,t)}+E\right)^2}\right]$$

Where $w=\dfrac{12v^2}{25\mu}$, E is a positive constant.

The value of the constant E is large to be in the neighborhood of the boundary conditions [5].

Results of the computer simulation are presented in table (1) where m=16 and table (2) where m=32 , here
$E=1000\,,\ \varepsilon=0.1\,,\ v=0.01\,,\ \mu=1\,,\ L=100, a=0, b=100\,,\ \Delta t=0.01\,,\ and\ t=1\,.$

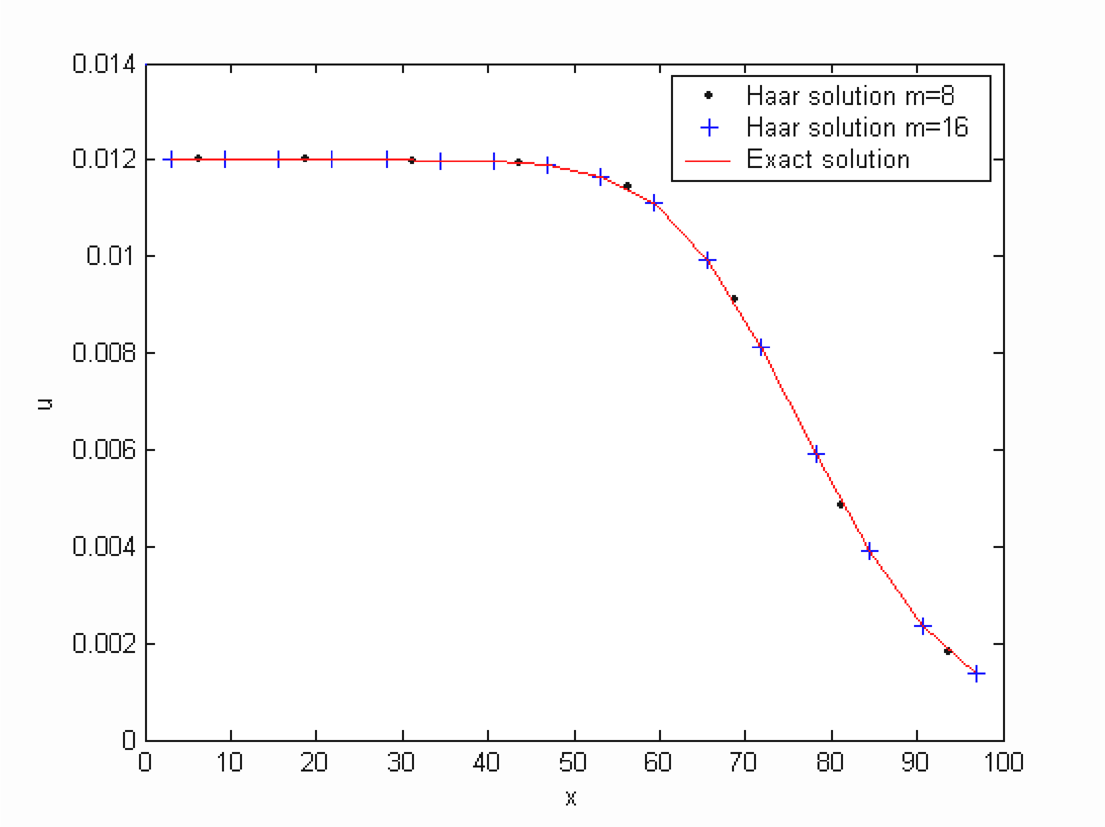
Table (1) Comparison of the numerical solution and the exact solution when m=16.

The value x	Wavelet solution	Exact solution	Absolute error
3.1250	0.01199997764226	0.01199997764242	1.5879e-013
9.3750	0.01199992214847	0.01199992214903	5.5916e-013
15.6250	0.01199972946932	0.01199972947127	1.9449e-012
21.8750	0.01199906349256	0.01199906349927	6.7108e-012
28.1250	0.01199678084055	0.01199678086345	2.2899e-011
34.3750	0.01198907690822	0.01198907698484	7.6625e-011
40.6250	0.01196379624936	0.01196379649701	2.4765e-010
46.8750	0.01188487863538	0.01188487938842	7.5303e-010
53.1250	0.01165870378318	0.01165870585304	2.0699e-009
59.3750	0.01109362558626	0.01109363045222	4.8660e-009
65.6250	0.00993794636355	0.00993795557229	9.2087e-009
71.8750	0.00810826334073	0.00810827671992	1.3379e-008
78.1250	0.00591798008513	0.00591799482872	1.4744e-008
84.3750	0.00389240546654	0.00389241818682	1.2720e-008
90.6250	0.00236381448149	0.00236382360458	9.1231e-009
96.8750	0.00136131972057	0.00136132550788	5.7873e-009

Table (2) Comparison of the numerical solution and the exact solution When m=32.

The value x	Wavelet solution	Exact solution	Absolute error
1.5625	0.01199998363630	0.01199998363639	8.9968e-014
4.6875	0.01199996945473	0.01199996945492	1.9388e-013
7.8125	0.01199994300063	0.01199994300102	3.8576e-013
10.9375	0.01199989368121	0.01199989368195	7.4184e-013
14.0625	0.01199980180339	0.01199980180479	1.4033e-012
17.1875	0.01199963082155	0.01199963082419	2.6310e-012
20.3125	0.01199931308204	0.01199931308694	4.9061e-012
23.4375	0.01199872376242	0.01199872377153	9.1102e-012
26.5625	0.01199763361276	0.01199763362961	1.6849e-011
29.6875	0.01199562420577	0.01199562423679	3.1018e-011
32.8125	0.01199193828139	0.01199193833817	5.6773e-011
35.9375	0.01198522110370	0.01198522120684	1.0313e-010
39.0625	0.01197308679413	0.01197308697962	1.8550e-010
42.1875	0.01195142203021	0.01195142235950	3.2929e-010
45.3125	0.01191333706306	0.01191333763773	5.7467e-010
48.4375	0.01184773395123	0.01184773493173	9.8050e-010
51.5625	0.01173765865149	0.01173766027914	1.6277e-009
54.6875	0.01155901673874	0.01155901934560	2.6069e-009
57.8125	0.01128083786318	0.01128084186427	4.0011e-009
60.9375	0.01086873624819	0.01086874208585	5.8377e-009
64.0625	0.01029276188437	0.01029276992137	8.0370e-009
67.1875	0.00953870290380	0.00953871328023	1.0376e-008
70.3125	0.00861857949441	0.00861859200431	1.2510e-008
73.4375	0.00757418134219	0.00757419540263	1.4060e-008
76.5625	0.00646994435853	0.00646995911073	1.4752e-008
79.6875	0.00537754295174	0.00537755746090	1.4509e-008
82.8125	0.00435942216286	0.00435943562643	1.3464e-008
85.9375	0.00345811147375	0.00345812335374	1.1880e-008
89.0625	0.00269370124351	0.00269371129444	1.0051e-008
92.1875	0.00206752746643	0.00206753568466	8.2182e-009
95.3125	0.00156849314067	0.00156849968137	6.5407e-009
98.4375	0.00117917815373	0.00117918325061	5.0969e-009

Fig. (2) Comparison of the numerical solutions when m=8 , m=16 and the exact solution



b-) we have solved equation (13) by using the equation (30) such that [5]:

$$\left.\begin{array}{ll} u(0,t)=1 & u(1,t)=0 \\ \dfrac{\partial u}{\partial x}(0,t)=\dfrac{\partial u}{\partial x}(1,t)=0 \end{array}\right\} \quad t \geq 0 \qquad \ldots(33)$$

$$u(x,0)=\frac{12v^2}{\varepsilon\,\mu}\left[1-\frac{e^{\frac{2v(xL+a)}{\varepsilon\mu}}}{\left(e^{\frac{v(xL+a)}{\varepsilon\mu}}+E\right)^2}\right] \qquad 0 \leq x \leq 1 \qquad \ldots(34)$$

Now, by substitute the boundary condition (33) in equations (25)-(30) we get:

$$c_m^T P_{i,3}(x)-\frac{x^2}{2}c_m^T D_{i,1}(x)-\frac{v\Delta t}{L^2}c_m^T P_{i,1}(x)+\frac{v\Delta t}{L^2}c_m^T D_{i,1}(x)+\frac{\mu\Delta t}{L^3}c_m^T h_m(x)$$

$$=-\frac{\varepsilon}{L}u(x,t_s)u'(x,t_s)+\frac{v}{L^2}u''(x,t_s)-\frac{\mu}{L^3}u'''(x,t_s) \qquad \ldots(30)$$

Where

$$u\ (x,t_{s+1}) = \Delta t\ c_m^T\, P_{i,3}(x) + u\ (x,t_s) - \frac{x^2 \Delta t}{2} c_m^T\, D_{i,1}(x)$$

$$u'(x,t_{s+1}) = \Delta t\ c_m^T\, P_{i,2}(x) + u'(x,t_s) - \Delta t c_m^T\, D_{i,1}(x)$$

$$u^{\bullet}(x,t_{s+1}) = c_m^T\, P_{i,3}(x) - \frac{x^2}{2} c_m^T\, D_{i,1}(x)$$

$$u''(x,t_{s+1}) = \Delta t\ c_m^T\, P_{i,1}(x) + u''(x,t_s) - \Delta t c_m^T\, D_{i,1}(x)$$

$$u'''(x,t_{s+1}) = \Delta t\ c_m^T\, h_m(x) + u'''(x,t_s)$$

This process is started with:

$$u(x\ ,t_s) = u(x\ ,0) = \frac{12\,v^2}{\varepsilon\,\mu}\left[1 - \frac{e^{\frac{2v\,(xL+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v\,(xL+a)}{\varepsilon\,\mu}} + E\right)^2}\right] \qquad 0 \le x\ \le 1$$

$$u'(x\ ,t_s) = u'(x\ ,0) = \frac{\partial}{\partial x}\left[\frac{12\,v^2}{\varepsilon\,\mu}\left[1 - \frac{e^{\frac{2v\,(xL+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v\,(xL+a)}{\varepsilon\,\mu}} + E\right)^2}\right]\right] \qquad 0 \le x\ \le 1$$

$$u''(x\ ,t_s) = u''(x\ ,0) = \frac{\partial^2}{\partial x^2}\left[\frac{12\,v^2}{\varepsilon\,\mu}\left[1 - \frac{e^{\frac{2v\,(xL+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v\,(xL+a)}{\varepsilon\,\mu}} + E\right)^2}\right]\right] \qquad 0 \le x\ \le 1$$

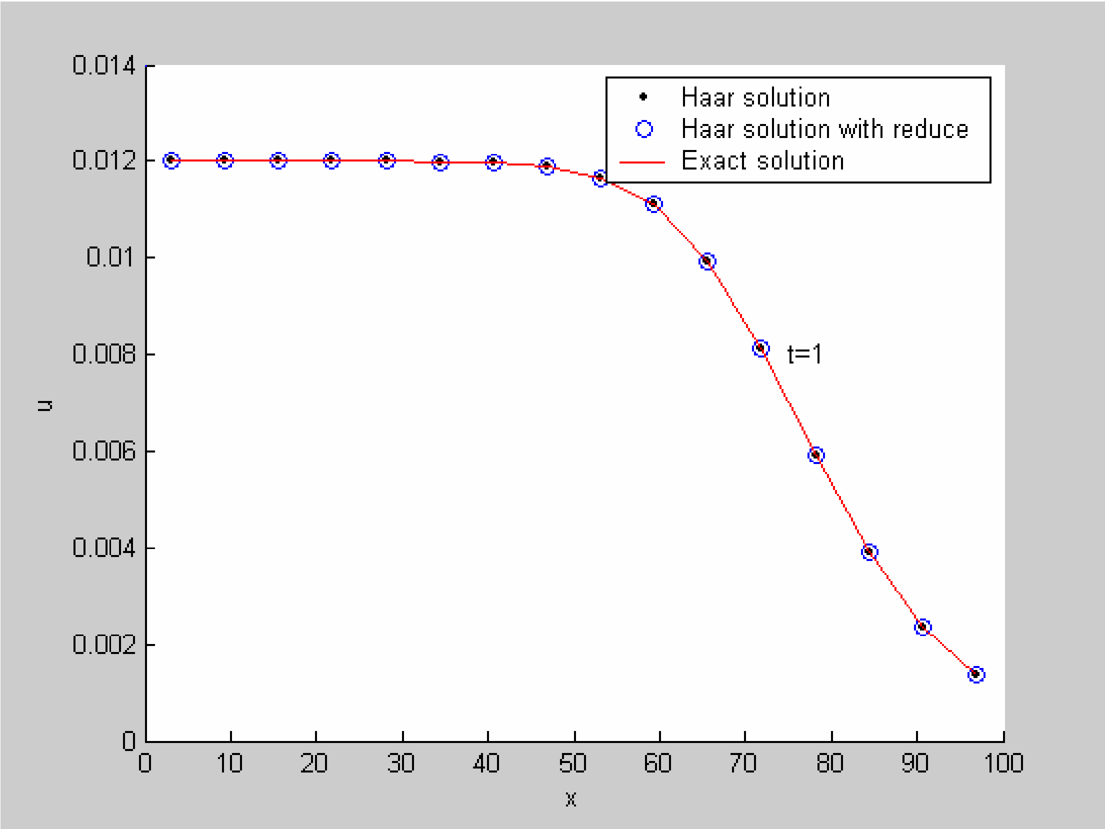
$$u'''(x\ ,t_s) = u'''(x\ ,0) = \frac{\partial^3}{\partial x^3}\left[\frac{12\,v^2}{\varepsilon\,\mu}\left[1 - \frac{e^{\frac{2v\,(xL+a)}{\varepsilon\,\mu}}}{\left(e^{\frac{v\,(xL+a)}{\varepsilon\,\mu}} + E\right)^2}\right]\right] \qquad 0 \le x\ \le 1$$

Results of the computer simulation are presented in table (3) where m=32 , here
$E = 1000$, $\varepsilon = 0.1$, $v = 0.01$, $\mu = 1$, $L = 100, a = 0, b = 100$, $\Delta t = 0.01$, *and* $t = 1$.

Table (3) Comparison of the numerical solution and the exact solution When m=32.

The value x	Wavelet solution	Exact solution	Absolute error
1.5625	0.01199998363580	0.01199998363639	5.8346e-013
4.6875	0.01199996945084	0.01199996945492	4.0790e-012
7.8125	0.01199994298994	0.01199994300102	1.1079e-011
10.9375	0.01199989366028	0.01199989368195	2.1668e-011
14.0625	0.01199980176882	0.01199980180479	3.5972e-011
17.1875	0.01199963076993	0.01199963082419	5.4256e-011
20.3125	0.01199931300994	0.01199931308694	7.7001e-011
23.4375	0.01199872366644	0.01199872377153	1.0509e-010
26.5625	0.01199763348949	0.01199763362961	1.4012e-010
29.6875	0.01199562405180	0.01199562423679	1.8499e-010
32.8125	0.01199193809330	0.01199193833817	2.4486e-010
35.9375	0.01198522087809	0.01198522120684	3.2875e-010
39.0625	0.01197308652757	0.01197308697962	4.5206e-010
42.1875	0.01195142171929	0.01195142235950	6.4020e-010
45.3125	0.01191333670441	0.01191333763773	9.3332e-010
48.4375	0.01184773354122	0.01184773493173	1.3905e-009
51.5625	0.01173765818723	0.01173766027914	2.0919e-009
54.6875	0.01155901621628	0.01155901934560	3.1293e-009
57.8125	0.01128083727934	0.01128084186427	4.5849e-009
60.9375	0.01086873559952	0.01086874208585	6.4863e-009
64.0625	0.01029276116747	0.01029276992137	8.7539e-009
67.1875	0.00953870211526	0.00953871328023	1.1165e-008
70.3125	0.00861857863082	0.00861859200431	1.3373e-008
73.4375	0.00757418040013	0.00757419540263	1.5002e-008
76.5625	0.00646994333460	0.00646995911073	1.5776e-008
79.6875	0.00537754184253	0.00537755746090	1.5618e-008
82.8125	0.00435942096495	0.00435943562643	1.4661e-008
85.9375	0.00345811018373	0.00345812335374	1.3170e-008
89.0625	0.00269369985798	0.00269371129444	1.1436e-008
92.1875	0.00206752598194	0.00206753568466	9.7027e-009
95.3125	0.00156849155396	0.00156849968137	8.1274e-009
98.4375	0.00117917646057	0.00117918325061	6.7900e-009

Fig. (3) Comparison of the numerical solutions and the exact solution When m=16.



7. Conclusions

In this paper, solving the nonlinear third-order KdV-Burger's equation by using Haar wavelet method was discussed. The fundamental idea of Haar wavelet method is to convert the differential equation into a group of algebra equations which involves a finite number of variables.

We found that Haar wavelet had good approximation effect by comparing with exact solution of KdV-Burger's equation at the same time. The bigger resolution J is obtained more accurate approximation in the solution, as note in table (1) when m=16 and the table (2) when m=32. Also when m=64 , m=128 , …, we can obtain the results closer to the exact values. Figure (2) shows the Comparison between numerical solution when m=8 and m=16 with exact solution.

We have also been reducing the boundary conditions used in the solution by using the finite differences method with respect to time and by using the notation (9) when x=L respect to space and the results were a high resolution as note in table (3) and Figure (3). Matlab language is using in find the results and figure draw, it's characteristic at high accuracy and large speed.

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