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Transformation Linear Membership Function by Using the Modified S- Curve

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Abstract:

In this paper, we concentrate on two cases of fuzzy linear programming problems (FLPP): LPP with coefficient of objective function and coefficient of available resources and constraint are trapezoidal and triangular of fuzzy numbers respectively. Though by using α- cut ,fuzzy numbers (fuzzy triangular and fuzzy trapezoidal) can be transformed into interval numbers and by taking one type of non-linear membership function this meant logistic function, we propose here the "modified S-curve function by simplex method ".

Key words: Fuzzy linear programming, fuzzy triangular, fuzzy trapezoidal, fuzzy number, modified S-curve function, fuzzy simplex method.

تحويل دالة الانتماء الخطية باستخدام منحني S المعدلة

م. شورش عمر عبدالله   د.عبدالرحيم خلف راهي

المستخلص

في هذا البحث ، نركز على إثنين من مشاكلِ البرمجة الخطية الضبابية(FLPP): LPP بمعاملِ (دالة الهدف) ومعامل (موارد متاحة وقيد) رباعي ومثلثية من الأعداد الضبابية على التوالي. مع ذلك بإستعمال α - القطع، (رباعية ضبابية ومثلثية ضبابية)بالاعداد ضبابية تتحول الى الفترة المغلقة ، الواحد من نوعِ دالة الانتماء غير الخطية يستخدم هنا دالة اللوجستيكية، نستخدم في هذ البحث "منحني S المعدلة بالطريقة Simplex".

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Introduction:

Fuzzy sets theory was first introduced by "Prof. Lotfi Zadeh" in 1965. Successful applications of fuzzy sets theory on controller systems in 80 decades led to the development of this theory in other fields such as simulation, artificial intelligence, operations research, management and many industrial applications.

In the real world, many applied problems are modeled as mathematical programming and it may be necessary to formulate these models with uncertainty. Many problems of these kinds are linear programming with fuzzy parameters. The first formulation of fuzzy linear programming is proposed by Zimmermann (1978).

After the pioneering works on fuzzy linear programming, several kinds of Fuzzy Linear Programming Problems (FLPP) have appeared in the literature and different methods have been proposed to solve such problems.

One convenient approach for solving the (FLPP) is based on the concept of comparison of fuzzy numbers by using the simplex method. Usually in such methods authors define a crisp model which is equivalent to the (FLPP) and then use optimal solution of the model as the optimal solution of the (FLPP).

We have extended their results and have established the simplex problem of the linear programming with modified S-curve fuzzy variables problem and hence deduce some simplex results. These results will be useful for post optimality analysis.

1- Fuzzy Set: [1],[3]

Let X be a universe set, whose generic elements are denoted x ,membership in a subset A of X is often viewed as a characteristic function μ_A from X to {0 ,1} such that

$$\mu_A(x) = \begin{cases} 1 & iff \quad x \in A \\ 0 & iff \quad x \notin A \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

If the set {0, 1} is allowed to be the real interval [0, 1], A is called a fuzzy set (Zadeh, 1965[10]). μ_A (x) is the grade of membership of x in A . The closer the value of μ_A (x) is to 1, the more x belong to A.

There is another expression of fuzzy sets:-

1- A is completely characterized by the set of pairs A= {(x, μ_A (x)), x $\in$X}.
2- When X is a finite set $\{x_1,...,x_n\}$, a fuzzy set on X is expressed as:

$$A = \mu_A(x_1)/x_1 + ... + \mu_A(x_n)/x_n = \sum_{i=1}^{n} \mu_A(x_i)/x_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

3-When X is infinite, the following is written:

$$A=\int_X \mu_A(x)/x \qquad \text{……………….. (3)}$$

2- Membership Function:[1],[2],[3]

A subset A of universal set X is called membership function if we can defined by the formally:

$$\mu_A(x)=\begin{cases}1 & \text{iff} \quad x\in A\\ 0 & \text{iff} \quad x\notin A\end{cases}$$

; We can say that the function μ_A maps the elements in the universal set X to the set {0, 1}, $\mu_A : X \to \{0,1\}$.

3- Support of a Fuzzy Set:[3]

Let A be a fuzzy set in X .Then the support of A, denoted by S (A) is the crisp set given by $S(A)=\{x\in X : \mu_A(x)>0\}$. ……………. (4)

4- Normal Fuzzy Set:[2],[5]

The height of A is defined as h(A) = sup $\mu_A(x)$, if h(A) =1 then the fuzzy set A is called a normal fuzzy set ; otherwise, it is called sub normal ,if 0< h(A) <1 , then the sub normal fuzzy set A can be normalized , i.e. ,it can be made normal by redefining the membership function as $\mu_A(x)$ / h(A) , $x\in X$.

5- Convex Set:[4]

Assuming universal set X is defined in n dimensional Euclidean Vector space $\Re^n$. If all the α - cut sets are convex, the fuzzy set with these α - cut sets is convex, in other words, if the relation: $\mu_A(A)\geq Min[\mu_A(x),\mu_A(y)]$

Where $\forall x,y\in\Re^n and \lambda x+(1-\lambda)y\in\Re^n;0\leq\lambda\leq 1$.

Holds, the fuzzy set A is convex.

6- Fuzzy Number:[2],[4],[5]

Let A be a fuzzy set in R, then A is called a fuzzy number, if satisfies the following conditions:-

1- A is normal.
2- A is convex.
3- μ_A is upper semi continuous .
4- The support of A is bounded.

7- α - cut:[6],[7]

Let A be a fuzzy set in X and $\alpha\in$ (0, 1], the α - cut of the fuzzy set A is the crisp set A_α given by:

$$A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}. \qquad \text{……………… (5)}$$

8- Linear Membership Function:

Linear membership function represents the value of membership elements into fuzzy set the form of a straight line, which are important shapes of linear membership functions:

a- Triangular Function: [3],[4]

It is the function to user rumor; it has three parameters α, β, γ and can be expressed as follows $\mu_T : X \to [0,1]$:

$$\mu_T(x,\alpha,\beta,\gamma) = \begin{cases} 0 & \text{if} \quad x < \alpha \\ \dfrac{x-\alpha}{\beta-\alpha} & \text{if} \quad \alpha \leq x \leq \beta \\ \dfrac{\gamma-x}{\gamma-\beta} & \text{if} \quad \beta \leq x \leq \gamma \\ 0 & \text{if} \quad x > \gamma \end{cases} \qquad \text{……………… (6)}$$

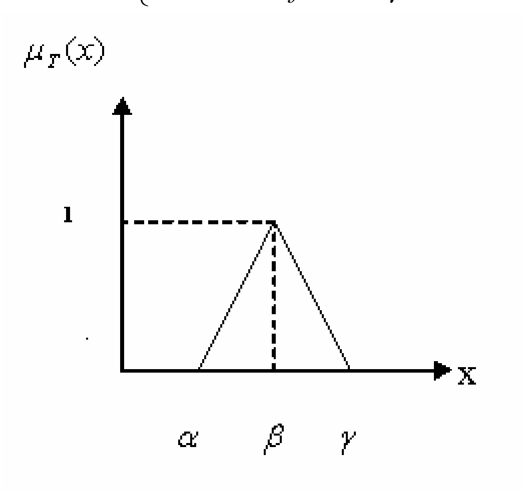

Fig (2-1) Triangular function

b- Trapezoidal function: [3],[4],[6]

It is the function which contains four parameters $\alpha, \beta, \gamma, \delta$ and can be expressed as follows: $\mu_{Tr} : X \to [0,1]$

$$\mu_{Tr}(x,\alpha,\beta,\gamma,\delta)=\begin{cases} 0 & if \quad x<\alpha \\ \dfrac{x-\alpha}{\beta-\alpha} & if \ \alpha \le x \le \beta \\ 1 & if \ \beta \le x \le \gamma \\ \dfrac{\delta-x}{\delta-\gamma} & if \ \gamma \le x \le \delta \\ 0 & if \quad x>\delta \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

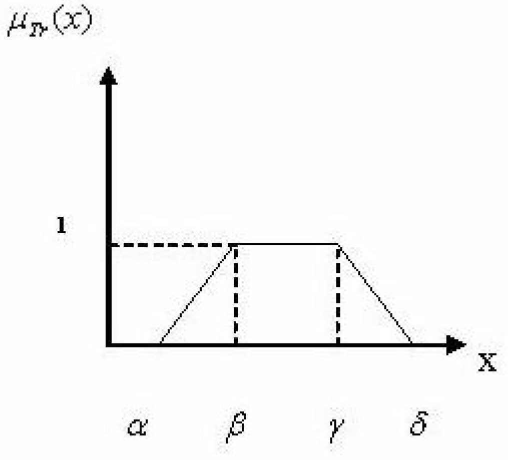

Fig (2-2) Trapezoidal function

9- Non Linear Membership Function:

There are many types of non linear membership function in this take one type is follows:

a- Logistic Functions: [5],[6],[7]

The logistic function for the non – linear membership function and α (0< α <∞) is a fuzzy parameter which measures the degree of vagueness. If $\alpha \to 0$ indicates crisp, the fuzziness is very large if $\alpha \to \infty$. This function can be expressed as:

$$\mu(x)=\begin{cases} 1 & x<x^a \\ \dfrac{w}{1+ue^{\alpha x}} & x^a<x<x^b \\ 0 & x>x^b \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where μ (x) : is the degree of member ship of x ,(0< μ (x) < 1)
x^a ,x^b : the upper and the lower value of x.
α : fuzzy parameter that determines the shape of the membership function , (0< α <∞).
u , w : constants .

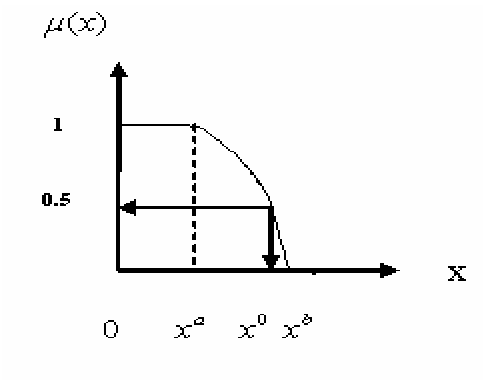

Fig (Logistic function)

There are many types of logistic function in this take one type is follows:

i - Modified S – Curve Function:

This function is a special case of logistic function of the specified values (α , w ,u) and 0.001 $\leq \mu(x) \leq$ 0.999. It can be defined:

$$\mu(x)=\begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \dfrac{w}{1+u\,e^{\alpha[\frac{x-x^a}{x^b-x^a}]}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

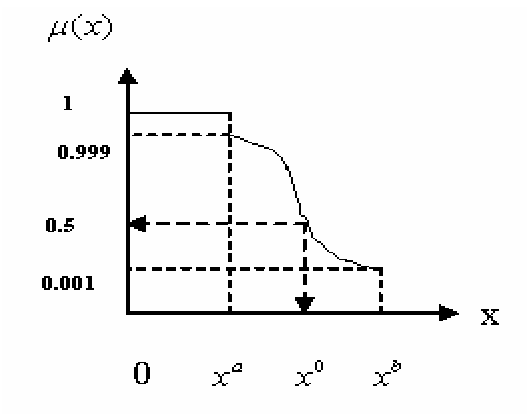

Fig (Modified S- curve function)

10- Fuzzy Linear Programming Model:

The formulation of FLP model has the following form:

i- All coefficients are fuzzy

$$\begin{aligned} &Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j \\ &s.t \quad \sum_{j=1}^{n} \tilde{a}_{ij}\, x_j \leq \tilde{b}_i \quad ,i = 1,\ldots,m\ ,\ x_j \geq 0; j = 1,\ldots,n \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Where $\tilde{Z}$: the value of the objective function.

x_j : the vector of decision variables.

$\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_j$: the fuzzy coefficients of fuzzy LP model.

ii- One coefficient is fuzzy

$$Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j$$
$$s.t \quad \sum_{j=1}^{n} a_{ij}\, x_j \le b_i \quad , i = 1,\ldots,m\ ,\ x_j \ge 0 \qquad \ldots\ldots\ldots\ldots\ldots (11)$$

iii- Double coefficients are fuzzy

$$Max\ \ Z = \sum_{j=1}^{n} c_j x_j$$
$$s.t \quad \sum_{j=1}^{n} \tilde{a}_{ij}\, x_j \le \tilde{b}_i \quad , i = 1,\ldots,m\ ,\ x_j \ge 0 \qquad \ldots\ldots\ldots\ldots\ldots (12)$$

11- Fuzzy Simplex Approach:[6],[7]

This method is used for solving fuzzy LPP for any interval as the coefficient fuzzy model takes various aspects and depends on calculating fuzzy membership function of coefficient and from calculating fuzzy coefficient model on the basis of membership function for all coefficient from coefficients model, it can be represented by the following steps:

1- The membership function is selected (linear or non- linear) of fuzzy coefficient programming models and calculate the value of fuzzy coefficient and in that method a type from many types of non linear logistic is used, modified S- curve membership function that depends on two factors , first is the accepted level (degree) Mu ($0.0010 \le Mu \le 0.9990$), and the other is the fuzzy factor (α) ,($1 \le \alpha \le 42$) and the coefficient of FLP is calculated model related the both Mu and α and fuzzy factor , where the membership function of the coefficient of objective function in FLPP as follows :

a- Membership $\tilde{c}_j$

$$\mu_{c_j} = \begin{cases} 1 & c_j < c_j^{\,a} \\ 0.999 & c_j = c_j^{\,a} \\ \dfrac{w}{1 + u\, e^{\alpha\left[\frac{c_j - c_j^{\,a}}{c_j^{\,b} - c_j^{\,a}}\right]}} & c_j^{\,a} < c_j < c_j^{\,b} \\ 0.001 & c_j = c_j^{\,b} \\ 0 & c_j > c_j^{\,b} \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots (13)$$

Where μ_{c_j} : membership of $\tilde{c}_j$;

$c_j^{\,b}$, $c_j^{\,a}$: the upper and the lower of fuzzy objective function .

And to obtain the value $\tilde{c}_j$.

$$\mu_{\tilde{c}_j} = \frac{w}{1 + u e^{\alpha[\frac{c_j - c_j^a}{c_j^b - c_j^a}]}}$$

$$e^{\alpha[\frac{c_j - c_j^a}{c_j^b - c_j^a}]} = \frac{1}{u}[\frac{w}{\mu_{c_j}} - 1]$$

$$\alpha\ [\frac{c_j - c_j^a}{c_j^b - c_j^a}] = \ln\frac{1}{u}[\frac{w}{\mu_{c_j}} - 1]$$

$$c_j = c_j^a + [\frac{c_j^b - c_j^a}{\alpha}]\ln\frac{1}{u}[\frac{w}{\mu_{c_j}} - 1]$$

Since c_j is fuzzy

$$\tilde{c}_j = c_j^a + [\frac{c_j^b - c_j^a}{\alpha}]\ln\frac{1}{u}[\frac{w}{\mu_{c_j}} - 1] \qquad \text{…………. (14)}$$

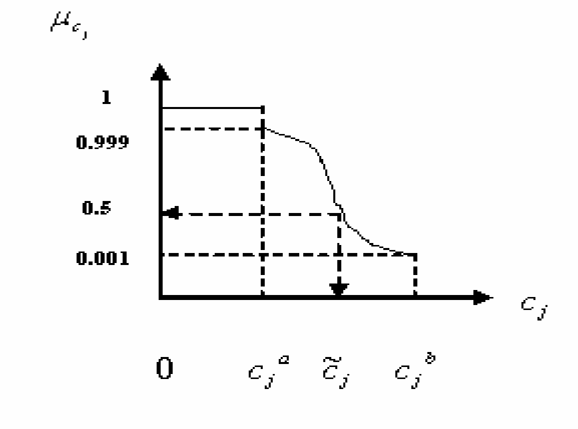

Fig (Membership μ_{c_j} *of* c_j)

Similarly the value b_i fuzzy and value a_{ij} fuzzy can be obtained.

b- Fuzzy b_i

$$\tilde{b}_i = b_i^a + [\frac{b_i^b - b_i^a}{\alpha}]\ln\frac{1}{u}[\frac{w}{\mu_{b_i}} - 1] \qquad \text{………….. (15)}$$

c- Fuzzy a_{ij}

$$\tilde{a}_{ij} = a_{ij}^a + [\frac{a_{ij}^b - a_{ij}^a}{\alpha}]\ln\frac{1}{u}[\frac{w}{\mu_{a_{ij}}} - 1] \qquad \text{..……….. (16)}$$

2 – The fuzzy LPP can be written by the coefficient that obtained as above.

Max $Z = \tilde{c}x$

S .t $\tilde{A}x = \tilde{b}$

3 - Consider the FLP

Ax=b and x $\geq$ 0,

Where A is m * n matrix and b is an m vector .Now, suppose that rank (A, b) = rank (A) =m. Partition after possibly rearranging the columns of A as [B, N], B m *m is non zero square matrix of rank (B) = (m) .The point $x = (x_B^T,\ \ x_N^T)^T$ where $x_B = B^{-1}b, x_N = 0$ is called a basic solution consider the FLP problem as is defined:

$$\text{Max } \tilde{z} = \tilde{c}_B x_B + \tilde{c}_N x_N$$
$$\text{S .t } B x_B + N x_N = b \qquad \ldots\ldots\ldots\ldots\ (17)$$
$$x_B , x_N \geq 0$$

Where B: basic matrix.

N: non basic matrix.

x_B: Component of basic variables.

x_N: Component of non basic variables.

$$x_B + B^{-1} N x_N = B^{-1} b$$
$$x_B = B^{-1} b - B^{-1} N x_N$$

So that the objective functions as:

$$\tilde{z} + (\tilde{c}_B B^{-1} N - \tilde{c}_N) x_N = \tilde{c}_B B^{-1} b$$
$$\tilde{z} = \tilde{c}_B (B^{-1} b + B^{-1} N x_N) + \tilde{c}_N x_N \qquad \ldots\ldots\ldots\ldots\ldots(18)$$

A basic feasible solution to the system, where $x_B = B^{-1}\tilde{b}$, $and\, x_N = 0$.

Then, the corresponding fuzzy objective values $\tilde{z} = \tilde{c}_B B^{-1} b$, and if $x_B \geq 0$ the basic feasible solution of the system,

$$\tilde{z} = \tilde{c}_B x_B$$

And then the above FNLP problem is rewritten in the following tableau format:

Basis	$\tilde{z}$	x_B	x_N	R.H.S
$\tilde{z}$	1	0	$\tilde{c}_B B^{-1} N - \tilde{c}_N$	$\tilde{c}_B B^{-1} b$
x_B	0	1	$B^{-1} N$	$B^{-1} b$

The above tableau gives us all the required information to proceed with the simplex method. The fuzzy cost row in the above tableau is $\tilde{\gamma} = (\tilde{c}_B B^{-1} a_j - \tilde{c}_j)_{j \neq B_i}$, which consists of the $\tilde{\gamma}_j = \tilde{z}_j - \tilde{c}_j$ for the non basic variables.

According to the optimality condition for these problems one would be at the optimal solution if $\tilde{\gamma} \geq 0$, for all $j \neq B_i$. On the other hand, if $\tilde{\gamma}_k < 0, for\, all\, k \neq B_i$ then $x_{B_r}\ with\ x_k$ may be exchanged .Then the vector $\gamma_k = B^{-1} a_k$ is computed. If $\gamma_k \leq 0, then\ x_k$ can be increased indefinitely, and then the optimal objective is unbounded.

Example 1: [6]

$$
\begin{aligned}
&Max\ \ \widetilde{Z} = (5,8,2,5)x_1 + (6,10,2,6)x_2 \\
&s.t \quad 2x_1 + 3x_2 \le 6 \\
&\qquad 5x_1 + 4x_2 \le 10 \\
&\qquad x_1, x_2 \ge 0
\end{aligned}
$$

Where (5, 8, 2, 5) and (6, 10, 2, 6) are fuzzy trapezoidal.

Example 2: [4]

$$
\begin{aligned}
&Max\ \ Z = 5x_1 + 4x_2 \\
&s.t \quad (4,2,1)x_1 + (5,3,1)x_2 \le (24,5,8) \\
&\qquad (4,1,2)x_1 + (1,0.5,1)x_2 \le (12,6,3) \\
&\qquad x_1, x_2 \ge 0
\end{aligned}
$$

Where the coefficient of available resources and constraint are fuzzy triangular**.**

12- Formulation to Transform Fuzzy Interval: [7]

If $\widetilde{M} = (m, \gamma, \beta)$ is a triangular fuzzy number then $\widetilde{M}_\alpha = [\gamma(\alpha - 1) + m, m - \beta(\alpha - 1)]$ and if $\widetilde{M} = (m^L, m^R, \gamma, \beta)$ is a trapezoidal fuzzy number then $\widetilde{M}_\alpha = [\gamma(\alpha - 1) + m^L, m^R - \beta(\alpha - 1)]$.

Assume that the $\alpha = 0.6$.

The examples above after transformation are as follows:

Example 1 as the form

$$
\begin{aligned}
&Max\ \ \widetilde{Z} = [4.2, 10]x_1 + [5.2, 12.4]x_2 \\
&s.t \quad 2x_1 + 3x_2 \le 6 \\
&\qquad 5x_1 + 4x_2 \le 10 \\
&\qquad x_1, x_2 \ge 0
\end{aligned}
$$

Example 2 as the form

$$
\begin{aligned}
&Max\ \ Z = 5x_1 + 4x_2 \\
&s.t \quad [3.2, 4.4]x_1 + [3.8, 5.4]x_2 \le [22, 27.2] \\
&\qquad [3.6, 4.8]x_1 + [0.8, 1.4]x_2 \le [9.6, 13.2] \\
&\qquad x_1, x_2 \ge 0
\end{aligned}
$$

13- Solving problem:

By the above formulation, we transformed the two examples into fuzzy interval, by the following steps construct the mathematical model of fuzzy linear programming problem of examples by writing the program (MATLAB) to outcome the decision variable value and the objective function value, illustrated as follows:

First Step:

By Selecting the membership function of fuzzy coefficient in the model through the above examples, there in this study one type(modified S-curve membership function) from many types of non linear logistic function be explained in formula (11) since this function characterizes from flexibility in describing uncertain in fuzzy coefficients of LPP through containing fuzzy factor , whereas selecting the membership function of (coefficient of objective function) by depending on the constant value of modified S-curve membership function of logistic function.

$u = 0.001001001$

$w = 1$

$\alpha = 13.81350$

Second Step:

By using the program (MATLAB) with (α = 13.81350), the value of fuzzy coefficient ($\tilde{c}_j$) is calculated by modified S-curve function.

Third Step:

The FLP model is written by formula (11) which indicates the fuzzy coefficient value that is obtained by the above step.

Forth Step:

The optimal objective function value (Z^*) is calculated with respect to different accepted degrees Mu , ($0.0010 \leq$ Mu $\leq$ 0.9990) by increasing (0.0499) for fuzzy factor of modified S- curve function (α = 13.8135) , and the result obtained is illustrated in table (A) and figure (A) : -

Table (A)

(Optimal value with decision variable of example 1(coefficient of objective function is fuzzy))

Mu/Z^*	X_1	X_2	Z^*
0.0010	0.8571	1.4286	26.2857
0.0509	0.8571	1.4286	21.8886
0.1008	0.8571	1.4286	21.0742
0.1507	0.8571	1.4286	20.5670
0.2006	0.8571	1.4286	20.1842
0.2505	0.8571	1.4286	19.8676
0.3004	0.8571	1.4286	19.5909
0.3503	0.8571	1.4286	19.3394
0.4002	0.8571	1.4286	19.1041
0.4501	0.8571	1.4286	18.8783
0.5000	0.8571	1.4286	18.6571
0.5499	0.8571	1.4286	18.4359
0.5998	0.8571	1.4286	18.2102
0.6497	0.8571	1.4286	17.9749
0.6996	0.8571	1.4286	17.7234
0.7495	0.8571	1.4286	17.4467
0.7994	0.8571	1.4286	17.1301
0.8493	0.8571	1.4286	16.7473
0.8992	0.8571	1.4286	16.2401
0.9491	0.8571	1.4286	15.4259
0.9990	0.8571	1.4286	11.0286

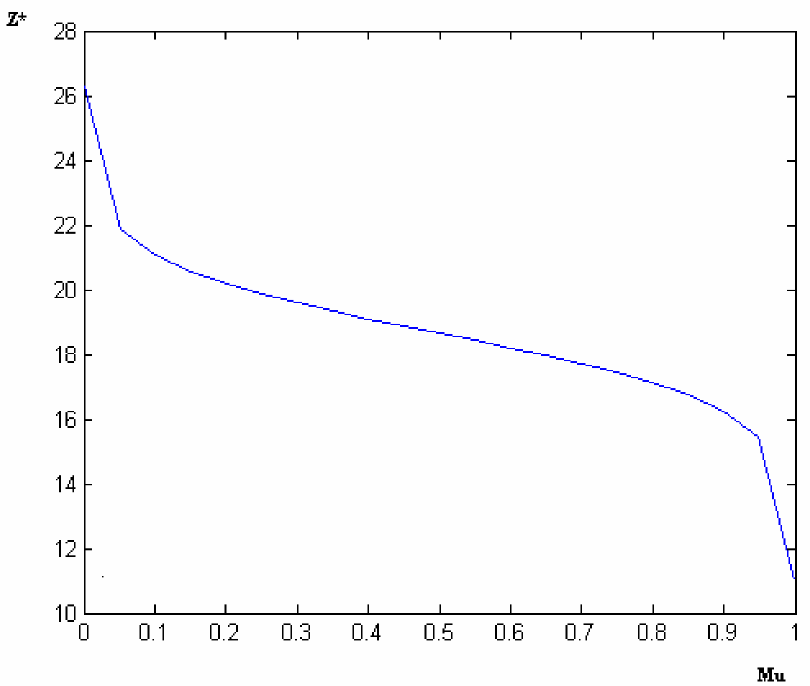

Fig (A)

(Z^* with the accepted degree Mu at(coefficient of objective function is fuzzy))

1- The objective function value Z^* decreases by increasing the accepted degree (0.0010 $\leq$Mu $\leq$0.8493), and the highest value of the objective function (Z^* =26.2857) at the higher level of accepted degree (Mu = 0.0010) and the lower value of the objective function (Z^* =11.0286) at the lower level of the accepted degree (Mu= 0.9990).

2- The different volume between one value and another of objective function decreases at the accepted degree (0.0010 $\leq$Mu $\leq$0.5000) whereas (ΔZ^*= 0.2212),(0.4501- 0.5000) and this increases (ΔZ^*= 0.2257)at the accepted degree (0.5499 – 0.5998) and this difference increases at the accepted degree (Mu $\geq$ 0.5998) whereas (ΔZ^*= 4.3973)at the accepted degree (0.9491- 0.9990).

3- With respect to the decision variable values of these case, the coefficient of objective function suffered from fuzzy problem, and the decision variables (X_1, X_2) have the same value at all levels of the accepted degree with increase of the accepted degree (0.0010 $\leq$Mu $\leq$0.9990).

4- From figure (A) it can be concluded that the optimal value (Z^*) decreases whenever the accepted degree increases.

And to study the effect of fuzzy factor on optimal value Z^*, the optimal value Z^* is calculated of example 1 at different degree from fuzzy factor (α) by adding (2) with different accepted degree contains the range (0.0010 $\leq$Mu $\leq$0.9990) and this contains (21) volume by increasing (0.0499) , the result obtained is illustrated in table (B) and figure (B) ,the following is concluded:-

Table (B)

(Z_* with accepted degree and fuzzy factor of different value at(coefficient of objective function is fuzzy))

20	18	16	14	12	10	8	6	4	2	fuzzy factor of different value at(coefficient of objective function is fuzzy)) α
26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	0.0010
23.2487	22.9112	22.4894	21.9471	21.2244	20.2152	18.7264	16.4737	13.6731	11.8880	0.0509
22.6862	22.2862	21.7863	21.1436	20.2873	19.0943	17.3566	14.9053	12.5202	11.4518	0.1008
22.3358	21.8969	21.3483	20.6431	19.7039	18.3980	16.5205	14.0412	12.0361	11.2986	0.1507
22.0713	21.6031	21.0178	20.2654	19.2637	17.8742	15.9032	13.4631	11.7675	11.2205	0.2006
21.8526	21.3601	20.7444	19.9531	18.8999	17.4425	15.4050	13.0391	11.5962	11.1731	0.2505
21.6614	21.1476	20.5054	19.6800	18.5819	17.0665	14.9810	12.7107	11.4775	11.1413	0.3004
21.4877	20.9546	20.2882	19.4319	18.2932	16.7264	14.6070	12.4468	11.3902	11.1185	0.3503
21.3250	20.7738	20.0849	19.1996	18.0231	16.4098	14.2685	12.2290	11.3234	11.1013	0.4002
21.1690	20.6005	19.8899	18.9769	17.7642	16.1079	13.9561	12.0456	11.2705	11.0879	0.4501
21.0161	20.4306	19.6988	18.7586	17.5108	15.8141	13.6629	11.8886	11.2277	11.0772	0.5000

See table (B)

42	40	38	36	34	32	30	28	26	24	22
26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857	26.2857
24.8395	24.7672	24.6873	24.5985	24.4992	24.3876	24.2610	24.1164	23.9495	23.7548	23.5248
24.5716	24.4859	24.3912	24.2860	24.1683	24.0360	23.8860	23.7146	23.5168	23.2861	23.0134
24.4048	24.3107	24.2068	24.0913	23.9622	23.8170	23.6524	23.4643	23.2473	22.9941	22.6949
24.2789	24.1785	24.0676	23.9444	23.8067	23.6517	23.4761	23.2754	23.0439	22.7737	22.4545
24.1747	24.0692	23.9525	23.8229	23.6780	23.5150	23.3303	23.1192	22.8757	22.5915	22.2557
24.0837	23.9736	23.8519	23.7167	23.5655	23.3955	23.2029	22.9827	22.7286	22.4321	22.0818
24.0009	23.8867	23.7604	23.6201	23.4633	23.2869	23.0870	22.8585	22.5949	22.2873	21.9239
23.9235	23.8054	23.6748	23.5298	23.3677	23.1853	22.9786	22.7424	22.4698	22.1518	21.7760
23.8492	23.7274	23.5927	23.4431	23.2759	23.0878	22.8746	22.6309	22.3498	22.0218	21.6342
23.7764	23.6509	23.5122	23.3581	23.1859	22.9922	22.7726	22.5217	22.2322	21.8944	21.4951

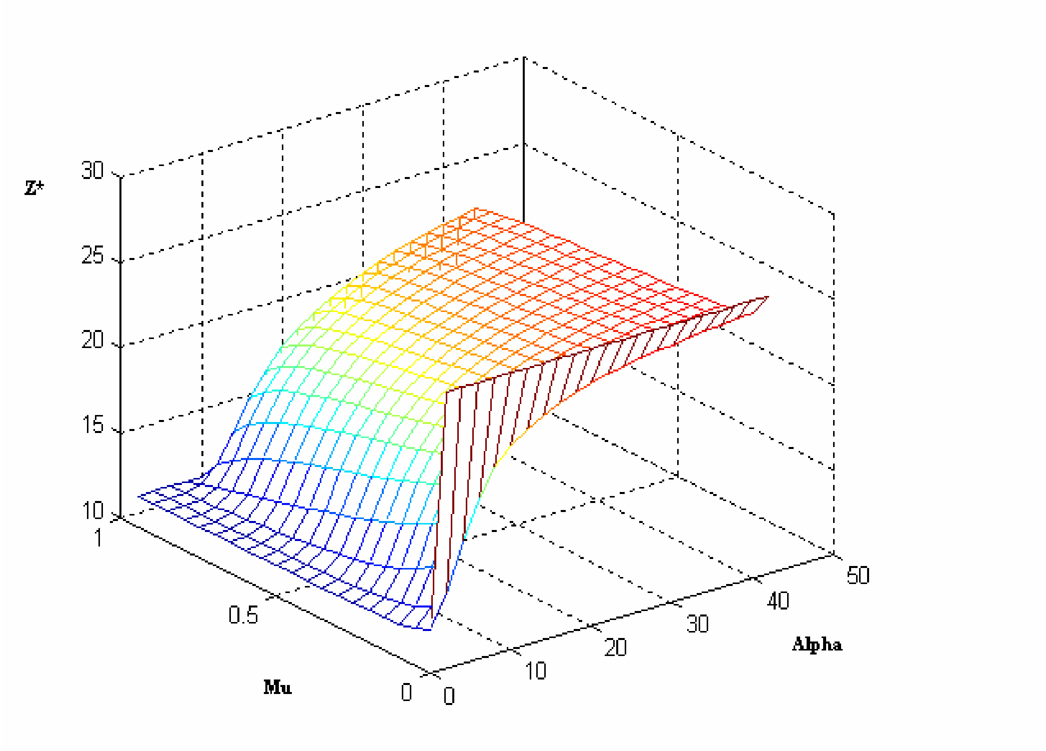

Fig (B)

(Z^* with the accepted degree and fuzzy factor of different value at(coefficient of objective function is fuzzy))

1- The optimal value of objective function (Z^*=26.2857) at the lower level of the accepted degree (Mu = 0.0010) and (Z^*=10.6837) at the higher level at the accepted degree (Mu= 0.9990) at all levels of fuzzy factor studied ($2 \leq \alpha \leq 42$) by adding (2).

2- The optimal value Z^* increases by increasing fuzzy factor ($2 \leq \alpha \leq 42$) and this indicates that there is a relation between Z^*, α at all degrees from the accepted degree (0.0509 $\leq$ Mu $\leq$ 0.9491).

3- The optimal value Z^* at the accepted degree (Mu= 0.0010) is very close in all levels of fuzzy factor ($2 \leq \alpha \leq 42$), and at the accepted degree (Mu = 0.9990) the value of Z^* has the same value at ($2 \leq \alpha \leq 30$) and this decreases at ($\alpha \geq 32$).

4- The optimal value Z^* decreases at fuzzy factor (α=2) by increasing the accepted degree (0.0010 $\leq$ Mu $\leq$ 0.9990) and the different volume between one value and another of Z^* at fuzzy factor (α=2) decreases by increasing the accepted degree (0.0010 $\leq$ Mu

$\leq$0.9990) whereas (ΔZ^*= 14.3977) at (0.0010 – 0.0509) and this difference decreases at (Mu $\geq$ 0.0509) whereas(ΔZ^*=0.0025) at (0.9491– 0.9990).

The optimal value Z^* decreases at (α=20) by increasing the accepted degree(0.0010 $\leq$Mu $\leq$0.9990) , and the different volume between one value and another of objective function at fuzzy factor (α= 20) decreases by increasing the accepted degree (0.0010 $\leq$Mu $\leq$0.5000) whereas (ΔZ^*= 0.1529)at (0.4501– 0.5000) and this different increases at (0.5499 $\leq$Mu $\leq$0.9990) whereas(ΔZ^*= 7.7421)at (0.9491–0.9990). And with respect to fuzzy factor ($\alpha \geq$ 30) which has the same different of fuzzy factor (α= 30).

5- From figure (B) it can be concluded that there is a relation between Z^*, α and Mu. In general the optimal value Z^* increases by increasing the accepted degree .The decision maker can make the higher value of Z^* was be profit is (26.2857).

a- Coefficient of Available Resources and Constraint are Fuzzy:

In this case ,the selection and calculating the membership function of fuzzy coefficient in the model by the steps (first and second)in cases (above) would be ,either the third step that is written as the FLP model by depending on the formula (12) to outcome with the set of decision variable values and objective function values at the coefficient of available resources and constraint of the model which suffer from fuzzy problem either coefficient of objective function is non fuzzy (normal), whereas calculating the value $Z^*$and decision variables value to different accepted degree (0.0010 $\leq$Mu $\leq$0.9990)by increasing value (0.0499) at fuzzy factor α=13.8135, and illustrated in table (C) and figure (C) the following can be concluded:-

Table (C)
(Objective function value with example 2 at (coefficient of available resources and constraint are fuzzy))

Mu/Z^*	X_1	X_2	Z^*
0.0010	1.6802	3.6680	23.0729
0.0509	1.6760	3.8281	23.6924
0.1008	1.6757	3.8606	23.8212
0.1507	1.6757	3.8813	23.9039
0.2006	1.6757	3.8972	23.9676
0.2505	1.6758	3.9106	24.0212
0.3004	1.6759	3.9223	24.0687
0.3503	1.6760	3.9332	24.1124
0.4002	1.6761	3.9434	24.1539
0.4501	1.6762	3.9532	24.1940
0.5000	1.6764	3.9630	24.2338
0.5499	1.6765	3.9729	24.2740
0.5998	1.6767	3.9830	24.3155
0.6497	1.6769	3.9937	24.3593
0.6996	1.6772	4.0052	24.4067
0.7495	1.6775	4.0180	24.4595
0.7994	1.6779	4.0328	24.5208
0.8493	1.6785	4.0509	24.5962
0.8992	1.6794	4.0754	24.6985
0.9491	1.6811	4.1158	24.8684
0.9990	1.6978	4.3597	25.9281

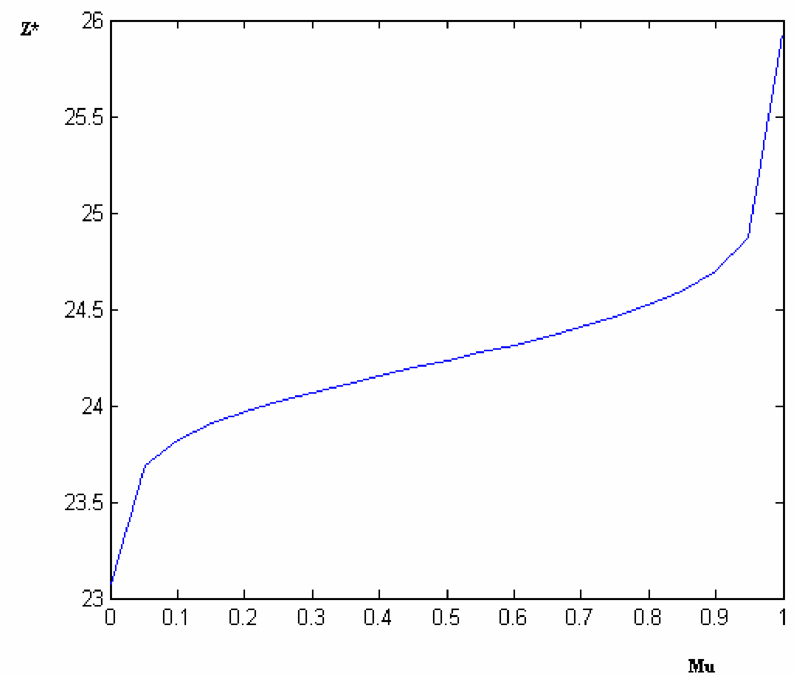

Fig (C)

(Z^* with the accepted degree Mu at (Coefficient of available resources and constraint are fuzzy))

1- The value of objective function (Z^*) increases by increasing the accepted degree, the small value of objective function (Z^*= 23.0729) at the lower level of the accepted degree (Mu= 0.0010) and this value increases to reach at the higher value of objective function (Z^*= 25.9281) at the higher level of the accepted degree (Mu = 0.9990).
2- The different volume between one value and another of objective function decreases at the accepted degree (0.0010 $\leq$Mu $\leq$0.4501) whereas (ΔZ^*= 0.0398) at (0.4501- 0.4002) and this difference increases at (Mu $\geq$ 0.5000) whereas (ΔZ^*= 1.0597) at (0.9990 – 0.9491).
3- With respect to this case ,the coefficient of constraint and available resources suffer from fuzzy problem , observe that of (X_1)

decreases whenever increasing the accepted degree (0.0010 $\leq$Mu $\leq$0.2006) and this increases at the accepted degree (0.2505 $\leq$Mu $\leq$0.9990), with respect to the decision variable (X_2) increases by increasing the accepted degree (0.0010$\leq$Mu $\leq$0.9990).

4- From figure (C) it can be concluded that whenever the accepted degree increases the optimal value (Z^*) increases.

And to study the effect of fuzzy factor on optimal value of objective function, the best value $Z^*$is calculated to example 2 at different degree from fuzzy factor (α) by adding (2) at different dgree the accepted degree contains range (0.0010 $\leq$Mu $\leq$0.9990) and contains (21) volume by increasing (0.0499) and the result obtained is illustrated in table (D) and figure (D) as follows:-

See table (D)

Table (D)

(Z^* with accepted degree and fuzzy factor of different value at(coefficient of available resource and constraint are fuzzy))

α/Mu	2	4	6	8	10	12	14	16
0.0010	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729
0.0509	25.6999	25.2601	24.6510	24.2213	23.9624	23.7971	23.6833	23.6005
0.1008	25.8143	25.5391	24.9809	24.4768	24.1556	23.9503	23.8100	23.7083
0.1507	25.8551	25.6617	25.1748	24.6416	24.2810	24.0492	23.8913	23.7773
0.2006	25.8761	25.7312	25.3096	24.7679	24.3782	24.1257	23.9540	23.8303
0.2505	25.8889	25.7761	25.4113	24.8729	24.4603	24.1902	24.0067	23.8747
0.3004	25.8975	25.8074	25.4917	24.9644	24.5332	24.2474	24.0533	23.9140
0.3503	25.9036	25.8306	25.5574	25.0468	24.6004	24.3002	24.0963	23.9502
0.4002	25.9083	25.8485	25.6124	25.1229	24.6640	24.3503	24.1370	23.9843
0.4501	25.9119	25.8627	25.6592	25.1944	24.7256	24.3989	24.1764	24.0174
0.5000	25.9149	25.8742	25.6997	25.2625	24.7865	24.4472	24.2155	24.0501

42	40	38	36	34	32	30	28	26	24	22	20	18
23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729	23.0729
23.2636	23.2735	23.2844	23.2966	23.3103	23.3257	23.3433	23.3636	23.3871	23.4147	23.4477	23.4878	23.5374
23.3003	23.3121	23.3252	23.3399	23.3563	23.3749	23.3961	23.4205	23.4489	23.4823	23.5223	23.5709	23.6313
23.3233	23.3364	23.3509	23.3671	23.3853	23.4059	23.4294	23.4565	23.4880	23.5251	23.5696	23.6237	23.6911
23.3408	23.3549	23.3704	23.3878	23.4074	23.4295	23.4547	23.4839	23.5178	23.5578	23.6057	23.6642	23.7370
23.3554	23.3702	23.3867	23.4050	23.4257	23.4491	23.4759	23.5067	23.5426	23.5851	23.6359	23.6980	23.7754
23.3682	23.3837	23.4009	23.4202	23.4419	23.4664	23.4945	23.5268	23.5645	23.6091	23.6626	23.7278	23.8094
23.3798	23.3960	23.4139	23.4340	23.4566	23.4822	23.5114	23.5452	23.5846	23.6311	23.6869	23.7552	23.8405
23.3908	23.4075	23.4262	23.4470	23.4705	23.4970	23.5274	23.5625	23.6034	23.6518	23.7099	23.7810	23.8699
23.4013	23.4187	23.4380	23.4595	23.4838	23.5113	23.5428	23.5792	23.6216	23.6718	23.7321	23.8059	23.8984
23.4117	23.4296	23.4495	23.4718	23.4969	23.5254	23.5580	23.5956	23.6395	23.6915	23.7540	23.8305	23.9265

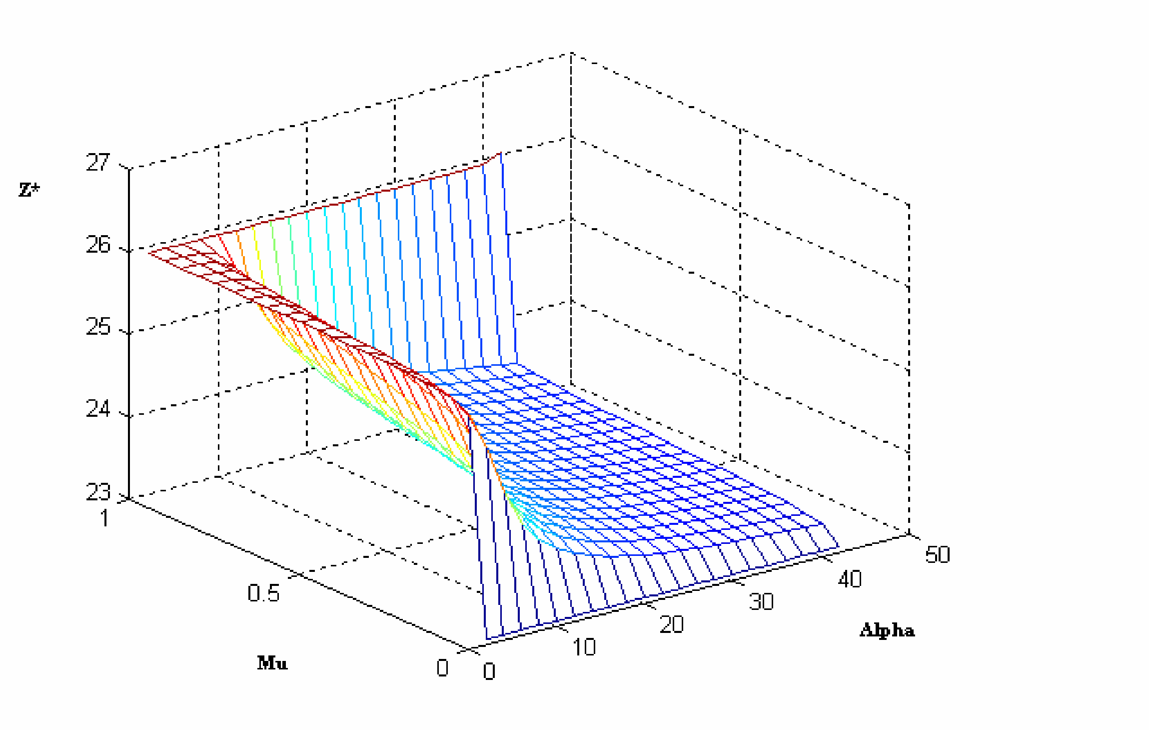

Fig (D)

(Z^* with the accepted degree and fuzzy factor of different value at(coefficient of available resource and constraint are fuzzy))

1- The optimal value of objective function (Z^*= 23.0729) at the lower level of the accepted degree (Mu = 0.0010) and (Z^*= 26.0229) at the higher level of the accepted degree (Mu = 0.9990), at all levels of fuzzy factor studied ($2 \le \alpha \le 42$) by adding (2).

2- The optimal value Z^*decreases by increasing fuzzy factor ($2 \le \alpha \le 42$) and this indicates that there is reflex relation between Z^* , α at the accepted degree ($0.0509 \le$ Mu ≤ 0.9491).

3- The optimal value Z^* has the same value at fuzzy factor ($2 \le \alpha \le 42$) at (Mu = 0.0010). The optimal value Z^* at (Mu = 0.9990) have the same value at fuzzy factor ($2 \le \alpha \le 38$) and this value increases at ($\alpha \ge 40$).

4- The value Z^* at fuzzy factor (α= 2) (Z^*= 23.0729) this value increases till it reaches (Z^*= 25.9281) ,the different value between one value and another decreases by increasing the accepted degree($0.0010 \le$ Mu ≤ 0.9990)whereas (ΔZ^*= 2.6270)at the accepted degree (0.0509 – 0.0010) ,this increases it reaches (ΔZ^*=

0.0007) at the accepted degree(0.9990 – 0.9491). If (α= 20) the different value between one value and another decreases by increasing at the accepted degree ($0.0010 \le$ Mu ≤ 0.5000) whereas (ΔZ^*=0.0246) and this difference increases by increasing at the accepted degree ($0.5499 \le$ Mu ≤ 0.9990) whereas (ΔZ^*= 1.7148) at (0.9990 – 0.9491). And with respect to another fuzzy factor similarly to fuzzy factor α= 20.

5- From figure (D) it can be concluded that there is a relation between Z^*, α and Mu. The optimal value increases whenever increasing the accepted degree increasing is in fuzzy factor.

14- Performance Measure:

In this paper study (2) circumstances of fuzzy that suffered by examples is studied through applying FLP where the fuzzy coefficient is calculated by using modified S- curve non linear membership function which is characterized by existing the fuzzy factor(α), an interval optimal value Z^* is obtained with respect to α , Mu of all cases under study . To obtain the optimal that the decision maker depends on under fuzzy circumstance (uncertain), comparison among these cases were done by using the performance measure

AM = max Z^* / ΔZ^*

Where ΔZ^* = max Z^* - min Z^*

max Z^* : higher value of Z^*.

min Z^*: least value of Z^* .

Table (E)
(Performance measure)

Case	Fuzzy coefficient	max Z^*	min Z^*	ΔZ^*	AM = max Z^* / ΔZ^*
1	c_j	26.2857	10.6837	15.602	1.6848
2	a_{ij} , b_i	26.0229	23.0729	2.95	8.8213

From table (E) we conclude the following:

Comparison between cases one and two, then the case two is the best among them and the coefficient of available resources and constrain are suffered from fuzzy problem, this case has one coefficient from normal (non fuzzy).This means that (c_j), and the higher profit of this case is (26.0229), can be made by decision maker, where the performance measure is (8.8213) .This is very useful for the decision maker to make a specific decision.

15) Conclusions:

An industrial application of FLP through the modified S- curve membership function has been investigated by using a set of data collected from two examples. The problem of fuzzy has been defined .Two possible cases were identified depending upon sets of fuzzy and non fuzzy coefficients. Necessary equations in each case have been formulated; profits and satisfactory level have been computed using FLPP. It can be concluded that the model contains double coefficient from fuzzy problem is the best of the case have one coefficient suffered from fuzzy problem.

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