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Simple Sliding Mode Controller with Adaptive Fuzzy Saturation Function for Nonlinear Single Input-Single Output System

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Abstract

Sliding mode control algorithm that uses fuzzy saturation function is designed in this paper for nonlinear system. The fuzzy saturation function is suggested to improve the accuracy and the robustness of the sliding mode control which are partially lost when using a fixed boundary layer. The fuzzy saturation function is simple, in the sense that both the membership functions and the rule base are simple. The overall control algorithm has stability assurance for the closed-loop controlled system; therefore, it may be applied to control different systems, in this paper this algorithm is applied on nonlinear SISO system with 10%parameter uncertainty and nonlinear disturbance. Simulation results show that the developed algorithm has good control performance with negligible chattering.

Keywords: Saturation function, fuzzy, SISO, sliding mode control, nonlinear system.

مسيطر مضبب بسيط لدالة اشباع مبهمة لمنضومات غير خطية ذات دخل مفرد وخرج مفرد

الخلاصة

البحث يتضمن استخدام خوارزمية لمسيطر مضبب عن طريق دالة اشباع مبهمة (fussy saturation function لتحسين الدقة و المتانة لهذا المسيطر و اللتان فقدتا بشكل جزئي نتيجة لوجود طبقة محددة حول (sliding mode) . لقد اعتبرت دالة الاشباع المبهمة بسيطة لكون المعادلات والقواعد الاساسية المستخدمة بسيطة .هذا النوع من المسيطرات يضمن استقرارية للمنضومة لذا فهو يطبق على منضومات مختلفة , ففي هذا البحث تم تطبيق المسيطر المضبب على منضومة غير خطية ذات دخل واحد وخرج واحد وبوجود عدم موثوقية للعناصر بنسبة 10% اضافة الى وجود مؤثرات خارجية غير خطية. وقد اضهرت النتائج ان هذه الخوارزمية اعطت اداء جيد من حيث اهمال (chattering) الذي كان يحدث نتيجة استخدام الطريقة المالوفة للمسيطر المضبب(SMC).

I. Introduction

A Sliding Mode Controller (SMC) is a Variable Structure Controller (VSC). Basically, a VSC includes several different continuous functions that can map plant state to a control surface, and the switching among different functions is determined by plant state that is represented by a switching function [1].

SMC yields good response and robustness to model uncertainties, however, it comes with high control activity, which is called control chattering. In general, chattering is highly undesirable, since it may excite high-frequency dynamic neglected in the course of modeling, and it may cause wearing in the actuators [2]. A common solution to this problem is to attempt to smooth the signum function to obtain a

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continuous approximation. The signum function is seen to be relay-like in nature. The ideal relay characteristic is often impossible to implement. One possible answer is to replace the sigum function by a sigmoid-like function to induce pseudo-sliding instead of ideal sliding [3]. Slotine proposed a technique to change the dynamics near to the sliding surface in order to avoid a real discontinuity and at the same time to preserve the sliding mode properties [4]. This technique introduces boundary layer on both sides of the sliding surface in order to avoid the chattering effect in the control signal. The sliding variable will reach within the vicinity of the sliding surface. However, the accuracy and robustness of the sliding mode are partially lost [5].

Recently, combinations of fuzzy control and SMC approaches have achieved superior performance [6], [7], [8], [9]. Briefly, for instance, Hwang and Lin [7] developed a non adaptive fuzzy controller, and Wu and Liu [9] used the switching manifold as a reference, where sliding modes are used to determine the optimal values of parameters in fuzzy control rules. Ohtani and Yoshimura [8] also presented a fuzzy control law using the concept of sliding mode, , where fussy rules are tuned by learning. A self-organizing and adaptive fuzzy sliding mode controller was proposed by Hsu and Fu [6], in which, however, there are two main problems existing in the design: 1) how to construct suitable fuzzy control rules and 2) if the system dimension increases, the number of fuzzy rules becomes too large for real-time implementation.

In this paper, a simple adaptive fuzzy SMC algorithm is established, where the proposed control law contains an equivalent control term and fuzzy SMC. The fuzzy SMC, associated with effective adaptive laws, updates the system parameter values on-line and also approximates unknown systems dynamics, while the equivalent control term is designed to satisfy the sliding condition so as to enhance the system robustness. The control rules and membership functions used in the fuzzy SMC algorithm are very simple. Furthermore, by embedding the fuzzy SMC with the proposed control law, it provides an ability to compensate the effect of uncertainties, disturbances, and unordered system dynamics.

The paper is organized as follows. The problem under investigation is first formulated in Section II. In Section III, a conventional variable structure controller is established. The basic framework and derivation of the fuzzy SMC design for SISO system is briefly described in Sections IV and V, respectively. Simulation results on second order system with nonlinear disturbance are presented in Section VI, with conclusions given in Section VII.

II. Problem Formulation

For a class of single-input and single-output (SISO) nonlinear system, its mathematical function can be described as [10];

$$x^{(n)} = f(x) + b(x)u(t) + d(t)$$
$$y = x \qquad \text{….(1)}$$

where $x = (x, \dot{x}, \ldots, x^{(n-1)})^T \in R$ is the state vector of the system, $u(t) \in R$ and $y(t) \in R$ are the input and output of the system, $f(x)$ is the unknown nonlinear function of the system, $b(x)$ is the control gain, and $d(t)$ is the unknown external disturbance . The controller design problem is as follows. Given the desired trajectories $x_d, \dot{x}_d, \ddot{x}_d$ with some (or all) system parameters being unknown, derive a control law for the torque (or force) input to force the system state vector to track a desired state vector $x_d = (x_d, \dot{x}_d, \ldots, x^{(n-1)})^T$ ($x_d \in R^n$ is bounded) in the presence of model uncertainties and external disturbances, if not exactly then closely.

The following assumptions and specific properties of the above dynamics will be used in our controller design:

Assumption 1: the system function $f(x)$ and the control gain $b(x)$ are not exactly known, but can be estimate as $\hat{f}(x)$ and $\hat{b}(x)$. Thus the estimation errors on $f(x)$ and $b(x)$ are assumed to be bounded by some known functions:

$$\left|\hat{f} - f\right| \le F \qquad \ldots\ldots(2)$$

$$b^{-1} < \frac{\hat{b}}{b} \le b \qquad \ldots\ldots(3)$$

where

$$\hat{b} = \sqrt{b_{\min} b_{\max}} \qquad \ldots..(4)$$

and the control gain margin, b , is defined as:

$$b = \sqrt{\frac{b_{\max}}{b_{\min}}} \qquad \ldots..(5)$$

Assumption 2: the external disturbance $d(t)$ is bounded with $d(t) \le D(t)$, where $D(t)$ is a known function.

III Sliding Mode Control

Let the tracking error be

$$e = x_d - x$$
$$\dot{e} = \dot{x}_d - \dot{x} \qquad \ldots\ldots(6)$$

Then to apply the SMC strategy, we first define a sliding surface as follows,

$$s = le + \dot{e} \qquad \ldots\ldots(7)$$

where l is a strictly positive constant.

The control signal $u(t)$ is calculated in such way that the closed-loop system reaches a pre-defined sliding surface $s(t)$ and remains on this surface[11].

To keep the $s(e, \dot{e})$ at zero, the control law is designed to satisfy the following sliding condition (Lyapunov function):

$$V = \frac{1}{2} s^2 \ge 0 \qquad \ldots\ldots (8)$$

Its time derivative becomes $\dot{V} = \dot{s}s$ and the control u is chosen such that

$$\dot{s}s \le -h|s| \qquad \ldots..(9)$$

where h is a positive constant that guarantees the system trajectories hits the sliding surface in a finite time. The control signal $u(t)$ required for the system to remain on the sliding surface is called the equivalent control which is given by the requirement $\frac{ds}{dt} = 0$, and since the system function $f(x)$ and the control gain $b(x)$ are estimated, then the equivalent control is

$$u_{eq} = \hat{b}^{-1}[\, l\dot{e} + \ddot{x}_d - \hat{f}(x, \dot{x})] = \hat{b}^{-1}\tilde{u}_{eq} \qquad \ldots.(10)$$

and the control law is now chosen as follows

$$u = \hat{b}^{-1}[\tilde{u}_{eq} + k\,\mathrm{sgn}(s)] \qquad \ldots..(11)$$

Where sgn(.) is the signum or the sign function, k is sliding gain that satisfying the sliding condition[12];

$$k \ge b(F + h) + (b - 1)|u_{eq}| \qquad \ldots..(12)$$

As explained before, using a sign function often ceases a chattering in practice. One solution is to introduce thin boundary layers (BL) neighboring the sliding surfaces. The thin BL is defined as [13];

$$B(t) = \{x_i, |s(x,t)| \le \Phi\} \qquad \ldots. (13)$$

where Φ is the boundary layer thickness, which reduces the chattering of the control input, as illustrated in Fig.1 in the error phase space. To remedy the control discontinuity in the boundary layer, the signum function sgn(s) in (11) is replaced by a saturation function of the form [14]:

$$sat(s) = \begin{cases} \mathrm{sgn}(s), & |s| \ge \Phi \\ \frac{s}{\Phi}, & |s| < \Phi \end{cases} \qquad \ldots.(14)$$

This amounts to a reduction of the control gain inside the boundary layer and results in a smooth control signal [15]; Unfortunately, boundary layer controllers do not guarantee asymptotic stability but rather uniform ultimate bounded ness [16, 17]. As a consequence, there exists a trade-off between the smoothness of control signals and the control accuracy. In the next section, a simple boundary layer width modification technique is introduced to improve tracking precision.

IV. Fuzzy Sliding Mode Controller

In this section, a simple adaptive fuzzy sliding surface is explained to develop the SMC by using a fuzzy saturation function instead of the standard saturation function. The fuzzy saturation function is designed by

$$fsat(s)=\begin{cases}\text{sgn}(s), & |s|\ge\Phi\\ k_{fuz}, & |s|<\Phi\end{cases} \quad \text{…..(15)}$$

where k_{fuz} is equal to

$$k_{fuz}=k_m\frac{s}{\Phi} \quad \text{…..(16)}$$

where the gain K_m is used to modified the boundary layer thickness, Φ, and k_{fuz} will be estimated via employed the inference mechanism of fuzzy control theory by following the standard procedure of fuzzy controllers design, which consists of fuzzification, control rule base establishment, and defuzzification.

A. Fuzzification

As shown in Fig. 2, the fuzzy controller employs one input, the sliding function s and one control output, k_{fuz} .

The positive scaling factor k_s , and k_k , are used for the input s and the output k_{fuz} , respectively. The membership functions for the input and output of the fuzzy controller are chosen as shown in Fig. 3 and Fig.4,

B. Fuzzy Control Rules

The quantitative *if-then* rules of fuzzy controller can be described as:

R₁ : if s is N_s then k_{fuz} is N_s

R₂ : if s is P_s then k_{fuz} is P_s

where N_s and P_s are linguistic terms of antecedent fuzzy set, they mean Negative and Positive respectively. We can use a general form to describe these fuzzy rules:

Ri : if s is A_i then k_{fuz} is B_i , i = 1,2….(17)

where A_i and B_i are a trapezoidal-shaped fuzzy values of the linguistic variables s_N and k_{fuz} in the universe of discourse from *-N* to *N* and from *–M* to *M* in respectively, see Fig. 3 and Fig. 4, The fuzzy implication rule of Mamdanî's min-min-max method is used for fuzzy reasoning.

B-Defuzzification

In the defuzzification step, the output k_{fuz} of fuzzy inference was calculated by centroid formula of Eq.(18) at the bottom of the page.

$$k_{fuz}=\frac{\sum membership\ \ value\ \ of\ \ input\times output\ \ correspo}{\sum membership\ \ val}$$

….(18)

VI. Simulation Study

In order to illustrate the effectiveness of the proposed fuzzy SMC algorithm, a simple second order dynamic system is simulated

$$\ddot{x}=f(x,\dot{x})+u(t)+d(t) \quad \text{….. (19)}$$

to force the system output x to track the desired trajectory $x_d=\sin(pt/2)$ ….(20)

In this simulation, the unknown system function is assumed to be

[15]; $f(x,\dot{x})=-(|\sin(t)|+1)\dot{x}^2\cos(3x)$ (21)

the $\hat{f}(x)$ is assumed to be estimate with 10% uncertainty as

$$\hat{f}(x)=-0.9f(x,\dot{x}) \quad (22)$$

and $\hat{b}(x)$ is assumed to estimate as 0.9 (i.e. with 10% uncertainty from the actual value which is one).
and the nonlinear disturbance is

$d(t) = 0.1\sin(10t)$ …..(23)

Fig.5 displayed the output response state x_1 and x_2 of this system with the reference versus time for the two controller, the conventional SMC and the proposed FSMC. The control parameter for both controller are given in table (1) and the value of N=1 therefore the length of the universe of discourse for the input membership functions is from -1 to 1, while the lengthof the universe of discourse for the output membership functions is from -2.5 to 2.5 because the value of *M*=2.5(which is the large than the value of Φ). In above figures, the states show a very little difference in their overall behavior and the system response with FSMC track the desired state vector x_d in the presence of 10% model uncertainties and external nonlinear disturbances very closely, while with conventional SMC the state x_1 and x_2 track the desired position and velocity with steady state error and with chattering problem in the response of state x_2. (See Fig .5b) .

Fig.6 shows the signals of the equivalent controller, u_{eq} ,sliding controller $k\,\mathrm{sgn}(s)$ for both conventional SMC and the proposed FSMC, in addition to the total control input signal . Fig.6a show the equivalent controller with the conventional SMC contains chattering due to the effect from the sliding controller which produces discontinuous switching to the sliding control signal. perfect tracking performance are obtained with no chattering in either the equivalent or the fussy sliding controller signal and hence in the total input control signal with the proposed FSMC, where more smooth and less value signal than he conventional SMC we obtain by the the proposed controller.

Fig.7 shows the tracking errors for the tested system with both controllers. Although the tracking errors is large with conventional SMC, perfect tracking performance (the tracking error tend to reach zero) is obtained especially in state x_2 when using the proposed controller.
The sliding function is shown in Fig.8, this figure shows that the *s* reached to zero when it enter the boundary layer with the FSM while it suffer from chattering with the conventional SMC.

VII. Conclusions

In this paper, we have proposed a simple adaptive fuzzy sliding mode controller for robust stabilization of uncertain nonlinear SISO system. Our method uses a fuzzy saturation function that includes a fuzzy expression instead of the terms s/Φ. The proposed scheme has a reaching motion of sliding mode at faster time and it is not dependent on the system model. The simulation results have shown that the proposed method controlled the SISO second order system with parameter uncertainty and nonlinear disturbance. The proposed controller is simple, so it can be applied for different mechanical system such as induction machine and robotic systems.

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Table (1) The parameter for the controllers 616-620, 1991.

SMC type	K	Φ	l	k_s	k_k
Conventional SMC	8	---	2	---	---
FSMC	8	0.5	2	2	1

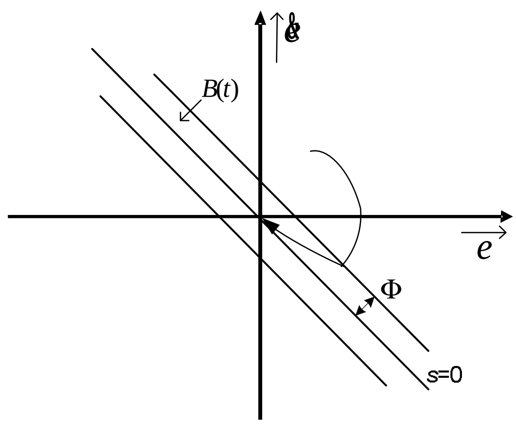


Figure (1) The sliding surface and the boundary layer

Figure (2) Fuzzy controller

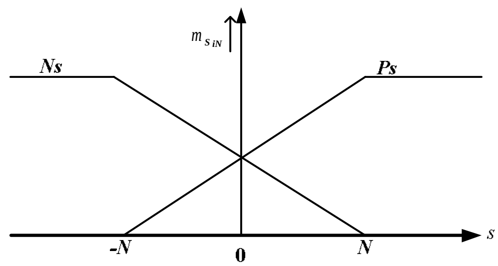


Figure (3) Input membership functions

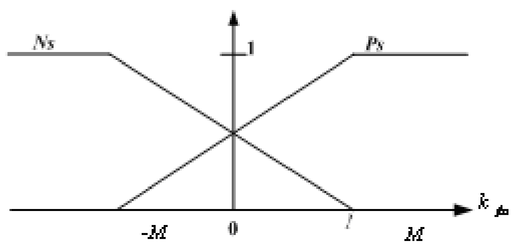
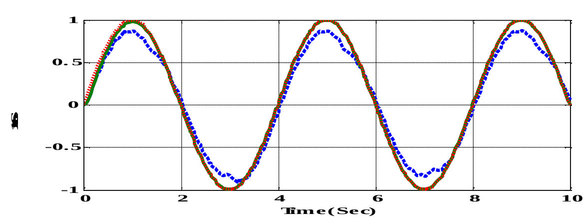


Figure (4) Output membership functions



a-the response of state x_1

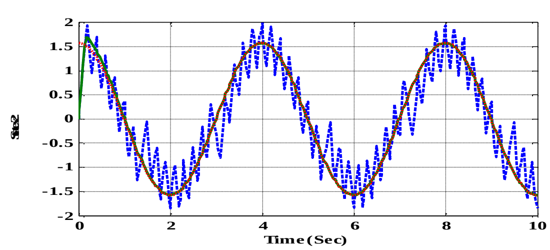
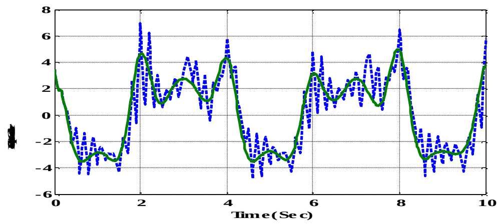
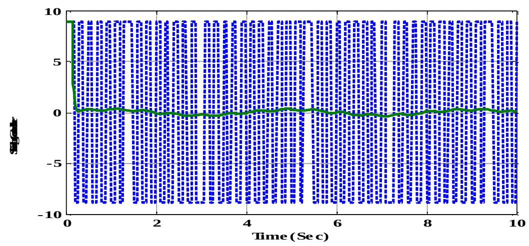


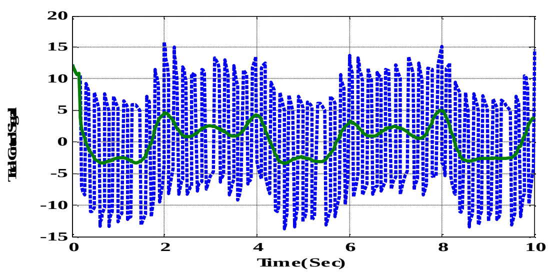
Figure (5) Desired trajectories (dote) and output responses of the conventional SMC (dashed), the proposed FSMC (solid).



a-the equivalent control signal.

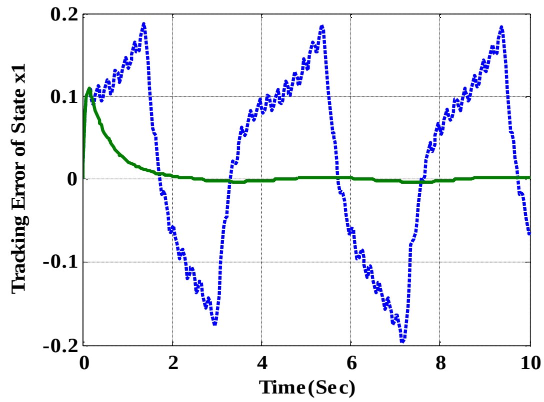


b-the sliding control signal.

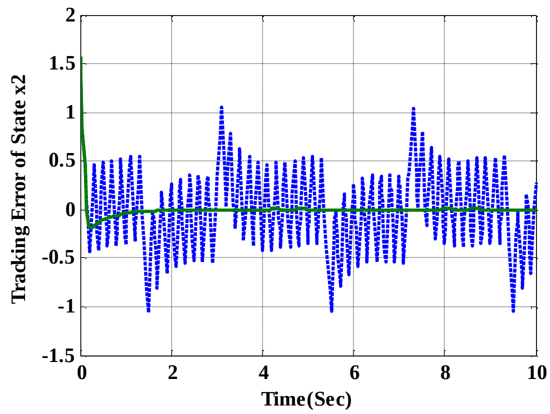


C-the total control signal.

Figure (6) the control signals with the conventional SMC (dashed), the proposed



a- state x_1 error signal.



b- state x_2 error signal.

Figure (7) Tracking error signals with the conventional SMC (dashed).

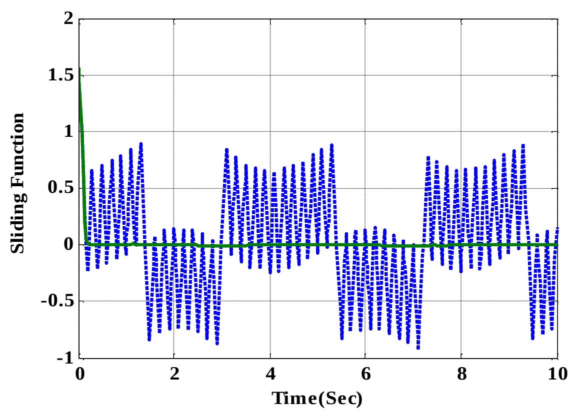


Figure (8) Sliding function.