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Dynamic Analysis Of Soil-Structure Interaction Problems Considering Infinite Boundaries

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Abstract:

One of the limitations of the usage of the finite element method in dynamic soil-structure interaction arises when it is used for the modelling of an infinite domain if nothing is done to prevent from artificial reflections at the mesh boundary; errors are introduced into the results. To handle reflections, different artificial boundaries have been proposed. The aim of such boundaries is to make them behave as nearly as possible as if the mesh extends to infinity. In this paper they are known as transmitting, absorbing or silent boundaries. A brief description to two different approaches of absorbing boundaries is made, first by using infinite elements and the second by using viscous boundaries method. For this purpose the computer program named "MIXDYN" is modified in this study to "Mod-MIXDYN" by adding mapped infinite element model to the finite elements models of the program to be used for dynamic analysis of soil-structure interaction problems. A new derivation of the mapped functions is made in this study for the cases when the infinite direction is extended to the left horizontally (at negative ξ direction) and down vertically (at negative η direction).

Two verification problems are solved to compare the results of the modified program with the results of other software, namely ANSYS and OpenSees representing other types of elements (dashpot elements) modelling boundaries as viscous boundary.

It was found that the transmitting boundary absorbs most of the incident energy. The distinct reflections observed in the "fixed boundaries" case disappear in the "transmitted boundaries" case. This is true for both cases of using viscous boundaries or mapped infinite elements. The viscous boundaries are more effective in absorbing the waves resulting from dynamic loads than mapped infinite elements. This is clear when comparing the results of both types with those of transient infinite elements.

التحليل الديناميكي لمسائل التداخل بين المنشأ و التربة مع الأخذ بنظر الاعتبار التخوم غير المحددة

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الخلاصة:

إن واحدة من التحديدات على استعمال طريقة العناصر المحددة في التحليل الديناميكي للتداخل بين المنشأ و التربة تظهر عندما تستعمل الطريقة في تمثيل المجال غير المحدد إذا لم يتخذ إجراء لمنع الارتدادات في تخوم شبكة المسألة حيث ستظهر أخطاء في النتائج. و للتعامل مع هذه الارتدادات، أقترحت أنواع مختلفة من التخوم الاصطناعية. إن الهدف من هذه التخوم هو جعلها تتصرف قدر الإمكان كما لو أن الشبكة تمتد إلى المالانهاية. في هذا البحث يطلق على هذه التخوم "الناقلة" أو "الممتصة" أو "الساكنة". يعرض البحث وصفا ملخصا لطريقتين مختلفتين للتخوم الممتصة، الأولى باستعمال العناصر المتناهية و الثانية باستعمال طريقة التخوم اللازجة. و لهذا الغرض تم تحوير برنامج الحاسبة المسمى "MIXDYN" ليصبح "Mod-MIXDYN" بعد إضافة نموذج العنصر المتناهي المنظم إلى نماذج العناصر المحددة للبرنامج الذي سوف يستعمل لتحليل مسائل التداخل الديناميكي بين التربة و المنشأ. و قد أضيف في هذه الدراسة اشتقاق جديد للدوال المنظمة للحالات التي يمتد فيها الاتجاه المتناهي باتجاه أفقي يسارا (بالاتجاه السالب للمحور ξ) و شاقوليا للأسفل (بالاتجاه السالب للمحور η).

و قد تم تحليل مسألتين لمقارنة نتائج البرنامج المحور مع نتائج برنامجين جاهزين هما ANSYSو OpenSees اللذين يتبنيان نوعين آخرين من العناصر المتناهية (عناصر الإخماد) التي تمثل التخوم على أنها تخوم لزجة. و قد وجد أن التخم الناقل يمتص معظم الطاقة المتولدة، و أن الانعكاسات البعيدة التي تمت ملاحظتها في حالة التخوم المقيدة تختفي في حالة التخوم الناقلة. و وجد أن هذه الحالة تصح في حالتي التخوم اللازجة و حالة العناصر المتناهية المنظمة. و وجد أن التخوم اللازجة أكثر كفاءة في امتصاص الموجات الناتجة عن الأحمال الديناميكية من العناصر المتناهية. و يبدو هذا واضحا عند مقارنة نتائج هذين النوعين من التخوم مع نتائج العناصر المتناهية العابرة.

Introduction:

Two important characteristics that distinguish the dynamic soil-structure interaction system from other general dynamic structural systems are the unbounded nature and the nonlinearity of the soil medium. Generally, when establishing numerical dynamic soil-structure interaction models, the following problems should be taken into account [Zuo, 2002].:

1. Radiation of dynamic energy into the unbounded soil.
2. The hysteretic nature of soil damping.
3. Separation of the soil from the structure.
4. Possibility of soil liquefaction under seismic loads.
5. Other inherent nonlinearities of the soil and the structure.

Details of the analytical techniques also vary according to the nature of the excitation. These can be divided into two broad categories [Wolf, 1988]:

1. Cases where the excitation is applied directly to the structure (e.g. wind, waves, unbalanced masses in rotating machinery, airplane impact or an explosion in the surrounding atmosphere).
2. Situations where the excitation is applied to the structure through the soil (e.g. earthquakes, underground explosion and presupposed elastic waves entering the computational domain or vibrations arising from pile driving, traffic and various other machines).

However, due to the complexity of dynamic soil-structure interaction, numerical modelling of this phenomenon still remains a challenge. Various kinds of analytical formulations and computer programs have been developed to solve the complex problem. There still exist many difficulties to cover in one model all the problems listed above. Current models usually stress one or several of these problems [Zuo, 2002] .

Infinite Elements

It is convenient to classify infinite elements as of static or dynamic type, as the methods needed for the two types are quite different. For static infinite elements, mapped and decay function type which can be used for some dynamic problems will be discussed.

- **Static Infinite Elements**

The succeeding infinite element formulations have followed two main lines of development. These have been [Bettess, 1977, 1992]:-

a. Mapping of the element from finite to infinite domain.
b. Using decay functions in conjunction with the ordinary finite element shape function.

- **Mapped Infinite Elements**

Many of the infinite elements proposed have used the idea of mapping, or can be cast in that form. In 1977, Ungless and Anderson used a term of form 1/(1+r) (r is the radial direction) in three dimensional elasticity applications. Medina and Penzin (1982) adopted the same approach. The first explicit stated mapping was by Beer and Meek (1981) , who used a mapping which included the terms $\boldsymbol{\xi_0}$:

$2\,(\xi_i+½)\,\xi$ for $\xi < 0$

$2(\xi_i+½)\,\xi\,/(1-\xi)$ for $\xi > 0$

They split the mapping into two parts, that from $\boldsymbol{\xi} = -1$ to $\boldsymbol{\xi} = 0$

and from $\xi = 0$ to $\xi = 1$. The procedure used was fairly complicated.

Beer and Meek also used a standard Gauss-Legendre numerical integration. They found that a simple 2 x 2 integration was beneficial, as a higher order integration tended to make the infinite domain elements too stiff [Beer and Meek, 1981].

Curnier (1983) characterized the two methods (decay function and mapping) as "descent shape function" and "ascent shape function", respectively. It was shown that the two methods can be made equivalent, depending upon the choice of shape function.

Pissanetzky (1984) used a similar approach of Beer and Meek (1981) but he carried out the integration in the infinite domain, and so had to modify the Gauss-Legendre abscissae and weights.

Okabe (1983) gave various possible shape functions for infinite domains, based on what he calls "The generalized Lagrange family for the cube".

The form in which Zienkiewicz mapping was originally given was simplified and systematized by Marques and Owen (1984), who worked out and tabulated the mapping function for large range of commonly used infinite elements, [Bettess, 1992].

Zienkiewicz Mapped Infinite Elements:

There is no doubt that the Zienkiewicz approach (1983) leads to a clarification and simplification of this class of method. The mapping functions and derivatives are given in Table (1) for two-dimensional quadratic serendipity mapped infinite element shown in Figure (1), [Bettess, 1992]. In this table, M refers to the mapping functions while ξ and η are the local coordinates.

A precisely analogous procedure to derive these mapping functions is described in detail by Dawood (2006).

Using the same procedure, mapping functions of the infinite elements in two cases will be derived ; the first when the infinite element extends to infinity in the negative ξ direction and in the second case, the infinite element extends to infinity in the negative η direction as shown in Figure (2).
The mapping functions and their derivatives for these two cases are derived here and shown in Tables (2) and (3), respectively.

In this paper, this type of mapped infinite element has been added to the finite element models of the computer program (Mod-MIXDYN).

Applications:

In this section, the computer program named "MIXDYN" (Owen and Hinton, 1980) is modified to (Mod-MIXDYN) by adding extra code to apply additional type of mapped infinite elements to it. This type is the 5-noded coding of mapped infinite element presented by Selvadurai and Karpurapu in 1988, [Karpurapu, 1988].

The program Mod-MIXDYN is coded in Fortran language and implemented on a Pentium-IV personal computer.

In order to check the validity and the accuracy of the mentioned program modifications in analyzing soil-structure interaction problems considering infinite boundaries, two verification examples are considered for this purpose. The results of the modified program are compared with the results of other program software called ANSYS and OpenSees representing other types of elements for modelling infinite boundaries using viscous boundary method.

Verification Problem No. (1)

A research at the University of Washington using the program (OpenSees) (which is a finite element tool developed by Berkley University), zero-length dashpot elements with viscous components normal and tangent to a given boundary are used to simulate the transmitting condition [u.washington.edu Website]. The dashpot coefficients are determined in terms of the material properties of the semi-infinite domain, as shown in Figure (2).

As a verification problem, the results of the above mentioned research on that website are used in this study to assess this problem. A simple 1-Dimensional case is analyzed using the program (ANSYS) in addition to the program (OpenSees). The 1-D condition is enforced constraining both sides of the model to move the same amount. The analysis is performed using fixed boundary condition at the bottom. The model details are shown in Figure (3) and the loading function is drawn in Figure (4).

Figure (5) shows the finite element mesh of this problem. The mesh, material properties and analysis information are listed in Tables (4) and (5).

Figures (6) and (7) present the time history of the vertical displacement at top node as predicted by the program (ANSYS) considering fixed and viscous boundaries, respectively.

Figures (8) and (9) present the time history of the vertical displacement at mid-node for the same conditions.

Figures (10) to (13) show the time history of the vertical displacement at top and mid- nodes as predicted by the program (OpenSees).

A comparison of recorded displacements at the top and middle nodes shows that the transmitting boundary absorbs most of the incident energy. The distinct reflections observed in the "fixed" case disappear in the "transmitted" case.

A comparison between Figures (6) to (9) and Figures (10) to (13) show that the results of the program (ANSYS) adopting fixed boundary with dashpot elements are in good agreement with those of the program OpenSees which adopts transmitting viscous boundaries, using zero-length dashpot elements.

Verification Problem No. (2):

In this case, a half-space with an open rectangular mine shown in Figure (14) is considered. This case was solved by Vardoulakis et al. (1987) using Laplace domain BEM and by Yerli et al. (1998) using transient infinite elements (TIE).

It is assumed that 15.24 cm thick concrete lining is added on the surface of the open-mined space so that the inside dimensions of the opening remain the same as in the unsupported case.

The material properties of the half-space are shown in Table (6).

Under the effect of this loading condition, the plane strain problem is solved by the finite element method with the coupling of finite and infinite elements.

The finite element mesh is shown in Figure (15), while the finite element mesh including infinite elements is shown in Figure (16).

In Figure (17), the finite element mesh with viscous boundaries is drawn. Table (7) lists the required information for the mesh of the problem.

The problem is analyzed using the program (Mod-MIXDYN) and also by the program (ANSYS) for two conditions; fixed and viscous boundaries. Vertical displacements at Points A, B, and C are presented in Figures (18) and (19) adopting viscous boundaries and Figures (20) and (21) adopting fixed boundaries.

For comparison purposes, the results obtained by Vardoulakis et al. (1987) and Yerli et al. (1998) are presented in Figure (22).

It is seen that the vertical displacement of point A is in very good agreement with the results of both Vardoulakis et al. (1987) and Yerli et al. (1998).

For Point B, displacements agree with those of Yerli et al. (1998) rather than with those of Vardoulakis et al. (1987).

However, for the displacement at point C, there are big differences between the present results and those of Vardoulakis et al. (1987). But the present results are in good agreement with Yerli et al. (1998) when considering transmitting boundaries. Both the amplitude and the sign of the displacement at point C are different from the results of Vardoulakis et al. (1987).

Because of these discrepancies, this problem was also solved with two alternative methods. The first one is Fourier domain boundary element method BEM developed by Mengi et al. (1994). Using this method, an unsupported case of an underground opening problem is compared with the infinite elements. The second method is FEM with standard viscous boundaries. It was observed that all of the results by BEM and by FEM with viscous boundary conditions are in good agreement with those obtained by Yerli et al. (1998) formulation and hence with the present formulation too.

Conclusions:

A dynamic finite-element analysis is carried out for soil-structure interaction problems considering transmitting boundaries. Two types of boundaries are considered: viscous boundaries and mapped infinite elements. The results are compared for three cases; the first one using finite elements only, the second using 5-node mapped infinite elements and the third one using viscous boundaries. The computer program named “MIXDYN” (Owen and Hinton, 1980) is modified in this study to “Mod-MIXDYN” by adding 5-node coding of infinite element presented by Selvadurai and Karpurapu (1988). A new derivation of the shape and mapped functions is made in this study for the cases when the infinite direction is extended to the left and down. The following conclusions are drawn:

1) The transmitting boundary absorbs most of the incident energy. The distinct reflections observed in the "fixed boundaries" case disappear in the "transmitted boundaries" case. This is true for both cases

of using viscous boundaries or mapped infinite elements.

2) The viscous boundaries are more effective in absorbing the waves resulting from dynamic loads than mapped infinite elements. This is clear when comparing the results of both types with those of transient infinite elements.
3) The results of the program (ANSYS) adopting fixed boundary with dashpot elements are in good agreement with those of the program OpenSees which adopts transmitting viscous boundaries, using zero-length dashpot elements.

References:

1. Beer, G. and Meek, J. L., ”Infinite Domain Elements”, International Journal for Numerical Methods in Engineering, Vol. 17, P.P. 43-52, 1981.
2. Bettess, P., “Infinite Elements.”, International Journal for Numerical Methods in Engineering, Vol. 11, 1977.
3. Bettess, P., "Infinite Elements”, First Edition, Penshaw Press, 1992.
4. Curnier, A., “A Static Infinite Element”, International Journal for Numerical Methods in Engineering. Vol. 19, p.p. 1479-1488, 1983.
5. Dawood, S. H., “Dynamic Analysis of Soil-Structure Interaction Problems Considering Infinite Boundaries”, M.Sc. thesis, University of Technology, Building and Construction Engineering Department, 2006.
6. Karpurapu, G. R., ”Composite Infinite Element Analysis of Unbounded Two-Phase Media”, Computational Mechanics Publications / Adv. Eng. Software, Vol.10, No. 4, 1988.
7. Marques, J. M. M. C. and Owen, D. R. J., “Infinite Elements in Quasi-static Materially Non-linear Problems”, Computers and Structures, Vol.18, No. (4), P.P. 739-751, 1984.
8. Medina, F., and Penzien, J., “Infinite Elements for Elastodynamics.”, Earthquake Engineering and Structural Dynamics, Vol. 10, 1982.
9. Mengi, Y., Tanrikulu, A.H., and Tanrikulu, A.K., “Boundary Element Method for Elastic Media: An Introduction”, METU Press, Ankara, Turkey, 1994.
10. Okabe, M., “Generalized Lagrange Family for the Cube with Special Reference to Infinite Domain Interpolations”, Comp. Meth. Appl. Mech. Eng., Vol. 38, No. (2), P.P. 153-168, 1983.

11. Owen, D. R. J. and Hinton, E., "Finite Elements in Plasticity: Theory and Practice", Pineridge Press Limited, 1980.
12. Pissanetsky, S., “A Simple Infinite Element”, Compel, 3, No. 2, Boole Press, Dublin, p.p. 107-114, 1984, (As cited by Bettess, 1992).
13. Vardoulakis, I. G., Beskos, D. E., Leung, K. L., Dasgupta, B., and Sterling, R. L., “Computation of Vibration Levels in Underground Space.” , International Journal Rock Mech. Min. Sci. and Geomech. Abstracts, Vol. 24, No. (5), P.P. 291-298, 1987.
14. Wolf, J. P., "Soil Structure-Introduction Analysis in Time Domain", Prentice Hall, 1988.
15. Yerli, H. R., Temel, B. and Kiral, E., “Transient Infinite Elements for 2D Soil-Structure Interaction Analysis”, Journal of Geotechnical and Geo-environmental Engineering, ASCE, Vol. 124. No. 10, P.P. 976-988, October 1998.
16. Zienkiewicz, O.C., Emson, C.R.I., and Bettess, P., “A Novel Boundary Infinite Element”, International Journal for Numerical Methods in Engineering”, Vol. 19, P.P. 393-404, 1983.
17. Zuo, D., "Numerical Modeling of Dynamic Soil-Structure Interaction during Earthquakes", Term paper, from the Internet, 2002.

Table (1): Serendipity eight-node two dimensional infinite element, [Bettess, 1992]. The mapping functions and derivatives.

Node i	ξ_i	η_i	*Mi*	$\partial Mi/\partial\xi$	$\partial Mi/\partial\eta$
6	0	1	$(1+\xi)(1+\eta)\ /2(1-\xi)$	$(1+\eta)/(1-\xi)^2$	$(1+\xi)/2(1-\xi)$
7	-1	1	$(-1-\xi-\xi\eta+\eta^2)\ /(1-\xi)$	$(-2-\eta+\eta^2)/(1-\xi)^2$	$-\xi+2\eta/(1-\xi)$
8	-1	0	$2(1-\eta^2)/\ (1-\xi)$	$2(1-\eta^2)/(1-\xi)^2$	$-4\eta/(1-\xi)$
1	-1	-1	$(-1-\xi+\xi\eta+\eta^2)\ /(1-\xi)$	$(-2+\eta+\eta^2)/(1-\xi)^2$	$(\xi+2\eta)/(1-\xi)$

2	0	-1	$(1+\xi)(1-\eta)\ \ /2(1-\xi)$	$(1-\eta)/(1-\xi)^2$	$-(1+\xi)/2(1-\xi)$

Table (2): Serendipity eight-node two dimensional infinite element extending to infinity representing case a. The mapping functions and derivatives for element extending to negative ξ direction, as derived in this study.

Node i	ξ_i	η_i	*Mi*	∂Mi/∂ξ	∂Mi/∂η
2	0	-1	$(1-\xi)(1-\eta)\ \ /2(1+\xi)$	$-(1+\eta)/(1+\xi)^2$	$(1-\xi)/2(1+\xi)$
3	1	-1	$-1+\xi-\xi\eta+\eta^2)\ \ /(1+\xi)$	$2-\eta-\eta^2)/(1+\xi)^2$	$-\xi+2\eta/(1+\xi)$
4	1	0	$2(1-\eta^2)/\ \ (1+\xi)$	$-2(1-\eta^2)/(1+\xi)^2$	$-4\eta/(1+\xi)$
5	1	1	$-1+\xi+\xi\eta+\eta^2)\ \ /(1+\xi)$	$2+\eta-\eta^2)/(1+\xi)^2$	$(\xi+2\eta)/(1+\xi)$
6	0	1	$(1-\xi)(1+\eta)\ \ /2(1+\xi)$	$(1+\eta)/(1+\xi)^2$	$(1-\xi)/2(1+\xi)$

Table (3): Serendipity eight-node two dimensional infinite element extending to infinity representing case b. The mapping functions and derivatives for element extending to negative η direction, as derived in this study.

Node i	ξ_i	η_i	*Mi*	∂Mi/∂ξ	∂Mi/∂η
4	1	0	$(1+\xi)(1-\eta)\ \ /2(1+\eta)$	$(1-\eta)/2(1+\eta)$	$-(1+\xi)/(1+\eta)^2$
5	1	1	$(-1+\eta+\xi\eta+\xi^2)\ \ /(1+\eta)$	$(\eta+2\xi)/(1+\eta)$	$2-\xi^2+\xi/(1+\eta)^2$
6	0	1	$2(1-\xi^2)/\ \ (1+\eta)$	$-4\xi/(1+\eta)$	$-2(1-\xi^2)/(1+\eta)^2$
7	-1	1	$(-1+\eta-\xi\eta+\xi^2)\ \ /(1+\eta)$	$(-\eta+2\xi)/(1+\eta)$	$2-\xi-\xi^2/(1+\eta)^2$
8	-1	0	$(1-\xi)(1-\eta)\ \ /2(1+\eta)$	$-(1-\eta)/2(1+\eta)$	$-(1-\xi)/(1+\eta)^2$

Property	Values in	
	U.S. customary units	SI metric units
Modulus of elasticity, E	288000 lb/ft^2	13795.2 kN/m^2
Poisson's ratio, μ	0.3	0.3
Density, γ	100 lb/ft^3	15.71 kN/m^3
Mass density, ρ	3.105590062 (lb-sec^2)/ft^4	1.601916998 (kN-sec^2)/m^4

Table(4):The finite element mesh of problem No.(1).

Type of information		ANSYS (Viscous boundaries)
No. of nodes		1361
No. of elements		400
Infinite elements		-
Dashpot elements		80
No of steps	100	Δt = 0.00005 sec.
	220	Δt = 0.0005 sec.
	100	Δt = 0.0025 sec.
	127	Δt = 0.005 sec.
C_t (lb-sec.)/ft^3		587
C_n (lb-sec.)/ft^3		947
Total No. of time steps		547
Time at the end of excitation		1.0 sec.

Table (5) Mesh information for problem No. (1).

Table (6) Material properties of the soil and concrete for problem No. (2) (from Yerli et al., 1998).

Material Properties	Soil	Concrete
Shear modulus, G (N/m^2)	470.24×10^6	10622.0×10^6
Poisson's ratio, μ	0.10	0.17
Density, γ (kg/m^3)	2048	2263

Table (7) Mine problem No. (2) mesh information.

Type of information	Mod-MIXDYN		ANSYS Viscous boundaries
	Fixed bounbaries	Infinite elements	
No. of nodes	696	740	828
No. finite of elements	208	208	208
Infinite elements	-	43	-
Dashpots	-	-	88
Total No. of nodal points with fixed degrees of freedom	44	44	44
Total No. of time steps	375	375	375
Time step length (sec.)	0.0002	0.0002	0.0002

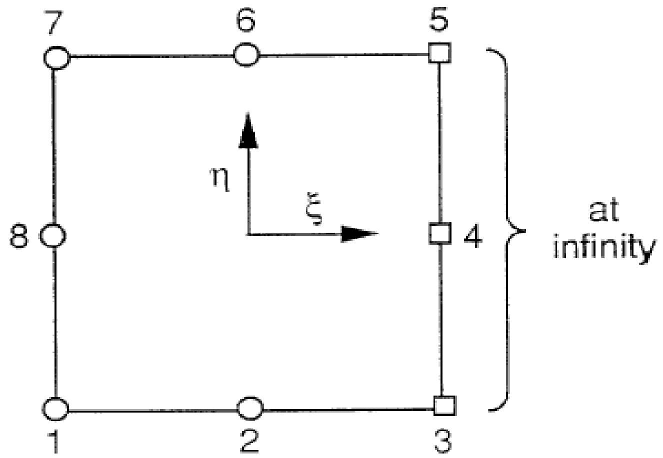


Fig. (1): Serendipity infinite element nodal numbering, element extending to infinity at positive ξ direction, [Bettess, 1992].

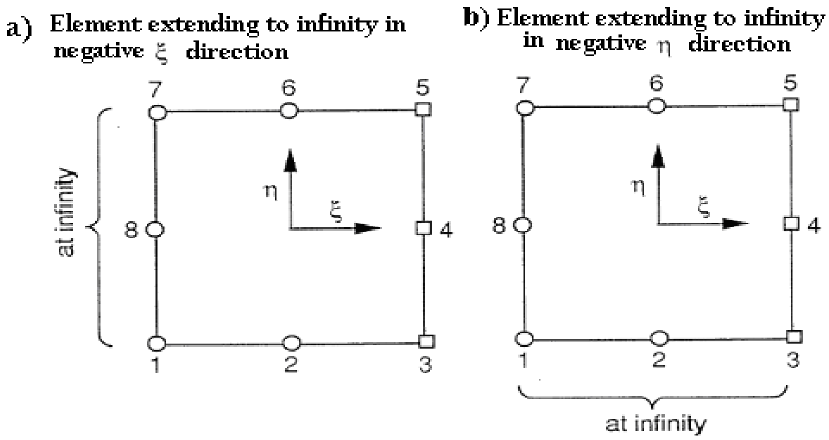


Fig. (2): Serendipity infinite element nodal numbering, element extending to: a) Negative ξ direction.
b) Negative η direction.

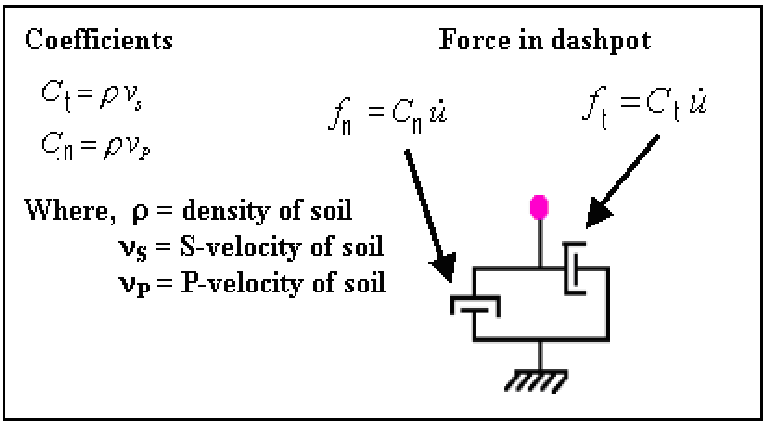
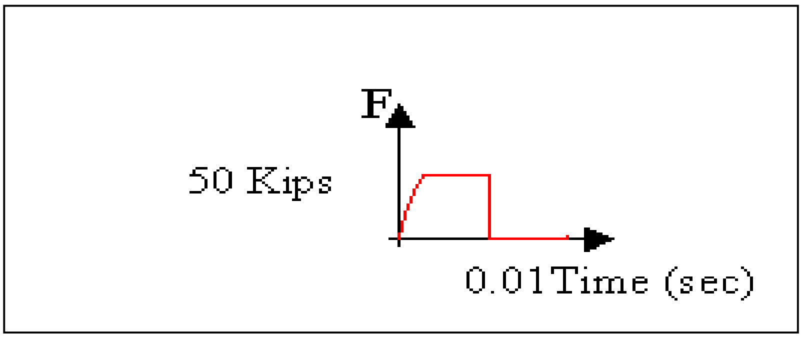


Fig. (3): Zero-length dashpot element [u.washington.edu Website].



Load P = (lb)	50000*sin(1570.796)*t	0 < t < 0.001 sec.
	50000	0.001 < t < 0.004 sec.
	0	0.004 < t < 1 sec.

Fig. (4): The loading function for problem No. (1) , [u.washington.edu Website].
Note: 1 kip = 4.448 kN

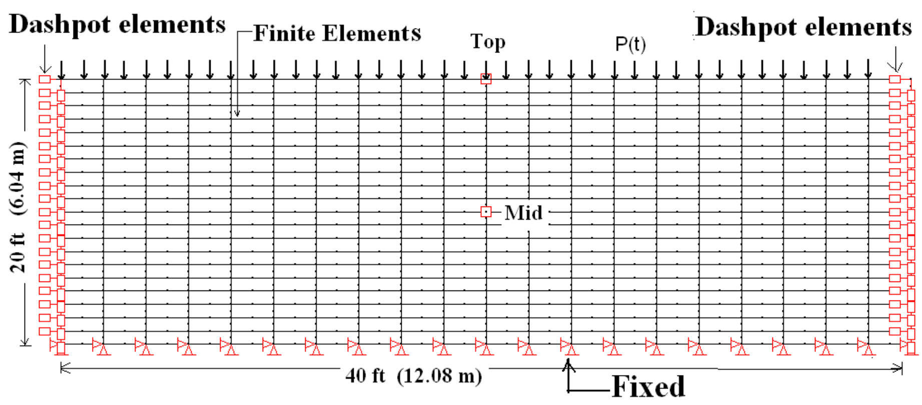


Fig. (5): The finite element mesh of problem No. (1).

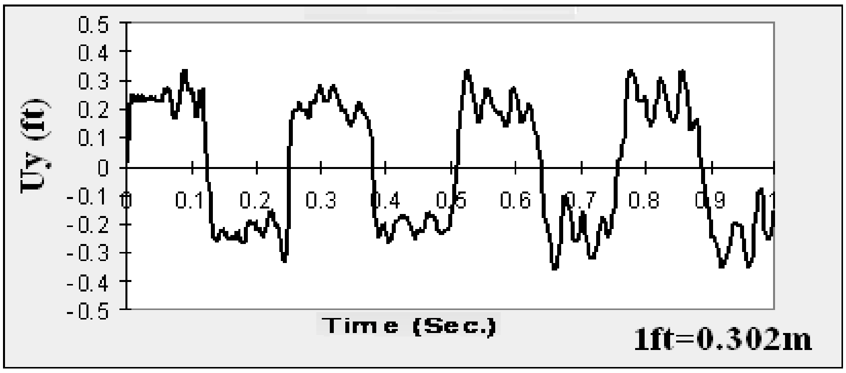


Fig. (6): Vertical displacements at top node considering fixed boundary predicted by (ANSYS).

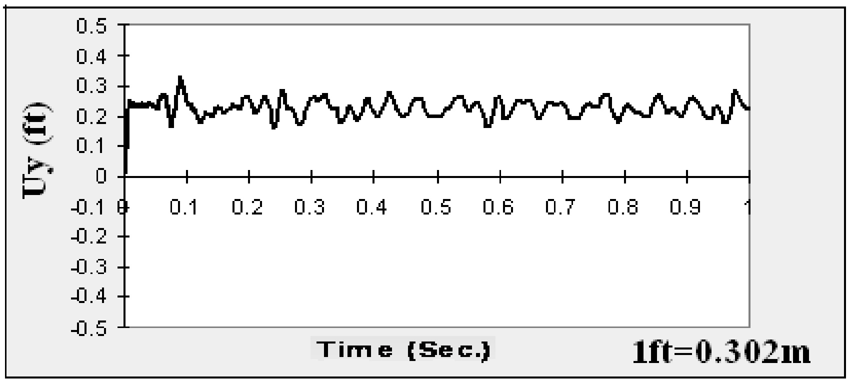


Fig. (7): Vertical displacements at top node considering viscous boundary (VB) predicted by (ANSYS).

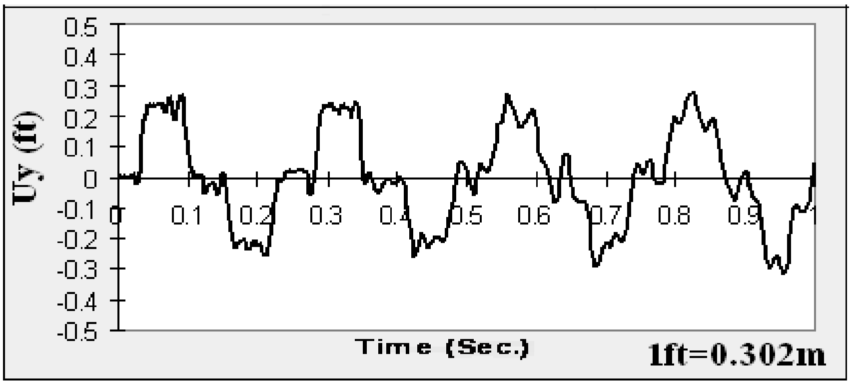


Fig. (8): Vertical displacements at mid-node considering fixed boundary (ANSYS).

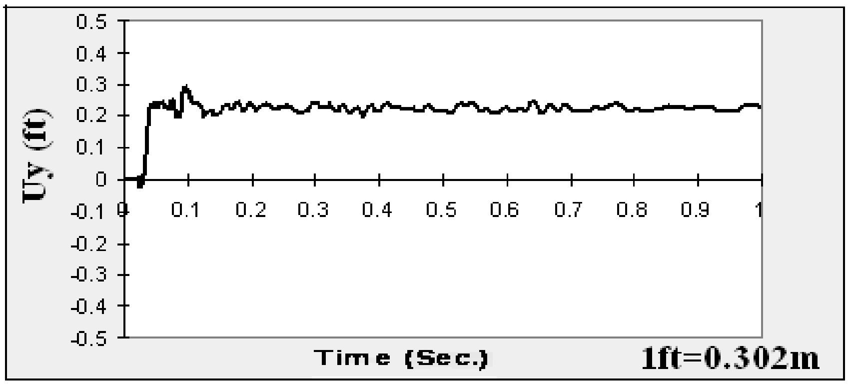


Fig. (9): Vertical displacements at mid-node considering viscous boundary (VB) predicted by (ANSYS).

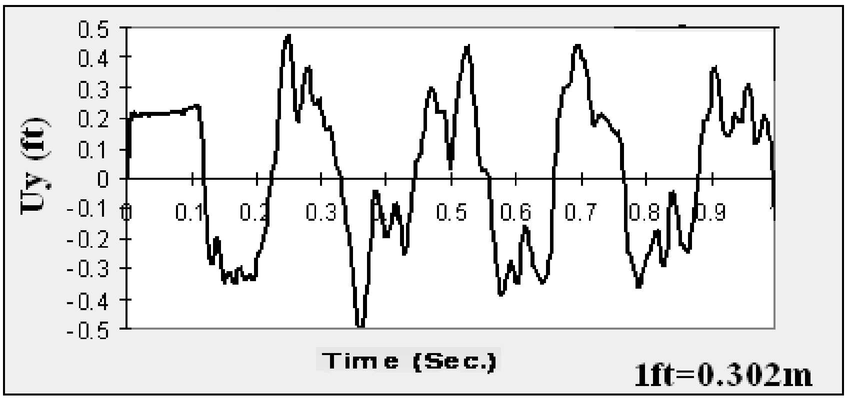


Fig. (10): Vertical displacements at top node as predicted by the program (OpenSees) for fixed boundary.

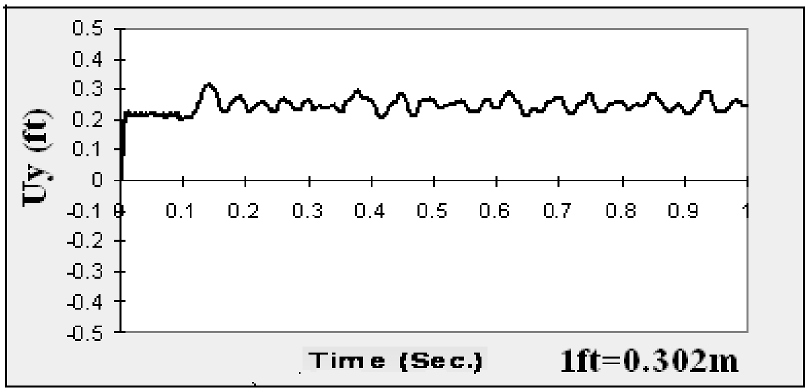


Fig. (11): Vertical displacements at top node as predicted by the program (OpenSees) for viscous boundary (VB).

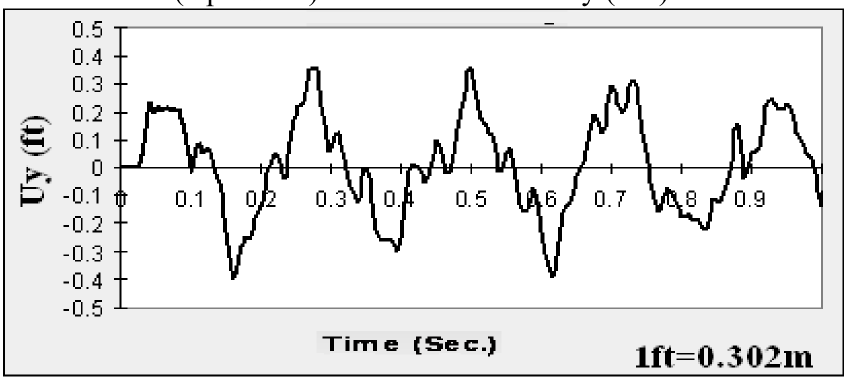


Fig. (12):Vertical displacements at mid-node as predicted by the program (OpenSees) for fixed boundary.

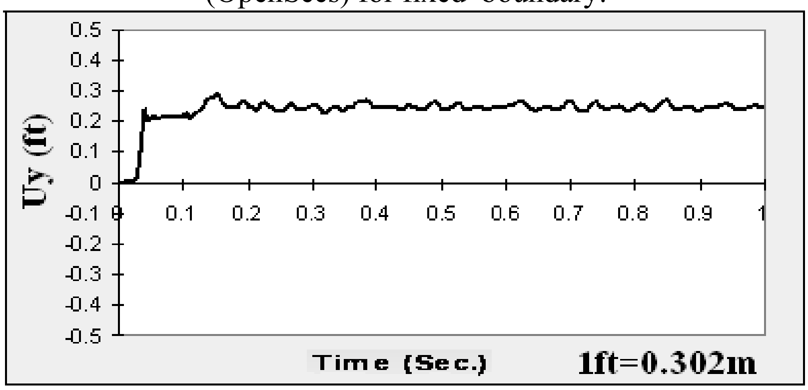


Fig. (13): Vertical displacements at mid-node as predicted by the program (OpenSees) for viscous boundary (VB).

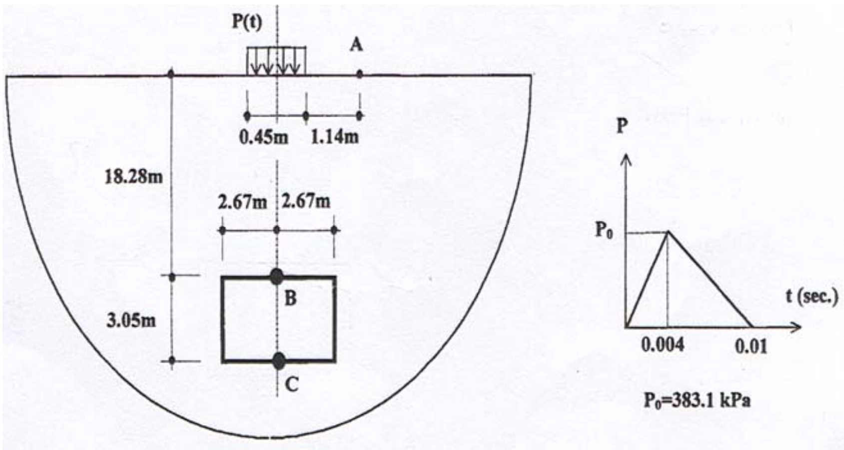


Fig. (14): Underground opening and forcing function of problem No. (2).

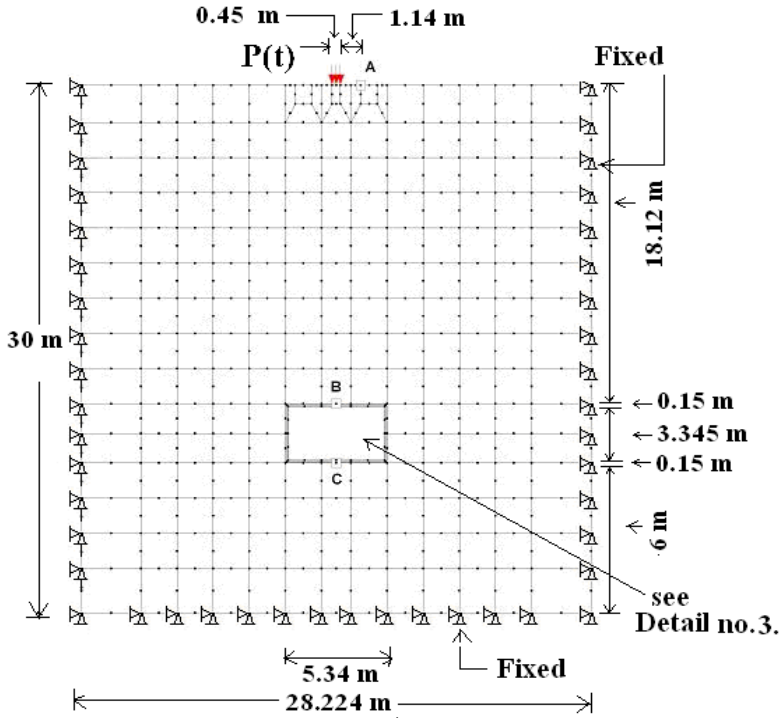
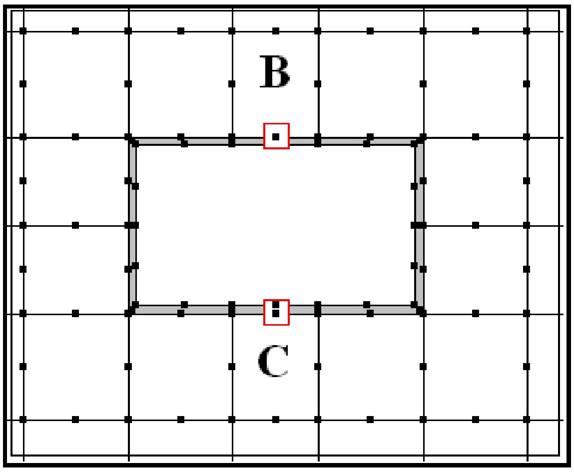


Fig.(15): Finite element mesh (fixed boundaries) for problem No.(2).



Detail No. (3).

Fig.(15): (Continued).

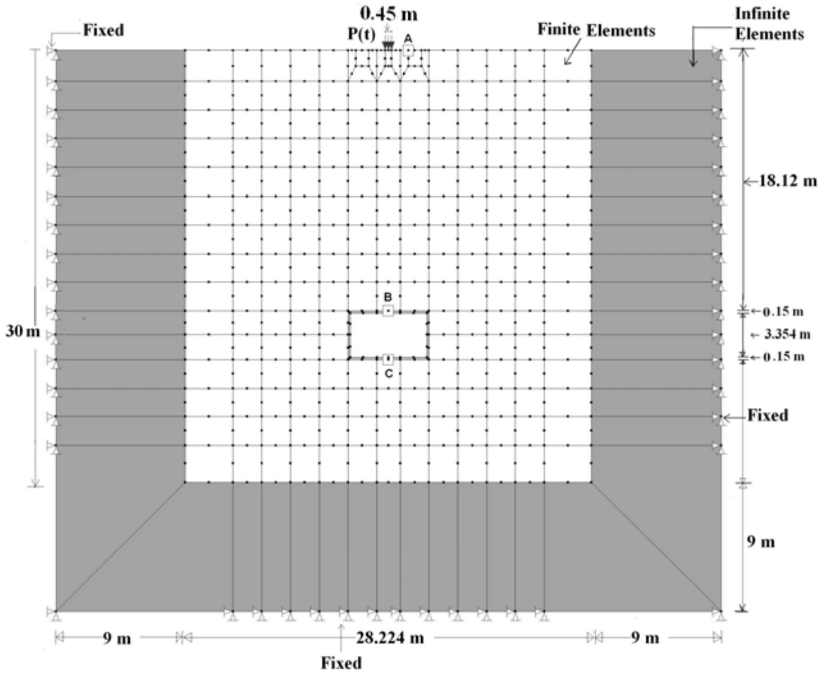


Fig.(16): Finite and infinite elements mesh for problem No. (2).

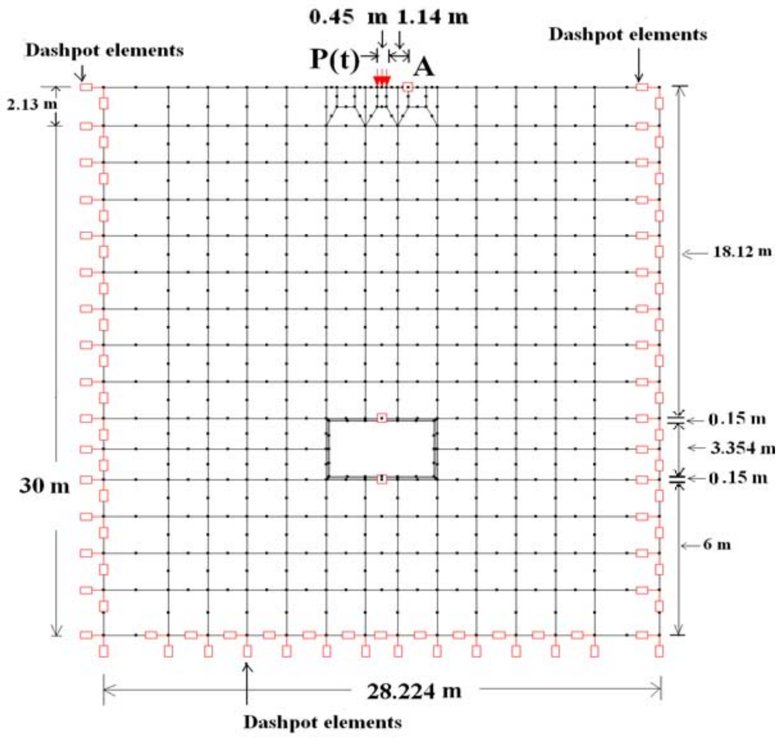


Fig. (17): Finite and dashpot elements mesh for problem No. (2).

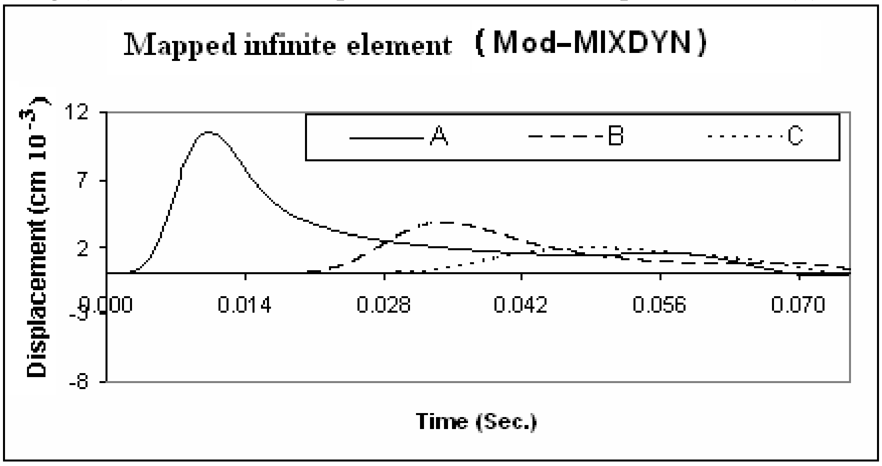


Fig. (18): Displacement versus time at points A, B and C using mapped infinite element (MIE) as predicted by (Mod-MIXDYN).

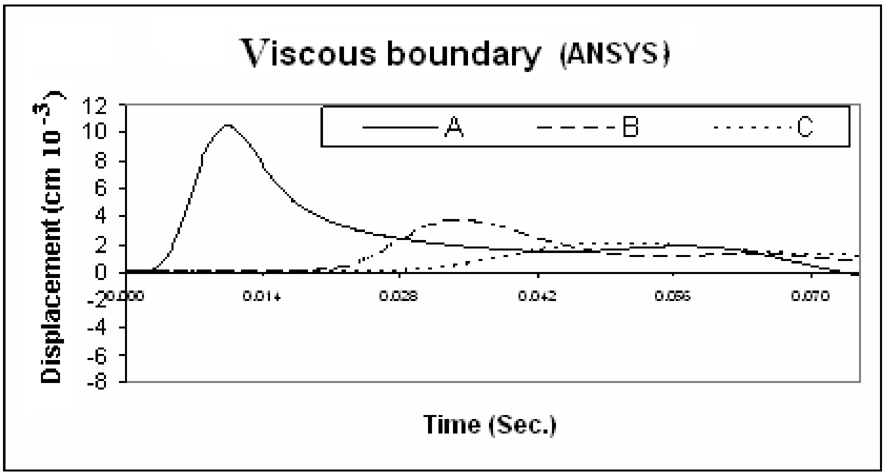


Fig. (19): Displacement versus time at points A, B and C considering viscous boundary (VB) as predicted by (ANSYS).

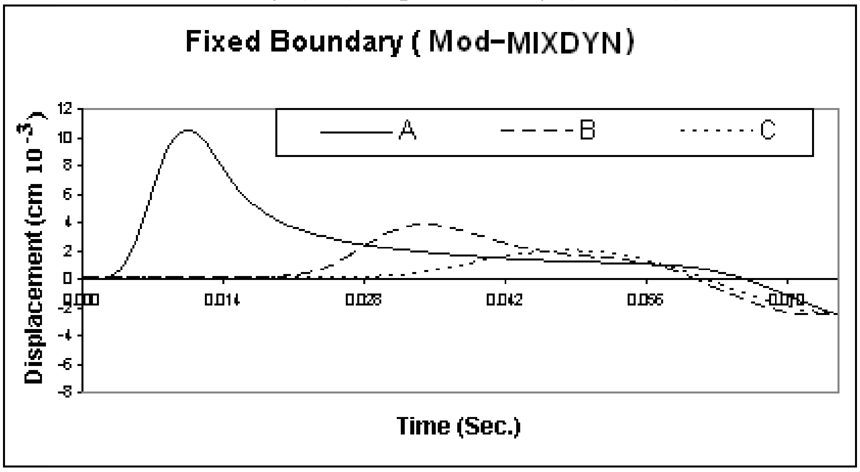


Fig. (20): Displacement versus time at points A, B and C considering fixed boundaries as predicted by (Mod-MIXDYN).

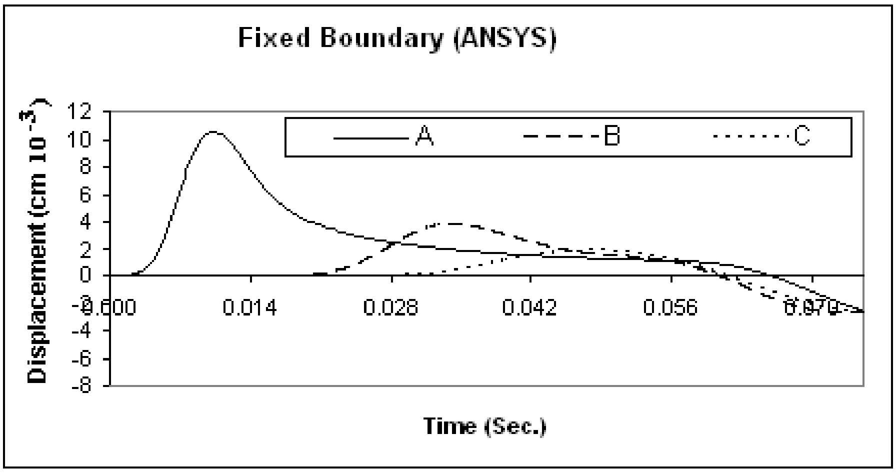


Fig. (21): Displacement versus time at points A, B and C considering fixed boundaries as predicted by (ANSYS).

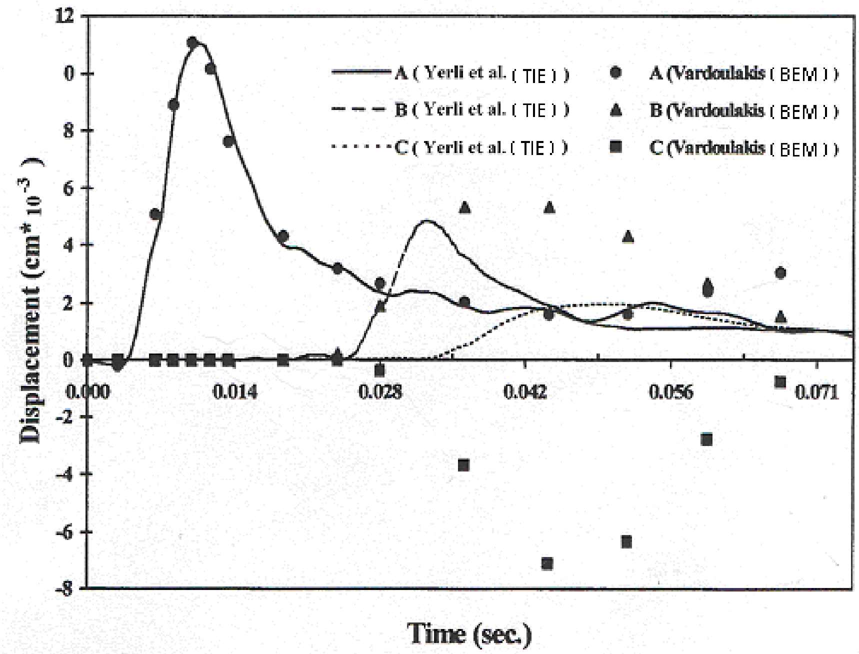


Fig. (22): Displacement versus time at points A, B and C , [Yerli et al., 1998].