Applicable Studies of the Slow Electrons Motion in Air with Application in the Ionosphere

I. G. Faiadh*, M. I. Ismaeel*, F. G. Hamudy* & S. S. Dawood*

Received on: 15/12/2009
Accepted on: 4/11/2010

Abstract

In this study, the motions of slow electrons in air and nitrogen with it's mixtures are studies by details. We solved numerically Boltzmann transport equation to calculate the parameters E, V_d and D/μ have been possible to deduce expression empirical formula from which may be derived the drift velocity, collisional frequencies, electronic temperature and mean energy loss per collision. These results applied to the ionosphere allow electronic collisional frequencies to be ready found in the ionosphere below 94 km, from the pressure of the air. The collision cross-sections of the molecules of air and nitrogen when the electronics temperature is less than about 2600°K.

The addition results are presented for electrons drifting through air in a steady state of motion under the action of the electric field, such as, K_T, U, Q, Q_0, ω, ω/P,η, ηK_T , and W/D for both Maxwell and Druyvesteyn distribution laws. This parameters are graphically as a functions for their variables.

The results appeared excellent agreements with the experiments and theoretical data.

Keywords: Boltzmann transport equation, electron energy distribution, interaction of radio waves.

دراسات تطبيقية لحركة الإلكترونات البطيئة في الهواء وإمكانية تطبيقها في طبقة الآيونسفير

الخلاصة

في هذا العمل، تمت دراسة حركة الإلكترونات واطئة الطاقة في الهواء وغاز النيتروجين مع مزيج $(Ar + N_2)$ (نيتروجين + آركون) من خلال حل معادلة الانتقال لبولتزمان عددياً لحساب المجال الكهربائي، سرعة انجراف الإلكترونات والطاقة المميزة للإلكترونات، ومن خلال هذه المعلمات تم استنتاج علاقة وضعية لحساب التردد التصادمي للإلكترونات ودرجة الحرارة الإلكترونية ومعدل الطاقة المفقودة للإلكترونات عند كل تصادم مع جزيئات الوسط.

تم تطبيق هذه النتائج أعلاه في طبقة الآيونسفير التي تسمح بترددات تصادمية إلكترونية في الآيونسفير لارتفاع دون 94 كيلومتر، من ضغط الهواء. المقاطع العرضية التصادمية لجزيئات الهواء والنيتروجين عندما تكون درجة حرارة اقل منها بحدود 2600 درجة كلفن.

تم في هذا البحث أيضاً حساب معلمات إضافية أخرى لانجراف الإلكترونات خلال الهواء تحت تأثير المجال الكهربائي مثل K_T، U، Q، Q_0، ω، ω/P، η، ηK_T و W/D لحالة كل من قانوني توزيع السرع لماكسويل ودروفستن. تمت جدولة هذه النتائج ورسمها كدوال أزاء متغيراتها.أظهرت هذه النتائج عند مقارنتها مع المعطيات العملية والنظرية المنشورة تطابقاً جيداً.

*Ministry of Science & Technology / Baghdad

https://doi.org/10.30684/etj.29.3.3
2412-0758/University of Technology-Iraq, Baghdad, Iraq

1-Introduction

The properties of slow electrons in air had been investigated by many authors, the agreement between their measurements is not always close, and there was need for a fresh investigation of electronic motion in air with modern vacuum techniques and an apparatus of improved design[1].

The distribution of collisional frequencies in height could be determined with range of height by use of radio-waves alone, but that it is preferable to derive it by making use of the distribution of pressure and temperature that have been directly measured in rocked flights. This distribution in height of the collisional frequencies of electrons with the molecules of the air in the lower E-region of the ionosphere is of importance for assessing the absorption suffered by a radio-wave which traverses this region. The collisional cross section of air molecules in collisions with electrons as a function of the speeds of the electrons is necessary because of the importance of the physical quantities in related fields of study, which is calculated, such as, for instance, the properties of high-frequency discharges, microwaves discharge and ionosphere, it is desirable that measurements of them as possible[2].

In this work we use of combination of various average of the agitational speeds U of the electrons, such as $\overline{U}$, $\overline{U}\,\overline{U^{-1}}$ in the form of dimensionless units from which the physical quantities of interest are calculated. A matter of fundamental importance for the theory of wave interaction (ionospheric cross-modulation) when the electronic energy does not greatly exceed the thermal energy of the gas [3].

Nomenclature and Definitions:

E electric field strength, V/cm
M molecular mass
k Boltzmann constant
°K Temperature degree in Kelvin unit
$f(\varepsilon)$ energy distribution function, $eV^{-2/3}$
N,n gas number density, cm^{-3}
$q_m(\varepsilon)$ momentum transfer cross section, cm^2
$q_j(\varepsilon)$ electronic excitation cross section, cm^2
ε_j excitation energy, eV
$q_{-j}(\varepsilon)$ cross section for collisions, cm^2
V_d electron drift velocity, cm/s
$V_{d(EM)}$ Empirical formula for V_d
$V_{d(EMP)}$ Empirical formula for V_d
D diffusion coefficient, cm^2/s
μ electron mobility $cm^2/V.s$
D/μ diffusion coefficient to mobility ratio eV
$\bar{u}$ electron energy average, eV
K_T, K_1 Townsend's energy factor, eV
$K_{T(EM)}$ Empirical formula for K_T
$K_{T(EMP)}$ Empirical formula for K_T
$\nu=\nu_1=\nu_2$ collisional frequency of the electrons s^{-1}
E/P electric field strength to gas pressure ratio, $V\ cm^{-1}\ Torr^{-1}$
U agitational speed of an electron
$\overline{U}$ mean speed of the electron
ℓ electrons mean free path
e/m specific charge of the electron
Q agitational energy of an electron
A collisional cross-section, cm^2
ω mean power supplied to an electron, watt
Q_0 energy of thermal agitation of a gas molecules, erg
ΔQ average energy lost by an electron in a collision with a molecule
$\nu/p=\nu/p_1=\nu/p_2$ collisional frequency of the electrons to the gas pressure ratio
$\nu/n=\nu/n_1=\nu/n_2$ collisional frequency of the electrons to

the molecular concentration ratio

ω/P mean power supplied to the electron to the gas pressure ratio

η proportion of the energy Q lost in a collision

W/D drift speed of the electrons to the diffusion coefficient ratio

D diffusion coefficient of the electrons.

2-Boltzmann transport equation:

The particular form of the Boltzmann equation derived to include the effect of the molecular energy distribution of electron in a gas subject to an electric field has been treated by many authors [4-5]:

$$\frac{E^2e^2}{3}\frac{d}{d\varepsilon}\left(\frac{\varepsilon}{Nq_m(\varepsilon)}\frac{df}{d\varepsilon}\right)+\frac{2m}{M}\frac{d}{d\varepsilon}\left(\varepsilon^2 Nq_m(\varepsilon)f(\varepsilon)\right)$$
$$+\frac{2mKT}{M}\frac{d}{d\varepsilon}\left(\varepsilon^2 Nq_m(\varepsilon)\frac{df}{d\varepsilon}\right)$$
$$+\sum_j[(\varepsilon+\varepsilon_j)f(\varepsilon+\varepsilon_j)Nq_j(\varepsilon+\varepsilon_j)$$
$$-\varepsilon f(\varepsilon)N_j q_j(\varepsilon)]$$
$$+\sum_j[(\varepsilon-\varepsilon_j)f(\varepsilon-\varepsilon_j)Nq_{-j}(\varepsilon-\varepsilon_j)$$
$$-\varepsilon f(\varepsilon)N\ q_{-j}(\varepsilon)]=0 \quad \ldots\ldots (1)$$

The solution of this equation gives the steady state energy distribution function of a swarm of electrons drifting and diffusing through a gas at temperature T under the influence of an uniform electric field E, which an electron gains the excitation energy from a molecular. The distribution function is normalized through the relation [6]:

$$\int_0^\infty \varepsilon^{\frac{1}{2}} f(\varepsilon)d\varepsilon = 1$$

An alternative procedure that has been adopted is to compare experimental and calculated values of electron drift velocities and diffusion coefficient. These coefficients are comparatively straight forward integrals of functions of the functions of the momentum transfer cross section and the energy distribution function. Written in terms of the energy distribution function of equation (1) these integrals are[7]:

$$V_d = \frac{-eE\sqrt{2/m}}{3N}\int_0^\infty \frac{\varepsilon}{q_m(\varepsilon)}\frac{df}{d\varepsilon}d\varepsilon \quad \ldots\ldots\ldots (2)$$

$$D = \frac{(\frac{2}{m})^{\frac{1}{2}}}{3N}\int_0^\infty \frac{\varepsilon f(\varepsilon)}{q_m(\varepsilon)}d\varepsilon$$

From which the ratio of diffusion coefficient to mobility $\mu = V_d/E$ is found to be:

$$D/\mu = -\int_0^\infty \frac{\varepsilon f(\varepsilon)d\varepsilon}{q_m(\varepsilon)} \Big/ \int_0^\infty \frac{\varepsilon}{q_m(\varepsilon)}\frac{df}{d\varepsilon}d\varepsilon \quad \ldots. (3)$$

We are evaluating the drift velocity V_d, E and diffusion coefficient to mobility D/μ using equations (2) and (3) after prepared the momentum transfer cross sections, electronic cross section, excitation cross section and ionization potentials fed it to the program as, input data to solve the numerically Boltzmann transport equation (1), [8].

The obtained results from the above, such as, V_d, E and D/μ had be fed to the our “computer program” which constructed to this purpose. This results are compared with recent theoretical and experimental data. This values had been obtained according to the available conditions in the reference [2].

3-Townsend's energy factor K_1 & K_T:

We know that the D/μ is a measure of the electron average energy in electron volts(eV) = $\bar{u}/e$; hence the name characteristic energy for this ratio. The ratio of the characteristic energy to its thermal equilibrium value is [9]:

$\frac{KT_g}{e}$ = thermal equilibrium ………(4)

From equation[9]:

$$\frac{D}{\mu} = \frac{2\bar{u}}{3e} \quad ……………(5)$$

where

$$\bar{u} = \frac{3KT_g}{2}$$

By dividing eq.(5) to eq.(4) yields:

$$\frac{e}{KT_g}\frac{D}{\mu} = K_1 \quad …………(6)$$

where K_1 is called Townsend's energy factor.

Frequent use of this factor is found in the literature, particularly by authors of the Australian School[9].

A stream of electrons, having already acquired a steady state of motion in a uniform electric field E, the temperature of electron T_e exceeds the temperature of the gas T_g. When T_g is fixed, then the ratio is:

$$\frac{T_e}{T_g} = K_T \quad …………(7)$$

where K_T ($K_T=K_1/1.14$) is called Townsend's energy factor which is a function of the E/P (electric field strength to the gas pressure ratio).

4-Electron mean speed distribution U :

There are two laws of distribution of the speeds U of the electron, those of Maxwell and of Druyvesteyn, because the smaller values of K_T are of greater interest, it will be assumed that Maxwell's is the correct law [2].

The root-mean-square speed is:

$K_1 = K_T$ (Maxwell's law distribution)

$K_1 \neq K_T$ (Druyvesteyn law distribution)

$= 1.14\ K_T$

$K_T = 0.877\ K_1$

$$(\overline{U^2})^{\frac{1}{2}} = 1.15\times10^7 (K_1)^{\frac{1}{2}} \text{ cm/s}$$

(Maxwell) …….(8)

$$= 1.08\times10^7 (K_T)^{\frac{1}{2}} \text{ cm/s}$$

(Druyvesteyn) …….(9)

The mean speed of the electron is:

$$\overline{U} = 1.06\times10^7 (K_1)^{\frac{1}{2}} \text{ cm/s}$$

(Maxwell) . …… (10)

$$= 1.02\times10^7 (K_T)^{\frac{1}{2}} \text{ cm/s}$$

(Druyvesteyn) …… (11)

5-The electron's collisional frequency :

To find the collisional frequency of electrons drifting through air in a steady state of motion under the action of an uniform and constant electric field as a function of E/P, it is necessary first to determine the drift velocity V_d as a function of E/P, in this situation, we find the drift velocity from the equation (2) by using numerically transport equation solution.

The slow electrons are scattered isotropically in collision with molecules as assumption[2]:

$$V_d = \frac{2}{3}E\frac{e}{m}\overline{\ell U^{-1}} \quad …………(12)$$

$$= \frac{2}{3}E\frac{e}{m}\left[\frac{\bar{\ell}}{\overline{U}}\overline{U}\,\overline{U^{-1}}\right] \quad …… (13)$$

Where:

$$\nu = \frac{\overline{U}}{\ell} \quad …… (14)$$

Substitute eq.(14) into eq.(13) yield:

$$V_d = \frac{2}{3}E\frac{e}{m}\frac{1}{\nu}\left[\overline{U}\,\overline{U^{-1}}\right] \quad …… (15)$$

Multiplied the right side of eq.(15) by factor P/P yield:

$$V_d = \frac{2}{3}\frac{e}{m}\frac{E}{P}\frac{P}{\nu}\left[\overline{U}\,\overline{U^{-1}}\right] \quad \ldots\ldots (16)$$

$$\frac{\nu}{P} = \frac{\frac{2}{3}\frac{e}{m}\frac{E}{P}\left[\overline{U}\,\overline{U^{-1}}\right]}{V_d} \quad \ldots\ldots\ldots (17)$$

We can find [1]:

$$\left[\overline{U}\,\overline{U^{-1}}\right] = 1.27 \text{ (Maxwell)} \quad \ldots\ldots\ldots (18)$$

$$= 1.18 \text{ (Druyvesteyn)} \quad (19)$$

Substitute eq.(18) and eq.(19) into eq.(17) yield:

$$\frac{\nu}{P}_1 = 1.49\times10^{15}\frac{E/P}{V_d} \text{ Maxwell)} \quad (20)$$

$$= 1.38\times10^{15}\frac{E/P}{V_d} \text{ (Druyvesteyn)} \quad \ldots (21)$$

Where E is expressed in V cm^{-1} , as above we indicate that the V_d is a function of E/P, it follows from eq.(17) that $\frac{\nu}{P}$ may also be found as a function of E/P . But K_T is also a function of E/P . Whence, $\frac{\nu}{P}$ can be represented as a function of K_T is that for $K_T < 9$ the relationship is linear to $K_T = 1$. Thus the experimental results justify the following representation:

$$\frac{\nu}{P}_2 = 9.36\times10^{7} K_1 \quad \text{(Maxwell)} \quad \ldots\ldots\ldots (22)$$

$$= 9.36\times10^{7} K_T \quad \text{(Druyvesteyn)} \quad \ldots\ldots (23)$$

Where P in unit of mm Hg and $K_T <$ 9; K_1=K_T.

We can see from eq.(22) the pressure is measure of the molecular concentration n according to a standard temperature T_g=288 °K. When T_g=288 °K and P = 1 mm of mercury, n = n_1 = 3.35 × 10^{16} cm^{-3}; consequently:

$$P = \frac{n}{3.35\times10^{16}} \quad \ldots\ldots\ldots (24)$$

from the above we can find:

$$K_T = \frac{T_e}{T_g} = \frac{T_e}{288} \quad \ldots\ldots\ldots (25)$$

$$T_e = \frac{2Q}{3K} \quad \ldots\ldots\ldots (26)$$

The collisional cross-section (A) isn't affected by the agitational motions of the molecules, it follows that (A) depends in a specified gas upon the temperature T_e of the electrons only and not explicitly upon the temperature T_g of the gas. It is ν depends only upon n and T_e [2].

The relationship may be deduced from either of equation (22) the first: substitute the eqs. (24-26) into eq.(22) yields:

$$\frac{\nu}{\frac{n}{3.35\times10^{16}}} = 9.36\times10^{7}\frac{T_e}{288}$$

$$\nu = 9.7\times10^{-12} nT_e \quad \ldots\ldots (27)$$

$$= 9.7\times10^{-12} n\frac{2Q}{3K}$$

$$= 4.68\times10^{4} nQ \quad \ldots\ldots (28)$$

$$\frac{\nu}{n} = 4.68\times10^{4} Q \quad \ldots\ldots\ldots (29)$$

where:

$$Q = \frac{1}{2}m\overline{U^2} \quad \ldots\ldots (30)$$

Substitute eq.(30) into eq.(29) yield:

$$\frac{\nu}{n} = 4.68\times10^{4}\times\frac{1}{2}m\overline{U^2}$$

$$\frac{\nu}{n} = 2.34\times10^{4} m\overline{U^2} \quad \ldots\ldots\ldots\ldots (31)$$

$$\nu_1 = 2.34\times10^{4} nm\overline{U^2} \text{ (Maxwell)}(32)$$

$$\nu_1 = 2.34\times10^{4} nm\overline{U^2} \quad \text{(Druyvesteyn)}..(33)$$

Substitute eqs. (10-11) into eq.(31) yield:

$$\frac{\nu}{n_1} = 2.39\times10^{-9} K_1 \quad \text{(Maxwell)} \qquad \ldots\ldots (34)$$

$$\frac{\nu}{n_1} = 2.22\times10^{-9} K_T \quad \text{(Druyvesteyn)} \qquad \ldots(35)$$

The second of equation (22) leads to the same results, i.e., $K_T < 9$ when T_g = 288 °K. For nitrogen a similar behavior of ν is:

$$\nu = 1.04\times10^{-11} nT_e$$

$$= 1.04\times10^{-11} n \frac{2Q}{3K}$$

$$= \frac{2\times1.04\times10^{-11} nQ}{3\times1.38\times10^{-16}}$$

$$= 5.02\times10^{4} nQ \qquad \ldots\ldots\ldots (36)$$

$$\frac{\nu}{n} = 5.02\times10^{4} Q$$

$$= 5.02\times10^{4} \times \frac{1}{2} m\overline{U^2}$$

$$\nu_2 = 2.51\times10^{4} nm\overline{U^2} \quad \text{(Maxwell)} \qquad \ldots (37)$$

$$\nu_2 = 2.51\times10^{4} nm\overline{U^2}$$

(Druyvesteyn) (38)

$$\frac{\nu}{n_2} = 2.57\times10^{-9} K_1 \quad \text{(Maxwell)} \qquad \ldots\ldots (39)$$

$$\frac{\nu}{n_2} = 2.38\times10^{-9} K_T \quad \text{(Druyvesteyn)} \qquad \ldots(40)$$

6-The dependence of the collisional cross-section A upon $\overline{U}$:

We can define the mean free path ℓ is the reciprocal of the product the molecular concentration n time the collision cross-section of the molecular, i.e. [3]:

$$\ell = \frac{1}{nA} \qquad \ldots\ldots\ldots\ldots(41)$$

From eq.(14) the collisional frequency of the electrons is:

$$\nu = \frac{\overline{U}}{\ell}$$

From eq.(41) we can find that in air:

$$\nu = \frac{\overline{U}}{(nA)^{-1}}$$

$$A = \frac{\nu}{n}\frac{1}{\overline{U}} \qquad \ldots\ldots\ldots(42)$$

Substitute eq.(27) into eq.(42) yield:

$$A = 9.70\times10^{-12} \frac{T_e}{\overline{U}} \qquad \ldots\ldots\ldots(43)$$

Substitute eq.(25) into eq.(43) yield:

$$A = 9.70\times10^{-12} \times 288 \frac{K_T}{\overline{U}} \qquad \ldots\ldots(44)$$

Substitute eq.(10) into eq.(44) yield:

$$A = 2793.6\times10^{-12} \frac{\frac{(\overline{U})^2}{(1.06\times10^{7})^2}}{\overline{U}}$$

$$= 2.48\times10^{-23}\, \overline{U} \quad \text{(Maxwell)} \ldots(45)$$

$$= 2.68\times10^{-23}\, \overline{U} \quad \text{(Druyvesteyn)} \qquad \ldots(46)$$

From the eq.(45) we can say the collisional cross-section is proportional to the mean speed of the electrons and to the inverse of the de Broglie wave-length and tends to zero as $\overline{U}$ approaches zero.

For my case which is to the nitrogen gas and Ar-N_2 mixture at 300 °K we can find from eq.(43) the following:

$$A = 9.70\times10^{-12} \frac{T_e}{\overline{U}}$$

Substitute eq.(25) into eq (43) obtained:

$$A = 9.70\times10^{-12} \frac{T_g K_T}{\overline{U}} \qquad \ldots\ldots\ldots(47)$$

$$A = 9.70\times10^{-12}\times300\frac{K_T}{\overline{U}} \quad ......(48)$$

Substitute eq.(10) and eq.(11) into eq (48) obtained:

$$A = 9.70\times10^{-12}\times300\frac{\frac{(\overline{U})^2}{(1.06\times10^7)^2}}{\overline{U}}$$

$$= 2.58\times10^{-23}\,\overline{U} \text{ (Maxwell)}...(49)$$

$$= 2.79\times10^{-23}\,\overline{U} \text{ (Druyvesteyn)} \quad .(50)$$

7-Electrons collisional frequencies in the lower E-region of the Ionosphere:

The chemical composition of the atmosphere is the same as that at the ground at heights less than 94 km [2]. From equation (27) we see that, when electrons are in thermal equilibrium in the ionosphere Te = Tg, the collisional frequencies of the electrons are given by the eq.(27) is:

$$\nu = 9.70\times10^{-12}\,nT_e$$

Where, the pressure of the air is:

$$P = nKT_e \quad ....(51)$$

By combine the eq.(24-27) and eq.(51) obtained:

$$\nu = 9.36\times10^{7}\,P\ (\sec^{-1}) \quad ....(52)$$

This equation indicate that the collisional frequency is a valueless to be estimated in terms of P and independently of T_e .

The values of the pressure P and temperature T_e in the lower regions of the ionosphere are found by rocket data [10], which it is found:

P = (210, 106, 54, 23, 10, 4.3, 1.9, 0.9, 0.43)×10^3 Torr ....(53)

Substitute the values of pressure in eq.(53) into eq.(52) obtained:

ν = (196.56, 99.216, 50.54, 21.52, 9.36, 4.02, 1.77, 0.84, 0.4)×10^5 s^{-1} ......(54)

8-The dependence of K_T on the mean power ω supplied to an electron

When electrons drift through a gas in a steady state of motion in a constant and uniform electric field E, the temperature T_e of electrons exceeds the temperature of gas T_g. The ratio $T_e/T_g = K_T$ is a function of E/P. For air at temperature T_g = 288 °K, it follow that T_e = 288 K_T. For same applications an alternative representation of K_T as a function of ω/P, where ω is the mean power supplied to an electron had been gained from the electric field E to each electron of a group drifting at speed V_d through the gas is [2]:

$$\omega = EeV_d \quad ....(55)$$

Multiply two sides of equation (55) by the factor 1/P obtained:

$$\frac{\omega}{P} = eV_d\,\frac{E}{P} \quad 0 \quad ....(56)$$

From the above, the values of K_T and ω/P that corresponding to each value of E/P because K_T and ω/P are single-valued functions of E/P.

9-Expression for K_T and V_d in terms E/P

We can express:

$$K_{T(EMP)} = f(\frac{E}{P}) \quad ....(57)$$

From eq.(22) for instance:

$$\frac{\nu}{P} = F(K_T) \quad ....(58)$$

From eq.(16):

$$V_{d(EMP)} = \frac{2}{3}\frac{e}{m}\frac{E}{P}\frac{P}{\nu}\left[\overline{U}\,\overline{U^{-1}}\right] \quad .. (59)$$

Substitute eq.(57) into eq.(58) and this later equation into eq.(59) obtained:

$$V_{d(EMP)} = \frac{\frac{2}{3}\frac{e}{m}\left[\overline{U}\,\overline{U^{-1}}\right]\frac{E}{P}}{F\left[f(\frac{E}{P})\right]} \quad ... (60)$$

Substitute eq.(18) for Maxwell's law speed U distribution into eq.(60) obtained:

$$V_{d(EMP)} = \frac{1.49\times10^{15}(\frac{E}{P})}{F\left[f(\frac{E}{P})\right]} \quad \dots (61)$$

We can obtained for air at T_g = 288 °K and K_T < 9 from eq, (22) the following:

$$\frac{\nu}{P} = aK_T \quad \dots\dots\dots\dots(62)$$

Where a = 9.36 × 10^7 [2]

From the above, we can let:

$$K_{T(EMP)} = b(\frac{E}{P})^{\alpha} \quad 4 < K_T < 9 \quad \dots. (63)$$

With the same range of values K_T obtained:

$$V_{d(EMP)} = \frac{1.49\times10^{15}(\frac{E}{P})^{(1-\alpha)}}{ab}$$

(4 < K_T < 9) …....(64)

The relationship (63) accurately represented K_T when α = 1/2 and b = 13.3, the formula (64) reduce to:

$$V_{d(EMP)} = 1.23\times10^{6}(\frac{E}{P})^{\frac{1}{2}} \quad \dots\dots. (65)$$

both K_T and V_d are a function of E/P.

The polynomial interpolation of the experimental curve show that K_T is represented by:

$$K_{T(EM)} = 1 + c(\frac{E}{P}) \quad K_T < 2 \quad \dots. (66)$$

and

$$V_{d(EM)} = \frac{1.49\times10^{15}(\frac{E}{P})}{a\left[1+\frac{cE}{P}\right]} \quad \dots (67)$$

In air with K_T < 2, c = 33. The deduction of eq.(68) from the behavior of K_T is a more reliable procedure.

10-The average energy lost by an electron in a collision

The average equilibrium energy of the electron depends upon the balance between the energy gained from the electric field and the energy-loss mechanism. The average energy ΔQ lost by an electron in a collision with a molecule is a function of the energy Q of the electron and of the agitational energy Q_0 of a molecule. If Q is not to change with time, mean power ω must be supplied to an electron, equal to the rate energy is lost by collisions. The relation of the equilibrium state is [2, 11].

$$\nu\Delta Q = \omega \quad \dots\dots\dots (68)$$

Substitute eq.(55) into eq.(68) obtained:

$$\Delta Q = \frac{EeV_d}{\nu} \quad \dots\dots\dots (69)$$

From eq.(15) we can find:

$$\nu = \frac{\frac{2}{3}\frac{Ee}{m}\left[\overline{U}\,\overline{U^{-1}}\right]}{V_d} \quad \dots\dots\dots\dots (70)$$

Substitute eq.(70) into eq.(69) obtained:

$$\Delta Q = \frac{\frac{3}{2}mV_d^2}{\left[\overline{U}\,\overline{U^{-1}}\right]} \quad \dots\dots\dots (71)$$

From eq.(18) and eq.(19) substitute the values into eq.(71) obtained:

$$\Delta Q = 1.08\times10^{-27}V_d^2 \quad \text{(Maxwell)} \quad \dots (72)$$

$$\Delta Q = 1.16\times10^{-27}V_d^2 \quad \text{(Druyvesteyn)} \quad .. (73)$$

From the above equation the ΔQ represent as a function of E/P.

We can find the proportion of the energy Q lost in a collision which is η defined as:

$$\eta = \frac{\Delta Q}{Q} \quad \dots\dots\dots\dots\dots (74)$$

Substitute eq.(71), and eq.(30) into eq.(74) yield:

$$\eta = \frac{\frac{3}{2}\frac{mV_d^2}{\left[\overline{U}\,\overline{U^{-1}}\right]}}{\frac{1}{2}m\overline{U2}}$$

$$\eta = \frac{3V_d^2}{\left[\overline{U}\,\overline{U^{-1}}\right]\overline{u^2}} \quad \text{………… (75)}$$

By using eq.(8), (9), (18) and (19) into eq.(75) yield:

$$\eta = 1.79\times10^{-14}\frac{V_d^2}{K_1} \quad \text{(Maxwell)} \quad \text{…… (76)}$$

$$\eta = 1.68\times10^{-14}\frac{V_d^2}{K_T} \quad \text{(Druyvesteyn)} \quad \text{(77)}$$

According to eq.(74) we can write:

$$\eta K_T = \frac{\Delta Q}{Q_0} \quad \text{…………… (78)}$$

where

$$Q_0 = \frac{Q}{K_T} \quad \text{…………… (79)}$$

Substitute eq.(79) into eq.(78) yield:

$$\eta K_T = \frac{\Delta Q}{Q}K_T \quad \text{…………… (80)}$$

Substitute eq.(74) and eqs. (76-77 into (80) obtained:

$$\eta K_T = \eta K_T$$

$$\eta = 1.79\times10^{-14}V_d^2 \quad \text{(Maxwell)} \quad \text{…… (81)}$$

$$\eta = 1.68\times10^{-14}V_d^2 \quad \text{(Druyvesteyn)} \quad \text{.(82)}$$

From eqs. (74) and (78), η, ηK_T are shown as a function of K_T and dependence of ΔQ upon $Q = K_T Q_0$ when $K_T < 3$ is of especial importance for the theory of radio-wave interaction.

11-Techniques for studying the motion of very slow electrons in air

One of the methods of examining collision phenomena between low energy electrons and gas molecules is to measure the ratio of the drift velocity W to the diffusion coefficient D of an electron swarm moving under the influence of an electric field E in a gas at pressure P [2].

The experimental results are always given either in term of D/μ or in term of K_1. We can find W/D in term of K_1 which is [2]:

$$K_1 = \frac{\frac{eE}{KT_g}}{\frac{W}{D}}$$

$$\frac{W}{D} = \frac{\frac{eE}{KT_g}}{K_1} \quad \text{……… (83)}$$

where

$$K_1 = \frac{3}{2}\left[\frac{\overline{U}}{\overline{U^2}\,\overline{U^{-1}}}\right]K_T \quad \text{……… (84)}$$

From Ref. [12] we can find:

$$\left[\frac{\overline{U}}{\overline{U^2}\,\overline{U^{-1}}}\right] = \frac{2}{3} \quad \text{(Maxwell)} \quad \text{…… (85)}$$

Substitute eq.(85) into eq.(84) obtained:

$K_1 = K_T$ (Maxwell's law distribution) ... .(86)

In Druyvesteyn distribution, as for instance,

$K_1 \neq K_T$

$K_1 = 1.14$ ………… (87)

Substitute eq.(86), and eq.(87) into eq.(83) yield:

$$\frac{W}{D} = \frac{\frac{eE}{KT_g}}{K_1}$$

$$\frac{W}{D} = \frac{e}{KT_g}\frac{E}{K_1} \quad \text{(Maxwell)} \quad \text{… (88)}$$

$$\frac{W}{D} = \frac{e}{KT_g}\frac{E}{1.14K_T} \quad \text{(Druyvesteyn)} \quad \text{.. (89)}$$

Since, W/D is a pressure-dependent quantity [13].

12-The calculations:

We calculate the physical quantities of the motions of slow electrons in the air and nitrogen with it's mixture in the lower E-region which transverse this region by using the precedent derivative relations for both Maxwell's and Druyvesteyn's law distribution [14].

To achieve above precedent equations; we constructed below "computer program" appendix A to calculate it. This program receive input data (E, V_d and D/μ) from the numerically transport equation solution (1-3) [15]. We use in the computer program the following information:-

- The molecular concentration n, in air = 3.35×10^{16} cm^{-3}, [2].
- The gas temperature °T, in air = 288 °K, [2].
- The molecular concentration n, in pure Nitrogen = 2.67×10^{19} cm^{-3} (from the work)
- The molecular concentration n, in Argon (99.9%) – Nitrogen (0.1%) mixture = 1.4345×10^{19} cm^{-3}
- The molecular concentration n, in Argon (99%) – Nitrogen (1%) mixture = 1.431×10^{19} cm^{-3}
- The molecular concentration n, in Argon (95%) – Nitrogen (5%) mixture = 1.4154×10^{19} cm^{-3}
- The gas temperature °T, in pure Nitrogen and it's mixture = 300 °K (from the work)

The obtained results are graphically in figures (1-35).

13-Results and Discussion

In this work we calculated many physical quantities which is tabulated for air and nitrogen with its mixture, this results applied to the ionosphere allow collisional frequencies to be found in the ionosphere below 94 km. The obtained results which is compared with experimental value and theoretical data in a good agreement [2].

Figs.(1-3) are showing the dependence Townsend's energy factor upon the ratio E/P of the electric field strength to the gas pressure for both Maxwell and Druyvesteyn law in air and pure-nitrogen gas. Which is seen the increasing k_1, k_T with E/P increasing, since k_1, k_T depend on E/P in air and N_2, which relate to air and N_2 at a temperature of 288 ^oK and 300 ^oK for air and N_2 respectively. It follows that T_e=288K_T.

Figs.(4-7) appear the collisional frequencies ν_1, ν_2 of the electrons as a function of the electric field strength to the gas pressure ratio, E/p for both Maxwell and Druyvesteyn law in air and pure nitrogen, which is seen increasing ν_1, ν_2 with increasing E/P.

Figs.(8-14) since drift velocity is a function of the E/P, it follows from eq.(20) that ν/p may also be found as a function of E/P. But k_1, k_T is also a function of E/P, consequently $\nu_{/p}$ can be represented as a function of k_1, k_2. The dependence of ν/p upon k_1,k_2 in air at 288°k is shown in figure 4, this mean, k_T<9 the relationship is linear, the curve is a straight line through the origin, this mean, (ν=0 at k_T=0), from eqs(41-42) we can say, the mean free path of an electron is $e=(nA)^{-1}$ and $\nu=nA\overline{U}$. Since the ν/p depend on K_T, K_1 in air and N_2 at T_g=288 ^oK and 300 ^oK respectively are shown in fig.(8) for K_T<9 the relationship is linear; the curve is a straight line through the origin.

It is evident ν depends only upon n and T_e. The eqs.(27-35) deduced first from eq.(22-23) and the second of eqs.(22-23) clearly leads to the same results. This valid for k_T<9. A similar behavior of ν is observed in nitrogen for eqs.(36-40).From this, it's cleared in figs.(8-14).

Fig.(15) shows the collisional cross-section (A) is proportional to the mean speed of the electrons, this mean, to the inverse of the de Broglie wave length, and tends to zero as $\overline{U}$ Approaches zero.

Fig.(16) from the figures (1-3) we see the Townsend's energy factor as a function of E/P. For application an alternative representation of k_T, k_1 as a function of ω/p. this factor k_1, k_T for electrons in air at temperature 288°k may be found. For ionospheric applications it would be an advantage if similar curves for other temperature were available, but in the absence of these experimental data it will be assumed that the chief influence of the temperature is upon the molecular concentration n corresponding to any specified pressure p. The curve is then generalized merely by designating the scale of abscissa as $(T_g/288)(\omega/p)$ instead of ω/p, leaving K_T unchanged.

Fig.(17) from eqs.(55-56), the mean power ω supplied by the electric field E to each electron of a group drifting at speed V_d through the gas. Because k_T, k_1 and ω/p that corresponding to each value of E/P. From this figures we can see increasing of ω/p with increasing of E/P, because the electrons gain the more energy from the electric field.

Figs.(18-19) shows the empirical formulae for k_T and V_d for E/P, which is increasing of the $k_{T(EMP)}$, $k_{T(EM)}$, $V_{d(EMP)}$ and $V_{d(EM)}$ with E/P increasing. The curve representing eq.(66) plotted for K_T<2 they don't diverge appreciably, for the range ($1\leq K_T\leq 2$) this usefull for the radio wave interaction applicable at $K_{T(EM)}$. For $K_{T(EMP)}$ plotted as a function of E/P at ranges K_T<9 as shown in eq.(63) fig.(18). It follows for $V_{d(EMP)}$ at eq.(64) fig(19).

Figs.(20-21) the deduction of eq.(67) from the behavior of K_T was a more reliable procedure for $V_{d(EM)}$, fig.(21).

Figs.(22-27) represents the average energy ΔQ lost by an electron in a collision with molecule is a function of the energy Q of the electron and of the agitational energy $Q_\circ$ of a molecule. Figs.(22-23) show in the state of thermal equilibrium the gain compensate the losses and ΔQ is zero. But figs(24-27) there is a losses in the electron energy with increasing of Q, E/P and V_d , from the above when Q exceeds $Q_\circ$ the losses exceed the gains and ΔQ is not zero.

Figs.(28-31) represents the mean power ω supplied to an electron as a function of the electron drift speed V_d and of the electric field E respectively. From the equation $\omega = eEV_d$, we can say for the figures, increasing of the mean power ω supplied by the electric field to each electron of a group drifting at V_d through the gas according to the above equation.

Figs.(32-33) in figures 32,33 η and ηk_T according to the equation (78) are show as a function of k_1,k_T, which is mean dependence of ΔQ on $Q=k_TQ_\circ$ when k_T<3 is of especial importance for the theory of radio-wave interaction, at over the range of values of k_1, k_T that occur in the ionosphere when radio-wave interaction takes place.

Figs.(34-35) shows the W/D as a function of the k_1, k_T for both Maxwell and Druyvesteyn in air and nitrogen gas, which is indicated increasing in W/D when k_1,k_T increasing.

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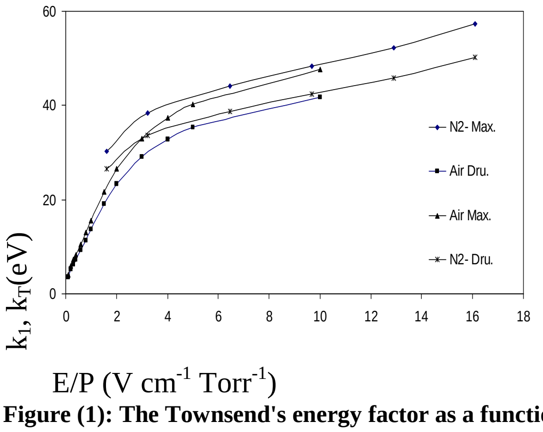


Figure (1): The Townsend's energy factor as a function of the ratio E/P of the electric field strength to the pressure P of the gas for both Maxwell and Druyvesteyn law in air and pure N_2 gas.

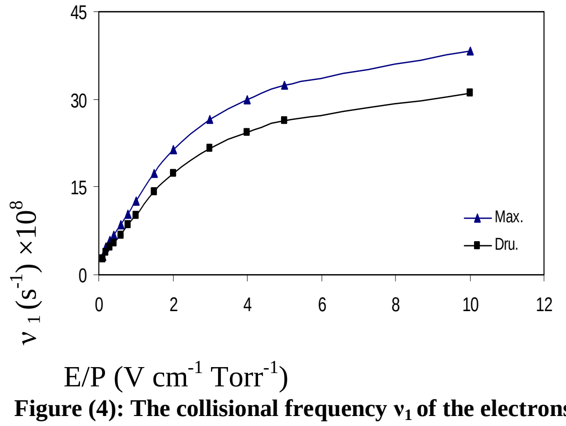


Figure (4): The collisional frequency ν_1 of the electrons as a function of the E/P, for both Maxwell and Druyvesteyn law in air.

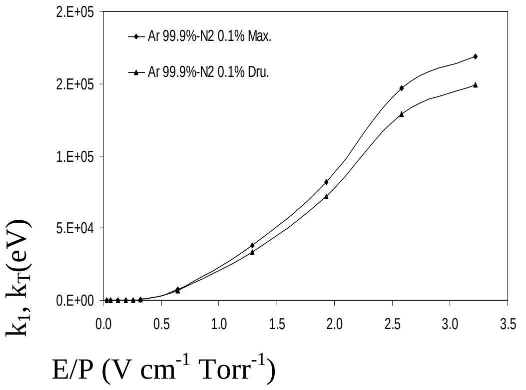


Figure (2): The Townsend's energy factor as a function of the ratio E/P of the electric field strength to the pressure P of the gas for both Maxwell and Druyvesteyn law in Ar-N_2 mixture

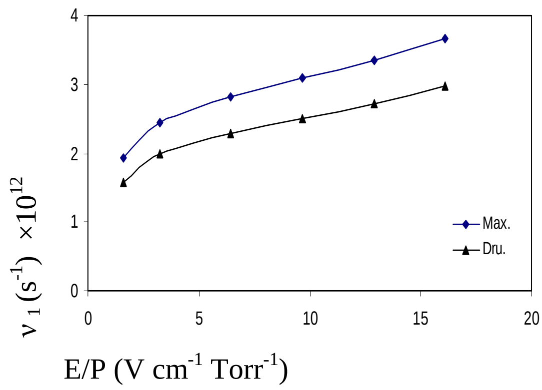


Figure (5): The collisional frequency ν_1 of the electrons as a function of the E/P, for both Maxwell and Druyvesteyn law in N_2.

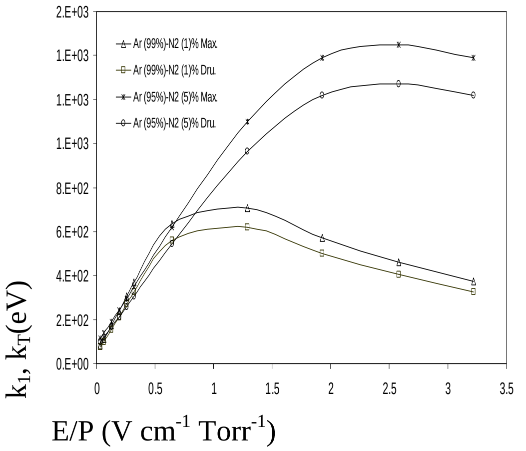


Figure (3): The Townsend's energy factor as a function of the ratio E/P of the electric field strength to the pressure P of the gas for both Maxwell and Druyvesteyn law in Ar-N_2 mixture.

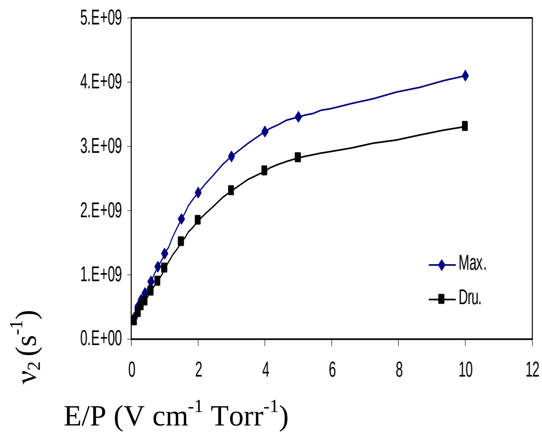


Figure (6): The collisional frequency ν_2 of the electrons as a function of the E/P, for both Maxwell and Druyvesteyn law in air.

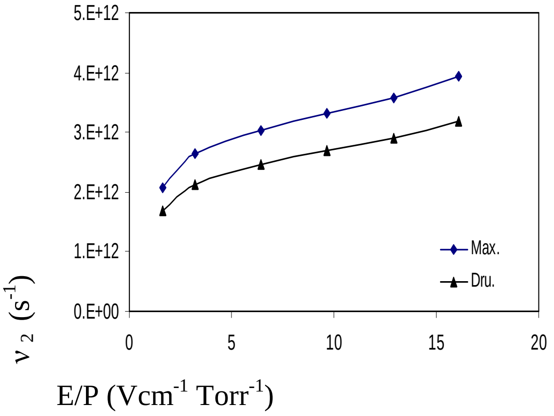


Figure (7): The collisional frequency ν_2 of the electrons as a function of the E/P, for both Maxwell and Druyvesteyn law in N_2.

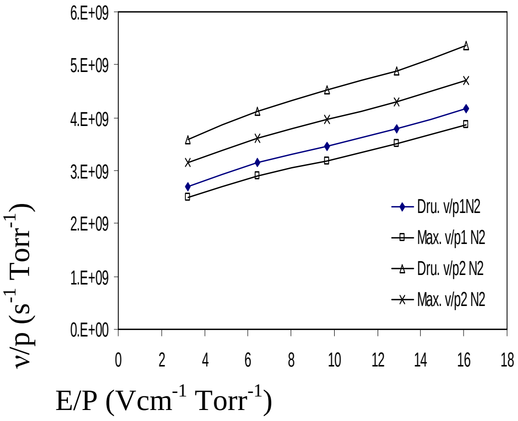


Figure (10): The Collisional frequency ν/p of the electrons as a function of the E/P for Maxwell and Druyvesteyn Law in N2.

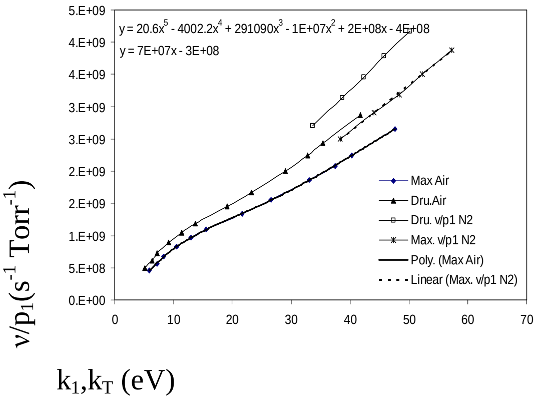


Figure (8): The Collisional frequency ν/p_1 of the electron as a function of the Townsend's energy factor for Maxwell and Druyvesteyn Law in air and pure N_2 gas.

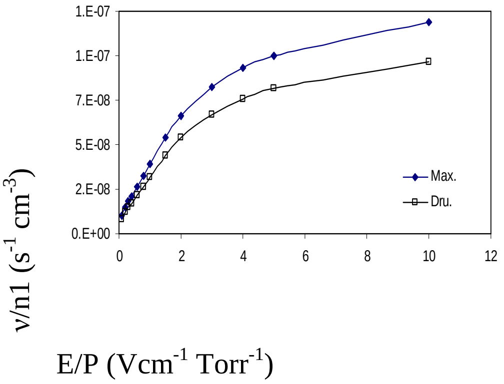


Figure (11): The collisional frequency ν/n_1 of the electrons as a function of the E/P, for both Maxwell and Druyvesteyn law in air

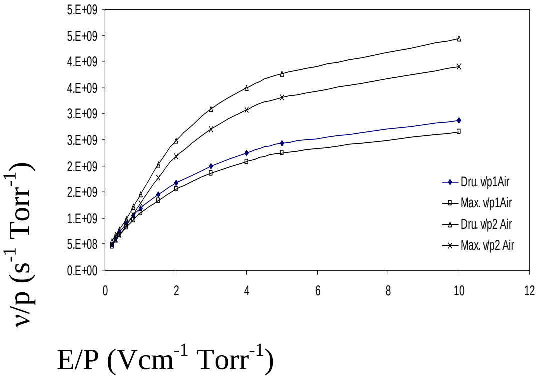


Figure (9): The Collisional frequency ν/p of the electrons as a function of the E/P for Maxwell and Druyvesteyn Law in air.

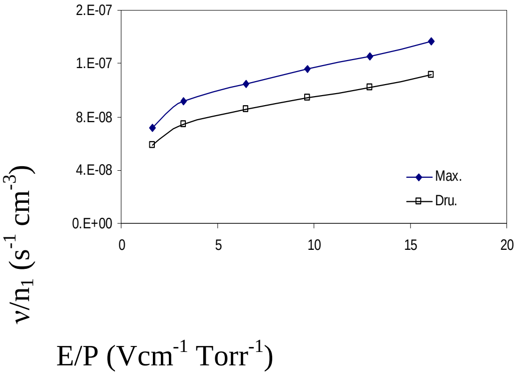


Figure (12): The collisional frequency ν/n_1 of the electrons as a function of the E/P, for both Maxwell and Druyvesteyn law in N_2 gas.

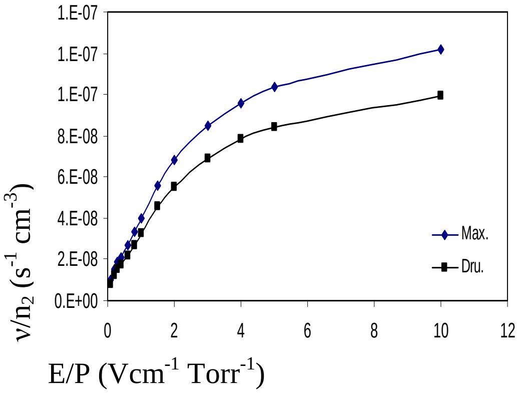


Figure (13): The collisional frequency v/n_2 of the electrons as a function of the E/P, for both Maxwell and Druyvesteyn law in air.

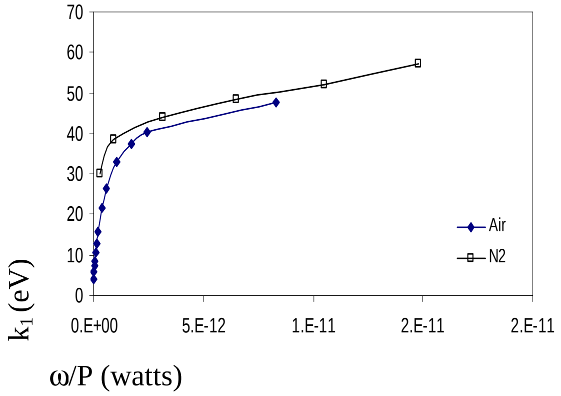


Figure (16): The Townsend's energy factor as a function of the ratio of the mean power supplied ω to the pressure P, for Max. law in air and pure N_2 gas.

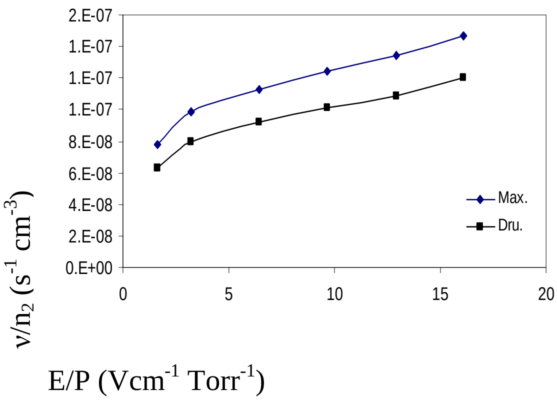


Figure (14): The collisional frequency v/n_2 of the electrons as a function of the E/P, for both Maxwell and Druyvesteyn law in N_2 gas.

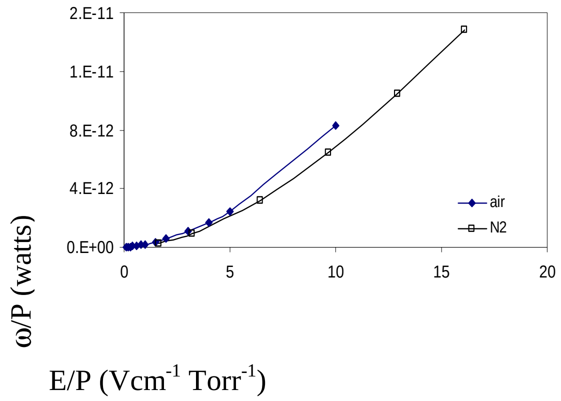


Figure (17): The mean power supplied to the gas pressure ratio as a function of the E/P for Max. law in air and N_2 gas.

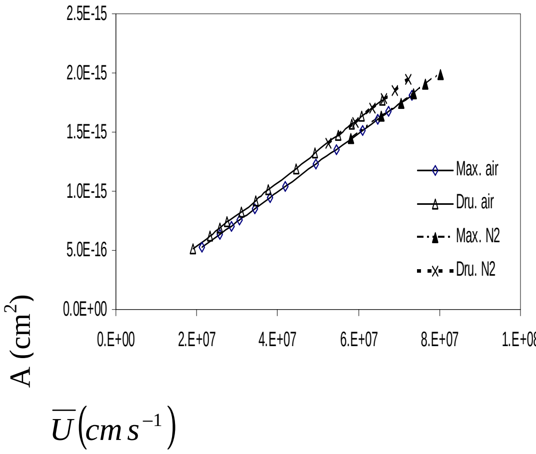


Figure (15): The collisional cross-section of a molecule for electrons as a function of the mean speed of the electrons for both Maxwell and Druyvesteyn law in air and pure N_2.

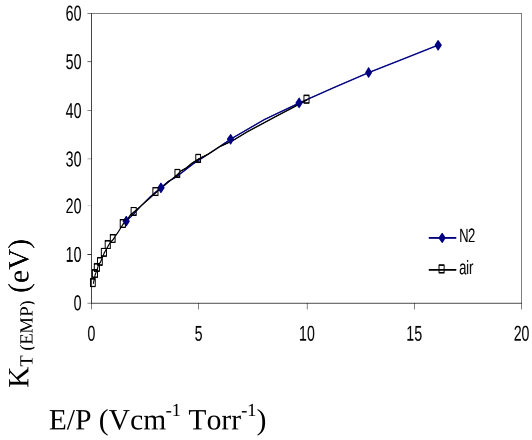


Figure (18): The Townsend's energy factor as a function of the E/P for Max. law in air and pure N_2 gas.

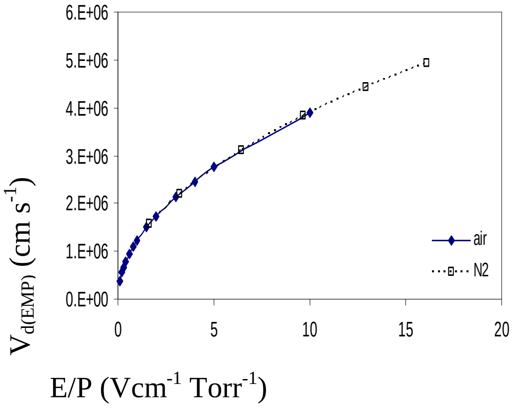
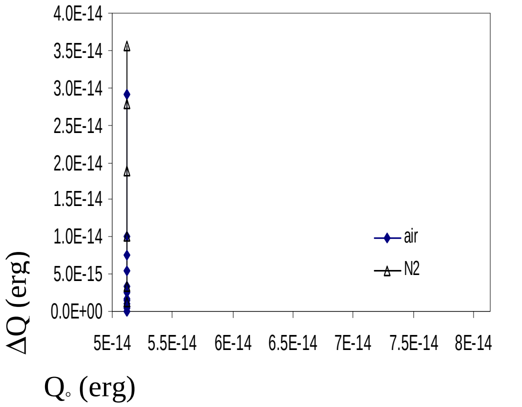


Figure (19): The drift speed of the electron group as a function of the E/P for Max. law in air and pure N_2.



Figure(22): The average energy ΔQ lost by an electron in a collision as a function of the agitational energy Q◦ of a molecule for Max. law in air & N_2.

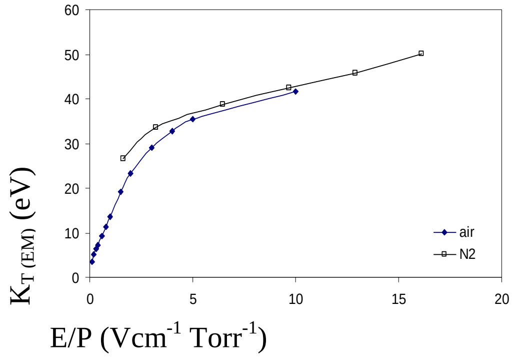


Figure (20): The Townsend's energy factor as a function of the E/P for Max. law in air and pure N_2.

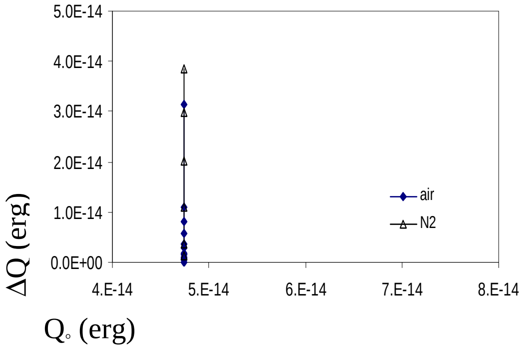


Figure (23): The average energy ΔQ lost by an electron in a collision as a function of the agitational energy Q◦ of a molecule for Druy. law in air & N_2.

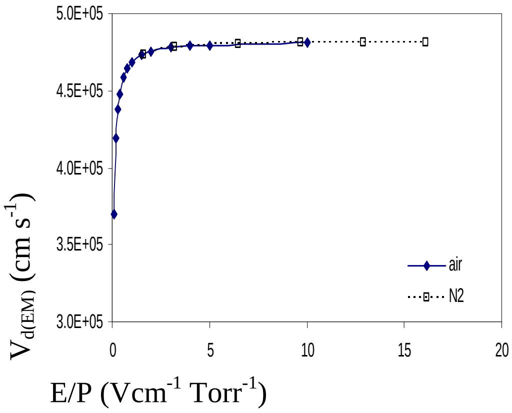


Figure (21): The drift speed of the electron group as a function of the E/P for Max. law in air and pure N_2.

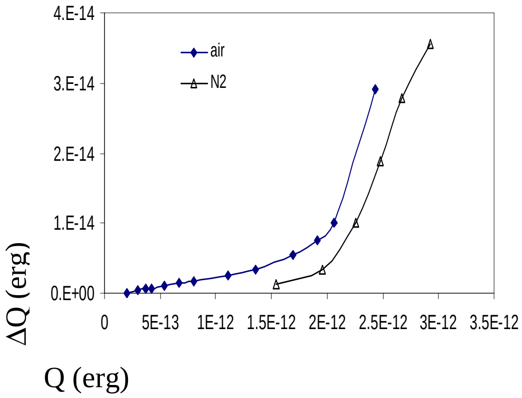


Figure (24): The average energy ΔQ lost by an electron in a collision as a function of the agitational energy Q of an electron for Max. law in air & N_2.

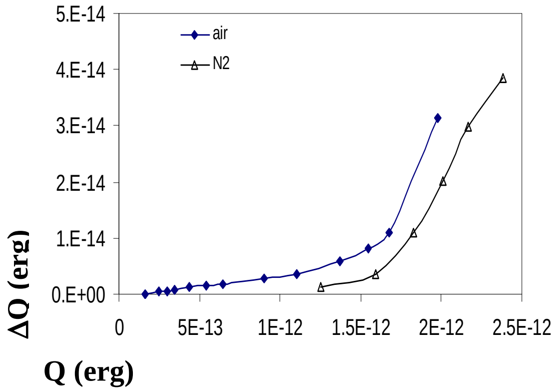


Figure (25): The average energy ΔQ lost by an electron in a collision as a function of the agitational energy Q of an electron for Druy. law in air & pure N_2.

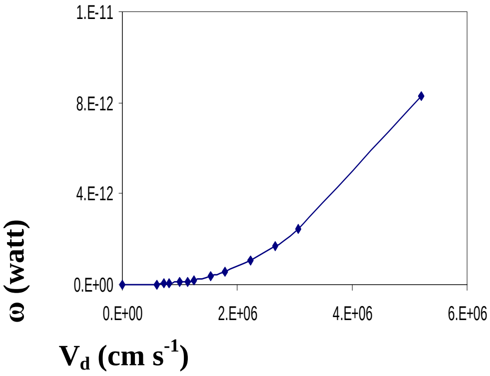


Figure (28): The mean power supplied to an electron as a function of the electron drift speed for Max. law in air.

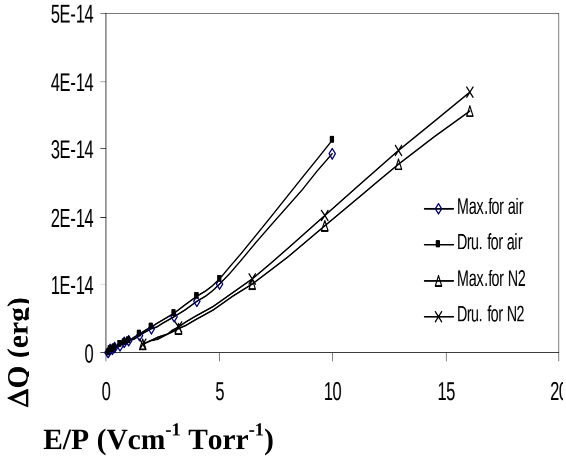


Figure (26): The average energy ΔQ lost by an electron in a collision as a function of the E/P for both Max. and Druy. law in air & pure N_2.

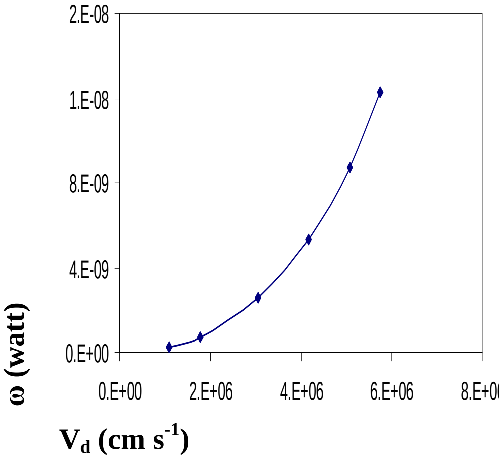


Figure (29): The mean power supplied to an electron as a function of the electron drift speed for Max. law in N_2.

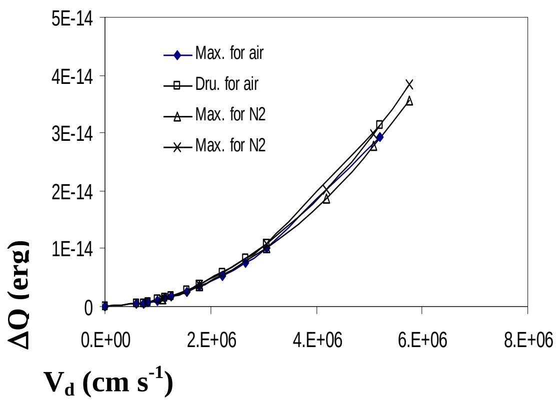


Figure (27): The average energy ΔQ lost by an electron as a function of the drift speed of the group electron for both Max. and Druy. law in air & pure N_2 gas.

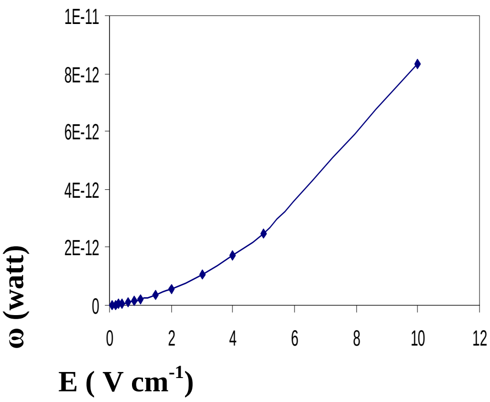


Figure (30): The mean power supplied to an electron as a function of the electric field E for Max. law in air.

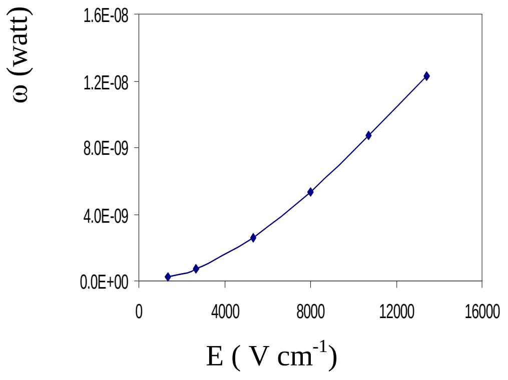


Figure (31): The mean power supplied to an electron as a function of the electric field E for Max. law in N_2 gas.

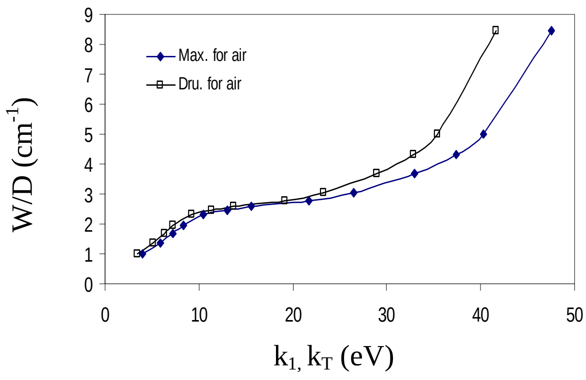


Figure (34): The ratio of the electrons drift speed to diffusion coefficient as a function of the Townsend's energy factor for both Max. & Druy. Law in air.

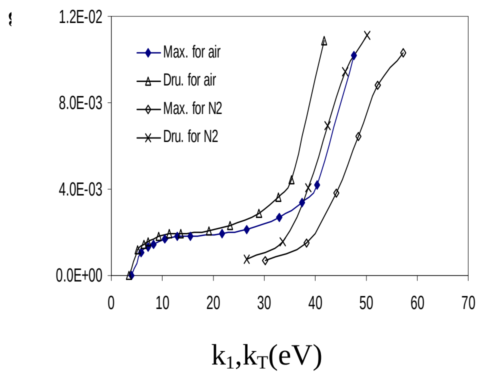


Figure (32): The proportion of the energy lost in a collision η as a function of the Townsend's energy factor for both Max. & Druy. In air & pure N_2 gas.

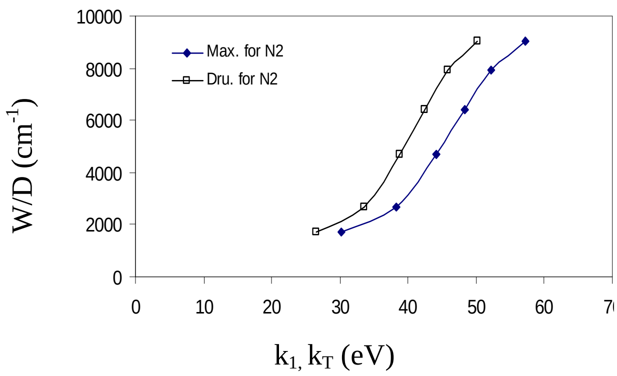


Figure (35): The ratio of the electrons drift speed to diffusion coefficient as a function of the Townsend's energy factor for both Max. & Druy. Law in N_2 gas.

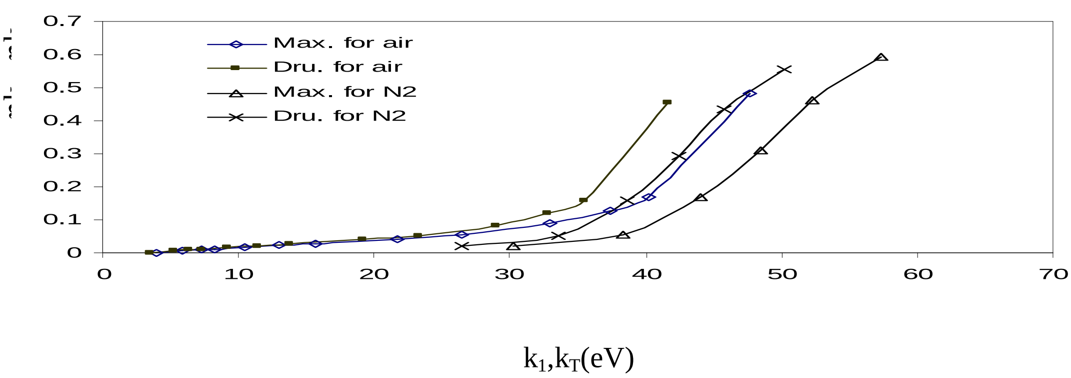


Figure (33): The mean energy lost by an electron per collision as a function of the Townsend's energy factor for both Max. & Druy. in air & pure N_2 gas.

Appendix (A)

The program list:

```
DIMENSION TITLE(20), EOP(20), VD(20), DOMU(20),E(20)
DIMENSION AK1(20), AKT(20), UA(20,2), VOP1(20,2),VOP2(20,2)
DIMENSION VON1(20,2), VON2(20,2), V1(20,2),V2(20,2)
DIMENSION AAIR(20,2), AN2(20,2), Q(20,2),Q0(20,2)
DIMENSION W(20), WOP(20), AKTEMP(20),VDEMP(20),AKTEM(20),VDEM(20)
DIMENSION DLTAQ(20,2), ZETA(20,2), ZETAKT(20,2),WOD(20,2)
CHARACTER*4 TITLE
OPEN(1,FILE='TABLE5.INP')
OPEN(2,FILE='TABLE5.OUT')
EC=1.602E-19
AK=1.380E-23
EM=9.109E-28
READ(1,1000)(TITLE(I),I=1,20)
WRITE(2,1000)(TITLE(I),I=1,20)
READ(1,*)NUMBER,AN,TG
WRITE(2,1001)NUMBER,AN,TG
WRITE(2,1003)
DO 10 I=1,NUMBER
READ(1,*,END=9999)EOP(I),VD(I),DOMU(I),E(I)
VD(I)=VD(I)*1.0E+5
WRITE(2,1002)I,EOP(I),VD(I),DOMU(I),E(I)
10 CONTINUE
1000 FORMAT(20A4)
1002 FORMAT(I5,5E10.3)
1003 FORMAT(/50('*')/,' No. ', ' E/P ',' Vd ',' D/Mu ',' E ', &/50('*'))
9999 DO 20 I=1,NUMBER
AK1(I)=EC*DOMU(I)/(AK*TG)
AKT(I)=AK1(I)/1.14
UA(I,1)=(1.06E+7)*SQRT(AK1(I))
UA(I,2)=(1.02E+7)*SQRT(AKT(I))
VOP1(I,1)=(1.49E+15)*(EOP(I)/VD(I))
VOP1(I,2)=(1.38E+15)*(EOP(I)/VD(I))
VOP2(I,1)=(9.36E+07)*AK1(I)
VOP2(I,2)=(9.36E+07)*AKT(I)
VON1(I,1)=(2.39E-09)*AK1(I)
VON1(I,2)=(2.22E-09)*AKT(I)
V1(I,1)=(2.34E+04)*AN*EM*UA(I,1)**2
V1(I,2)=(2.34E+04)*AN*EM*UA(I,2)**2
VON2(I,1)=(2.57E-09)*AK1(I)
VON2(I,2)=(2.38E-09)*AKT(I)
V2(I,1)=(2.51E+04)*AN*EM*UA(I,1)**2
V2(I,2)=(2.51E+04)*AN*EM*UA(I,2)**2
AAIR(I,1)=(2.48E-23)*UA(I,1)
AAIR(I,2)=(2.68E-23)*UA(I,2)
AN2(I,1)=(2.58E-23)*UA(I,1)
AN2(I,2)=(2.79E-23)*UA(I,2)
Q(I,1)=0.5*EM*UA(I,1)**2
Q(I,2)=0.5*EM*UA(I,2)**2
Q0(I,1)=Q(I,1)/AK1(I)
Q0(I,2)=Q(I,2)/AKT(I)
W(I)=E(I)*EM*VD(I)
WOP(I)=EC*EOP(I)*VD(I)
AKTEMP(I)=13.3*SQRT(EOP(I))
VDEMP(I)=(1.23E+06)*SQRT(EOP(I))
AKTEM(I)=1.0+33.0*EOP(I)
VDEM(I)=(1.49E+15*EOP(I))/(9.36E+07*AKTEM(I))
DLTAQ(I,1)=1.08E-27*VD(I)**2
DLTAQ(I,2)=1.16E-27*VD(I)**2
ZETA(I,1)=(1.79E-14*VD(I)**2)/AK1(I)
ZETA(I,2)=(1.68E-14*VD(I)**2)/AKT(I)
ZETAKT(I,1)=1.79E-14*VD(I)**2
ZETAKT(I,2)=1.68E-14*VD(I)**2
WOD(I,1)=(EC*E(I)/(AK*TG*AK1(I)))
WOD(I,2)=(EC*E(I)/(AK*TG*1.14*AKT(I)))
20 CONTINUE
WRITE(2,2001)
DO 30 I=1,NUMBER
WRITE(2,2000)I,AK1(I),AKT(I),(UA(I,J),J=1,2),(VOP1(I,J),J=1,2),(VOP2(I,J),J=1,2)
30 CONTINUE
WRITE(2,2002)
DO 40 I=1,NUMBER
WRITE(2,2000)I,(VON1(I,J),J=1,2),(V1(I,J),J=1,2),(VON2(I,J),J=1,2),(V2(I,J),J=1,2)
40 CONTINUE
WRITE(2,2003)
DO 50 I=1,NUMBER
WRITE(2,2000)I,(AAIR(I,J),J=1,2),(AN2(I,J),J=1,2),(Q(I,J),J=1,2),(Q0(I,J),J=1,2)
50 CONTINUE
WRITE(2,2004)
DO 60 I=1,NUMBER
WRITE(2,2000)I,W(I),WOP(I),AKTEMP(I),VDEMP(I),AKTEM(I),VDEM(I)
60 CONTINUE
61 WRITE(2,2005)
DO 70 I=1,NUMBER
WRITE(2,2000)I,(DLTAQ(I,J),J=1,2),(ZETA(I,J),J=1,2),(ZETAKT(I,J),J=1,2),(WOD(I,J),J=1,2)
70 CONTINUE
WRITE(2,2006)
2000 FORMAT(I5,8E10.3)
2001 FORMAT(/85('*')/,' No. ',' k1 ',' kT ','Ave.U Max.','Ave.U Dru.',' v/p1 Max.',' v/p1 Dru.',' v/p2 Max.',' v/p2 Dru.',/85('*'))
2002 FORMAT(/85('*')/,' No. ',' v/n1 Max.',' v/n1 Dru.',' v1 Max.',' v1 Dru.',' v/n2 Max.',' v/n2 Dru.',' v2 Max.',' v2 Dru.',/85('*'))
2003 FORMAT(/85('*')/,' No. ',' Aair Max.',' Aair Dru.',' AN2 Max.',' AN2 Dru.',' Q Max.',' Q Dru.',' Q0 Max.',' Q0 Dru.',/85('*'))
2004 FORMAT(/85('*')/,' No. ',' W Max. ',' W/P Max. ','Kt(EMP) Max.',' Vd(EMP)
```

```
Max.',' Kt(EM) Max.',' Vd(EM)
Max.',/85('*'))
2005 FORMAT(/85('*')/,' No. ','DltaQ Max
','DltaQ Dru ',' Eta Max. ',' Eta Dru.
','EtakT1 Max.','EtaKt Dru.',' W/D Max.','
W/D Dru.', /85('*'))
2006 FORMAT(/85('*'))
STOP
END
```