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Approximate Solution for Linear Time- Delayed Improper Integral Equation Using Orthogonal Polynomials

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ABSTRACT

In this paper we adopt the collocation method based on orthogonal polynomials (Laguerre, Hermite) to solve linear time delayed improper integral equation approximately. Some examples are given to illustrate the high accuracy and the efficiency of the proposed numerical techniques.

Keywords: Improper integral Equation, Time delayed, Orthogonal polynomials.

الحل التقريبي للمعادلات التكاملية المعتلة الخطية ذات زمن متباطئ باستخدام متعددات الحدود المتعامدة

الخلاصة

في هذا البحث تم تبني طريقة التجميع بالاعتماد على متعددات الحدود المتعامدة (Laguerre, Hermite) كأساس لحل المعادلات التكاملية المعتلة الخطية ذات زمن متباطئ تقريبيا. بعض الأمثلة أعطيت لبيان الدقة العالية والكفاءة للتقنية العددية المقترحة.

INTRODUCTION

The integral equation is called improper integral equation if one or both of it's limits are infinite. Many problems of electromagnetic scattering problem boundary integral equation [3,12] lead to improper integral equation. Many researchers have developed the approximate method to solve improper integral equation using Galerkin method with Laguerre polynomials as a bases function [1] while Sloan [2] used quadrature methods for solving integral equation of the second kind over infinite intervals.

The general form of the improper integral equation is:-

$$f(s)=g(s)+\int_{0}^{\infty}k(s,t)\,f(t)dt \qquad ...(1)$$

or $$f(s)=g(s)+\int_{-\infty}^{\infty}k(s,t)\,f(t)dt \qquad ...(2)$$

where g(s) is continuous function and the kernel K(s,t) might has singularity in the region $D=\begin{Bmatrix}(s,t):0\le s,t<\infty\\(s,t):-\infty<s,t<\infty\end{Bmatrix}$ and f(s) is to be determined

1- The linear time- delayed improper integral equation [2,4,11] (LT-DIIE)

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The general form of (LT-DIIE) can be written as follows

$$f(s-t) = g(s) + \int_{0}^{\infty} k(s,t) f(t) dt \qquad ...(3)$$

$$f(s-t) = g(s) + \int_{-\infty}^{\infty} k(s,t) f(t) dt \qquad ...(4)$$

Where (τ>0) is a appositive integral called time delayed

2-ORTHOGONAL BASES POLYNOMIALS [5]

Orthogonal polynomials and their properties have a major role in both pure and applied mathematics as well as in a numerical computation. Some important properties that will be needed throughout this paper, some of these properties which are:-

2.1 Laguerre Polynomials [1,6,7]

The Laguerre polynomials denoted by $L_n(s)$ are important sets of orthogonal polynomial over the interval [0,∞).

Consider Laguerre base polynomials as $\{L_0(s), L_1(s), ..., L_n(s)\}$
Where

$$L_n(s) = \sum_{m=0}^{n} \frac{(-1)^m}{m!} \binom{n}{m} s^m \qquad ...(5)$$

With the following properties

$$(L_m(s), L_n(s)) = \int_{0}^{\infty} e^{-s^2} L_m(s) L_n(s) ds = 0 \quad m \neq n$$

And

$$\| L_m(s) \| = 1 \quad m = 0,1,2,...$$

2.2 Hermite polynomials [6,9,10]

The Hermite polynomials $H_n(s)$ are a set of orthogonal polynomials over the domain (-∞,∞). It is well known that the general form of Hermite polynomials is

$$H_n(s) = n! \sum_{k=0}^{[n/2]} \frac{(-1)^k}{k!(n-2k)!} (2s)^{n-2k} \qquad ...(6)$$

With the following properties

$$(H_m(s), H_n(s)) = \int_{-\infty}^{\infty} e^{-s^2} H_m(s) H_n(s) ds = 0 \quad m \neq n$$

And

$$\| H_m(s) \| = 1 \quad m = 0,1,2,...$$

3. COLLOCATION METHOD

Collocation method has been applying for along time and Kantorovich gave a general scheme for defining and analyzing the collocation method to solve the linear operator equations [7, part II].

To solve the equation approximately

$$f(s-t)=g(s)+\int_D k(s,t)f(t)dt \qquad s\in D=\{[0,\infty)\,or\,(-\infty,\infty)\}$$

We usually choose a finite dimensional family of polynomials (Laguerre and Hermite) that is believed to contain a function $f_n(s)$ close to the exact solution f(s).

3.1 Solving (LT-DIIE) by Collocation Method With Aid of Laguerre Polynomials

Consider the linear time delay improper integral equation

$$f(s-t)=g(s)+\int_0^\infty k(s,t)f(t)dt \qquad ...(8)$$

By approximating f(s) into linear combination of Laguerre polynomials

$$f(s)=f_n(s)=\sum_{i=0}^{n}c_iL_i(s) \qquad ...(9)$$

and substituting into equation (8), yield

$$\sum_{i=0}^{n}c_iL_i(s-t)=g(s)+\int_0^\infty K(s,t)(\sum_{i=0}^{n}c_iL_i(t))dt \qquad \ldots(10)$$

For which we have the residue equation

$$R_n(s)=\sum_{i=0}^{n}c_i(L_i(s-t)-\int_0^\infty K(s,t)L_i(t)dt)-g(s)$$

Let $m_j(s)=\int_0^\infty K(s,t)L_i(t)dt$ j=0,1,2,...

So, in this method the collocation points $s_0,s_1,s_2,...,s_n$ are on the interval $[0,\infty)$ such that $s_j=\dfrac{j}{(1+n)}$ j=0,1,...,n

Hence we have $R_n(s_j)=0$ j=0,1,...,n

This leads to

$$\sum_{i=0}^{n}c_i(L_i(s_j-t)-m_i(s_j)=g(s_j) \qquad ...(11)$$

Equ(11) can be seen as a system of n+1 equations in n+1 unknown coefficients c_i , i=0,1,...,n

This system can easily be written in matrix formal as AC=G where

$$A=\begin{bmatrix} L_0(s_0-t)-m_0(s_0) & L_1(s_0-t)-m_1(s_0) & \cdots & L_n(s_0-t)-m_n(s_0) \\ L_0(s_1-t)-m_0(s_1) & L_1(s_n-t)-m_1(s_1) & \cdots & L_n(s_1-t)-m_n(s_1) \\ \vdots & \vdots & \ddots & \vdots \\ L_0(s_n-t)-m_0(s_n) & L_1(s_n-t)-m_1(s_n) & \cdots & L_n(s_n-t)-m_n(s_n) \end{bmatrix} \quad G=\begin{bmatrix} g(s_0) \\ g(s_1) \\ \vdots \\ g(s_n) \end{bmatrix}$$

By using Guass elimination to determine the values c_i which satisfy equ(9).

3.2 Solving (LT-DIIE) Using Collocation Method With Aid of Hermite Polynomial

We will use Hermite polynomials as a bases function to approximate f(s) in the equation

$$f(s-t) = g(s) + \int_{-\infty}^{\infty} k(s,t)\, f(t)\,dt \qquad \ldots(12)$$

Such that

$$f(s) \cong f_n(s) = \sum_{i=0}^{n} c_i H_i(s) \qquad \ldots(13)$$

Substituting equ(13) into equ(12) we get

$$\sum_{i=0}^{n} c_i H_i(s-t) = g(s) + \int_{-\infty}^{\infty} k(s,t) H_i(t)\,dt \qquad \ldots(14)$$

So, we have the residue equation

$$R_n(s) = \sum_{i=0}^{n} c_i (H_i(s-t) - \int_{-\infty}^{\infty} k(s,t) H_i(t)\,dt - g(s) \qquad \ldots(15)$$

Let $\quad p_i(s) = \int_{-\infty}^{\infty} k(s,t) H_i(t)\,dt$

The collocation points $s_0, s_1, \ldots, s_n$ on the interval $(-\infty, \infty)$ are

$$s_j = \frac{j}{1+n} \qquad j=0,1,\ldots,n$$

Hence $\quad R_n(s_j) = 0 \quad$ j=0,1,…,n

This leads to

$$\sum_{i=0}^{n} c_i (H_i(s_j - t) - p_i(s_j)) = g(s_j) \qquad \ldots(16)$$

This system can be written in matrix formal as BC=G

Where

$$B=\begin{bmatrix} H_0(s_0-t)-p_0(s_0) & H_1(s_0-t)-p_1(s_0) & \cdots & H_n(s_0-t)-p_n(s_0) \\ H_0(s_1-t)-p_0(s_1) & H_1(s_n-t)-p_1(s_1) & \cdots & H_n(s_1-t)-p_n(s_1) \\ \vdots & \vdots & \ddots & \vdots \\ H_0(s_n-t)-p_0(s_n) & H_1(s_n-t)-p_1(s_n) & \cdots & H_n(s_n-t)-p_n(s_n) \end{bmatrix} \quad G=\begin{bmatrix} g(s_0) \\ g(s_1) \\ \vdots \\ g(s_n) \end{bmatrix}$$

By solving this system by using Gauss elimination to get the values of c_i, which satisfy equ(13).

NUMERICAL EXAMPLE

Example 1

Consider the following delay infinite integral equation

$$f(s-1)=(s-1)^2-\frac{\sqrt{\pi}}{2}+\int_{-\infty}^{\infty} t^2 e^{-t} f(t)dt$$

with exact solution $f(s)=s^2$

By using Hermite polynomials we get

$$f(s)\cong f_2(s)=c_0H_0(s)+c_1H_1(s)+c_2H_2(s)=c_0+2sc_1+(4s^2-2)c_2$$

$$R(s)=c_0(1-\sqrt{\pi})+2(s-1)c_1+(4(s-1)^2-2)c_2$$

So

$$B=\begin{bmatrix} (1-\sqrt{\pi}) & 0 & -2 \\ (1-\sqrt{\pi}) & 2 & 2 \\ (1-\sqrt{\pi}) & 4 & 14 \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}=\begin{bmatrix} -\frac{\sqrt{\pi}}{2} \\ 1-\frac{\sqrt{\pi}}{2} \\ 4-\frac{\sqrt{\pi}}{2} \end{bmatrix}$$

Hence

$$c_0=\frac{1}{2},\ c_1=0,\ c_2=\frac{1}{4}$$

The approximate solution is $f_2(s)=s^2$

Example 2

Consider the following delay infinite integral equation

$$f(s-1)=\cos(s-1)-\tfrac{1}{2}e^{-s}+\int_{0}^{\infty} e^{-t-s} f(t)dt \qquad \ldots(1)$$

with exact solution $f(s)=\cos(s)$

using Laguerre polynomial

$$f(s)\cong f_n(s)=\sum_{i=0}^{n} c_i L_i(s)$$

Substitute it in the above equation we get

$$\sum_{i=0}^{n} c_i L_i(s-1) = \cos(s-1) - \tfrac{1}{2}e^{-s} + \int_0^{\infty} e^{-t-s} \sum_{i=0}^{n} c_i L_i(t)dt$$

So

$$R(s) = \sum_{i=0}^{n} c_i L_i(s-1) - \int_0^{\infty} e^{-t-s} \sum_{i=0}^{n} c_i L_i(t)dt$$

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By applying the proposed algorithm the solution of eq(1) for different values of n for arbitrary final time the values of c_i displayed in tables(1).
Table (2) presents a comparison between the exact and numerical solution obtained by collocation method with aid of Laguerre polynomial for $t \in [0,1]$ depending on least square error (L.S.E).

CONCLUSIONS

Collocation method as an approximate method for solving linear time delayed improper integral equation using orthogonal polynomials was proposed. The method based on Laguerre and Hermite polynomials. From the numerical results in table(1) it is clear that using these functions to approximate the solution produce accurate results as n increases and the numerical solution convergent to the correct one as the length of series increase.

REFERENCES

[1] Niklong,N.M.A. Z.K.Eshkuvotov,M.Yaghobifar, and M.Hasan "Numerical Solution of infinite Boundary Integral Equation by using Galerkins Method with Laguerre polynomials " Departement of mathematics, University putra Malysia 2000.

[2]Ian H Sloan "Quadrature Method for Integral Equation of the second kind over Infinite Intervals"Mathematics of computation volume 36,number 154 April 1981.

[3] Sanikidz, D.G "On the numerical solution of a class of singular integral equation on an infinite interval " Differential Equations, vol 41, No 9, pp.1353-1358,2005.

[4] Asmaa A.Aswhad "Approximate Solution of vDelay Differential Equation using the collocation method Based on Bernstien polynomials " Department of mathemqatics, Ibn AL Haitham , college of Education , University of Baghdad 2009 .

[5] "Orthogonal polynomials",http:11 Math word. Wo/ fram . com/orthogonal polynomials Html.

[6] W.A.AL.Salam, Some Characterization of Laguerre and Hermite polynomials , Michigan Math. J. 10(1983),381-383.

[7] H.M. Srivastava and s.k.Chatlerjea, some operational representations for the Laguerre polynomials, Nat.Acad.sci-Letl.India 11(1988), 19-21.

[8] Hayat .A.Ali "Approximated solution of Linear Delay Integro Differential Equation " M.Sc Thesis Schoole of Applied Science University of technology 2006.

[9] "Coding Along Hermite polynomials for Gaussian Noise channel "E.Abbe,and .Zheng. 2009
[10] Dan Drake "The Combinatorics of associated Hermite Polynomials" Nankai university, Tianjin, China, 2007.
[11] S. T Colzodillas and C.Lizama" Bounded Mild solution of Perturbed Volterra Equation with Infinite Delay " Mathematics subject classification ,primary 45 No5;Secondary 43A60,47Do6,2000.
[12] M.Benchohra and S.K.Ntouyas"Existence Results on Infinite Integrals for Neutral Functional Differential and Integro-Differential Inclusion in Banach Spaces"Georgian Mathematical Journal Volume 7, number 4,609-625, 2000.
[13] H.Duenas and F.Marcellan,"The Laguerre Sobolev-type orthogonal polynomials,J.Approx Theory 1689241-440 MR 2581392,2010.
[14] J.Caetano" Unified approach to the SU(2) principle chiral Field Model at Finite Volume"Perimeter Institute for Theoretical Physics,water 100,Ontario N2j2wg,Canada arxiv:1012.2600vi[hep.th] 12 Dec 2010.

Table(1)

n	n= 5	n= 7	n= 10
c_1	0.4993	0.5116	0.5026
c_2	0.5913	0.4023	0.4760
c_3	-0.0806	0.7208	0.4252
c_4	0.5863	-1.2390	-0.7449
c_5	-0.6721	1.8094	1.9608
c_6	0	-1.8533	-4.1275
c_7	0	0.6536	5.2238
c_8	0	0	-4.6654
c_9	0	0	2.5444
c_{10}	0	0	-0.5966

Table (2)

t	n=5	n=7	n=10	Exact
0	0.9244	1.0054	0.9985	1.0000
0.1	0.9629	0.9990	0.9959	0.9950
0.2	0.9799	0.9835	0.9821	0.9801
0.3	0.9774	0.9588	0.9578	0.9553
0.4	0.9574	0.9247	0.9235	0.9211
0.5	0.9217	0.8814	0.8798	0.8776
0.6	0.9217	0.8293	0.8272	0.8253
0.7	0.8105	0.7688	0.7663	0.7648
0.8	0.7383	0.7005	0.6978	0.6967
0.9	0.6572	0.6250	0.6224	0.6216
1	0.5688	0.5433	0.5408	0.5403
L.S.E	0.0186	1.6235e-004	3.1797e-005	