Fuzzy Logic Control of Two Heated Tanks in Series

Dr. Duraid Fadhel Ahmed, Lecturer Dr. Abodulkahar Mahdi Mohammed, Lecturer Chem. Eng. Dept., University of Tikrit

Abstract

The mathematical modeling of two heated tanks system was developed based on the heat balance and used model parameters for tuning PID and Dahlin controller parameters. The system was studied by introducing step change in the temperature of the inlet stream and heat supply and measuring the temperature change of the tanks. In this paper, a rule-based controller that incorporates fuzzy logic controller has been designed and evaluated. Through simulation study by using MATLAB, it has been shown that the estimated parameters of the model are in good agreement with the experimental values. Also the proposed fuzzy logic controller has given an excellent tracking and regulation performance compared to that of the PID control and Dahlin controller systems.

Keyword: Mathematical modeling of Heating tanks in series, Matlab simulation, Fuzzy logic, PID controller, Dahlin controller.

السيطرة على خزانين مسخنين متصلين على التوالي باستخدام منطق الدليل الغامض

الخلاصة

تم دراسة الموديل الرياضي لنظام مكون من خزانين متصلين على التوالي احدهما مسخن بالاعتماد على الموازنة الحرارية وتم استخراج ثوابت جهاز التحكم من نوع تناسبي-تكاملي-تفاضلي و جهاز التحكم من نوع دهلن من الموديل الرياضي. تم تعريض النظام لتغيير درجي بكمية الحرارة الناتجة من المسخن الكهربائي ودرجة حرارة الماء الداخل إلى النظام وقياس درجات الحرارة للخزانين . وكذلك تناول البحث تصميم طريقة سيطرة حديثة وهي طريقة الدليل الغامض وتم مقارنة طرق السيطرة الثلاثة من خلال محاكاة نظام السيطرة باستخدام برنامج "الماتلاب" . وقد أظهرت النتائج إن طريقة الدليل الغامض و تم المريقة المريقين لأنها . وقد أظهرت النتائج إن طريقة الدليل الغامض هي أفضل الطرق وذات كفاءة عالية مقارنة مع الطريقتين لأنها . وقد تنبذب وأسرع بالوصول إلى القيمة المرغوبة.

Notations

- A,B,C,D: Matrices of model parameters.
 A_h: Area of heat transfer coefficient (m²).
 CE: Rate of change of error.
 C_P: Specific heat capacity, (kJ/kg.°C).
 E: Error signal.
 G(s): Transfer function.
 K_C: Controller gain.
 K_P: Steady state gain.
 L: Length of pipe (m).
 M: Mass of liquid in the tank (kg).
 m: Liquid mass flow rate (kg/sec).
 P: Vector of input variables.
- Q: Heat input of the electrical coil (kW).
- s: Laplace domain (1/sec).
- T: Temperature (°C).
- t: Time domain (sec).
- t_s: Sampling time (sec)
- U: Universal fuzzy set.
- U_h : Heat transfer coefficient (W/m². °C)
- u: Output of fuzzy controller.
- V: Velocity of liquid (m/sec).
- x: Vector of temperature of the tanks.
- y: Vector of measured variables

Greek letters

- β: Constant of the controller tuning equation.
- τ : Time constant of the system (sec). τ_D : Time delay (sec)

Introduction

Heated tanks in series are commonly used in the chemical industries including chemical reactors, distillation process, evaporators and crystallizers, etc. These equipments have constraints inherent to their operation such products as. specifications, safety, environmental effects and economic. Therefore, the temperature control is considered to be the heart of these equipments and must be selecting a very good method for the process control.

The dynamics and control of temperature of heated tanks have been treated extensively in the literature. A nonlinear model-predictive control law was applied to control the jacket temperature in polymerization reactor by Heemskerk ^[1]. The proposed control algorithm used an explicit process model implemented the elements of dynamic classical matrix control (DMC). Rajapalan and Seshadri^[2] applied a feedforward/feedback to control the temperature in a continuous tank reactor. The reactor stirred temperature was controlled through a typical cascade temperature control [3] and Guanzhong scheme. Lei proposed a feed/forward control and Smith prediction control of temperature for polymerizing kettle system. In their study, a simulation of the control problem has been generated and developed which enables the user to modify the tuning parameters.

William and Richard ^[4] designed a modern control technology to control the temperature of a heating tank. They concluded that if the temperature

- $\tau_{\rm C}$: Time constant of closed loop (sec)
- µ: Membership functions in fuzzy controller.

exceeds the variable software temperature limit, the controller turns off the one or more heating elements. Kenneth ^[5] implemented proportional band temperature controller in the water heater tank for conducting electric power to the electric resistance heating element.

Fuzzy control techniques have recently been applied to various complex industrial processes such as batch chemical reactors, blast furnaces, distillation columns, and neutralization process. Chow and Kuehn^[6] designed a fuzzy PI controller for temperature control in a furnace. Entries in the rule base are used to prevent integral windup and a fuzzy gain scheduler allows the controller to be tuned once and used over the whole operating temperature range of the system. Substantial improvements are shown for settling times when both large and small step changes in reference set point.

Pal et al.^[7] used a conventional on-off control and its corresponding fuzzy version for a small temperature process. It is observed that fuzzy control of temperature offers smother control than the conventional one. Hiroyuki and Takeshi^[8] applied a fuzzy control of the fermentation temperature in а bioprocess. They showed that the rules learned by the fuzzy clustering method perform well. Their results provided support for the use of fuzzy clustering algorithms in process control.

In this paper, the dynamic behavior of two heated tanks in series was studied by theoretically and experimentally to find the model parameters for tuning PID and Dahlin controller parameters. Three control methods, PID, Dahlin and fuzzy controls were simulated by using MATLAB software to choose the best method for temperature control of the system.

Theory

Mathematical Model

The theoretical model of two heated tanks in series as shown in Figure (1) is derived by using energy balance under the following assumptions:

- The density and heat capacity of liquid are constant.
- Perfect mixing is assumed in the tanks.
- The heat supply through heating coil may change.
- The rate of heat transfer from the coil to the tank is calculated by the following equation:

Rate of heat flow in – Rate of heat flow out + Rate of heat generated in the coil = Rate of heat accumulated in the tank(2)

Heat balance on the first tank:

$$mC_{P}T_{1}(t) - mC_{P}T_{2}(t) + Q = MC_{P}\frac{dT_{2}}{dt}$$
....(3)

<u>Case (1)</u>: At step change in the temperature of the inlet stream (T₁) equal to Δ T₁ with (Q) remains constant, Eq. (3) becomes:

$$MC_{P} \frac{dT_{2}}{dt} + mC_{P}T_{2}(t) = mC_{P}T_{1}(t)....(4)$$

Taking Laplace transformation of Eq. (4), gives:

<u>Case (2)</u>: At step change in heat of the coil (Q) to ΔQ with the temperature of the inlet stream (T₁) remains constant, then the transfer function of the process becomes:

Where $K_{P1} = 1/mC_P$

The transfer function of the pipe between two tanks is:

 $T_3(t) = T_2(t - \tau_D)$ (8)

Taking Laplace transformation of Eq. (8) with expression $T_2(t-\tau_D)$ in term of Taylor series, gives:

$$T_3(s) = T_2(s) EXP(-\tau_D s)$$
(9)

Where $\tau_D = L/V$

Heat balance on the second tank:

Following the same steps as with first tank and a heat balance on second tank, we get the following equations:

$$MC_{P} \frac{dT_{4}}{dt} + mC_{P}T_{4}(t) = mC_{P}T_{3}(t).....(11)$$

The overall transfer function of the two heated tanks in series system is:

When $\tau_1 = \tau_2 = \tau$, the overall transfer function of system becomes:

Control Methods <u>PID and Dahlin Controllers</u>

This section presents several special aspects of the design for computer control system. These methods require sampling the continuous process signals and quickly calculating and manipulating these signals bv an algorithm in the computer, and then updating this output signal and holding it constant until next update. Smith C., and Armando B. C.^[9] stated that <u>Dahlin</u> et al. developed the feedback algorithm with dead compensation and applied it in computer control. Details of tuning PID and Dahlin controllers can be found in reference (9). The tuning formulas for these control schemes are as follows:

for PID and Dahlin controllers

for PID and Dahlin controllers

for PID controller :

$$K_{c} = \frac{(1-q)(\beta_{1}-2\beta_{1}\beta_{2}+\beta_{2})}{K_{P}(1-\beta_{1})(1-\beta_{2})[1+N(1-q)]} \dots (17)$$

for Dahlin controller

Where $\beta_1 = e^{-t_s/\tau_1}$, $\beta_2 = e^{-t_s/\tau_2}$, $q = e^{-t_s/\tau_C}$ and $N = \tau_D / t_s$

Fuzzy Control

A fuzzy control system was developed based on fuzzy mathematics, which is a branch of applied mathematics. The fuzzy mathematics has broad applications in many fields including statistics and numerical analysis, systems and control engineering, pattern recognition, signal and image processing, and biomedical engineering. Fuzzy control provides effective solutions for nonlinear and partially unknown processes, mainly because of its ability to combine information form different sources, such available mathematical models, as experience of operators, process measurements, etc. Like other control mechanisms, fuzzy logic control is essentially a feedback control system.

Fuzzy Set Basic Operation

The theory of sets and the concept of a set itself constitute a foundation of modern mathematics. As far as one considers mathematical and simulation models of application problems, on deals with mathematics and the set theory at the base of mathematics. The space which fuzzy sets are working in is called the universal set. Then a fuzzy subset (A) of universal set (U) is characterized by a membership function $(\mu_A(u))$ which is assigns to each element (U \exists u). This function determines if the element of the universal set does or does not belong to this subset A. Hence the function may

$$\mu_{A}(u) = \begin{cases} 1 & \text{if and only if } u \in A \\ 0 & \text{if and only if } u \notin A \end{cases} \dots \dots (19)$$

have two values: true or false or in numbers, 1 or 0.^[10]

The main operations used are defined as follow:

• The intersection of the fuzzy subsets (A) and (B) of the universal set (X) is denoted by:

with characteristic function define by:

$$(A \cap B)$$

$$\mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u)) \dots (20)$$

This corresponds to the logical "AND" operation.

• The union of the fuzzy subsets (A) and (B) of the universe set (U) is dented by:

$$(A \cup B)$$

with characteristic function define by:

$$\mu_{A\cup B}(u) = \max(\mu_A(u), \mu_B(u))$$
(21)

This corresponds to the logical "OR" operation.

The complement of a fuzzy subset (A) of the universe set (U) is denoted by this corresponds to the logical"NOT" operation.

 $\mu_A(u) = 1 - \mu_A(u)$ (22)

In fuzzy set theory the characteristic function is usually called the membership function.

Design of Fuzzy Logic Controller

The decision-making activities of a process operator in a regulation control task are shown in the dotted block in Figure (2); for the purposes of this work this activity is expressed as a fuzzy relationship or algorithm, relating significant observed variables to the control actions. In the case of single input-single output regulation tasks which are the subject of this study, the process operator is assumed to respond to the system error (E) and its rate of change (CE), the result of a control decision being a change in the control valve setting (CU). The values for each membership function are labeled $\mu(x)$, and are determined by the original measured signal x and the shapes of the membership functions. A common fuzzy classifier splits the signal x into five fuzzy levels as follows ^[10]: -

a) LP: x is large positive

b) MP: x is medium positive

- c) S: x is small
- d) MN: x is medium negative
- e) LN: x is large negative

A five level defuzzifier block will have inputs corresponding to the following five actions:

a) LP: Output signal large (positive)

- b) MP: Output medium (positive)
- c) S: Output signal small

d) MN: Output signal medium (negative)

e) LN: Output signal large (negative).

The defuzzifier combines the information in the fuzzy inputs to obtain a single crisp (non-fuzzy) output variable. There are a number of ways of doing. The simplest and most widely used method is called the center of Gravity Method. It works as: If the fuzzy levels LP...LN have membership values that are labeled $\mu 1...\mu 5$, then the crisp output signal u is defined as: -

The complete procedures of the fuzzy controller design can be described as follow:

1. Choose a suitable scaled universe of set (U) of;

$$-L \leq (E_i, CE_i) \leq L$$

where L and –L represent the positive and negative ends respectively.

2. The calculation of the error and its rate of change, from the fuzzy logic control point of view the calculations of error (E) and its rate of change (CE) are as below:

 $E_i = (Measured value)_i - Set value$

CE_i=Instant error–Previous error

- **3.** Both E_i and CE_i are multiplied by the same scale factor of the universe of set.
- 4. Choose a membership function, such as number of classes to described all the values of the linguistic variable on the universe, the position of different membership functions on the universe of discourse, the width of the membership functions and the shape of a particular membership function.
- **5.**Calculate the applicability degree. At this the degree to which the whole condition part (all the inputs) satisfies the rule is calculated. This degree is called the degree of applicability of the condition part. It is denoted as β :
- $\beta = \min(\mu E(u), \mu CE(u))$
- **6.**The fuzzy decision rules are developed linguistically to do a particular control task and are implemented as a set of fuzzy conditional statements of the form:
- " IF E is PB AND CE is NB THEN NS Action " Table (1) shows the fuzzy rules conclusions. The seven fuzzy sets definition generates (49) rules fuzzy controller.
- **7.**Choice of the defuzzification procedure. The defuzzification goal in Mamdani type fuzzy controllers is to produce a crisp output taking the fuzzy output obtained after rules processing. The center of gravity (COG) method is used (Equation (23)).
- **8.**Fuzzy Controller program: The fuzzy controller can be programmed in C, Fortran, Basic, Matlab, or virtually any other programming language. Suppose that we let the computer variable x1 denote E(t), which we call the first input, and x2 denote CE(t), which we will call the second input. Using these definitions, consider the program for a

fuzzy controller that is used to compute the fuzzy controller output given its two input:

- Obtain x1 and x2 values. (Get inputs to fuzzy controller).
- Compute μ₁(i) and μ₂(j) for all i, j. (Find the values of all membership functions given the values for x1 and x2 and linguistic-numeric value i, j.
- Compute $\beta(i, j) = \min (\mu_1(i), \mu_2(j))$ for all i, j. (Find the values for the premise membership functions for a given x1 and x2 using the minimum operation).
- Compute $U_A(i,j) = area$ (Rule (i,j), $\beta(i,j)$) for all i, j. (Find the area under the membership functions for all possible implied fuzzy sets, where area = w(h-(h2/2)).
- Let num.=0, den.=0 (Initialize the center of gravity numerator and denominator values).
- For i=0 to 7
- For j=0 to 7(cycle through all areas to determine COG).
- (compute numerator for COG)
- den.=den.+UA(i,j)
- (compute denominator for COG)
- Next j
- Next i
- Output u crisp=num./den.
- (Output the value computed by the fuzzy controller)
- Go to step 1.

Simulation of Control Methods

The simulation technique is based on the software tool MATLAB to solve the ordinary differential equations which represent the system behavior. During the digital simulation of the three methods of control, the controlled variables (T_2 , T_4) are calculated and from the response of these variables we find the best conditions of the control system.

The process is described by Eq. (1) and (11) and can be converted into perturbation variables by inspection as the following:

$$\frac{dT_2}{dt} = \frac{m}{M}T_1 - \frac{m}{M}T_2 + \frac{1}{Mcp}Q \dots (24)$$
$$\frac{dT_4}{dt} = \frac{m}{M}T_2 - \frac{m}{M}T_4 \dots (25)$$

To solve these equations in MATLAB we put them into state variables as follows:

$$\frac{dx}{dt} = A x + B P \dots (26)$$

$$y = C x + D P \dots (27)$$

$$x = \begin{bmatrix} T_2 \\ T_4 \end{bmatrix}, \quad p = \begin{bmatrix} T_1 \\ Q \end{bmatrix}, \quad A = \begin{bmatrix} -\frac{m}{M} & 0 \\ \frac{m}{M} & -\frac{m}{M} \end{bmatrix},$$
$$B = \begin{bmatrix} \frac{m}{M} & \frac{1}{MCp} \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

The open loop response to step change in the temperature of the inlet stream and in the heat of the coil is calculated from these matrices by MATLAB program. The closed loop response is calculated by using Eq. (15) to (18) with the conventional equations for PID and Dahlin methods to estimate the output of the controller for manipulating the power of the coil. The Fuzzy controller output is calculated by applying the steps in section (2.2.2.2), and software tool MATLAB to obtain the closed loop response to a step change of inlet stream temperature.

Experimental Work

Laboratory apparatus consists of two square tanks in series with 0.2 m

width and 0.2 m height. The first tank is heated by 1.2 kW electrical heater and two tanks are provided the mixer at a rotor speed of 500 rpm. The apparatus is four provided temperature measurements to measure the temperature of the all streams and 0.5 in. diameter, 12 in. long of pipe to connect the two tanks. The water at ambient temperature is fed to the system by a pump and flow rate is measured by independently calibrated rotameter.

The range of flow is (0-10) lit./min. of water at 20°C. A schematic diagram of the apparatus is shown in Figure (1). Eight runs were carried out, four runs for step change in temperature of the inlet stream and four runs for step change in heat of coil. For each run, the tanks were filled with water at ambient temperature. The unsteady state step change in inlet temperature runs were conducted by using the three ways valve to change inlet stream from cold water at ambient into hot water at 30°C at 2 lit./min.

The temperature of the tanks are recorded every 20 sec. and the measurement procedure was continued until steady state was reached. This run is repeated for different temperatures of hot water 35, 40 and 45°C. The unsteady state step change in heat of coil runs were conducted by turn on the electrical heater at flow rates 2 lit./min of feed. The temperature of the tanks are recorded every 20 sec. and the measurement procedure was continued until steady state was reached. This run is repeated for different flow rates (4,6,8) lit./min of feed.

Results and Discussion

The dynamic behavior of two heated tanks in series was determined by the step change in the temperature of the inlet stream and the heat supplied by the electrical coil at different flow rates. The simulated and actual responses to disturbances under different conditions are shown in the Figures (3) to (6). It can be seen that the simulation response is faster than the experimental response due to other small lags of the thermocouple and transmitting signal which are always present in an experimental case.

The fuzzy controller presented is applied for the heated tanks system. In order to a certain the advantages offered by the fuzzy control strategy, simulation results are also presented for PID and Dahlin methods. A simulation study was carried out to establish the effectiveness of the proposed methods in controlling the temperature and to predict the dynamic process behavior with tuning the parameters of the controllers. The parameters of the dynamic behavior and the best controller settings concluded from the simulation and are founded in the Table (1) and (2).

The results obtained for the control system are shown in the Figures (7) to (10). In both cases the fuzzy and conventional (PID and Dahlin) control system were adjusted to obtain the best response possible and the results are comparable. The Control over a range of operating conditions showed that the conventional control system was difficult to adjust and good control responses could not be achieved with the same controller settings due to changes in process dynamics. The figures showed that the response of Dahlin controller reaches the set point faster and with a sharper response than PID controller because the Dahlin algorithm makes a larger initial change in the controller output than PID and Dahlin algorithm waits for the error to respond before it takes action again.

The results obtained with the fuzzy controller were much better than those conventional methods. The fuzzy

controller gave good control at all operating points with a rapid response and small amount of overshoot. These responses show the improvement in controlling the heated tanks system using fuzzy logic by shorting the time requires for reaching the set point and eliminating the oscillation in the response.

Conclusions

From the present study, the following conclusions are drawn regarding the control of two heated tanks in series:

- The simulation response is faster than the experimental response due to lags of the thermocouple and transmitters of the signal.
- The fuzzy controller has been successfully used to stabilize the controlled system and to achieve good control performance for disturbances in the inlet variables.
- The performance of the PID and Dahlin control methods were oscillatory, while the performance of the fuzzy controller could dampen the oscillation, fairly well. In these tests, when the criterion was the controller's ability to damp the oscillations and to react quickly to the changes in the process flow, the fuzzy controller was the best controller.

References

- Heemskerk A. H., "Model-predictive control of polymerization reactor ", Chem. Eng. Sci., Vol. 35, No. 1/2, p. 439, 1980.
- 2- Rajagopalan T. and Seshadri V." Feedforward control temperature of the continuous stirred tank reactor ", Int. J. Control, Vol. 16, No. 5, p. 903, 1972.
- **3-** Lei F. and Guanzhong D., "Research on control problem with long delay in polymerizing of PVC", Intelligent

control and Automation, Vol. 2, Issue June, p. 7716-7719, 2006.

- 4- William E. M., and Richard M., "Control and method for operating an electric water heater", US patent No. 7221862, 2007.
- 5- Kenth A. B., " Proportional band temperature control for multiple heating elements", , US patent No. 6374046, 2002.
- 6- Chow M. Y., and Kuehn R. T., " Fuzzy control of temperature in a semiconductor processing furnace", The 24th annual conference, Vol. 3, Issue 31 August, p. 1774-1779, 1998.
- 7- Pal S., Sen S. K., Sakar G., and Anish D., "Studies on a PC-based fuzzy and non-fuzzy on-off controller for a temperature process", IEEE Computer Society, Vol. 1, p. 345-366, 2008.
- 8- Hiroyuki H., and Takeshi K., " Fuzzy control of bioprocess", Vol. 89, No. 5, p. 401-408, 2000.
- **9-** Smith C., and Armando B. C., " Principles and practice of automatic process control", John wiley & Sons Inc., p. 650-699, 1997.
- 10- Passino M. K., and Stephan Y., "Fuzzy control", Addison-Wesley Longman Inc., 1998.



Figure (1) Schematic diagram of experimental and simulated two heated tanks in series.



Figure (2) Block diagram of the fuzzy control system.



Figure (3) Comparison between the simulated and experimental response of first tank temperature to step change in inlet temperature from 22 to 30°C.



Figure (4) Comparison between the simulated and experimental response of first tank temperature to step change in heat of coil.



Figure(5) Comparison between the simulated and experimental response of second tank temperature to step change in inlet temperature from 22 to 30°C.



Figure (6) Comparison between the simulated and experimental response of second tank temperature to step change in heat of coil.



Figure (7) Temperature response of first tank under three control methods for step change in inlet temperature from 22 to 30°C.



Figure (8) Temperature response of first tank under three control methods for step change in inlet temperature from 22 to 40°C.



Figure (9) Temperature response of second tank under three control methods for step change in inlet temperature from 22 to 30°C at set point of 26°C



Figure (10) Temperature response of secondtank under three control methods for step change in inlet temperature from 22 to 40° C at set point of 26° C.

Table (1): Fuzzy controller rules for the two heated tanks in series

NCB	NCM	NCS	ZC	PCS	РСМ	РСВ	CEE
NUS	NUM	NUB	NUB	NUB	NUB	NUB	PEB
NUS	NUM	NUM	NUB	NUB	NUB	NUB	PEM
PUS	PUS	NUZ	NUM	NUM	NUM	NUM	PES
PUM	PUM	PUS	NUZ	NUS	NUM	NUM	ZE
PUM	PUM	PUM	PUM	NUZ	NUS	NUS	NES
PUB	PUB	PUB	PUB	PUM	PUS	NUZ	NEM
PUB	PUB	PUB	PUB	PUM	PUS	NUZ	NEB

Table (2): Best parameters of the dynamics and control system

K _{p1}	$ au_{\mathrm{D}}$	τ	m	Dahlin Controller			PID Controller		
Sec.ºC/kJ	(sec)	(sec)	(lit./min)	$\tau_{\rm D}$,min	τ _I ,min	K _C	$\tau_{\rm D}$,min	τ_{I} ,min	K _C
7.18	1.9	240	2	2.1	8.3	208	2.01	8.3	31.2
3.6	0.95	120	4	1.04	4.2	208	1.04	4.2	52
2.4	0.63	80	6	0.7	2.8	208	0.7	2.8	67
1.8	0.48	60	8	0.5	1.96	208	0.5	1.96	78.3