SPEED ESTIMATION USING EXTENDED KALMAN FILTER TECHNIQUE Ayad Kasem Hussen Collage of Electronic & Electrical Techniques

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ABSTRACT

This paper presents a state estimation technique for speed sensorless field oriented control of induction motors. The theoretical basis of each algorithm is explained in detail and its performance is tested with simulations using MATLAB package VER.6.3.

A stochastical nonlinear state estimator, Extended Kalman Filter (EKF) is presented. The motor model designed for EKF application involves rotor speed, dq-axis stator currents. Thus, using this observer the rotor speed and rotor fluxes are estimated simultaneously. Different from the widely accepted use of EKF, in which it is optimized for either steady- state or transient operations, here using adjustable noise level process algorithm the optimization of EKF has been done for both states; the steady-state and the transient-state of operations.

KEYWORDS

Induction motor, Kalman filter, estimation, simulation.

INTRODUCTION

In controlling AC machine drives speed transducers such as tacho-generators, revolvers, or digital encoders are used to obtain speed information.

Especially, in defective and aggressive environments, the speed sensor might be the weakest part of the system. This would degrade the system's reliability and reduces the advantage of an induction motor drive system. This has led to a great many speed sensorless vector control methods^[1]. On the other hand, avoiding use of additional algorithms and means added sensor computational complexity that requires high-speed processors for real time applications. As digital signal processors have become cheaper, and their performance greater, it has become possible to use them for controlling electrical drives as a cost effective solution. Some relatively new fully digitized methods, used for speed sensorless field-oriented control, utilize this enhanced processing capacity^[2-4].

Usually sensorless control is defined as a control scheme where no mechanical parameters like, speed and torque, are measured. Traditional vector control systems use the method of flux and slip estimations based on measurements of the phase currents and DC link voltage of the inverter, but this has a large error in speed estimation particularly in the low-speed range. MRAS (model reference adaptive system) techniques are also used to estimate the speed of an induction motor ^[5-7]. These also have a speed error in low-speed range and settle to an incorrect steady-state value. In recent years, non-linear observers are used to estimate induction motor parameters and states ^[8-12].

GENERAL THEORY ON OBSERVERS

Estimation of unmeasurable state variables is commonly called observation. A device (or a computer program) that estimates or observes the states are called a state-observer or simply an observer An observer can be used to estimate states which cannot be measured, or where the measurements are corrupted by noise. If a system can be described in discrete time as:

$$\mathbf{x}(\mathbf{k}+1) = \mathbf{A}\mathbf{x}(\mathbf{k}) + \mathbf{B}\mathbf{u}(\mathbf{k}) \tag{1}$$

$$y(k) = C x(k)$$

and the system is observable, i.e. the observability matrix, M_o , has full rank 1, the states can be estimated by (2) where,

$$M_{o} = \begin{bmatrix} CF \\ CF^{2} \\ . \\ .CF^{n} \end{bmatrix}$$

 $\hat{\mathbf{x}}(\mathbf{k}+1) = \mathbf{A}\,\hat{\mathbf{x}}(\mathbf{k}) + \mathbf{B}\,\mathbf{u}(\mathbf{k})$

(3)

$$\hat{\mathbf{x}}(k+1) = \mathbf{A} \,\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{L}(\mathbf{y}(k+1) - \mathbf{C} \,\hat{\mathbf{x}}(k+1))$$
 (2)

The error of the observer is defined by:

$$\mathbf{e}(\mathbf{k}) \, \stackrel{_{\frown}}{=}\, \mathbf{x}(\mathbf{k}) - \hat{\mathbf{x}}(\mathbf{k})$$

where **L** is the observer gain

e(k+1) = (A-LCA)e(k)

Kalman filter

When applied to a physical system, the observer described above will be under the influence of two noise sources; process noise (thermal noise) and measurement noise (quantization noise). Considering these two noise sources, Eq.1 can be rewritten as:

$$\mathbf{x}(\mathbf{k}+1) = \mathbf{A}\mathbf{x}(\mathbf{k}) + \mathbf{B}\mathbf{u}(\mathbf{k}) + \mathbf{G}\mathbf{v}(\mathbf{k})$$
(4)

$$\mathbf{y}(\mathbf{k}) = \mathbf{C}\mathbf{x}(\mathbf{k}) + \mathbf{w}(\mathbf{k})$$

where v(k) is the process noise and w(k) is the measurement noise.v(k) and w(k) will be regarded as zero mean, uncorrelated white noise sequences with covariances V1(k) and V2(k). The objective of the Kalman algorithm is to determine a gain matrix, **L**, which minimizes the mean square of the error, e. This can be achieved with the following algorithm,

$$\hat{\mathbf{x}}(\mathbf{k})(k|n) = \mathbf{E} \{ \mathbf{x}(\mathbf{k}) | \mathbf{y}(1), \mathbf{y}(2), \mathbf{y}(3), ..., \mathbf{y}(n) \}$$
(5)

$$\mathbf{Q}(k+1) = \mathbf{E}\left\{ \mathbf{e}(k+1), \mathbf{e}^{\mathrm{T}}(k+1) \right\}$$

State estimate time update:

$$\hat{\mathbf{x}}(\mathbf{k})(\mathbf{k}|\mathbf{k}-1) = \mathbf{A}(\mathbf{k}-1) \ \hat{\mathbf{x}}(\mathbf{k})(\mathbf{k}-1|\mathbf{k}-1) + \mathbf{B}(\mathbf{k}-1) \ \mathbf{u}(\mathbf{k}-1)$$
 (6)

Covariance Time update:

$$\mathbf{Q}(k) = \mathbf{A}(k-1) \ \mathbf{Q}(k-1) \ \mathbf{A}^{\mathrm{T}}(k-1) + \mathbf{B}(k-1) \ \mathbf{V}_{1}(k-1) \ \mathbf{B}^{\mathrm{T}}(k-1)$$
(7)

Kalman Gain Matrix

$$\mathbf{L}(\mathbf{k}) = \mathbf{Q}(\mathbf{k}) \mathbf{C}^{\mathrm{T}}(\mathbf{k}) [\mathbf{C}(\mathbf{k}) \mathbf{Q}(\mathbf{k}) \mathbf{C}^{\mathrm{T}}(\mathbf{k}) + \mathbf{V}_{2}(\mathbf{k})]^{-1}$$
(8)

State estimation measurement update:

$$\hat{\mathbf{x}}(\mathbf{k})(\mathbf{k}|\mathbf{k}) = \hat{\mathbf{x}}(\mathbf{k})(\mathbf{k}|\mathbf{k}-1) + \mathbf{L}(\mathbf{k})[\mathbf{y}(\mathbf{k}) - \mathbf{C}\,\hat{\mathbf{x}}(\mathbf{k})(\mathbf{k}|\mathbf{k}-1)]$$
(9)

If anything but x kept constant, the covariance matrix will converge towards the solution to the discrete Riccati equation:

$$Q(k) = A(k)Q(k)A^{\tau}(k) + G_{\nu}(k)V_{1}G_{\nu}^{T} - L^{1}(k)C(k)Q(k)A^{T}(k)$$
(10)

where

$$L'(k) = A(k)Q(k)C^{\tau}(k)[C(k)Q(k)C^{T}(k) + V_{2}(k)^{-1}$$
(11)

Since the variables in Riccati equation (10) are matrices, it is rather complicated to solve symbolically.

Extended Kalman Filter

An Extended Kalman Filter is a recursive optimum stateobserver that can be used for the state and parameter estimation of a non-linear dynamic system in real time by using noisy monitored signals that are distributed by random noise. This assumes that the measurement noise and system noise are uncorrelated. In the first stage of the calculations, the states are predicted by using a mathematical model (which contain previous estimates) and in the second stage; the predicted states are continuously corrected by using a feedback correction scheme. This scheme makes use of actual measured states, by adding a term to the predicted states (which is obtained in the first stage). The additional term contains the weighted difference of measured and estimated output signals. Based on the deviation from the estimated value, the EKF provides an optimum output value at the next input instant. In an induction motor drive the EKF can be used for the real-time estimation of the rotor speed, but it can also be used for state and parameter estimation. For this purpose the stator voltages and currents are measured and, for example, the speed of the machine can be obtained by the EKF quickly and precisely ^[14].

Motor Model for EKF

The model for induction motor developed in stationary reference frame and used in ^[13], and ^[9] is given below:

$$\frac{d}{dt} \begin{bmatrix} i_{s}^{s} \\ i_{qs}^{s} \\ \psi_{dr}^{s} \\ \psi_{qr}^{s} \\ w_{r}^{s} \end{bmatrix} = \begin{bmatrix} -\frac{K_{R}}{K_{L}} & 0 & \frac{L_{m}R_{r}}{L_{r}^{2}K_{L}} & \frac{L_{m}w_{r}}{L_{r}K_{L}} & 0 \\ 0 & -\frac{K_{R}}{K_{L}} & -\frac{L_{m}w_{r}}{L_{r}K_{L}} & \frac{L_{m}R_{r}}{L_{r}^{2}K_{L}} & 0 \\ \frac{L_{m}}{T_{r}} & 0 & -\frac{1}{T_{r}} & -w_{r} & 0 \\ 0 & \frac{L_{m}}{T_{r}} & w_{r} & -\frac{1}{T_{r}} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{s}^{s} \\ w_{qr}^{s} \\ w_{r}^{s} \end{bmatrix} + \frac{1}{K_{L}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{ds}^{s} \\ V_{qs}^{s} \end{bmatrix}$$
(12)
$$\begin{bmatrix} i_{s}^{s} \\ i_{qs}^{s} \\ \vdots_{qs}^{s} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{ds}^{s} \\ i_{qs}^{s} \\ w_{dr}^{s} \\ w_{r}^{s} \end{bmatrix}$$
(13)

where, *Lr*, *Ls*, *Lm* are rotor, stator and main inductance. *Tr*, *and Ts* are rotor and stator time constants.

The application of Eq.12 to the EKF will give not only the rotor speed, but also the rotor flux-linkage components (and consequently the angle and modulus of the rotor flux-linkage space-vector will also be known). This is useful for high performance field-oriented drive implementations.

Discretized Augmented Machine Model

The motor equations are converted to the standard form:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{t}} = \mathbf{A} \, \mathbf{x} + \mathbf{B} \, \mathbf{u} \tag{14}$$
$$\mathbf{y} = \mathbf{C} \, \mathbf{x}$$

And then discretized for the digital implementation of EKF as:

$$\mathbf{x}(\mathbf{k}+1) = \mathbf{A}_{\mathbf{b}} \mathbf{x}(\mathbf{k}) + \mathbf{B}_{\mathbf{d}} \mathbf{u}(\mathbf{k})$$
(15)

$$y(k) = C_{b} x(k)$$

 \mathbf{A}_{b} and \mathbf{B}_{d} matrices in the (15) are discretized system and input matrices, respectively. They are:

$$\mathbf{A}_{b} = \mathbf{e}^{\mathrm{AT}} \approx \mathbf{I} + \mathbf{AT} + (\mathrm{AT})^{2}/2 \tag{16}$$

$$\mathbf{B}_{d} = \int_{0}^{T} e^{A\xi} \mathbf{B} \, d\zeta \approx \mathbf{B} T + ABT^{2}/2$$

$$\mathbf{C}_{b} = \mathbf{C}$$
(17)

where T is the sampling time. When the last terms in (16) and (17) are ignored, then very short sampling times, they require, are attainable to have a stable and accurate discretized model.

However, a better approximation is obtained with the given second-order series expansion at (16) and (17). In general to achieve an adequate accuracy, the sampling-time should be appreciably smaller than the characteristic time-constants of the machine. The final choice for this should be based on obtaining adequate execution time of the full EKF algorithm and also satisfactory accuracy and stability. The second-order technique obviously increases the computational time. If the second-order terms are neglected in (16) and (17) then the discrete form of matrices become:

$$\mathbf{A}_{b} = \mathbf{e}^{\mathbf{A}\mathbf{T}} \approx \mathbf{I} + \mathbf{A}\mathbf{T}$$
(18)

$$\mathbf{B}_{d} = \int_{0}^{T} e^{A\xi} \mathbf{B} \, d\zeta \approx \mathbf{B} T$$

$$\mathbf{C}_{b} = \mathbf{C}$$

$$\mathbf{A}_{d} = \begin{bmatrix} 1 - T \frac{K_{R}}{K_{L}} & 0 & T \frac{L_{m}R_{r}}{L_{r}^{2}K_{L}} & T \frac{L_{m}W_{r}}{L_{r}K_{L}} & 0 \\ 0 & 1 - T \frac{K_{R}}{K_{L}} & -T \frac{L_{m}W_{r}}{L_{r}K_{L}} & T \frac{L_{m}R_{r}}{L_{r}^{2}K_{L}} & 0 \\ T \frac{L_{m}}{T_{r}} & 0 & 1 - T \frac{1}{T_{r}} & -T W_{r} & 0 \\ 0 & T \frac{L_{m}}{T_{r}} & T W_{r} & 1 - T \frac{1}{T_{r}} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(20)$$

$$\mathbf{B}_{d} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \mathbf{C}_{d} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$
(21)

$$\mathbf{x}(\mathbf{k}) = \begin{bmatrix} i_{ds}^{s}(\mathbf{k}) & i_{qs}^{s}(\mathbf{k}) & \psi_{ds}^{s}(\mathbf{k}) & \psi_{qs}^{s}(\mathbf{k}) & \mathbf{w}_{r}(\mathbf{k}) \end{bmatrix}^{\mathrm{T}}$$
(22)

 $\mathbf{u}(\mathbf{k}) = \begin{bmatrix} V_{ds}^{s}(\mathbf{k}) & V_{qs}^{s}(\mathbf{k}) \end{bmatrix}^{T}$

By considering the system noise $\upsilon(k)(v)$ is the noise vector of states), being zero-mean white-Gaussian and independent of x(k) with a covariance matrix Q, the system model becomes:

$$\mathbf{x}(\mathbf{k}+1) = \mathbf{A}_{\mathbf{b}} \mathbf{x}(\mathbf{k}) + \mathbf{B}_{\mathbf{d}} \mathbf{u}(\mathbf{k}) + \mathbf{v}(\mathbf{k})$$
(23)

By considering a zero-mean white-Gaussian measurement noise, $\Box \Box w(k)$ (noise in the measured stator currents) which is independent of y(k) and v(k) with a covariance matrix R, the output equation becomes :

$$\mathbf{y}(\mathbf{k}) = \mathbf{C} \, \mathbf{x}(\mathbf{k}) + \mathbf{w}(\mathbf{k}) \tag{24}$$

Determination of the Noise and State Covariance Matrices

To be more specific, the goal of the Kalman filter is to obtain unmeasurable states (i.e. covariance matrices Q, R, P of the system noise vector, measurement noise vector, and system state vector (x) respectively). In general, by means of noise inputs, it is possible to take computational inaccuracies, modeling errors, and errors in measurements into account in modeling the system. The filter estimation (\hat{x}) is obtained from the predicted values of the states (x) and this is corrected recursively by using a correction term, which is product of the Kalman gain (L) and the deviation of the estimated measurement output vector and the actual output vector $(y - \hat{y} \Box)$. The Kalman gain is chosen to result in the best possible estimated states.

Thus filtering algorithm contains basically two main stages, a prediction stage and a filtering stage. During the prediction stage, the next predicted values of the state x(k+1) are obtained by using a mathematical model (state variable equations) and also the previous values of the estimated states.

Furthermore, the predicted-state covariance matrix (P) is also obtained before the new measurements are made and for this purpose the mathematical model and also the covariance matrix of the system (Q) are used. In the second stage which is the filtering stage, the next estimated states $\hat{x}(k+1)$, are obtained from the predicted estimate x(k+1) by adding a correction term $L(y-\hat{y})$ to the predicted value. This correction term is a weighted difference between the actual output vector (y) and the predicted output vector (\hat{y}), where L is the Kalman gain. Thus the predicted state-estimate (and also covariance matrix) is corrected through a feedback correction scheme that makes use of actual measured quantities. The Kalman gain is chosen to minimize the estimation error variance of the states to be estimated. The computations are realized by using recursive relations.

A critical part of the design is to use correct initial values for the various covariance matrices. These can be obtained by considering the stochastic properties of the corresponding noises. Since these are usually not known, in most cases they are used as weight matrices, but it should be noted that sometimes-simple qualitative rules could be set up for obtaining the covariance in the noise vectors.

The system noise covariance matrix (Q) is [5x5], and the measurement noise covariance matrix (R) is [2x2] matrix, so in general this would require the knowledge of 29 elements. However, by assuming that the noise signals are not correlated, both Q and R are diagonal, and only 5 elements must be known in Q and 2elements in R.However, the parameters in $\alpha \square \square$ and β \square axes are the same, which means that the first two elements of the diagonal are equal (q11=q22), the third and fourth elements in the diagonal of Q are equal (q33=q44), so Q =diag

(q11,q11,q33,q33,q55) contains only 3 elements which have to be known. Similarly, the two diagonal elements in R are equal (r11=r22), thus R=diag (r11, r11). It follows that in total only 4 noise covariance elements needs to be known.

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q} \mathbf{i}_{ds}^{s} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{Q} \mathbf{i}_{qs}^{s} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{Q} \psi_{dr}^{s} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{Q} \psi_{qr}^{s} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{Q} w_{r} \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} \mathbf{R} \mathbf{i}_{ds}^{s} & 0 \\ 0 & \mathbf{R} \mathbf{i}_{qs}^{s} \end{bmatrix}$$
(25)

Starting values of the state vector xo and the starting values of the noise covariance matrices Qo and Ro are set together with the starting value of the state covariance matrix Po, where P is the covariance matrix of the state vector. The starting state covariance matrix can be considered as diagonal matrix, where all elements are equal. The initial values of the matrices reflect the degree of knowledge of the initial states: the higher their value, the less accurate is any available information on the initial states. Thus the new measurement data will be more heavily weighted and the covariance speed of the estimation process will increase. However, divergence problem or large oscillations of the state estimates around the true value may occur when too high initial covariance values are chosen. A suitable selection allows us to obtain satisfactory speed convergence, and avoid divergence problems or unwanted oscillations.

The accuracy of the state estimation is affected by the amount of information that the stochastic filter can extract from its mathematical model and the measurement data processing. Some of the estimated variables, especially unmeasured ones, may indirectly and weakly be linked to the measurement data, so only poor information is available to the EKF. After deciding how to initialize the covariance matrices, the next step is prediction of the state vector.

EKF Algorithm

a. prediction of the state vector

Prediction of the state vector at sampling time (k+1) from the input u (k), state vector at previous sampling time, $X_{k|k}$, by using Ad and Bd is obtained from

$$\mathbf{x}_{k|k+1} = \mathbf{A}_{b} \mathbf{x}_{k|k} + \mathbf{B}_{d} \mathbf{u}(k)$$
(26)

$$\mathbf{x}_{k|k+1} = \mathbf{F}(k+1,k,\mathbf{x}_{k|k},\mathbf{u}(k))$$
(27)

Where

$$\mathbf{F} = \begin{bmatrix} (1 - T\frac{K_{R}}{K_{L}})i_{ds}^{s} + T\frac{L_{m}R_{r}}{L_{r}^{2}K_{L}}\psi_{dr}^{s} + T\frac{L_{m}w_{r}}{L_{r}K_{L}}\psi_{qr}^{s} + T\frac{1}{K_{L}}V_{ds}^{s} \\ (1 - T\frac{K_{R}}{K_{L}})i_{qs}^{s} - T\frac{L_{m}R_{r}}{L_{r}^{2}K_{L}}\psi_{dr}^{s} + T\frac{L_{m}w_{r}}{L_{r}K_{L}}\psi_{dr}^{s} + T\frac{1}{K_{L}}V_{qs}^{s} \\ T\frac{L_{m}}{T_{r}}i_{ds}^{s} + (1 - T\frac{1}{T_{r}})\psi_{dr}^{s} - Tw_{r}\psi_{qr}^{s} \\ T\frac{L_{m}}{T_{r}}i_{qs}^{s} + (1 - T\frac{1}{T_{r}})\psi_{qr}^{s} + Tw_{r}\psi_{dr}^{s} \\ W_{r} \end{bmatrix}$$
(27)

$$\mathbf{h} \stackrel{\circ}{=} \mathbf{C}_{d} \mathbf{x}_{k|k+1} = \begin{bmatrix} \mathbf{i}_{ds}^{s} \\ \mathbf{i}_{qs}^{s} \end{bmatrix}$$
(28)

The notations $\mathbf{x}_{k|k+1}$ means that it is a predicted value at the (k+1)-th instant, and it is based on measurements up to k-th instant. In the following steps of the recursive EKF computation, covariance matrix of prediction is computed.

b. prediction covariance computation

The prediction covariance is updated by:

$$\mathbf{P}_{k+1|k} = \mathbf{M} \, \mathbf{P}_{k|k} \, \mathbf{M}^{\mathrm{T}} + \mathbf{Q} \quad , \quad \mathbf{M} = \frac{\partial}{\partial x} (\mathbf{F}) \Big|_{x = \hat{x}_{k|k}}$$
(29)

with

$$\frac{\partial}{\partial \mathbf{x}} (\mathbf{F}) = \begin{bmatrix} 1 - T \frac{\mathbf{K}_{\mathrm{R}}}{\mathbf{K}_{\mathrm{L}}} & 0 & T \frac{\mathbf{L}_{\mathrm{m}} \mathbf{R}_{\mathrm{r}}}{\mathbf{L}_{\mathrm{r}}^{2} \mathbf{K}_{\mathrm{L}}} & T \frac{\mathbf{L}_{\mathrm{m}} \mathbf{w}_{\mathrm{r}}}{\mathbf{L}_{\mathrm{r}} \mathbf{K}_{\mathrm{L}}} & T \frac{\mathbf{L}_{\mathrm{m}}}{\mathbf{L}_{\mathrm{r}} \mathbf{K}_{\mathrm{L}}} \psi_{\mathrm{qr}}^{\mathrm{s}} \\ 0 & 1 - T \frac{\mathbf{K}_{\mathrm{R}}}{\mathbf{K}_{\mathrm{L}}} & -T \frac{\mathbf{L}_{\mathrm{m}} \mathbf{w}_{\mathrm{r}}}{\mathbf{L}_{\mathrm{r}} \mathbf{K}_{\mathrm{L}}} & -T \frac{\mathbf{L}_{\mathrm{m}}}{\mathbf{L}_{\mathrm{r}} \mathbf{K}_{\mathrm{L}}} \psi_{\mathrm{dr}}^{\mathrm{s}} \\ T \frac{\mathbf{L}_{\mathrm{m}}}{\mathbf{T}_{\mathrm{r}}} & 0 & 1 - T \frac{1}{\mathbf{T}_{\mathrm{r}}} & -T \mathbf{w}_{\mathrm{r}} & T \frac{\mathbf{U}_{\mathrm{m}}}{\mathbf{L}_{\mathrm{r}} \mathbf{K}_{\mathrm{L}}} \psi_{\mathrm{dr}}^{\mathrm{s}} \\ 0 & T \frac{\mathbf{L}_{\mathrm{m}}}{\mathbf{T}_{\mathrm{r}}} & T \mathbf{w}_{\mathrm{r}} & 1 - T \frac{1}{\mathbf{T}_{\mathrm{r}}} & T \psi_{\mathrm{dr}}^{\mathrm{s}} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\ \frac{\partial}{\partial \mathbf{x}} \mathbf{h} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$
(31)

The next step is the computation of the Kalman filter gain matrix.

c. kalman gain computation

The Kalman filter gain (correction matrix) is computed as;

$$\frac{\partial h}{\partial x}$$

$$L_{k} = P_{k|k-1} N^{T} [NP_{k|k-1} N^{T} + R]^{-1} \text{ Where } N = x = \hat{x}_{k|k-1}$$
(32)

d. state vector estimation

The predicted state-vector is added to the innovation term multiplied by Kalman gain to compute state-estimation vector. The state-vector estimation (filtering) at time (k) is determined as:

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{L}_{k}(\mathbf{y}_{k} - \hat{\mathbf{y}}_{k})$$
(33)

Where

$$\hat{\mathbf{y}}_{k} = \mathbf{C}_{d} \mathbf{X}_{k|k-1} \tag{34}$$

e. estimation covariance computation

The last step is estimation covariance computation as ;

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{L}_{k}(\mathbf{y}_{k} - \hat{\mathbf{y}}_{k})$$
(35)

After all steps executed, set k=k+1 and start from the step-a to continue the computation recursively.

State Estimation Simulations with EKF

In this part, the state estimation performance of EKF is simulated. The simulation is implemented with Matlab. In this simulation input voltages and measured currents in stationary reference frame are produced by FOC simulation^[15].

In the simulation parameters of a 1-hp motor are used. Base excitation frequency is 60 Hz. The observable states in this model as mentioned previously:

 $\left\{\mathbf{i}_{ds}^{s}(k) \quad \mathbf{i}_{qs}^{s}(k) \quad \psi_{dr}^{s}(k) \quad \psi_{qr}^{s}(k) \quad \mathbf{w}_{r}(k)\right\}$

In Fig.1 speed reversal at no-load is given with reference speed. The estimated speed and the reference speed (linear) are plotted together.

Measurement and state covariances are chosen so that both the transient and steady state speed errors be optimized. One may error speed choose different covariance and obtain almost zero steady-state speed error with a poor transient speed estimation as shown in Fig. 2 or vice versa.

In the case of Fig.3 simulation, state covariance is decreased; the algorithm begins to behave such that the state space model gives more accurate estimates compared to measure values so it assigns less importance to the measurements. This causes a decrease in Kalman gain, which reduces the correction speed of the currents. In the extra time used for current correction the algorithm finds opportunity to decrease the steady-state error.

Low speed estimation performance of the EKF is also quite satisfactory and close to reference speed as shown in Fig (4)-(5). In Fig.6 rated mechanical load is applied to the motor between 0.75-1.5 sec. To verify the performance of EKF under loaded conditions. As shown above EKF works properly even under fully loaded case. One may decrease steady-state error to very low levels with appropriate state covariance optimized for steady state.

CONCLUSION

- The following points can be deduced from the previous results:
- The EKF shows high tracking performance for both high and low speed estimations and close to reference speed. The high performance is verified for four-quadrant speed.
- ✓ If R is large then L is small and the transient performance is faster. Moreover, if Q is large the L is large and the transient performance is lower.
- The steady-state error may be decreased to very low levels with appropriate state covariance optimized for steady-state case.

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Figure. 1 – High Speed, No-Load, Four Quadrant Speed Estimation with EKF (in (P/2)* [rad/sec])



Figure. 2– (Zoomed at steady state) High Speed, No-Load, Four Quadrant Speed Estimation with EKF at Steady State (in (P/2)* [rad/sec])



Figure. 3- High Speed, No-Load, Speed Estimation with EKF -Steady State Performance Optimized (in (P/2)* [rad/sec])



Figure. 4– Low Speed, No-Load, Four Quadrant Speed Estimation with EKF(in (P/2)*[rad/sec])



Figure. 6- High Speed, Full-Load, Speed Estimation with EKF (in (P/2)* [rad/sec])

الخلاصة

يتناول هذا البحث تقنية تحديد المتغيرات (State Estimation) الخاصة بعملية السيطرة للمجال الموجه للمحركات الحثية. الأسس النظرية لكل خوارزمية تم شرحها وأختبار أدائها بأستخدام المحاكاة ضمن الحقيبة البرمجية MATLAB شرحها وأختبار أدائها بأستخدام المحاكاة ضمن الحقيبة البرمجية (EKF الأصدار 6.3 . في هذا البحث تم استخدام مرشح كالمان الموسع (EKF) لتخمين المتغيرات اللاخطية وقد تضمن نموذج المحرك (لغرض تطبيق مرشح كالمان) على سرعة الجزء الدوار (Rotor) وتيارات الجزء الثابت للمحرك (تعرض تطبيق مرشح كالمان) على وهكذا فان قيم سرعة الجزء الدوار والفيض الناتج عن تيارات الجزء تحدد قيمها آنيا" باستخدام هذا الراصد (observer) . أن عمل المرشح تنظيم مستوى الضوضاء باستخدام خوارزميات مثالية (Optimization) . أن عمل المرشح المحرك الحرك

> الكلمات الدالة محرك حثى ، مرشح كالمان ، تخمين ، محاكاة