ESTIMATING LONGITUDINAL AND TRANSVERSE DISPERSION COEFFICIENTS IN OPEN CHANNEL

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ABSTRACT

Results are presented from a series of laboratory experiments conducted on an open channel. Twenty five data sets (L1-L25) have been measured to obtain the magnitude of the longitudinal dispersion coefficient and eight data sets (T1-T8) have been performed to estimate transverse mixing coefficient in channel. The method involves derivation of a new expression for the longitudinal and transverse dispersion coefficients which used routing concentration of pollutant in the advection- dispersion equation solution in water quality mathematical model. Values of longitudinal and transverse dispersion coefficients are compared with measured data and previous similar studies. These comparisons have been showed that tracer technique by using second moment is the most accurate prediction of the longitudinal and transverse dispersions coefficients than the other techniques.

KEYWORDS

Longitudinal dispersion coefficient, Transverse dispersion coefficient.

LIST OF SYMBOLS

b	width of the channel, (m)
C	Concentration of solute, (mg/L)
C_p	peak concentration, (mg/L)
D	Longitudinal dispersion coefficient, (m^2/s)
d	Depth of water, (m).
Q	flow of water, (m^3/s)
M	Mass of solute, (g)
m	frequency of data.
K_y	Transverse mixing coefficient, (m^2/s)
O	Observed Data
P	Predict Data
R	Hydraulic radius, (m).
s	slope of channel.
T	top width of water surface, (m).
t	time with each concentration. (s)
t_p	time to the peak concentration (s)
u	velocity of flow, (m/s)
u_*	shear velocity, (m/s)
σ_t^2	variances of the temporal concentration profiles, (s^2).
σ_y^2	variance of the transverse concentration profiles, (m^2).
x	distance of sampling station, (m)

INTRODUCTION

The prediction of dispersion coefficients in longitudinal, lateral and vertical directions of flow are of utmost importance when evaluating the time concentration distribution at any point in a stream. To evaluate these coefficients, it is required to measure field data (velocity, width, depth, etc.) at various locations along the cross section of the river. Dispersion coefficients represent all the mixing processes in the flow. (Fischer, 1979)[1]. Longitudinal dispersion coefficient can be estimated directly using tracer test technique (Rutherford, 1994)[2]. Several empirical and analytical equations for computing the longitudinal dispersion coefficient have been recommended by various investigations. These equations produce values of longitudinal dispersion coefficient which vary widely for the same flow conditions. In this study the dispersion coefficient in the flume of hydraulic laboratory of the Engineering at AL-Mustansirya University were measured as were the other elements of the mixing process. Several new data have been generated during this work (Majeed, 2006)[3]. Twenty five data sets (L1-L25) have been used to derive the longitudinal dispersion coefficient equation and eight sets (T1 – T8) have been used to evaluate transverse mixing coefficient in channel .

PREVIOUS WORK

To calculate the dispersion coefficient requires information on velocity and turbulent mixing that is often not known; it is therefore more common to estimate the coefficient from empirical equations or use tracer experiments by the second moment's and Chatwin method which are considered later.

[Fisher, 1979][1] demonstrated how the change of variance of measured concentration profiles could be used to calculate a longitudinal dispersion coefficient. This technique, called also the method of moment’s, is valid as long as the rate of change of variance is linear.

In the method concentration distributions of a tracer material are measured at two (or more) points along the channel and the dispersion coefficient is calculated from the rate of change of variance of the distributions, as bellow. [Fisher, 1979][1] [Rutherford, 1994][2] [Merle, 1997][4].

$$D=\frac{1}{2}u^2\,\frac{\sigma_t^{\,2}(x_2)-\sigma_t^2(x_1)}{\bar{t}_2-\bar{t}_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

$$u=\frac{x_2-x_1}{\bar{t}_2-\bar{t}_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

$$\bar{t}=\frac{\int_0^\infty t*C(t)*Q(t)*dt}{\int_0^\infty C(t)*Q(t)*dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(3)}$$

$$\sigma_t^2=\frac{\int_0^\infty (t-\bar{t})^2*C(t)*Q(t)*dt}{\int_0^\infty C(t)*Q(t)*dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(4)}$$

Where: D = longitudinal dispersion coefficient, (m^2/s).

σ_t^2 = variances of the temporal concentration profiles, (s^2).

u = cross-sectional flow velocity, (m/s).

x = distance of sampling station, (m)

t = time with each concentration.

$\bar{t}$ = mean residence time.

This equation is applied in equilibrium zone where the longitudinal variance of the cross-sectional average tracer concentration increases linearly with time .

Chatwin developed a method for determining longitudinal dispersion intended to address the problem of non-Fickian behavior. Technically, the Chatwin method is only really valid for impulse releases, but it does provide a reasonable approximation for longitudinal dispersion for pulse and continuous releases [Won, 1998][5] [Field, 2002][6].

$$\sqrt{t*\ln\left(\frac{C_p\sqrt{t_p}}{C\sqrt{t}}\right)}=\frac{x}{2\sqrt{D}}-\frac{u*t}{2\sqrt{D}} \quad \text{………………………………..…. (3)}$$

Chatwin plotted $\sqrt{t*\ln\left(\frac{C_p\sqrt{t_p}}{C\sqrt{t}}\right)}$ against time t. Using the transformation Gaussian tracer data plots as a straight line whose slope is proportional to the longitudinal dispersion coefficient. The peak concentration occurs where the transformed data changes sign. Where $\left(\frac{x}{2\sqrt{D}}\right)$ is the y intercept of the straight line fit to the early time data and $\left(\frac{u}{2\sqrt{D}}\right)$is the slope of the straight line fit to the early time data. [Guymer, 1998][7] [Field, 2002][6].

To estimate the longitudinal dispersion coefficients from empirical equations for previous studies, the following equations were proposed by :

McQuive and Keefer, 1974[8] $D = 0.058 * (Q / s * b)$

Fischer, 1975[9] $D = 0.011 * u^2 * T^2 / d * u_*$

Jain, 1976[10] $D = a * (u * b^2 / k_y)$, $0.001 < a < 0.016$

Liu, 1977[11] $D = b * (Q^2 / u_* * R^3)$, $b = 0.18 * (u_* / u_q)$

The above equations are used in present study to compare them with measured data sets.

[Rutherford, 1994][2] Presented a model of mixing in open channel flows from which the transverse mixing coefficient can be inferred from the degree of mixing, over the width, of a solute from a continuous source. The rate of change of variance does give a measure of the transverse mixing that includes the effects of the turbulent eddy diffusivity and secondary flows. Accordingly the transverse mixing coefficient was calculated from equation:

$$k_y = \frac{1}{2} \cdot u \cdot \frac{\sigma_y{}^2(x_2) - \sigma_y^2(x_1)}{x_2 - x_1} \qquad \text{……………………………………..} \; (4)$$

$$\sigma_y^2 = \frac{\int_0^\infty (y-\bar{y})^2 * C(y) * dy}{\int_0^\infty C(y) * dy} \quad \text{...} \quad (5)$$

Where σ_y^2 = variance of the transverse concentration profiles, (m^2).

RESULTS OF LONGITUDINAL DISPERSION COEFFICIENTS

To calculate the observed dispersion coefficients from the field data, both second moment and Chatwin methods were considered. These results are listed in Tables (1) and (2) .

The averages of the reliable measurements of the longitudinal dispersion coefficient taken at the narrow flow rate are given in Table (2) , Fig.(1) shows this data plotted against the product of the depth and shear velocity (evaluated at 16 m from the flume inlet). A linear regression fitted to the data gave the following relationship:

$$D = 17.018 * du_* + 0.0035 \quad \text{...........................} \quad (7)$$

With a coefficient of determination for the least squares fit $r^2 = 0.8008$. The regression implied that longitudinal dispersion could occur in the flume with no flow; this is clearly non logical and results from the limited range of experiments that were performed.

In general the longitudinal dispersion coefficient in this method with range of ($0.008361 - 0.03323 \ m^2/s$) in flow rate ($3.05$ and $10.05 \ L/s$) respectively.

By using Chatwin transformation, the data collected from all experiments at flow rates of (0.56-12.24 L/s) are plotted in Fig. (2). Different slopes are found at sampling positions 9m and 18m because the passage of a discrete cloud of solute along a channel from an instantaneous source as having three stages. Initially advection dominates the transport and the longitudinal variance of the solute cloud changes non-linearly whilst the skewness rises; this is known as the advective zone. After some time, the change of variance becomes linear and the skewness falls; this stage is known as the equilibrium zone. Eventually the spatial concentration profile becomes Gaussian in the Gaussian zone.

All Chatwin's data in table (1) for 9 and 18 m sampling station are plotted against the product of the depth and side wall corrected bed shear velocity in Fig.(3).

A linear regression fitted to the data gave the following relationship:

- For 9m, $D = 7.2753 * du_* + 0.0023$ With $r^2 = 0.9245$. ……(8)
- For 18m, $D = 10.378 * du_* + 0.0023$ With $r^2 = 0.8464$………(9)

In general the longitudinal dispersion coefficients in Chatwin's method take with range of;

- For 9m, ($0.0038 - 0.0176 \ m^2/s$) in flow rate (1.0 and $7.71 \ L/s$)

- For 9m, ($0.00457 - 0.021\ m^2/s$) in flow rate (1.0 and $8.7\ L/s$)

VERIFICATION OF LONGITUDINAL DISPERSION PRESENT STUDY

In order to test the behavior of the existing dispersion coefficient equations , 25 data sets measured in flume in the laboratory of the Engineering at Al-Mustansiriyah University [Majeed, 2006][3]. These data sets contain hydraulic and geometric parameters including channel width, mean depth, mean velocity, slope and tracer characteristics. These data sets were tested using six methods for predicting dispersion coefficient suggested by previous investigators. These included the dispersion equations proposed by [McQuivey and Keefer, 1974][8], [Jain, 1976][10], [Liu, 1977][11], [Fischer, 1975][9], Chatwin and [Fischer, 1979][1]. The dispersion coefficients were using the selected equations were compared with measured data and are shown in Fig.(4).

Fig.(4) shows that the use of Liu equation significantly underestimates observed values. The equations of Jain , Fischer (1975), and McQuivey and Keefer , also give low values, whereas second moment equation developed by Fischer (1979) which agree relatively well with observed values. Chatwin equation generally overestimates observed values.

To evaluate the difference between measured and predicted values of the dispersion coefficient more quantitatively , Median Relative Error, MRE [Thomann,1982 and Helsel, 2002][12] was used as an error measure.

$$MRE = \frac{\sum_{i=1}^{m} \frac{|O-P|}{O} * 100\%}{m} \quad \text{……………………..(12)}$$

Which O= Observed Data, P= Predict Data, and m= frequency of data.

Accuracy of each equation is listed in Table (3). Of the equation examined , second moment shown highest accuracy.

RESULTS AND VERIFICATION OF TRANSVERSE MIXING COEFFICIENT

The transverse mixing coefficients calculated from observed data (T1-T8) as shown in Table (4), it has been arranged as below;

- The position of outfall as the relative of the left bank side, column (2).
- The flow depth multiplies of the shear velocity, column (5).
- Values of Transverse mixing coefficient, k_y are calculated by Eqs.(4) and (5), column (6), (7) and (9).
- Coefficients of determination (r^2) for the least squares error fit used to know accurate for the rate of change of the variance of the transverse conc. profiles in column (8). As shown in Fig.(5).

- The non-dimensional transverse mixing coefficients k_y / du_*, column (10).

The transverse mixing coefficients observed are plotted against the product of the depth and shear velocity in Fig. (6). A linear regression for middle bank releases gave:

$$k_y = 0.1764 du_* + 2E^{-5}, \quad r^2 = 0.9848. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

It can be noted that the relationship between the mixing coefficients and product of the depth and bed shear velocity observed in the flume was very similar to that for straight natural open channel flows.

The coefficients of determination (r^2) for the least squares error fit used to find the rate of change of the variance of the transverse conc. profiles were generally between 0.994 and 0.9 indicate good values.

CONCLUSIONS

The results of this study , among the existing dispersion coefficient equation , Fischer ,1975 equation is not amenable to estimate the longitudinal dispersion coefficient because it underestimates significantly. Liu , Jain and McQuiveand Keefer predict also underestimates , whereas the tracer technique by Chatwin give results overestimates. Tracer technique by second moment predict relatively well . Twenty five data sets have been measured to obtain the magnitude of longitudinal dispersion coefficient This technique is given the following Eq.

$$D = 17.018 * du_* + 0.0035$$

The transverse mixing coefficient equation has been developed by using eight data sets using model presented by Rutherford, 1994 gives the Eq.

$$k_y = 0.1764du_* + 2E^{-5}$$

Results of above equations demonstrate that the new dispersion coefficients developed in this study is superior to existing equations in predicting dispersion coefficient more precisely in open channel.

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Table (1) Results of longitudinal dispersion coefficients

EXP. NO.	**Flow Discharge** $()L/s$	**Average Velocity** (m/s)	**Variance At (9m)** (s^2)	**Chatwin Method (9m)** $m^2/s\ D$	**Variance At (18m)** (s^2)	**Chatwin Method (18m)** $m^2/s\ D$	**Second Moment Mehtod** $m^2/s\ D$	**Shear Velocity* Depth** $()m^2/s$
L1	0.56	0.043375	8958.96	0.005089	35973.72	0.00643	0.125	0.000215
L2	0.9	0.05923	1648.44	0.004406	3947.4	0.005375	0.029824	0.000283
L3	1	0.062585	1445	0.003768	2735.208	0.004566	0.02109	0.000307
L4	1.1	0.06482	1157.904	0.005079	3729.6	0.006376	0.042657	0.000331
L5	1.24	0.06925	1090.05	0.004829	2567.34	0.008542	0.034211	0.000357
L6	3.05	0.11971	155.1852	0.006308	226.206	0.007176	0.008361	0.000609
L7	3.06	0.120235	113.7896	0.005935	246.402	0.007	0.013611	0.000609
L8	4.43	0.14864	80.0784	0.0086	132.53	0.008713	0.011592	0.000767
L9	4.58	0.149445	102.7224	0.008373	215.5068	0.010027	0.021554	0.0008
L10	5.1	0.15764	68.8716	0.008552	118.152	0.008629	0.012282	0.000868
L11	5.76	0.169485	35.0626	0.009586	94.1148	0.008119	0.015896	0.000938
L12	5.98	0.17163	52.002	0.00878	131.346	0.012861	0.02261	0.000973
L13	6.2	0.1738	79.5672	0.010635	128.257	0.013043	0.02655	0.001009
L14	6.26	0.175355	67.6224	0.010005	149.857	0.01319	0.027537	0.001009
L15	6.85	0.1831	99.9072	0.010754	177.484	0.01702	0.029083	0.001082
L16	7.71	0.19296	86.2452	0.012852	180.4824	0.01869	0.028074	0.001194
L17	8.02	0.19653	62.7012	0.012769	145.2384	0.019383	0.027133	0.001232
L18	8.7	0.204555	101.1	0.012498	264.1	0.020923	0.02954	0.00131
L19	9.08	0.209385	55.908	0.00985	94.4928	0.017594	0.02356	0.00135
L20	9.34	0.211335	55.044	0.013157	98.7444	0.016502	0.026454	0.00139
L21	9.65	0.21409	36.036	0.011535	81.728	0.01502	0.026021	0.00143
L22	10.05	0.21903	29.524	0.010523	80.2044	0.017483	0.033231	0.001471
L23	11.78	0.23491	30.65472	0.017622	53.1036	0.017378	0.0307	0.001679
L24	12.22	0.23965	36.8352	0.015214	70.0128	0.019181	0.02548	0.001722
L25	12.24	0.240045	31.15044	0.014092	49.0212	0.01851	0.02905	0.001722

Table (2) Average longitudinal dispersion coefficients for second moment method.

Exp.	Q (L/s)	D (m^2/s)	du* (m^2/s)	D/ du*
L6-L7	3.05 – 3.06	0.010986	0.000608948	18.04095
L8-L10	3.43 – 5.1	0.015142	0.000811985	18.64895
L11-L14	5.76 – 6.26	0.023148	0.000981975	23.57316
L15-L17	6.85 – 8.02	0.028097	0.001169388	24.0271
L18-L20	8.7 – 9.34	0.026518	0.001349881	19.64469
L21-L22	9.65 – 10.05	0.029615	0.001450252	20.42093
L22-L25	11.78 -12.24	0.028393	0.001708073	16.62302

Table (3) verification of longitudinal dispersion coefficients methods.

Eqs. of Dispersion Coefficient	Dispersion Coefficients m^2/s	MRE %
McQuivey &Keefer	0.3758	54.4
Calculated by Jain	0.26	48.6
Calculated by Liu	1.178	72
Calculated Fischer,1975	0.2478	63.59
Measured by chatwin	0.015	21.77
Measured By moment	0.02807	13.19
linear Fit of Moment	0.0238	7.95

Table (4) Transverse mixing coefficients.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
No.	Outfall m	Flow L/s	Velocity m/s	$*10^{-6}\ du_*$ m^2/s	dx m	$d\sigma_y^2$ m^2	r^2	$*10^{-6}\ k_y$ m^2/s	k_y/du_*
T1	0.425	3.05	0.1197	608.948	10	0.0214	0.99	128.41	0.211
T2	0.425	6.2	0.1738	1008.72	8	0.0194	0.994	210.9	0.209
T3	0.425	10.05	0.22	1470.58	12	0.0293	0.99	269	0.1829
T4	0.425	12.24	0.24	1722	2	0.0056	0.987	335.13	0.1946
T5	0.425	10.05	0.22	1470.58	10	0.0254	0.97	279.4	0.19
T6	0.425	10.05	0.22	1470.58	10	0.0254	0.97	279.4	0.195
T7	0.2	6.2	0.174	1009	8	0.0213	0.9	232.07	0.23
T8	0	6.2	0.174	1009	-	-	-	-	-

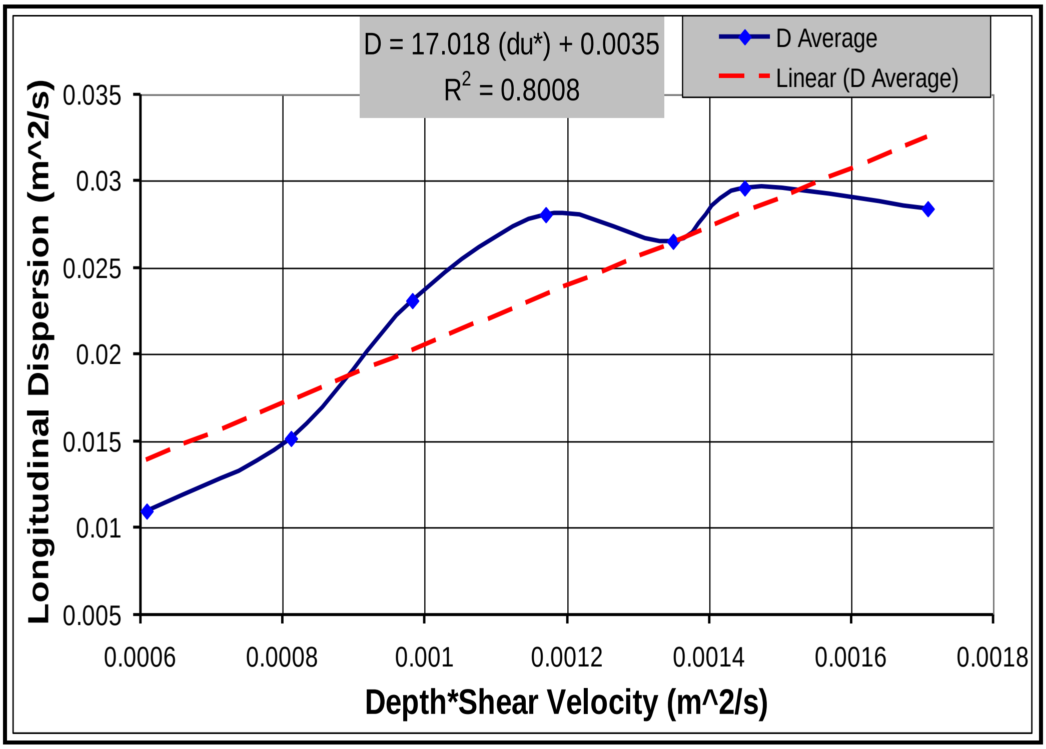

Fig. (1) longitudinal dispersion in the flume (Second moment's).

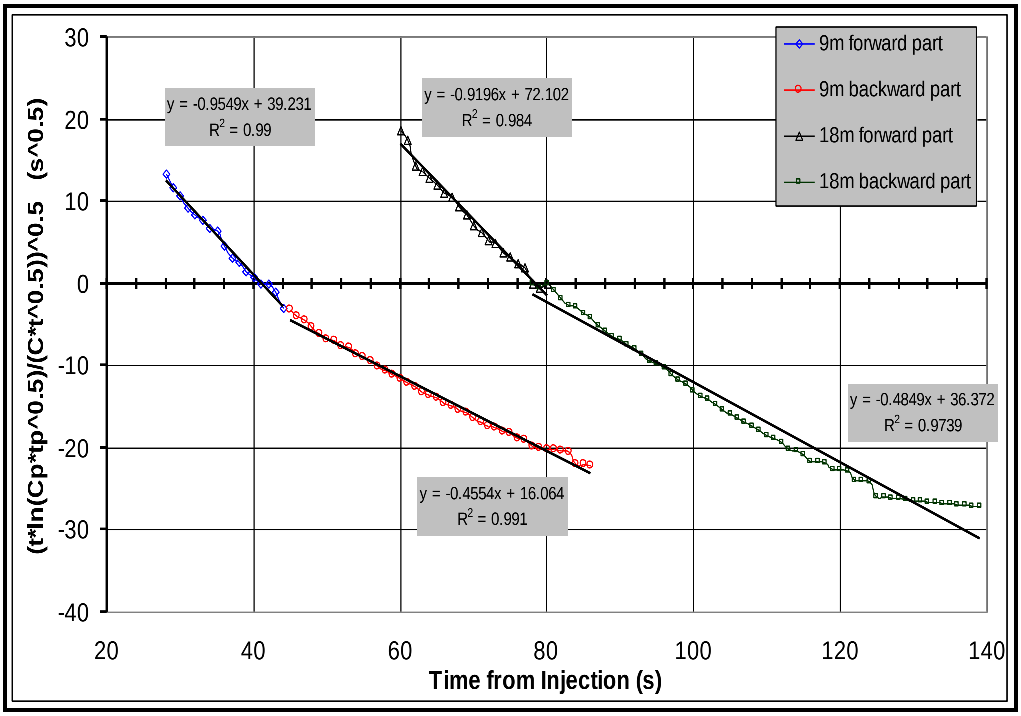

Fig. (2) Data plotted using Chatwin's transformation for $Q = 5.1L/s$.

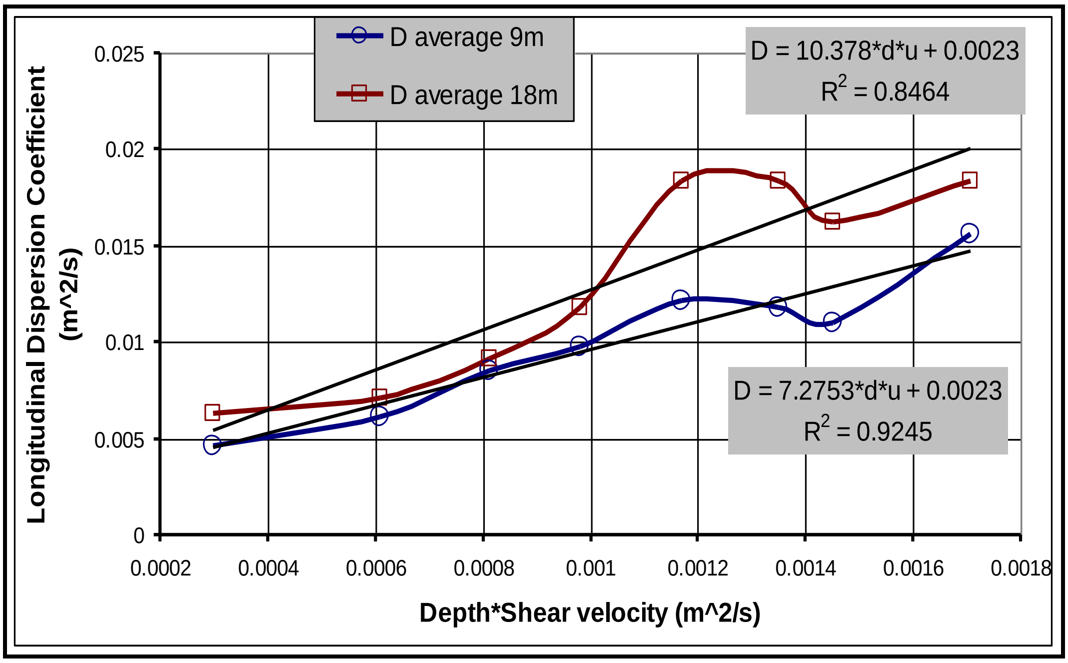

Fig. (3) longitudinal dispersion (Chatwin's method).

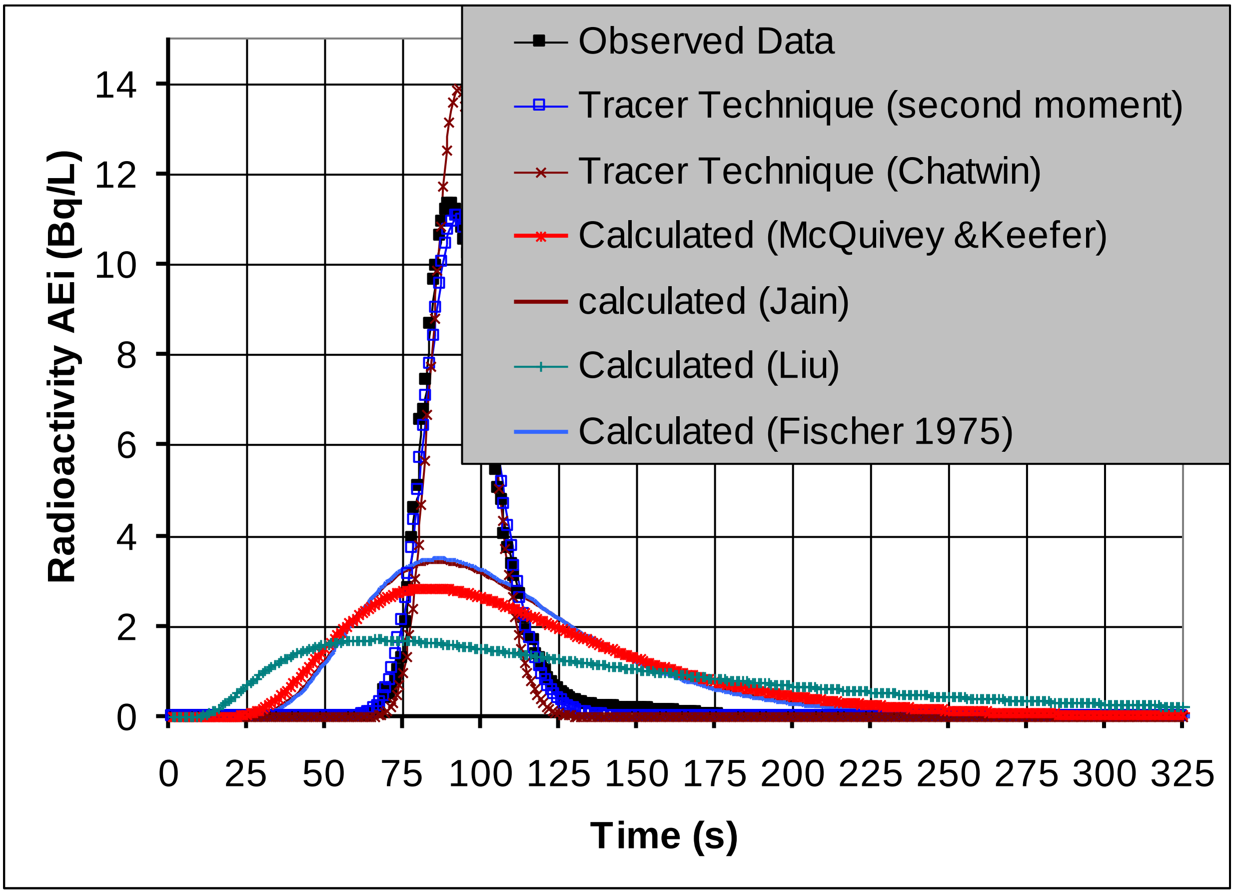

Fig. (4) Comparison between Observed and Empirical Equ..

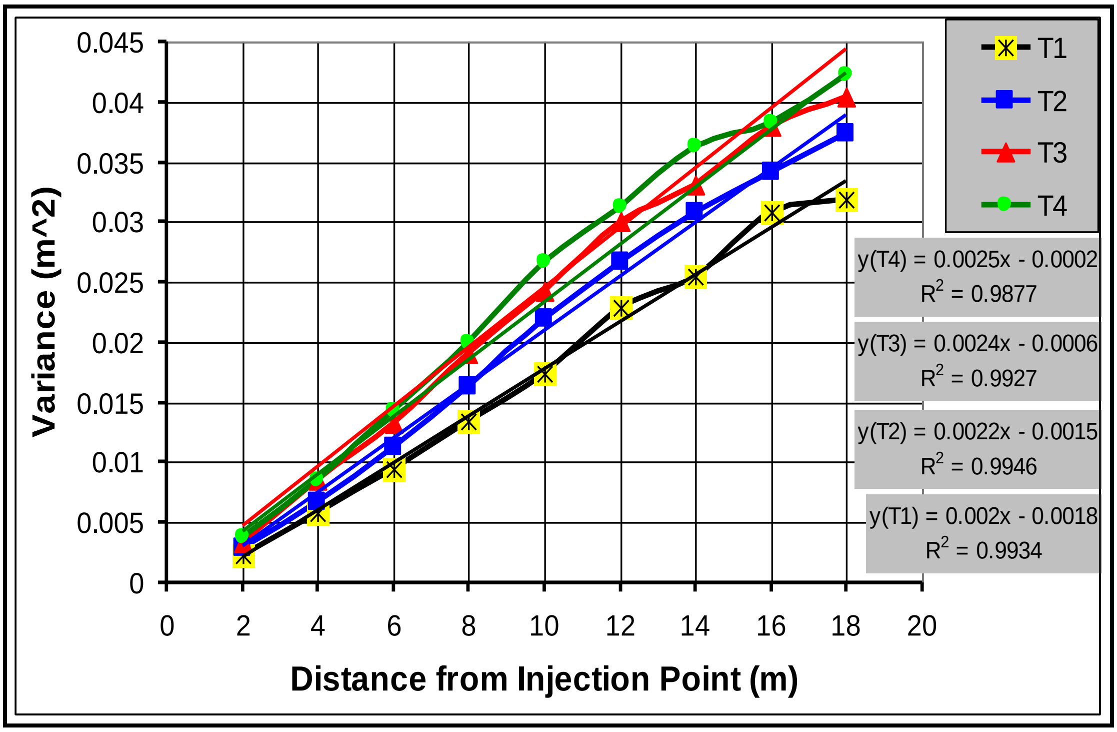

Fig. (5) Variance Distributions with respect to distance.

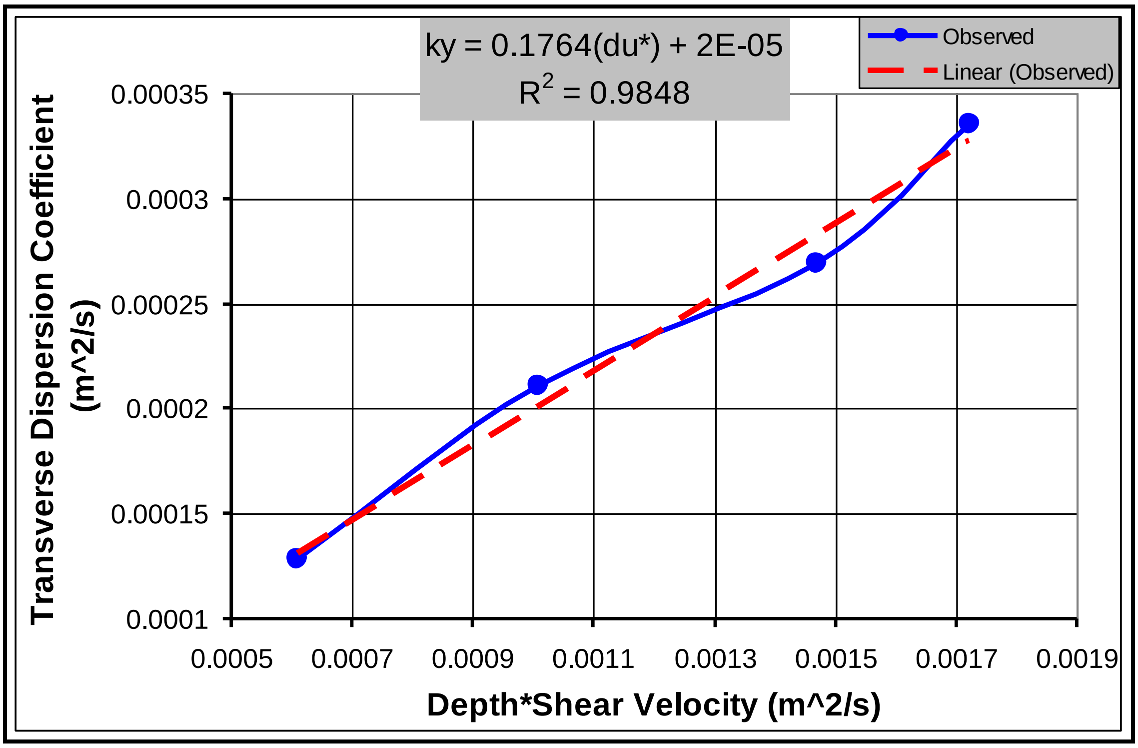

Fig. (6) Transverse mixing coefficients for mid release.

تقدير معاملات الانتشار الطولي والعرضي في القناة المفتوحة

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الخلاصة

أجريت عدة تجارب مختبرية في قناة مفتوحة وقدمت نتائجها. استعملت منها خمسة وعشرون مجموعة معلومات (L1-L25) مقاسة للحصول على معامل الانتشار الطولي و ثمانية مجاميع معلومات (T1-T8) لتقدير معامل الانتشار العرضي في القناة . شملت الطريقة اشتقاق تعبير جديد لمعاملات الانتشار الطولي والعرضي التي تستعمل في استنباع تركيز الملوث لحل معادلة الانتشار–الانتقال (Advection-Dispersion Eq.) في النموذج الرياضي لنوعية المياه (Water Quality Model) . قورنت معاملات الانتشار الطولي والعرضي مع المعلومات المقاسة المختبرية والدراسات السابقة المشابهة وبينت تلك المقارنات على أن طريقة الاثور(Tracer Technique by using second moment) باستخدام العزم الثاني هي الأكثر دقة في تنبأ معاملات الانتشار الطولي والعرضي من الدراسات السابقة.

الكلمات الدالة

معامل الانتشار الطولي ، معامل الانتشار العرضي.