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Study of the Resonance in Series Piping Systems with Oscillating Valve

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Abstract

In This search, the phenomena of resonance in series piping systems with oscillating valve have been studied. The one-dimensional flow, unsteady state flow, and partial differential equations have been solved by using transfer matrix method.

The details of the transfer matrix method, the derivation of the field matrices and point matrices, and procedures for determining the natural frequencies and frequency response of piping system are then presented. A computer program (Resonance) was developed to calculate the pressure and discharge oscillations at the valve, the phase angle between the pressure head and relative gate opening of the valve, the phase angle between the discharge and the relative gate opening of the valve and also the pressure and discharge oscillations at certain sections of the pipes. To verify the transfer matrix method, the results have been compared with impedance method and agreement has been found.

Keywords:-Oscillating Valve, Series Piping, Resonance

دراسة الرنين لمنظومة انابيب مربوطة على التوالي بوجود صمام متذبذب

الخلاصة

في هذا البحث ، سيتم دراسة ظاهرة الرنين لمنظومات من ألأنابيب مربوطة على التوالي ، الجريان ذو البعد الواحد ،الجريان الغير مستقر، والمعادلات التفاضلية تم حلها باستخدام طريقة المصفوفة ألأنتقالية.

تفاصيل طريقة المصفوفة ألأنتقالية وأشتقاق طريقة مصفوفات الحقل وطريقة مصفوفات النقطة وأجرائات تحديد الترددات الطبيعية وأستجابة التردد تم تقديمها. برنامج الحاسوب الألكتروني (رنين) طور لحساب الضغط المتذبذب والتدفق المتذبذب ، وزاوية فرق الطور بين الضغط وفتحة بوابة الصمام وزاوية فرق الطور بين التدفق وفتحة بوابة الصمام وكذلك الضغط المتذبذب والتدفق المتذبذب في مقاطع محددة من ألأنابيب. للتحقق من استخدام طريقة المصفوفة الأنتقالية ، النتائج تم مقارنتها مع طريقة impedance ووجد توافق جيدا بينهما.

الكلمات الدالة:- صمام متذبذب ،أنابيب مربوطة على التوالي، رنين

Notation & Symbols

A: Cross Section area of the Pipe(m^2)
A_v: Area of the Valve Opening(m^2)
a: Wave Speed(m/Sec)
C_D: Coefficient of discharge
C_L: Lift Force Coefficient
D: Diameter of Pipe(m)
F: Field Transfer Matrix
f: Darcy-Weisbach Friction
g: Acceleration(m/Ses^2)
H: Pressure Head(m)
H_o: Average Pressure Head(m)
h^*: Pressure Head Deviation from the Mean(m)
h_r: Pressure Head Ratio

P'_{ov}: The Extended Point Matrix for an Oscillating Valve
Re: Real Part of the Complex Variables
Q: Discharge(m^3/Sec)
Q_o: Average Dischrage(m^3/Sec)
q^*: Discharge Head Deviation from the Mean(m^3/Sec)
q_r: Discharge Head Ratio
Superscripts R: Right of the Section
T: Time(Sec)
T_{th}: Therotical Period(Sec)
U: Overall Transfer Matrix
ω: Frequency(Rad/Sec)
ω_n: Natural Frequency of mass-spring(Rad/Sec)
ω_{th}: Theortical Frequency (natural frequency) (Rad/Sec)
x: Distance(m)
Z_i: State Vector
τ: Instantaneous relative Gate Opening
τ_o: The Mean relative Gate Opening
τ^*: Deviation of The Valve Motion
υ: Kinematic viscocity of the fluid(m^2/sec)
Φ_m:The Phase Angle Between The Pressure Head & The Relative Gate Opening(Degree)
φ_m:The Phase Angle Between The Discharge & The Relative Gate Opening(degree)

Introduction

Fluid oscillations in systems may be analyzed conveniently by use of procedures borrowed from linear vibration theory and electrical transmission-line theory. By use of the method forced oscillatory motions are effectively treated, and the resonating characteristics of systems may be identified [1].

The pipeline system conveying high pressurized unsteady internal flow may experience severe transient vibrations due to the fluid-pipe interaction under the time-varying conditions imposed by the pump and valve operations, a set of fully coupled dynamic equations of motion for the pipeline system are developed to include the effect of the circumferential strain due to the internal fluid pressure [2].

A finite element model and its equivalent electronic analogue circuit has been developed for fluid transients in hydraulic transmission lines with laminar frequency-dependent friction. Basic equations are approximated to be a set of ordinary differential equations that can be represented in state-space form [3].

Impedance methods have been used extensively in steady state vibration problems. The specific application of these concepts to resonance in fluid systems is accomplished [4].

Steady oscillatory flow in a branched piping system with partial blockages is studied by using the frequency response method. The peak pressure frequency diagrams at the downstream end are developed with the partial blockage at different locations in the system by using the transfer matrix method [5].

The design for a side discharge valve for generating a pseudorandom binary sequence of pressure changes that are of a small magnitude in relation to the steady state head of the pipeline [6].

A new procedure utilizing transient state pressures to detect leakage in piping systems. Transient flow, produced by opening or closing a valve, is analyzed in the time domain by the method of characteristics and the results are transformed into the frequency domain by the fast Fourier transform [7].

The aim of this work is to obtain a mathematical model to find out the pressure and discharge oscillations at the valve, the phase angle between the pressure head and relative gate opening, the phase angle between the discharge and the relative gate opening and also to find out the pressure and discharge oscillations at certain sections of the pipe .A transfer matrix method is used to analyze and calculate these variables.

Development of Resonating Conditions

From fundamental mechanics that the natural frequency ,ω_n , of the spring-mass system shown in s(.1a) is equal $\frac{1}{2\pi}\sqrt{K/M}$,in which ω_n=natural frequency of the system in rad/Sec ,M=mass(kg),and K=spring constant. If a sinusoidal force having frequency ω (fig.1b) is applied to the mass, initially a beat develops (transient state) and then the system starts to oscillate (fig.1c) at the forcing frequency ω and with a constant amplitude. These oscillations, having constant amplitude, are called steady vibrations. The amplitude of the vibrations depends upon the ratio $\omega_r = \omega/\omega_n$. If the forcing frequency ω is equal to the natural frequency ω_n and the system is frictionless, then the amplitude of steady vibrations becomes infinite. The reason for this is that the total energy of the system keeps on increasing with each cycle because no energy is dissipated in the system [8].

Now let us consider a pipeline having a reservoir at the upstream end and a valve at the downstream end (fig.2a). Let us assume that the valve is initially in a closed position but that we open and close it sinusoidally at frequency ω staring at time t=0 (fig.2b). Similar to our spring-mass system, a beat develops first (transient state), and then the flow and pressure oscillate at a constant amplitude but with frequency ω (fig.2c). Such a periodic flow is termed steady-oscillatory flow [8].

Let us compare the characteristics of the steady-oscillatory flow in our simple hydraulic system with the steady vibrations of the spring-mass system. The displacement of the spring at the fixed end in our spring –mass system is zero. Similarly, the water level in the upstream reservoir of the hydraulic system is constant. Therefore, the amplitude of pressure node at the reservoir .In the spring-mass system, there is only one mass and one spring; therefore, there is only one mode of vibrations or one degree of freedom, and hence the system has only one natural frequency (or natural period).

Let us now consider another significant difference between our spring-mass and hydraulic systems. In the former, the source of energy is the external periodic force acting on the mass. In the hydraulic system, although the valve is the forcing function, it is not the source of energy .The valve is just controlling the efflux of energy from the system, whereas the upstream reservoir is the source of energy.

Once a discharge node is formed at the valve, opening or closing of the valve has no effect on the energy efflux, and thus the amplitude of the pressure oscillations does not increase further even though it is assumed that there is no energy dissipation in the system [8].

Forced Oscillations

Steady-oscillatory flows in piping systems may be caused by a boundary that acts as a periodic forcing function. The system oscillates at the frequency of the forcing function during forced oscillations.

A periodically opening and closing valve is an example of a periodic variation of the relationship between the pressure and the flow [8].

Mathematical Model

The terminology is used in the mathematical model [8] is:

Instantaneous and Mean Discharge and Mean Head

In a steady-oscillatory flow, the instantaneous discharge, Q, and the instantaneous pressure head, H, see fig. (3) Can be divided into two parts:-

$$Q = Q_o + q^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$H = H_o + h^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In which Q_o=mean discharge (m^3/Sec); q^*=discharge deviation from the mean (m^3/Sec); Ho=mean pressure head (m); and h^*=pressure head deviation from the mean (m).

Both h^* and q^* are functions of time, t, and the distance, x. It is assumed that h^* and q^* are sinusoidal in time, which, in practice, is often true or a satisfactory approximation.

By using complex algebra

$$q^* = \mathrm{Re}\left(q(x)e^{j\omega t}\right) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$h^* = \mathrm{Re}\left(h(x)e^{j\omega t}\right) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$j = \sqrt{-1}$;h & q are complex variables

Theoretical Period

For a series piping system,

$$T_{th} = 4\sum_{i=1}^{m} \frac{Li}{ai} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

And

$$\omega_{th} = \frac{2\pi}{T_{th}} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In which T_{th}=theoretical period (Sec);
ω_{th} =theoretical frequency(rad/Sec)
n=number of pipes; and a=velocity of water hammer waves (m/Sec).
The subscript i denotes quantities for the i_{th} pipe.

State Vectors and Transfer Matrices

A general system (fig. (4)) whose input variable $x_1, x_2, \ldots . x_n$ and output variables $y_1, y_2, \ldots .., y_n$ are related by the following n simultaneous equations:-

$$y_1 = u_{11}x_1 + u_{12}x_2 + \ldots\ldots + u_{1n}x_n$$
$$y_2 = u_{21}x_1 + u_{22}x_2 + \ldots\ldots + u_{2n}x_n$$
……………………………………..
……………………………………..
$$y_n = u_{n1}x_1 + u_{n2}x_2 + \ldots\ldots + u_{nn}x_n$$
………………………………………(7)

In the matrix notation, these equations can be written as

$$\{y\} = [U]\{x\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

In which U=transfer matrix

The general system of fig.(4) has n input and output variables. In hydraulic systems, however, the quantities of interest at the section i of a pipeline are usually h and q, which can be combined in the matrix form as

$$z_i = \begin{Bmatrix} q \\ h \end{Bmatrix}_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The column vector z_i is called the state vector at section i. The state vectors just to the left and to right of a section are designated by the superscripts L and R , respectively. For example, z_i^L refers to the state vector just to the left of the i_{th} section (fig.(5)).

A matrix relating two state vectors is called a transfer matrix. The upper-case letters F,P,and U are used to designate the transfer matrices.

Transfer matrices are of three types:-

1-*Field Transfer Matrix or field Matrix (F):-*

A field –transfer relates the state vectors at two adjacent sections of a pipe. for example in fig.(5)

$$z_{i+1}^L = F_i z_i^R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

In which F_i=field matrix for the i_{th} pipe.

2-*Point Transfer Matrix or Point Matrix (P)*

The state vectors just to the left and to the right of a discontinuity, such as at a series junction see fig.(6) or at a valve ,are related by a point-transfer matrix.

$$z_{i+1}^R = P_{sc} z_{i+1}^L \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

In Which P_{sc}=point matrix for a series junction

3-*Overall Transfer Matrix (U):-*

The overall transfer matrix relates the state vector at one end of a system, or a side branch, to that at the other end. For example if n+1 is the last section, then

$$z_{n+1}^{L} = Uz_{1}^{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

In which, U=overall transfer matrix. This is obtained by an ordered multiplication of all the intermediate field and point matrices as follows:-

$$\left.\begin{aligned} z_2^L &= F_1 z_1^R \\ z_2^R &= P_2 z_2^L \\ z_3^L &= F_2 z_2^R \\ &\ldots\ldots\ldots\ldots \\ z_i^L &= F_{i-1} z_{i-1}^R \\ z_i^R &= P_i z_i^L \\ &\ldots\ldots\ldots\ldots \\ z_n^R &= P_n z_n^L \\ z_{n+1}^L &= F_n z_n^R \end{aligned}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Elimination of $z_2^L, z_2^R, \ldots\ldots\ldots, z_n^L$, and z_n^R from eq.(13) yields

$$z_{n+1}^{L} = (F_n P_n \ldots\ldots F_i P_i \ldots\ldots\ldots F_3 P_3 F_2 P_2 F_1) z_1^R \quad .(14)$$

Hence, it follows from Eqs. (12) and (14) that

$$U = F_n P_n \ldots\ldots\ldots F_i P_i \ldots\ldots\ldots F_3 P_3 F_2 P_2 F_1 \ldots(15)$$

Block Diagrams

A block diagram is a schematic representation of a system in which each component ,or a combination of components , of the system is represented by a "black box". The box representing a pipeline of constant cross-sectional area, wall thickness, and wall material is characterized by a field matrix, while that representing a discontinuity in the system geometry is represented by a point matrix [8].

The number of the section is written below the circle and the left- and right-hand sides of the section are designated by written the letter L and R above the circle. For example, in fig.(7),i and i+1 denote the number of the sections, and L and R denote the left-and right-hand sides of the section.

Derivation of Transfer Matrices

To determine the resonating characteristics of a piping system by the method of transfer matrix, it is necessary that the transfer matrices of the elements of the system be known. The point matrices for a series junction and for valves are developed [8].

Field Matrices

<u>Single Conduit</u>

The field matrix for a conduit having a constant cross-sectional area, constant wall thickness, and the same wall material is derived in this section. In the derivation, the system is considered to be distributed, and the friction-loss term is linearized.

The continuity and dynamic equations describing the flow through closed conduits.

1-continuity Equation

$$\frac{\partial Q}{\partial x} + \frac{gA}{Q^2}\frac{\partial H}{\partial t} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

2-Dynamic Equation

$$\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{fQ^n}{2gDA^n} \quad \ldots\ldots\ldots\ldots(17)$$

In which A=cross-section area of the pipeline (m^2); g=acceleration due to gravity(m/Sec^2);D=inside diameter of the pipeline(m);f=Darcy-Weisbach friction factor;n=exponent of velocity in the friction losses term ;x=distance along the pipeline(m),measured positive in the downstream direction(see fig.(5)); and t=time(Sec).

As the mean flow and pressure head are time-invariant and as the mean flow is constant along a pipeline, $\frac{\partial Q_o}{\partial x}, \frac{\partial Q_o}{\partial t}$, and $\frac{\partial H_o}{\partial t}$ are all equal to zero .Hence ,it follows from eqs.1 and 2 that

$$\left.\begin{array}{l} \frac{\partial Q}{\partial x} = \frac{\partial q^*}{\partial x}; \frac{\partial Q}{\partial t} = \frac{\partial q^*}{\partial t} \\ \frac{\partial H}{\partial t} = \frac{\partial h^*}{\partial t}; \frac{\partial H}{\partial x} = \frac{\partial H_o}{\partial x} + \frac{\partial h^*}{\partial x} \end{array}\right| \ldots\ldots.(18)$$

However, since it is considering the friction losses, $\frac{\partial H_o}{\partial x}$ is not equal to zero

For turbulent flow,

$$\frac{\partial H_o}{\partial x} = -\frac{fQ_o^n}{2gDA^n} \ldots\ldots\ldots\ldots\ldots\ldots..\ldots(19)$$

and for laminar flow, $\frac{\partial H_o}{\partial x} = -\frac{32\upsilon Q_o}{gAD^2}$ ……… ……(20)

In which υ=kinematic viscosity of the fluid (m^2/Sec)

if $q^* << Q_o$,then

$$Q^n = \left(Q_o + q^*\right)^n \approx Q_o^n + nQ_o^{n-1}q^* \ldots...(21)$$

In which higher-order terms are neglected

It follows from eqs.16 through 21

$$\frac{\partial q^*}{\partial x} + \frac{gA}{a^2}\frac{\partial h^*}{\partial t} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

$$\frac{\partial h^*}{\partial x} + \frac{1}{gA}\frac{\partial q^*}{\partial t} + Rq^* = 0 \ldots\ldots\ldots\ldots.(23)$$

In which $R = \left(nfQ_o^{n-1}\right)/\left(2gDA^n\right)$ for turbulent flow and $R = (32\upsilon)/\left(gAD^2\right)$ for laminar flow.

Elimination of h^* from eqs.22 and 23 yields

$$\frac{\partial^2 q^*}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 q^*}{\partial t^2} + \frac{gAR}{a^2}\frac{\partial q^*}{\partial t} \ldots\ldots\ldots..(24)$$

Now, if it is assumed that the variation of q^* is sinusoidal with respect to t, then on the basis of eq.3, eq.24 takes the form

$$\frac{d^2 q}{dx^2} = \left(-\frac{\omega^2}{a^2} + \frac{jgA\omega R}{a^2}\right)q \ldots\ldots\ldots(25)$$

Or

$$\frac{d^2 q}{dx^2} - \mu^2 q = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

In which

$$\mu^2 = -\frac{\omega^2}{a^2} + \frac{jgA\omega R}{a^2} \ldots\ldots\ldots\ldots\ldots..(27)$$

The solution of eq.26 is

$$q = C1\sinh\mu x + C2\mathrm{Cosh}\mu x \ldots\ldots..(28)$$

In which C1 and C2 are arbitrary constants.

If h^* is also assumed sinusoidal in t, then by substituting eqs.28 and 4 into eq.22 and solving for h, we obtain

$$h = -\frac{a^2\mu}{jgA\omega}\left(C1\mathrm{Cosh}\mu x + C2\mathrm{Sinh}\mu x\right)\ldots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(29)$$

The field matrix relating the state vectors at the i_{th} and at the $(i+1)_{th}$ section of the i_{th} pipe (see fig.(5)) of length L_i is to be derived. It is known that at the i_{th} section (at x=0), $h = h_i^R$ and $q = q_i^R$ .Hence, it follows from eqs.28 and 29 that

$$\left.\begin{array}{l} C1 = -\frac{jgA_i\omega}{a_i^2\mu_i}h_i^R \\ \text{and} \\ C2 = q_i^R \end{array}\right| \ldots\ldots\ldots\ldots\ldots\ldots..(30)$$

In addition, at the $(i+1)_{th}$ section (at $x=L_i$), $h=h^L_{i+1}$ and $q=q^L_{i+1}$.
The substitution of these values of h and q ,and C1 and C2 from Eq.30 into Eqs.28 and 29 yields

$$q^L_{i+1}=(\text{Cosh}\mu_iL_i)q^R_i-\frac{1}{Zc}(\text{Sinh}\,\mu_iL_i)h^R_i$$
……………………………………(31)

$$h^L_{i+1}=-Zc(\text{Sinh}\,\mu_iL_i)q^R_i+(\text{Cos}\mu_iL_i)h^R_i$$
……………………………………(32)

In Which $Zc=\left(\mu_ia_i^2\right)\big/\left(j\omega gA_i\right)$

Equations 31 and 32 can be expressed in the matrix notation as

$$\begin{Bmatrix}q\\h\end{Bmatrix}^L_{i+1}=\begin{bmatrix}\text{Cosh}\mu_iL_i & -\frac{1}{Zc}\text{Sinh}\,\mu_iL_i\\ -Zc\sinh\mu_iL_i & \text{Cosh}\mu_iL_i\end{bmatrix}\begin{Bmatrix}q\\h\end{Bmatrix}^R_i$$
…………………………………..........(33)

Or

$$Z^L_{i+1}=F_iZ^R_i$$ ……………………..….(34)

Hence ,field matrix for the i_{th} pipe is

$$F_i=\begin{bmatrix}\text{Cosh}\mu_iL_i & -\frac{1}{Zc}\text{Sinh}\,\mu_iL_i\\ -Zc\text{Sinh}\mu_iL_i & \text{Cosh}\mu_iL_i\end{bmatrix}$$
…………………..………………(35)

If friction is neglected, i.e.,$R_i=0$, the F_i becomes

$$F_i=\begin{bmatrix}\text{Cos}b_i\omega & -\frac{j}{C_i}\text{Sin}b_i\omega\\ -j\text{Sin}b_i\omega & \text{Cos}b_i\omega\end{bmatrix}$$ ……(36)

In which $b_i=L_i\big/a_i$ and $C_i=a_i\big/(gA_i)$ .Note that b_i and C_i are constants for a pipe and are not functions of ω.

Point Matrices

The point matrix is required in the calculation of the overall transfer matrix for the system, which is then used to determine the resonant frequencies and / or frequency response of the system.

Series Junction

A junction of two pipes having different diameters (see fig. (6)), wall thicknesses, wall materials, or any combination of these variables is called a series junction.

It follows from the continuity equation that

$$q^R_i=q^L_i$$ …………………………..…..(37)

In addition

$$h^R_i=h^L_i$$ ……………………...…….(38)

If the losses at the junction are neglected, these two equations can be expressed in the matrix notation as

$$z^R_i=P_{sc}z^L_i$$ ……………………..…..(39)

In which the point matrix for the series junction is

$$P_{sc}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$ …………………..…..(40)

Since P_{sc} is a unit matrix.

Oscillating Valve Discharge into Atmosphere

The oscillating valve is used to control the efflux of energy from the system and it is used in pressurized piping system [8].

The point matrix for a valve can be derived by linearizing the gate equation. The instantaneous and mean discharge through a valve (fig.(8)) are given by the equations

$$Q^L_{n+1}=C_dA_v\left(2gH^L_{n+1}\right)^{1/2}$$ ………..….(41)

$$Q_o=\left(C_dA_v\right)_o\left(2gH_o\right)^{1/2}$$ ………..…..(42)

Division of Eq.41 by Eq.42 yields

$$\frac{Q^L_{n+1}}{Q_o}=\frac{\tau}{\tau_o}\left(\frac{H^L_{n+1}}{H_o}\right)^{1/2}$$ ……………..(43)

In which the instantaneous relative gate opening $\tau = \frac{(C_d A_v)}{(C_d A_v)_s}$,and the mean relative gate opening $\tau_o = \frac{(C_d A_v)_o}{(C_d A_v)_s}$.The subscript s denotes steady-state reference, or index , values

The relative gate opening may be considered to be made up of two parts,i.e.

$\tau = \tau_o + \tau^*$ ……………………..…..(44)

Substitutions of Eqs.1,2 and 44 into eq.43 yields

$$\left(1+\frac{q_{n+1}^{*L}}{Q_o}\right) = \left(1+\frac{\tau^*}{\tau_o}\right)\left(1+\frac{h_{n+1}^{*L}}{H_o}\right)^{1/2} \quad ....(45)$$

If the valve motion is assumed sinusoidal, then

$\tau^* = \mathrm{Re}\left(ke^{j\omega t}\right)$……………..………(46)

In which k=amplitude of the valve motion

By expanding eq.45, neglecting terms of higher order (this is valid only if $\left|h_{n+1}^{*L}\right| << H_o$),and substituting eqs.3, 4, and 46 into the resulting equation, we obtain

$$h_{n+1}^{L} = \frac{2H_o}{Q_o} q_{n+1}^{L} - \frac{2H_o k}{\tau_o} \quad …..………(47)$$

Since $h_{n+1}^{R} = 0$,on the basis of eq.47 , we can write

$$h_{n+1}^{R} = h_{n+1}^{L} + \frac{2H_o}{\tau_o} - \frac{2H_o}{Q_o} q_{n+1}^{L} \quad …..…(48)$$

In addition, from the continuity equation it follows that

$q_{n+1}^{R} = q_{n+1}^{L}$…………………………(49)

Equations 48 and 49 may be expressed in the matrix notation as

$$\begin{Bmatrix} q \\ h \end{Bmatrix}_{n+1}^{R} = \begin{bmatrix} 1 & 0 \\ \frac{-2H_o}{Q_o} & 1 \end{bmatrix} \begin{Bmatrix} q \\ h \end{Bmatrix}_{n+1}^{L} + \begin{Bmatrix} 0 \\ \frac{2H_o k}{\tau_o} \end{Bmatrix}$$

……………………………...……...(50)

The two matrix terms on the right-hand side may be combined as follows:

$$\begin{Bmatrix} q \\ h \\ 1 \end{Bmatrix}_{n+1}^{R} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-2H_o}{Q_o} & 1 & \frac{2H_o k}{\tau_o} \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} q \\ h \\ 1 \end{Bmatrix}_{n+1}^{L} \quad \ldots$$

……………………………..……...(51)

To combine the matrix terms in some cases the state vector is defined as

$$z_i^{/} = \begin{Bmatrix} q \\ h \\ 1 \end{Bmatrix}_i \quad ……………………..…...(52)$$

Because of the additional element with unit value, the column vector $z_i^{/}$ is called the extended state vector

On the basis of Eq.52, Eq.51 may be written as

$z_{n+1}^{/R} = P_{ov}^{/} z_{n+1}^{/L}$……………………..….(53)

In which $P_{ov}^{/}$ =the extended point matrix for an oscillating valve and is given by

$$P_{ov}^{/} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-2H_o}{Q_o} & 1 & \frac{2H_o k}{\tau_o} \\ 0 & 0 & 1 \end{bmatrix} \quad ………...(54)$$

Let $U^{/}$ be the extended overall transfer matrix relating the state vectors at the 1st and the (n+1) the section of the system,i.e.,

$z_{n+1}^{/L} = U^{/} z_1^{/R}$ …………………….....(55)

In addition

$z_{n+1}^{/R} = P_{ov}^{/} z_{n+1}^{/L}$……………………..….(56)

Hence

$z_{n+1}^{/R} = P_{ov}^{/} U^{/} z_1^{/R}$ ………………….…..(57)

Boundary Conditions

From fig(8)

At the 1st (constant head reservoir) $h_1^R = 0$

At the (n+1) section $h_{n+1}^{R} = 0$ & $q_{n+1}^{L} = q_{n+1}^{R}$

By substituting P'_{ov} from eq.54; multiplying the matrices P'_{ov} and U' ; and by using boundary conditions

$$q_1^R = -\frac{u_{23} - \frac{2H_o}{Q_o}u_{13} + \frac{2H_o k}{\tau_o}u_{33}}{u_{21} - \frac{2H_o}{Q_o}u_{11} + \frac{2H_o k}{\tau_o}u_{13}} \quad \text{....(58)}$$

$$q_{n+1}^L = u_{11}q_1^R + u_{13} \quad \text{..........................(59)}$$

In which $u_{11}, u_{12}, \ldots, u_{33}$ are the elements of the matrix, U'.By expanding eq.55 and noting that $h_1^R = 0$, it obtain

$$h_{n+1}^L = u_{21}q_1^R + u_{23} \quad \text{..........................(60)}$$

To determine the frequency response, the extended field and point matrices are first computed. Then, the extended overall transfer matrix is determined by multiplying the field and point matrices starting at the downstream end,i.e;

$$U' = F'_n P'_n \text{.........} P'_2 F'_1 \quad \text{.....................(61)}$$

The value of q_1^R is determined from eq.58, and q_{n+1}^L and h_{n+1}^L are computed from eqs.59 and 60.The absolute values of h_{n+1}^L and q_{n+1}^L are the amplitudes of pressure head and discharge fluctuations at the valve, and their arguments are, respectively. The phase angles between head and τ^* and between discharge and τ^*.

If there is no other forcing function except the oscillating valve at the downstream end of the system ,ordering field and point matrices may be used instead of the extended ones. In this case, $u_{13}=u_{23}=u_{31}=0$ and $u_{33}=1$ in eqs.58 through 60 [8].

The phase angle between the pressure head and the relative gate opening is

$$\phi_m = \tan^{-1}\left[\frac{\mathrm{Im}(h_{n+1}^L)}{\mathrm{Re}(h_{n+1}^L)}\right] \quad \text{................(62)}$$

The phase angle between the discharge and the relative gate opening is

$$\varphi_m = \tan^{-1}\left[\frac{\mathrm{Im}(q_{n+1}^L)}{\mathrm{Re}(q_{n+1}^L)}\right] \quad \text{................(63)}$$

The pressure head ration at the valve is

$$h_r = \frac{2\left|h_{n+1}^L\right|}{H_o} \quad \text{..............................(64)}$$

The discharge ratio at the valve is

$$q_r = \frac{2\left|q_{n+1}^L\right|}{Q_o} \quad \text{..............................(65)}$$

Procedure for Determining the frequency Response

The frequency response of piping systems may be determined as follows:

1-Draw the block diagram and then the simplified block diagram for the system.

2-calculate the overall transfer matrix by an ordered multiplication of the point and field matrices, starting at the downstream end. For this calculation, the block diagram of step 1 is very helpful.

3-Use the expression developed in this section to determine the frequency response.

4-If a frequency-response diagram is to be plotted, repeat step 2 and 3 by taking different frequencies.

Pressure and Discharge Variation along a Pipeline

It is necessary to determine the amplitudes of the discharge and pressure fluctuations along the length of the pipeline.

Suppose that the amplitudes of the discharge and pressure oscillations at the k_{th} section on the i_{th} pipe (fig.9a) are to be determined. Let the transfer matrix relating the state vectors at the first section of the first pipe and the first section of the i_{th} pipe be designated by W[8] ,i.e.,

$$\left(z_1^R\right)_i = w\left(z_1^R\right)_1 \text{ ....(66)}$$

And the field matrix relating the state vectors at the first and the k_{th} section of the i_{th} pipe by F_x ,i.e.,

$$\left(z_k^L\right)_i = F_x\left(z_1^R\right)_i \text{ ....(67)}$$

In these equations, the subscript within the parentheses refers to the pipe number. The matrix W is computed by multiplying the point and field matrices for the first (i-1) pipes (see fig.9b),i.e.,

$$W = P_i F_{i-1} P_{i-1} \text{..........} F_1 \text{ ....(68)}$$

And the matrix F_x is calculated by replacing L with x in eq.35.Note that the elements of the matrix W for a specified frequency are constants, while those of the matrix F_x depend upon the value of x as well.

It follows from eqs.66 and 67 that

$$\left(z_k^L\right)_i = S\left(z_1^R\right)_1 \text{ .... (69)}$$

In which

$$S = F_x W = F_x P_i F_{i-1} P_{i-1} \text{........} F_1 \text{ .... (70)}$$

The value of $\left(q_1^R\right)_1$ is calculated from eq.58.furthermore, it is known that $\left(h_1^R\right)_1 = 0$ substitutions of these values into the expanded form of eq.69 yields

$$\left(q_k^L\right)_i = s_{11}\left(q_1^R\right)_1 \text{ ....(72)}$$

And

$$\left(h_k^L\right)_i = s_{21}\left(q_1^R\right)_1 \text{ ....(73)}$$

The amplitude of the discharge and pressure fluctuations at any other section can be determined by proceeding in a similar manner.

Case Study

The details of the two series piping systems investigated are shown in fig.(10a), the friction is neglected in this case(R=0) and the valve motion is assumed sinusoidal[4] and the block diagram of this case is illustrated in fig.(10b).

Computer Program

By using (Q-Basic) language a computer program which is named (Resonance) is developed to solve numerical one-dimensional partial differential equations, unsteady state using transfer matrix method. The flow chart of the computer program is shown in fig.(11).

Results & Discussions

In this section, the numerical results are obtained by using transfer matrix methods .

The response spectrums are represented in a non dimensional form. The frequency ratio ω_r , is defined as ω/ω_{th}.

Fig.(12a) show the relation between the frequency response ratio (ω_r) and head ratio (h_r) and the results are compared with impedance method.

In the impedance method the pressure head ratio at the valve is

$$h_r = \frac{8Q_o\delta|X_s|}{\sqrt{4H_o^2 + (Q_oX_s)^2}} \text{ ....(74)}$$

In which δ =amplitude of oscillatory motion of valve; and X_s=real number denoting the imaginary part of the complex impedance.

And more details on impedance method it can be return to [4].

The agreement between the transfer matrix method and impedance method is reasonable

fig.(12b) show the relation between the frequency response ratio (ω_r) and discharge ratio (q_r).

The values of h_r and q_r determined by the transfer matrix method represent the amplitude of the swing from the minimum to maximum value. The frequency of the forcing function is designated by ω.

The oscillating valve is the forcing functions and the valve movement is taken as sinusoidal with $\tau_{o=}1$ and k=0.5.

The pressure head fluctuation does not continue to grow during the forced valve movement. The answer lies in the fact that although the excitation remains, its effect becomes very small since the discharge is reduced to a value near zero in the resonating conditions as shown in fig.(12a) that demonstrates that the maximum value of hr=2 at ω_r=3 and fig.(12b) that demonstrates that the minimum value of q_r=0 at ω_r=3. Fig.(13) shows the variation in the pressure head at section A & B and fig.(14) shows the variation in the discharge at section A & B.

The phase angle between the pressure head and the relative gate opening and the phase angle between the discharge and the relative gate opening are presented in table (1).

Conclusions

It is concluded from this paper that:-

1-The transfer matrix method was verified by comparing its results with impedance methods.

2-The frequency response diagram gives a useful aid in evaluating the frequency response of the system.

3- The pressure head fluctuation does not continue to grow during the forced valve movement as shown in fig. (12a).

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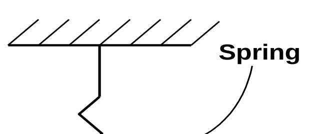
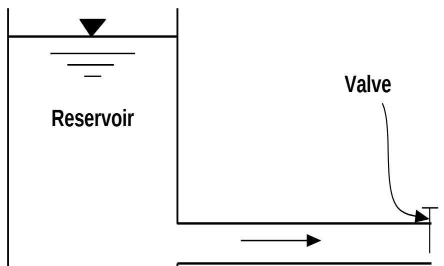
Table (1) Phase Angles (degree)

ω_r	Φ_m	φ_m
0.2	-93.17641	-3.183044
0.4	-96.73576	-6.742381
0.6	-101.2443	-11.25096
0.8	-107.9189	-17.92554
1	-120.2022	-30.20885
1.2	-150.2127	-60.21932
1.4	152.3755	62.38211
1.6	120.0508	30.05743
1.8	106.8753	16.88193
2	99.60407	9.610703
2.2	94.35714	4.363767
2.4	-90.46196	-0.4685906
2.6	-96.43787	-6.44449
2.8	-108.5118	-18.5184
3	-178.8325	-88.8391
3.2	108.8317	18.83833
3.4	96.55218	6.558803
3.6	90.53909	0.5457221
3.8	-94.28376	-4.290385
4	-99.51443	-9.521064
4.2	-106.7362	-16.74278
4.4	-119.7625	-29.76917
4.6	-151.6133	-61.61988
4.8	150.937	60.94361
5	120.474	30.48068
5.2	108.0496	18.05618
5.4	101.3251	11.33175
5.6	96.79534	6.801967
5.8	93.22669	3.233319
6	90.04095	4.757981E-2

Φ_m= The Phase Angle Between The Pressure Head & The Relative Gate Opening(Degree)

φ_m= The Phase Angle Between The Discharge & The Relative Gate Opening(Degree)





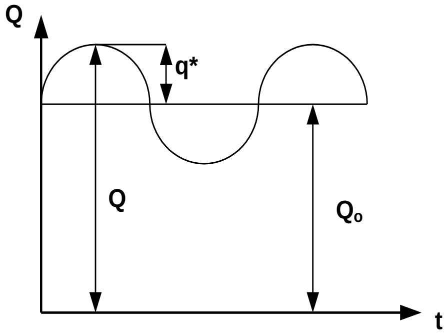


Figure (3).Instantaneous ,Mean ,and Oscillatory Discharge

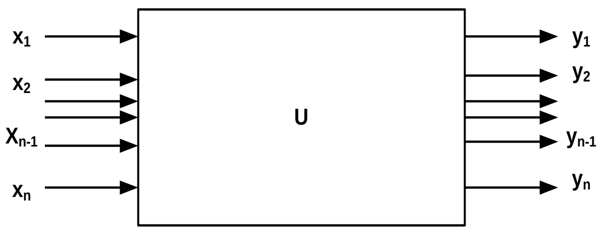


Figure (4).Block Diagram for One-Component System

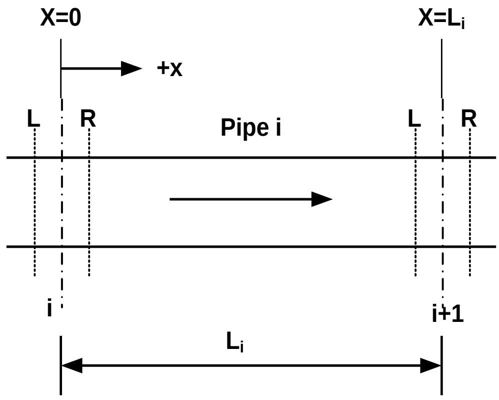


Figure (5).Single Pipeline

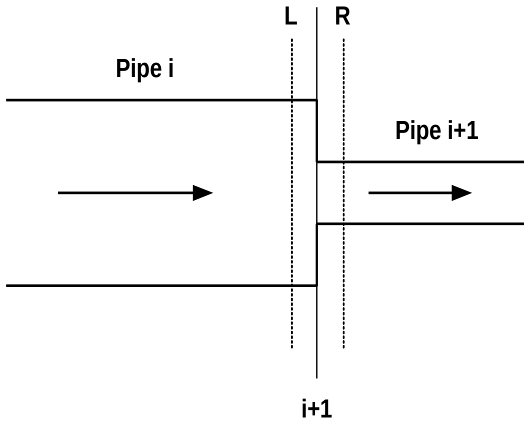


Figure (6).Series Junction

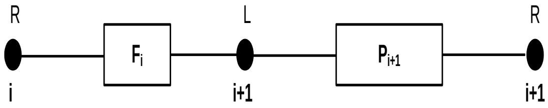


Figure (7).Block Diagram

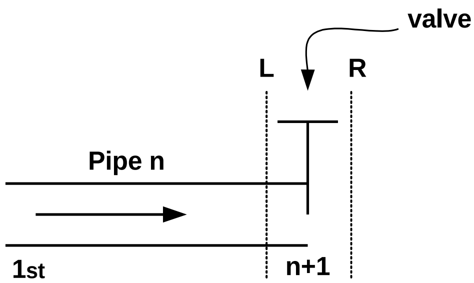
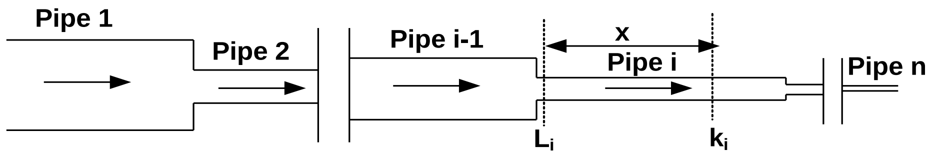
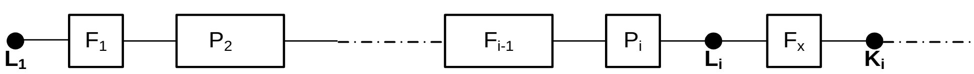


Figure (8).Valve at Downstream End of Pipeline



(a).Piping System



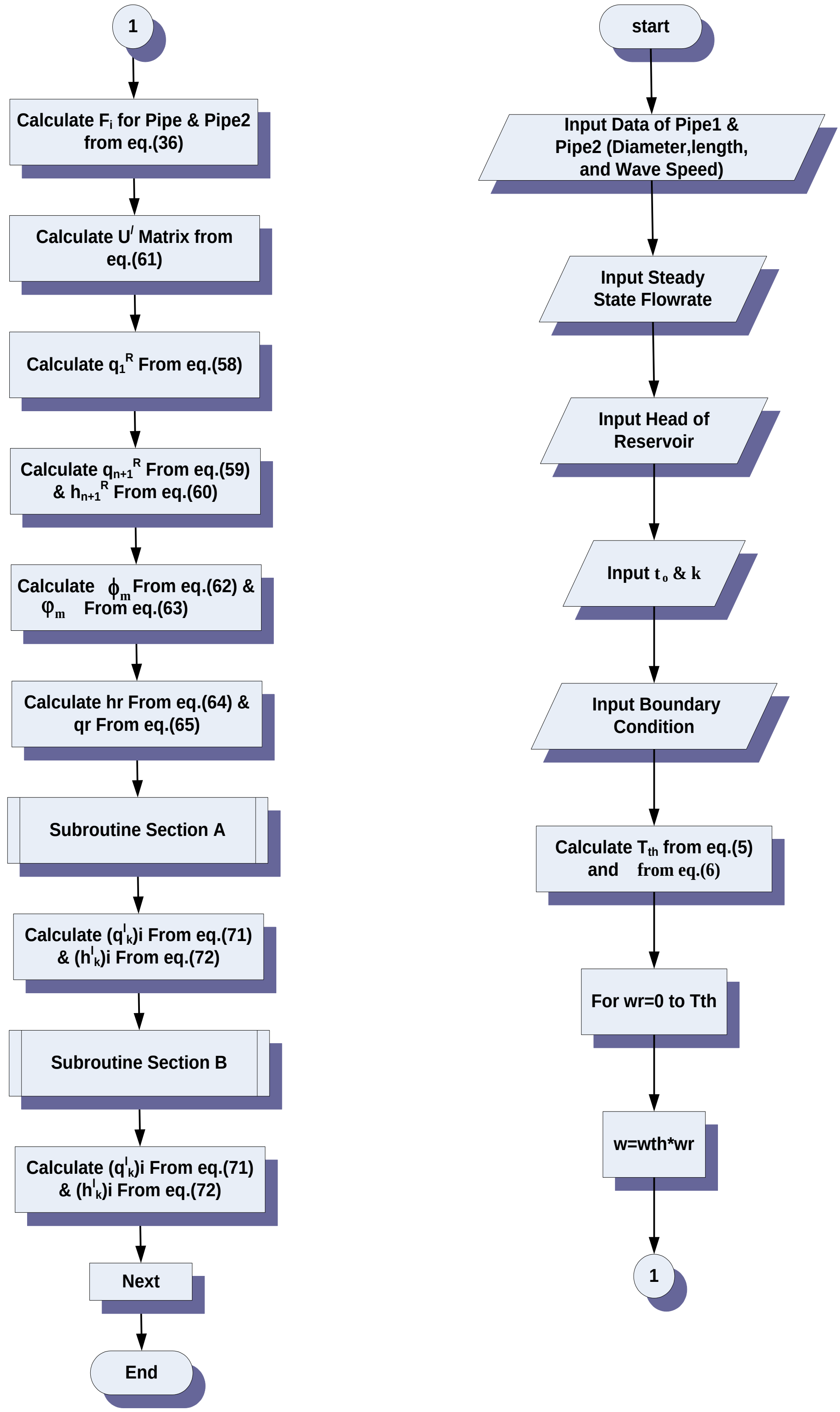


Fig.(11) Flow Chart of Computer Program

Resona