The Plasma Radiation Power Losses in Toroidal Fusion **Reactors** خسائر القدرة الإشعاعية للبلازما فى مفاعلات الاندماج الحلقية

Assistant Lecturer Ali Abdul Ameer Rashead Kerbala University/ Collage of Science/ Department of Physics

Abstract :

In this work we calculate the radiation power losses as a function of temperature (T) and as a function of density (n_e) . The radiation losses strongly influence the fusion power density and attainability of ignition. Radiation power loss include recombination radiation (P_R) , bremsstrahlung radiation (P_B), line radiation (P_L), and cyclotron radiation (P_C) for plasma in toroidal fusion reactors. We do these calculation because the radiation power losses decrease the net power we get from fusion reactors and losing the plasma particles so we must determine these losses to understanding how can we reduced these losses to increase the net fusion reactors power through confining the plasma particles as long as possible .

الخلاصة: في هذا العمل نقوم بحساب خسائر القدرة الإشعاعية كدالة لدرجة الحرارة (T) وكدالة للكثافة (ne)، الخسائر الإشعاعية تؤثر بقوة على كثافة القدرة من الاندماج وعلى إمكانية تحقيق الاتقاد. خسائر القدرة الإشعاعية تضم إشعاع إعادة الالتحام (P_A)، إشعاع الكبح (التباطؤ) (P_B)، الإشعاع بخط (P_L)، والإشعاع السايكلتروني (P_C)، في مفاعلات الاندماج الحلقية. نقوم بحسابات هذه الخسائر الإشعاعية لكونها تقلل من صافي القدرة المستحصلة من مفاعلات الاندماج وتؤدي الى فقدان جسيمات البلازما لذلك وجب تحديد هذه الخسائر لكي نفهم كيف يمكن تقليلها لزيادة صافي القدرة من مفاعلات الاندماج من خلال احتواء جسيمات البلازما لأطول فترة ممكنة.

Introduction:

Tokamak is a toroidal plasma confinement system the plasma being confined by a magnetic field: The principal magnetic field in a tokamak is the toroidal field however this field alone does not allow confinement of the plasma in order to have an equilibrium in which the plasma pressure is balanced by magnetic forces its necessary also to have poloidal magnetic field. In tokamak this field is produced mainly by currents in the plasma itself. A tokamak reactor would be considerably more complex than a non-thermonuclear reactor, the toroidal field coil are envisaged to be superconducting in order to avoid the vary large energy loss from ohmic heating in normal conductors Fig. 1. The blanket surrounding the plasma has two role: firstly it absorbs the 14MeV neutrons transforming their energy into heat which is then carried away by suitable coolant, secondly the blanket allows the breeding of tritium needed to fuel the reactor.

A plasma is an ionized gas composed of ions and electrons. These two components are strongly coupled because any substantial separation of charges within the plasma leads to a very large restoring force. Such separation can therefore only occur over short lengths. Small charge separations do arise as results of thermal fluctuations. In fusion reactor the ion orbits typically have a radius of a few millimeters and the electron orbits are smaller by the square root of the mass ratio. Although the precise behavior of the plasma is determined by the motion of the individual particles in the local electromagnetic field the constraints on the particle motions described above give the plasma fluid-like properties on lengths larger than the Larmor radii. Much of our understanding of fusion reactor is based on such fluid models of the plasma. The particle concentration in to fusion

reactor is typically $\sim 10^{20} m^{-3}$ which is about 10^{-5} of the particle concentration in the atmosphere. fusion reactor typically reach temperatures of several *keV* which corresponds to tens of millions of degrees Kelvin.

This is of order 10^5 times atmospheric temperatures and consequently the pressure in fusion reactor plasmas is comparable to that of the atmosphere. The outward force of the plasma pressure is balanced by the magnetic field. However the plasma energy density in a fusion reactor is small compare to that of magnetic field, typically $\sim 1\%$. The basic magnetic field is the toroidal field produced by coil outside the plasma. The poloidal field produced by the toroidal current is typically ten times smaller. Many processes in the plasma are determined by particle collisions. Collisions between ions and electrons give rise to an electrical resistance which leads to ohmic heating of the plasma. The shape and intensity of a continuous radiation spectrum from a plasma is determined by the interplay of the following processes: [bremsstrahlung produced by the interaction of an electron with ions, bremsstrahlung produced as a result of electron – electron interaction, recombination radiation produced by radiative capture of an electron by an atom, line radiation produced by collisional excitation of an orbital electron in a partially ionized impurity atom by a plasma electron causes the plasma electron to lose an amount of energy (ΔE) equal to the difference in energy levels of the ground and excited states].

The reaction rate for each of these processes is a function of the temperature and density however the relative role of each of the processes can vary for different spectral regions and for different electron temperatures. The variation of the coefficients and the complicated nature of the dependence of the radiation intensity on electron temperature means that under certain conditions the intensity of the radiation in some spectral range can be a rather weak function of the temperature on other hand the absolute intensity of that portion of the spectrum can with a high degree of accuracy be used to determine the density provided even relatively limited information is available on the temperature.

A pure hydrogen plasma emits electromagnetic radiation. Microscopically this is due to the acceleration of the charged particles. Because of their lighter mass the electrons undergo larger acceleration than the ions. Consequently they radiate much more strongly and only the electrons need be considered. The electrons are accelerated in two ways: firstly they are accelerated by collisions the resulting radiation being called "Bremsstrahlung" (P_B). Secondly they are subject to the acceleration of their cyclotron motion and the associated radiation is called "cyclotron" radiation (P_c). There are also further losses due to the atomic processes of line radiation (P_L) and recombination radiation (P_R), and in this work we will calculate these losses The results arranged in tables (1,2,3,4,5,6).

Theory:

<u>Radiation processes</u>: the fusion power density is primarily a function of ion temperature (T_i) is strongly coupled to (T_e) by coulomb collisions and (T_e) is limited by radiative power losses. Therefore radiation losses strongly influence the fusion power density and attainability of ignition. Radiation power loss processes include radiative recombination (P_R) , bremsstrahlung radiation (P_B) , line radiation (P_L) , and cyclotron radiation (P_C) . The total radiation power losses per unit volume is [1,2]:

$$P_{rad}(W/m^3) = P_R + P_L + P_B + k_C P_C....(1)$$

 (k_c) is the fraction of cyclotron radiation which is absorbed in the walls and not reabsorbed in the plasma. Reabsorbed of the other types of radiation is negligible.

Radiative recombination involving capture of a free electron by an ion with the binding energy subsequently released by emission of radiation as the resultant atom or ion de-excites to its ground

state. Whenever a charged particle is accelerated it will emit some radiation. For a given force electrons will have large accelerations because of their light mass and ions will have small accelerations. Therefore radiation by acceleration of ions is generally negligible.

Electrons are accelerated by electric and magnetic fields. The $(q\vec{v} \times \vec{B})$ force produces Larmor rotation around the magnetic field lines and consequent emission of cyclotron radiation (P_C) . The coulomb force $(q\vec{E})$ from ions and other electrons during coulomb collisions produces deflections and consequent emission of bremsstrahlung radiation (braking radiation) (P_B) .

Approach to coronal equilibrium: Let (n_{kj}) represent the concentration of atoms or ions of species k and ionization state j. Let (I_{kj}) represent the rate coefficient for ionization of (n_{kj}) and (R_{kj}) be the rate coefficient for recombination of (n_{kj}) with an electron. The rate of change of (n_{kj}) is given by [2]:

$$\frac{dn_{kj}}{dt} = n_e \Big[n_{k,j+1} R_{k,j+1} + n_{k,j-1} I_{k,j-1} - n_{kj} (R_{kj} + I_{kj}) \Big] \dots \dots \dots (2)$$

The recombination coefficients are functions only of (T_e) . For temperatures of a few (keV) or less electron impact ionization is dominant (fig. 1) and the ionization rates also are functions only of (T_e) . We will write particle balance equations to compute the population of each ionization state from which the radiative power losses can be calculated. Consider the case of atoms of a single impurity species incident on a plasma with constant (T_e) . Assuming that the atoms are initially neutral the initial conditions are $(n_{kj} = 0)$ for all except (j=0). For example the equations for helium are:

neutral
$$He: \frac{dn_{20}}{dt} = n_e (n_{21}R_{21} - n_{20}I_{20}), \quad n_{20}(0) = n_0$$

 $He^+: \frac{dn_{21}}{dt} = n_e [n_{22}R_{22} + n_{20}I_{20} - n_{21}(R_{21} + I_{21})], \quad n_{21}(0) = 0$
 $He^{++}: \frac{dn_{22}}{dt} = n_e (n_{21}I_{21} - n_{22}R_{22}), \quad n_{22}(0) = 0.....(3)$

(there may also be a source of (n_{22}) from fusion reactions.) this set of coupled ordinary differential equations may be solved by use of Laplace transformations or by integration on a digital computer. If we assume n_e = constant and divide by (n_e) then the left side of the equations may be written:

$$\frac{1}{n_e}\frac{dn_{kj}}{dt} = \frac{dn_{kj}}{d(n_e t)}\dots\dots(4)$$

And the solution becomes a function of the new variable $(n_e t)$. After a long time coronal equilibrium is established and the time derivatives go to zero. If the temperature is high enough most of the low Z ions will be completely stripped at equilibrium.

Each of radiation power terms in (eq. 1) except (P_C) is proportional to the electron density and to the density of impurity atoms ($n_k = n_{k0} + n_{k1} + \dots + n_{kk}$) so we can define a **radiation power parameter** for species k [2]:

$$Q_k(Wm^3) = (P_R + P_L + P_B)_k / n_e n_k$$
.....(5)

Which varies with time during approach to equilibrium and is also a function of (T_e) . Equilibrium is attained when $(n_e t \sim 10^{18} m^{-3} s)$ and the power radiated during approach to equilibrium is about an order of magnitude higher than the equilibrium value.

<u>Coronal equilibrium case</u>: at coronal equilibrium the relative contributions of the four terms in (eq. 5) vary with (T_e) . at low temperatures line radiation is the dominant process. As fewer bound electrons remain at higher temperatures recombination and bremsstrahlung become significant and the total radiated power decreases. At very high temperatures the radiation power parameter is dominated by bremsstrahlung and increases as $(T_e^{1/2})$.

The total radiation power lost from the plasma is found by summing (eq.5) over impurity species and combining with (eq.1) [2,3]:

$$P_{rad}(W/m^3) = \sum_{k} n_e n_k Q_k + k_C P_C \dots (6)$$

For a given impurity species the average values of ionic charge and charge squared are given by:

$$\left\langle Z \right\rangle_{k} = \frac{\sum_{j} n_{kj} Z_{kj}}{\sum_{j} n_{kj}}$$
$$\left\langle Z^{2} \right\rangle_{k} = \frac{\sum_{j} n_{kj} Z_{kj}^{2}}{\sum_{j} n_{kj}}.....(7)$$

Where $(Z_{kj}=j)$. As (T_e) increase more electrons become stripped off the ion and $\langle Z \rangle_k \rightarrow Z$ the atomic number of the nucleus. The total density of heavy particles in the plasma is :

According to the principle of "quasi neutrality" the electron density equals the density of positive charges per unit volume:

$$n_e = \sum_k n_k \langle Z \rangle_k \dots \dots \dots \dots (9)$$

We can define an effective value of Z for the plasma as a whole by :

 (Z_{eff}) is useful in estimating plasma resistivity and bremsstrahlung losses.

Bremsstrahlung: The power radiated by an electron undergoing an acceleration (a) is:

$$P = \frac{e^2}{6\pi\varepsilon_0 C^3} a^2 \dots (11)$$

The acceleration of an electron during a collision with an ion is caused by the coulomb force $(Ze^2/4\pi\varepsilon_0r^2)$ where (r) is their separation and (Ze) the charge on the ion. Because of the resulting (r^{-4}) dependence of (P) close collisions dominate. There is a cut-off for r at the quantum mechanical

distance $(d \approx \hbar/mv)$ so that the effective cross-section $(\sigma \approx \pi d^2)$ and the duration of collision is typically $(2d/\bar{v})$ where \bar{v} is an average electron velocity. Thus the energy lost per collision is [3]: $\delta E = 2Pd / \bar{v}$

Since the effective collision frequency is $(n_Z \sigma \overline{v})$ where (n_Z) is the density of charge Z ions, the bremsstrahlung power for an electron density (n_e) is [3]:

$$P_B \approx n_e n_Z \sigma \bar{\nu} \delta E....(13)$$

Taking $(3/2T_e = 1/2m_e \bar{v}^2)$ and substituting the numerical factor obtained from a full calculation then gives the radiation due to ions of charge Z as [3]:

Where g is the gaunt factor which gives a quantum mechanical correction. Under the conditions of interest $(g \approx 2\sqrt{3} / \pi)$, $(Z_{eff} = Z^2 n_z / n_e)$, we can calculate the constants and unified the unites to get:

$$P_B(MWm^{-3}) = 5.35 \times 10^{-44} Z^2 n_e n_Z T_e^{1/2}$$

or

Where (T_e) in keV. Bremsstrahlung Density power losses will be:

$$W_B(MW) = P_B.V.\dots(16)$$

Where V- plasma volume inside reactor .

<u>Cyclotron radiation</u>: The theory of cyclotron radiation losses in quite complex involving emission and absorption at harmonics of the cyclotron frequency in spatially varying plasma and magnetic field. However the basic elements can be simply described. The power radiation from a single non-relativistic electron is given by eq. 11 with the acceleration of cyclotron orbit ($a = \omega_{ce} \rho_e$). Putting $[\rho_e = (2T_e/m_e)^{1/2}/\omega_{ce}]$ would then give a radiation power density [3]:

$$P_{C} = (e^{4} / 3\pi\varepsilon_{0}m_{e}^{3}C^{3})B^{2}n_{e}T_{e}....(17)$$

Where (ε_0) is the permittivity of free space and (C) is the speed of light. Expressing (T_e) in keV we can calculate the constants and unified the unites (eq.17) become:

$$P_C(MWm^{-3}) \cong 6.21 \times 10^{-23} n_e T_e B^2 \dots (18)$$

Cyclotron Density power losses will be:

$$W_C(MW) = P_C.V....(19)$$

The magnetic field is reduced inside the plasma by plasma current. The resultant magnetic field is a function of the parameter:

$$\beta = \frac{plasma \ pressure}{vacuum \ magnetic \ field \ pressure} = \frac{p}{B^2 / 2\mu_0} \dots (20)$$

Where (B) is the magnetic induction without plasma.

Line radiation: collisional excitation of an orbital electron in a partially ionized impurity atom by a plasma electron causes the plasma electron to lose an amount of energy (ΔE) equal to the difference in energy levels of the ground and excited states. Normally the excited state will immediately decay to the ground state accompanied by the emission of a photon with a discrete frequency (v) where (hv=E). the power loss due to this "line radiation " can be approximated by [4]:

Line Density power losses will be:

 $W_L(MW) = P_L.V.\dots(22)$

Recombination radiation: Free plasma electrons will recombine with partially ionized impurity atoms to form an ion in a ground state lower by 1. the energy loss to the plasma is approximately the plasma electron thermal energy which is emitted in the form of a photon. This "recombination " radiation power loss can be approximated by:

Recombination Density power losses will be:

$$W_R(MW) = P_R.V.\dots(24)$$

Calculations and results:

From eqs. (15,19,20) the calculation of radiation power losses due to (Z_{eff}) was arranged in table (1), for toroidal fusion reactors (Tokamak), (Z_{eff} =1-10) ref. [5].

Table (1): Calculated Radiation	Power Losses which dependent	ndent on (Z_{eff}) at $(T_e=10 keV)$
	$(n_e=10^{20}m^{-3}), (Z_{imp}=1)^{-1}$	

Z _{eff}	$P_B (MWm^{-3})$	$P_L (MWm^{-3})$	$P_R (MWm^{-3})$
1	0.016918	0.0000569	0.00000013
2	0.033836	0.0001138	0.00000026
3	0.050754	0.0001707	0.00000039
4	0.067672	0.0002276	0.00000052
5	0.084590	0.0002845	0.00000065
6	0.101508	0.0003414	0.00000078
7	0.118426	0.0003983	0.00000091
8	0.135344	0.0004552	0.00000104
9	0.152262	0.0005121	0.00000117
10	0.169180	0.0005690	0.00000130

From eq. (17) the calculation of cyclotron radiation power losses due to the magnetic induction (B), the results was arranged in table (2), for toroidal fusion reactors (Tokamak), (B=1-10 Tasla) ref. [6].

B (Tasla)	P_C (MWm ⁻³)
1	0.0621
2	0.1242
3	0.1863
4	0.2484
5	0.3105
6	0.3726
7	0.4347
8	0.4968
9	0.5589
10	0.6210

Table (2): Calculated Cyclotron Radiation Loss At $(T_e=10 keV), (n_e=10^{20} m^{-3})$

From eq.(19,20) the calculation of radiation power losses due to (Z_{imp}) , the results was arranged in table (3), for toroidal fusion reactors (Tokamak) ($Z_{imp} = 1-10$) ref. [6].

Table (3): Calculated Radiation Power Losses which dependent on (Z_{imp}) at $(T_e=10keV)$, $(n_e=10^{20}m^{-3})$, $(Z_{eff}=1)$

Z_{imp}	$P_L (MWm^{-3})$	$P_R (MWm^{-3})$
1	0.0000569	0.00000013
2	0.0002276	0.00000208
3	0.0005121	0.00001053
4	0.0009104	0.00003328
5	0.0014225	0.00008125
6	0.0020484	0.00016848
7	0.0027881	0.00031213
8	0.0036416	0.00053248
9	0.0046089	0.00085293
10	0.0056900	0.00130000

We study the effect of density increasing on radiation power losses and we arranged results in tables(4,5,6) according to density values, the range of effective fusion plasma densities in toroidal fusion reactors (Tokamak) are between $(10^{19}-10^{21} \text{ m}^{-3})$.

From Eqs. (5,15,17,19,20) we calculate the radiation power losses and the radiation power parameter due to temperatures increasing (fusion reactors temperatures T=1keV-100keV) at density ($n_e = 10^{19} m^{-3}$), the results was arranged in table (4), For density ($n_e = 10^{20} m^{-3}$) the results was arranged in table (5), For density ($n_e = 10^{21} m^{-3}$) the results was arranged in table (6).

$n_{e}=10^{19} m^{-3}$							
T	P_B	P_{C}	P_L	P_R	P _{Total}		
(KeV)	(MWm^{-3})	(MWm^{-3})	(MWm^{-3})	(MWm^{-3})	(MWm^{-3})		
1	5.35×10^{-6}	0.0188	$1.8x10^{-6}$	$4.1x10^{-8}$	0.0188		
10	1.69×10^{-5}	0.188	$5.7x10^{-7}$	$1.3x10^{-8}$	0.188		
100	5.35×10^{-5}	1.880	1.8×10^{-7}	$4.1x10^{-9}$	1.880		
1000	1.69×10^{-4}	18.80	$5.7x10^{-8}$	$1.3x10^{-9}$	18.80		

Table (4): Calculated Radiation Power Losses at density ($n_e = 10^{19} m^{-3}$), (B=5.5 Tasla [7]).

Table (5): Calculated Radiation Power Losses at density $(n_e = 10^{20} m^{-3})$, (B=5.5 Tasla [7]).

$n_{e} = 10^{20} m^{-3}$							
T (KeV)	$\begin{array}{c} P_B \\ (MWm^{-3}) \end{array}$	$\frac{P_C}{(MWm^{-3})}$	$\frac{P_L}{(MWm^{-3})}$	$\frac{P_R}{(MWm^{-3})}$	P_{Total} (MWm ⁻³)		
1	5.35×10^{-4}	0.1880	0.00018	$4.1x10^{-6}$	0.1887		
10	1.69×10^{-3}	1.880	5.7×10^{-5}	$1.3x10^{-6}$	1.8817		
100	5.35×10^{-3}	18.80	1.8×10^{-5}	$4.1x10^{-7}$	18.810		
1000	1.69×10^{-2}	188.0	5.7×10^{-6}	$1.3x10^{-7}$	188.10		

Table (6): Calculated Radiation Power Losses at density ($n_e = 10^{21} m^{-3}$), (B=5.5 Tasla [7]).

$n_{e} = 10^{21} m^{-3}$							
T (KeV)	$\begin{array}{c} P_B \\ (MWm^{-3}) \end{array}$	$\frac{P_C}{(MWm^{-3})}$	$\frac{P_L}{(MWm^{-3})}$	$\frac{P_R}{(MWm^{-3})}$	P_{Total} (MWm ⁻³)		
1	0.05350	1.880	0.018	$4.1x10^{-4}$	1.8870		
10	0.16906	18.80	0.0057	$1.3x10^{-4}$	18.810		
100	0.5350	188.0	0.0018	$4.1x10^{-5}$	188.10		
1000	1.6906	1880.0	0.00057	$1.3x10^{-5}$	1881.7		

As a comparison we compare between the calculated density power radiation losses (Theo.) from Eqs. (16, 19, 22, 24). And values of density power radiation losses taken from reference (Ref.) for three kinds of fusion reactors (TFTR, IGNITOR, ITER) parameters, the results was arranged in table (7).

Table (7) : Theoretical Density Radia	ition Power Losses	s Calculation (Theo.) Compare With
Reference Radia	ation Power Losse	s Values (Ref.),	

Density Power	TF TR		IGNI TOR		IT ER	
Losses	Theo.	Ref. [8]	Theo.	Ref. [8]	Theo.	Ref. [8]
W_B (MW)	0.300	0.300	2.700	3.400	20.50	21.00
W_C (MW)	0.075	0.100	0.200	0.100	1.050	0.600
W_L (MW)	0.070	0.100	0.430	0.400	3.800	4.000
W_R (MW)	0.028		0.320		0.443	
$W_{RAD.}(MW)$	0.473	0.500	3.650	4.100	25.79	26.00
Plasma	2		2		2	
Volumes [8]	$V(m^{-3})=$	38.8	$V(m^{-3})=$	9.90	$V(m^{-3}) =$	820

TFTR: Tokamak Fusion Test Reactor. **IGNITOR:** Ignition Tokamak Reactor. **ITER:** International Tokamak Energy Reactor.

Discussion and Conclusion:

- 1. The radiation power loss from a plasma depends strongly upon effective ionic charge (Z_{eff}) as shown in table (1).
- 2. The cyclotron radiation power loss from a plasma depends strongly upon magnetic induction (B) as shown in table (2).
- 3. The radiation power loss from a plasma depends strongly upon the impurity concentration and impurity effective ionic charge (Z_{imp}) , when (Z_{imp}) increase the recombination losses increase more than line losses, as shown in table (3).
- 4. The radiation power loss and radiation power parameter from a plasma depends on increasing electron temperature and density increasing as shown in table (4, 5, 6).
- 5. we conclude form tables that the parameters (Z_{eff} , Z_{imp} , B, T, n) should be controlled to reduced the losses and to increase the net fusion reactors power through confining the plasma particles (energy) as long as possible.
- 6. At (T=100KeV) the losses becomes so much where the reactor fail in confining plasma energy as shown in tables (4,5,6), we conclude from calculations the most suitable temperatures are between (T=1-100KeV), and the most suitable density is $(n=10^{20} \text{ m}^{-3})$ for toroidal fusion reactors.
- 7. From table (7) the comparison between theoretical density power losses and density power losses values taken from reference prove that our calculations are so reasonable.
- 8. From results shown in tables we conclude that by reduced these losses we can increase the net fusion power through confining the plasma particles as long as possible.

References:

(1) أساسيات النظرية الكهر ومغناطيسية، ريتز –ملفورد، ترجمة يحيى عبد الحميد والدكتور رحمن رستم عبد الله، جامعة الموصل، 1988 .

- (2) Fusion Research, Dolan, Thomas James, vol. 1, Pergamon press, 1982.
- (3) **Tokamaks**, John Wesson, J.W.Conner, R.D. gill, R.J. Hastie, J. Hugill, oxford, Clare don press, 1987.
- (4) **Fusion Magnetic Confinement-Addendum** H.l.Berk ,Institute for fusion studies the university of Texas at Austin, USA, 1997.
- (5) *Nuclear Fusion, Hollmann, E.M, et al, 45, P-1055, 2005.*
- (6) Nuclear Fusion, Rozhansky, V.I., et al, 46, P-367-382, 2006
- (7) **Fusion Magnetic Confinement-Addendum,** H.l.Berk, Institute for fusion studies the University of Texas at Austin, USA, 1997.
- (8) **Fusion Alpha Parameters in Tokamak with high DT Fusion Rates,** R.V. Budny, Princeton university, Princeton, New Jersey, U.S.A., June- 2002.