# Forecasting Sales by Applying the Markov Improve Chain

Dr.Mohammad Mahmood Faqe
Lecturer
University of Sulaimani
College of Administration & Economics
Mohammad.faqe @univsul.edu.iq

Kosar Mahmood Hassan
Assistant professor
University of Sulaimani
College of Commerce
kosar.hassan@univsul.edu.iq

Sara Noori Mohammad Ali
Lecturer
University of Sulaimani
College of Administration & Economics
sara.mohammad @univsul.edu.iq

# التنبؤ بالمبيعات من خلال تطبيق سلسلة تحسين ماركوف

**أ.م.د.محمد محمود فقي حسي**
**جامعة السليمانية**
**كلية الادارة و الاقتصاد**

**م.كوسار محمود حس**
**جامعة السليمانية**
**كلية التجارة**

**م. سارا نوري محمد علي**
**جامعة السليمانية**
**كلية الادارة و الاقتصاد**

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## Abstract

Sales forecasting is the process of predicting how much money someone will make over a certain period of time. It is used to determine how many products you need to make, how much inventory you need to keep on hand, and when you will need to order more. The aim of this study is to forecast and determine the market share of rice using Markov chains model. The Markov chains model is one of the most important quantitative models that decision makers need to know, and its importance arises from its broad application areas, in which it has been successfully applied to predict a system over a given period of time. The results of this study showed that the market share is divided into three-time period. In the first period, the consumption ascending of the rice is as follows: other rice varieties, golden roster rice, ration card rice, Mahmood rice, and Kurdish rice, where as in the second period the consumption ascending of rice is as follows: other rice varieties, golden rooster rice, Mahmood rice and ration card rice, and Kurdish rice In the third period, the consumption ascending of the rice is as follows: other types of rice, Mahmood rice and golden rooster rice, Kurdish rice and ration card rice

***Keywords:*** *Markov chain, sales, forecasting, one-step transition probability matrix*

## المستخلص

التنبؤ بالمبيعات هو عملية توقع مقدار الأموال التي ستكسب خلال فترة زمنية معينة. يتم استخدامه لتحديد عدد المنتجات التي تحتاج إلى صنعها ، ومقدار المخزون الذي تحتاج إلى الاحتفاظ به في متناول اليد ، ومتى ستحتاج إلى طلب المزيد. الهدف من هذه الدراسة هو التنبؤ وتحديد الحصة السوقية للأرز باستخدام نموذج سلاسل ماركوف لأن نموذج سلاسل ماركوف هو أحد أهم النماذج الكمية التي يحتاج صناع القرار إلى معرفتها ، وتنبع أهميتها من حيث امكانية استخدامها بشكل واسع في جميع المجالات ، حيث تم تطبيقها بنجاح للتنبؤ بالنظام خلال فترة زمنية معينة. أظهرت نتائج هذه الدراسة الى أن الحصة السوقية تنقسم إلى ثلاث فترات زمنية. في الفترة الأولى صنف استهلاك الأرز تصاعدياً على النحو التالي: أصناف الأرز الأخرى ، أرز القائمة الذهبية ، أرز البطاقة التموينية ، أرز محمود ، أرز كردي ، في الفترة الثانية صنف استهلاك الأرز تصاعدياً على النحو التالي: أصناف الأرز الأخرى ، أرز الديك الذهبي ، أرز محمود وأرز البطاقة التموينية ، أرز كردي ، وفي الفترة الثالثة صنف الاستهلاك تصاعديا للأرز على النحو التالي: أنواع أخرى من الأرز ، أرز محمود ، أرز الديك الذهبي ، أرز كردي وأرز البطاقة التموينية.

***الكلمات الرئيسية:*** *سلسلة ماركوف ، المبيعات ، التنبؤ ، مصفوفة احتمالية الانتقال بخطوة واحدة*

## 1-1 Introduction

Due to erroneous supply and demand, businesses are ineffective. The forecast does not apply to the upcoming production because the influence has subsided and passed [7]. Forecasting benefits include helping with problem-solving and creating business plans. [2]. Sales forecasting is essential for developing business analytic solutions. Businesses need solid sales forecast to support their operational and sales processes.
Markov chain is a technique for producing future predictions that only need current data and little prior information giving businesses a theoretical basis on which to base their commodity sales strategy, by developing a Markov chain model that can predict commodity sales based on their properties. [8]. A mathematical model is then created from the information.

## 1-2 Research Objectives:

The research objectives lie in the following:

- describing the role and importance of using statistical measures in forecasting, and here we mention the Markov chains model;
- Highlighting the importance of sales in economic institutions;
- The possibility of applying this model in any resources, regardless of their size and type;
- forecasting and determining the market share of rice using Markov chains model

## 1-3 Research Importance:

The importance of this research paper can be presented by the following indicators:

- Clarifying the importance of statistical analysis in developing plans and making decisions by applying the statistical methods used in the forecasting process of the sales of some economic resources, and thus introducing a new method for analyzing data;
- Economic forecasting is one of the topics that gain great importance, because decision-makers through economic forecasting can draw economic and social policies for the coming periods;
- The importance of the Markov chain model which is one of the important statistical tools in forecasting modeling.

## 1-4 Literature Review

In 2015 (Ka Ching Chan ) Uses four mathematical models of the market share problem based on different underlying assumptions, the four homogeneous, time-varying models, a new time-varying model, and a new non-Markov model. The telecom industry is included to show that four models can be used to predict market share. In 2018 (Christos Aristeides Tsiliyannis) uses the retention probability sequence in the explicit expressions for the product return flow and age distribution, based on the Markov representation of inventory and flows. The model allows for random and unfixed early loss, uncertain demand and diversified use of reusable proceeds. Markov chain Monte Carlo simulation allows to evaluate the effectiveness of the prediction method. In 2021 (Haiying Chen, Haiyan Chen, Wei Zhang, Chaodan Yang, and Hongxiu Cui ) Introduce a mathematical model to forecast product marketing by creating a product state transition probability matrix, analysing and calculating it using Markov chain, after using the Markov analysis

method, a mathematical model can be generated based on market investigations and statistics. Finally in 2021 (XIA YUTONG) two major everyday examples to explain that in a Markov process, the probability of one state depends only on the next prior state. A mathematical model of the market forecast and weather forecast can be built by creating transition probability matrix, analysis and computing using Markov chain.

**1-5 Research Methodology:**

The purpose of this paper is to clarify the steps that need to be taken when using Markov chains model in forecasting sales due to their extreme importance. To achieve this goal, the research was divided into three points. The first is the theoretical basis on which the Markov chain method is based. The second is the use of statistical analysis to study some types of rice, and it was applied to a sample of 300 observations. Finally, the third is application of the Markov chains model in determining an efficient prediction model that allows the institution under study to adopt an accurate system for managing its various activities.

**1-6 Markov Chain Model**

A statistical model called the Markov chain was developed by Andrei A. Markov (1856–1922). In order to ascertain the characteristics of variables in the future, it explains a succession of prospective events, with the chance of each event relying on the probability of the one before it. Markov chains are popular and very simple methods for modelling random processes in statistics [4]. This model shows a set of processes where each experiment's occurrence depends only on the event that happened before it and is independent of preceding sequences of events. Since Markov's technique is widely used to analyse the movement of interest in an object, it is regularly used to evaluate marketing strategies. The objective is for business people to understand the parameters of what is happening and future projections for their companies. [6]. The following is a definition in mathematics: [8]

$$P_{ij} = P\{Z_{n+1} = j | Z_0 = i \ , \dots, Z_i = i_1, \dots, Z_n = i\} = P\{Z_{n+1} = j | Z_n = i\} \qquad (1)$$

Two main types of Markov chains are determined by various state spaces. The first type is a continuous-time Markov chain (CTMC), and the second is a discrete-time Markov chain (DTMC), which is a countable and measurable state.[9]

**1-7 Assumptions of Markov Chain**

Stochastic processes $\{Z_n : n \in T\}$ are characterized by Markov chain. According to the hypotheses listed below, Markov chains are incomplete because no other processes are present. [10]

1- This method discrete state space.
2- Parameter space (T) is discrete time if and only if $\{Z_n : n = 1, 2, \dots\}$.
3- The future state depends only on the current state and not on the past this is called a Markov property.If the process is not dependent on time, the process is referred to as a homogeneous - Markov chain and the chain is known as a stationary transition probability matrix. If the process is depended on time, the process is known as non-homogeneous Markov chain.

**1-8 Transition Probability Matrix**

The transition probability matrix is a key model finding the Markov chain model (MCM), which captures the characteristics of interaction between states. The matrix contains the probabilities of moving from one state to another state. One of the most significant studies on the MCM is one that examines how the MCM will behave in the future. This idea is known as convergence of the n-step transition probability matrix for n-steps to infinity. Finding the limit of the transition probability matrix to the power of n, where n is an infinite integer, is the mathematical definition of the convergence of the transition probability matrix [5].

Let the homogeneous Markov chain with a discrete infinite state space then

$$(2)P_{ij} = P\{Z_{n+1} = j|Z_n = i\}$$

A transition probability matrix is defined by

$$P = [p_{ij}] = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \cdots \\ p_{10} & p_{11} & p_{12} & \cdots \\ p_{20} & p_{21} & p_{22} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

Where each element satisfies the following conditions:

$$(3)P_{ij} \geq 0 \quad , \quad \sum_{j=0}^{\infty} P_{ij} = 1 \quad , \quad i = 0,1,2,\ldots$$

A square matrix $P_{ij}$ whose elements are satisfied the conditions above is called a stochastic matrix.[11]

State-transition diagram

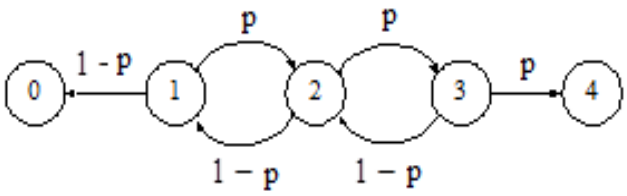


Figure (1): States-transition diagram

## 1-9 State the Probability Distribution [16]

Consider a Markov chain $\{Z_n, n = 0,1,2,\ldots\}$, where $Z_n \in S$. Suppose that we know the probability distribution of $Z_0$. More specifically, define the row vector $\pi^{(0)}$as

$$(4)\pi^{(0)} = [P(Z_0 = 1) \quad P(Z_0 = 2) \quad \ldots P(Z_0 = r)]$$

How can we obtain the probability distribution of $Z_1, Z_2, \ldots$? We can use the law of total probability. More specifically, for any $j \in S$, we can write

$$P(Z_1 = j) = \sum_{k=1}^{r} P(Z_1 = j|Z_0 = k)\, P(Z_0 = k) = \sum_{k=1}^{r} p_{kj} P(Z_0 = k)$$

If we generally define

$$\pi^{(n)} = [P(Z_n = 1) \quad P(Z_n = 2) \quad \dots P(Z_n = r)] \qquad (5)$$

we can rewrite the above result in the form of matrix multiplication

$$\pi^{(1)} = \pi^{(0)}P,$$

where $P$ is the state transition matrix. Similarly, we can write

$$\pi^{(2)} = \pi^{(1)}P = \pi^{(0)}P^2 \qquad (6)$$

more generally, we can write

$\pi^{(n+1)} = \pi^{(n)}P, \text{for } n = 0,1,2, \dots$

$$\pi^{(n)} = \pi^{(0)}P^n, \text{for } n = 0,1,2, \dots \qquad (7)$$

**1.10 Classification States of Markov Chain:** [11]

The states of can be classified based on the transition probability matrix $P_{ij}$

1. For some $n \geq 0$, $P_{ij}^n > 0$, and we write $i \to j$, state j is said to be accessible from state i.

We write $i \leftrightarrow j$ to represent the communication between two mutually accessible states i

and j. We say that the Markov chain is irreducible if all of the states are in communication with one another.

2. If $P_{jj} = 1$ , state j is said to be an absorbing state since once it is reached, it never leaves.

3. If $f_{ij} = P(T_j < \infty | X_0 = j) < 1$, where $T_j$ is the time of the first visit to state j after time 0,

state j is said to be non-recurring. This will occur mathematically when $\lim_{n\to\infty} P_{ij}^{(n)} = 0$

for every i. In this scenario, there is a chance of never going back to state j.

4.The probability of returning back to j is one starting from j. Positively recurring state j is one that exists repeatedly if

$$E(T_j | X_0 = j) < \infty \qquad (8)$$

and state j is called to be null recurrent if

$$E(T_j | X_0 = j) = \infty \qquad (9)$$

5. We define the period of state j to be

$$\delta(j) = \text{greatest common divisor}\{n \geq 1 : P_{jj}^{(n)} > 0\} \qquad (10)$$

If $\delta(j) > 1$, then state $j$ is called periodic. If $\delta(j) = 1$, then state $j$ is called a periodic. Whenever $P_{jj}^{(n)} > 0$ , $j$ is a periodic.[3]

## 2. Applications and Discussions

### 2.1. Data Analysis and Discussions

Data was collected through a questionnaire, forms (n=350) were randomly distributed to a sample of families in Sulaymaniyah Governorate. The questionnaire consisted of two parts, the first part related to the type of rice and the second part related to the reason for choosing this type of rice. This data is shown below:

**Table (1): Different types of rice**

| Rice Types | Mahmood rice | Ration card rice | Kurdish rice | Golden rooster rice | Other types of rice |
|---|---|---|---|---|---|
| Frequency | 97 | 66 | 110 | 54 | 23 |
| Percentage | % 27.7 | % 18.9 | % 31.4 | % 15.4 | % 6.6 |

The proportion of rice consumed by the sample size is shown in Table no. 1, with Kurdish rice coming in first place with a percentage of (31.4), and Mahmood rice coming in second place with a percentage of (27.7). Ration card rice, golden rooster rice, and various types of rice are served in the third, fourth, and fifth places.

Up to 5 factors can be used as a standard for this usage. These elements are:
$P_1$: Price $P_2$: Taste $P_3$: Rice type $P_4$: Quality $P_5$: Healthy
These five components can be compared in relation to the five different kinds of rice. 350 participants were interviewed for the analysis and each one indicated which type they liked (the factors behind using a particular type of rice). Table 2 displays how the respondents' responses varied based on the given criteria.

**Table (2): Responses of the study's sample participants**

| Rice Types | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | Total |
|---|---|---|---|---|---|---|
| Mahmood rice | 7 | 37 | 43 | 9 | 1 | 97 |
| Ration card rice | 38 | 19 | 4 | 5 | 0 | 66 |
| Kurdish rice | 10 | 31 | 4 | 41 | 24 | 110 |
| Golden rooster rice | 1 | 27 | 11 | 15 | 0 | 54 |
| Other types of rice | 1 | 6 | 11 | 5 | 0 | 23 |

According to Table 3, which is a data transfer produced by other species that already exist, each rice type is assigned a unique species. As an illustration, Mahmood rice has 60 consumers of whom 6 purchase other varieties, 11 consumers purchase ration cards rice, 24 consumers purchase Kurdish rice, and 19 purchase golden rooster rice. The other varieties of rice are also purchased from different suppliers.

**Table (3): Shopper conquest data for first Period**

| Name | Mahmood rice | Ration card rice | Kurdish rice | Golden rooster rice | Other types of rice | Acq |
|---|---|---|---|---|---|---|
| Mahmood rice | 0 | 11 | 24 | 19 | 6 | 60 |
| Ration Card rice | 10 | 0 | 11 | 0 | 5 | 26 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Kurdish rice | 10 | 12 | 0 | 10 | 0 | 32 |
| Golden rooster rice | 0 | 3 | 6 | 0 | 0 | 9 |
| Other types of rice | 23 | 4 | 20 | 11 | 0 | 58 |
| Loss | 43 | 30 | 61 | 40 | 11 | |

Each corporation has losses in addition to acquisitions, as illustrated in table 4. For instance, Mahmood rice lost 43 clients, of which 10 transferred to Kurdish rice, 10 to another variety, and 23 to yet another, this is in contrast will the losses in the other three rice varieties.

**Table (4): Shopper loses data on difference types of rice**

| Name | Mahmood rice | Ration card rice | Kurdish rice | Golden rooster rice | Other types of Rice | Loss |
|---|---|---|---|---|---|---|
| Mahmood rice | 0 | 10 | 10 | 0 | 23 | 43 |
| Ration card rice | 11 | 0 | 12 | 3 | 4 | 30 |
| Kurdish rice | 24 | 11 | 0 | 6 | 20 | 61 |
| Golden rooster rice | 19 | 0 | 10 | 0 | 11 | 40 |
| Other types of rice | 6 | 5 | 0 | 0 | 0 | 11 |
| Acq | 60 | 26 | 32 | 9 | 58 | |

Table 5 displays the consumer gain and loss for each rice variety. If this is the beginning of the period or the end of the previous period, N = new, and B = before if this is the next period. Mahmood rice last experienced a decline. 60 customer losses and 43 new purchases were the outcomes.

**Table (5): Transition for rice types for one period**

| Name | Before | Acquisition | Loss | New |
|---|---|---|---|---|
| Mahmood rice | 97 | 43 | 60 | 114 |
| Ration card rice | 66 | 30 | 26 | 62 |
| Kurdish rice | 110 | 61 | 32 | 81 |
| Golden rooster rice | 54 | 40 | 9 | 23 |
| Other types of rice | 23 | 11 | 58 | 70 |

As demonstrated in Table 6, some consumers were among those who did not alter their variety during the prior period. 241 people, including 54 Mahmood rice users, 36 ration card rice users, 49 Kurdish rice users, 14 golden rooster rice users, and 12 users of other forms of rice, continue to consume the old variety.

**Table (6): Transition table for type of rice**

| No. | Rice Types | Mahmood | Other types | Ration | Kurdish | Golden | Total | Percentage % |
|---|---|---|---|---|---|---|---|---|

| | | rice | of rice | card rice | rice | rooster rice | | |
|---|---|---|---|---|---|---|---|---|
| 1 | Mahmood rice | **54** | 23 | 10 | 10 | 0 | **97** | (54/97)× 100 =55.67 |
| 2 | Ration card rice | 11 | 4 | **36** | 12 | 3 | **66** | (36/66)× 100 =54.55 |
| 3 | Kurdish rice | 24 | 20 | 11 | **49** | 6 | **110** | (49/110)× 100 =44.55 |
| 4 | Golden rooster rice | 19 | 11 | 0 | 10 | **14** | **54** | (14/54)× 100 =25.93.36 |
| 5 | Other types of rice | 6 | **12** | 5 | 0 | 0 | **23** | (12/23)× 100 =52.17 |
| | Total | 114 | 70 | 62 | 81 | 23 | 350 | |

The results of the following percentage computation explain why some companies lost their market share.

$$\text{Mahmood rice} = \frac{114-97}{114} * 100 = 14.91\%$$
$$\text{Other types of rice} = \frac{70-23}{70} * 100 = 67.14\%$$
$$\text{Ration card rice} = \frac{62-66}{62} * 100 = -6.45\%$$
$$\text{Kurdish rice} = \frac{81-110}{81} * 100 = -35.8\%$$
$$\text{Golden rooster rice} = \frac{23-54}{23} * 100 = -134.78\%$$

From the above result, we found that ration card rice (-6.45%), Kurdish rice (-35.8%) and golden rooster rice (-134.78%) decreased in the next period. Mahmood rice (14.91%) and (67.14%) increased in the new period.

**2.2. Estimating Market Share**

When the prerequisites for data collection and loss are satisfied, the market share calculation can be finished. The market shares are compared using data from three different time periods. One can calculate market share for the first period (MSF), second period (MSS), and third period (MST). The following gives an example of how the matrix used to compute market share:

$$Rb = \begin{bmatrix} 97 \\ 23 \\ 66 \\ 110 \\ 54 \end{bmatrix}, FM = \begin{bmatrix} 54 & 23 & 10 & 10 & 0 \\ 6 & 12 & 5 & 0 & 0 \\ 11 & 4 & 36 & 12 & 3 \\ 24 & 20 & 11 & 49 & 6 \\ 19 & 11 & 0 & 10 & 14 \end{bmatrix}$$

The probability of transition is calculated using the formula below: $\text{T. P. M.}(\text{col}, \text{row}) = \frac{\text{FM}(\text{col},\text{row})}{\text{Rb}(\text{row})}$

$$T.P.M. = \begin{bmatrix} 0.56 & 0.24 & 0.10 & 0.10 & 0.00 \\ 0.26 & 0.52 & 0.22 & 0.00 & 0.00 \\ 0.17 & 0.06 & 0.55 & 0.18 & 0.05 \\ 0.22 & 0.18 & 0.10 & 0.45 & 0.05 \\ 0.35 & 0.20 & 0.00 & 0.19 & 0.26 \end{bmatrix}$$

The outcome is the company's transition probability (TP) at a specific time, as shown in the preceding matrix.

$$MSF = \begin{bmatrix} 97/350 \\ 23/350 \\ 66/350 \\ 110/350 \\ 54/350 \end{bmatrix} = \begin{bmatrix} 0.28 \\ 0.07 \\ 0.19 \\ 0.31 \\ 0.15 \end{bmatrix}$$

$$MSS = \begin{bmatrix} 0.56 & 0.24 & 0.10 & 0.10 & 0.00 \\ 0.26 & 0.52 & 0.22 & 0.00 & 0.00 \\ 0.17 & 0.06 & 0.55 & 0.18 & 0.05 \\ 0.22 & 0.18 & 0.1 & 0.45 & 0.05 \\ 0.35 & 0.20 & 0.00 & 0.19 & 0.26 \end{bmatrix} \times \begin{bmatrix} 0.28 \\ 0.07 \\ 0.19 \\ 0.31 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.22 \\ 0.15 \\ 0.22 \\ 0.24 \\ 0.21 \end{bmatrix}$$

$$MST = \begin{bmatrix} 0.56 & 0.24 & 0.10 & 0.10 & 0.00 \\ 0.26 & 0.52 & 0.22 & 0.00 & 0.00 \\ 0.17 & 0.06 & 0.55 & 0.18 & 0.05 \\ 0.22 & 0.18 & 0.1 & 0.45 & 0.05 \\ 0.35 & 0.20 & 0.00 & 0.19 & 0.26 \end{bmatrix} \times \begin{bmatrix} 0.22 \\ 0.15 \\ 0.22 \\ 0.24 \\ 0.21 \end{bmatrix} = \begin{bmatrix} 0.21 \\ 0.18 \\ 0.22 \\ 0.22 \\ 0.21 \end{bmatrix}$$

The comparison of market share calculations over three periods is shown in the image below. Calculations of the market shares show this in all three periods.

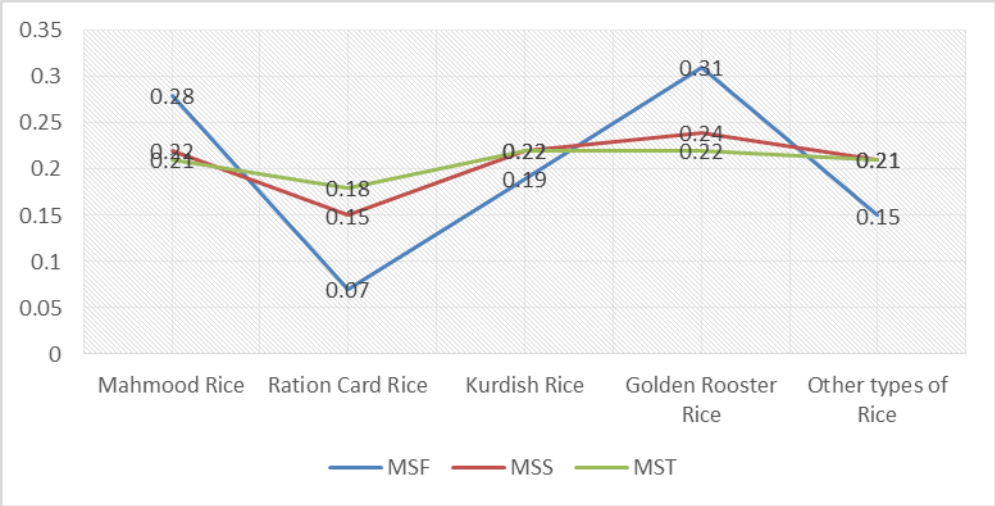


Figure (2): Comparing the value of market share over three periods

Figure 2 compares the results of the market share calculations for the three different periods. The consumption of various rice varieties climbed throughout all the three periods, while the consumption of ration card rice and Kurdish rice also increased, whereas Mahmood and Golden rooster rice decreased over the first and fifth periods, according to market share estimates. The outcomes of Markov chains can be used to assess the effectiveness of a company's marketing initiatives. Based on the estimated cost of customer acquisition, a business can develop the following strategy: While these results are not totally correct, they at least provide ideas for potential marketing plans.

## 3. Conclusions

Based on the analysis, this study has the following conclusions:

1. The Markov technique is particularly effective for predicting the organization's performance outcomes based on data collected over a period of time. The following rice varieties are used by customers in the MSF: Mahmood rice (consumption 0.28), ration card rice (0.19), Kurdish rice (0.31), golden rooster rice (Consumption 0.15) and other rice varieties (Consumption 0.07).
2. In MSS, we find that consumers of Golden rooster rice used the following types of rice with the following probabilities: Mahmood rice (0.22), ration card rice (0.22), Kurdish rice (0.24), Golden rooster rice (0.21), and other rice varieties (0.15).
3. In the MST, we observe that customers used the following rice varieties with the following probabilities: Mahmood rice (0.21), ration card rice (0.22), Kurdish rice (0.22), golden rooster rice (0.21) and other types of rice (0.18).
4. Consumer demand for the other rice varieties has increased, and purchases of government and Kurdish rice have also increased.
5. Consumption of Mahmood rice and Golden rooster rice decreases in the first and fifth periods.

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