

## Polyphase Spreading Sequences for DS-CDMA Wireless Application

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### Abstract

In this paper, some of polyphase signature sequences for direct sequence code division multiple access (DS/CDMA) system is proposed. The novel sets of polyphase spreading sequences is obtained through the multiplication of the polyphase chirp sequences by some other binary and polyphase sequences such as Barker & Willard binary sequences, generalized chirp-like (GCL) and orthogonal polyphase sequences. These sets of polyphase sequences are tested using Matlab version 6.5. The sets shows good correlation properties.

GCL-Chirp polyphase sequence show the best performance and a better correlation properties, for the length  $N=9, 16$  &  $25$  which are under test, the normalized peak side-lobe levels of the normalized auto-correlation function are  $0.15, 0.18$  &  $0.15$  for GCL-Single chirp, and  $0.11, 0.29$  &  $0.21$  for GCL-Double chirp respectively. On the other hand, the peak cross-correlation magnitudes are  $0.49, 0.32$  &  $0.55$  for GCL-Single chirp, and  $0.5, 0.38$  &  $0.45$  for GCL-Double chirp, respectively.

For the proposed new sets of GCL-Chirp and the Modulatable Chirp - polyphase sequences, there is no limitation in sequence length, but for the Barker-Chirp and Willard-chirp, the length is limited according to the limitation of the Barker and Willard sequences length.

### شفرات متعددة الأطوار لأنظمة DS/CDMA

#### الخلاصة

في هذا البحث في هذا البحث تم اقتراح شفرات متعددة الأطوار لأنظمة DS/CDMA. الشفرات الأصلية المقترحة هي عن طريق ضرب شفرة متعددة الأطوار نوع Chirp مع شفرات أخرى ثنائية مثل شفرة Barker و شفرة Willard وغير ثنائية (متعدد الأطوار) مثل شفرات Modulatable sequence و GCL. هذه الشفرات تم فحصها باستخدام برنامج Matlab إصدار 6.5 و تبين أن لها مواصفات تطابق جيدة.

بينت النتائج أن شفرة GCL-Chirp بأطوال 9، 16، 25 والتي كانت تحت الاختبار هي الأحسن من بين الشفرات الأخرى المقترحة حيث لسيها أحسن خواص تطابق correlation، دالة التطابق الذاتي auto-correlation كان قيمة أعلى فص جانبي هو 0.15, 0.18, 0.15 لنوع GCL-Single chirp وقيم 0.11, 0.29, 0.21 لنوع GCL-Double chirp على التوالي، من جهة أخرى فإن أعلى قيمة تطابق متباين cross-correlation لنفس الأطوال هي 0.49, 0.32, 0.55 للنوع GCL-Single chirp وقيم 0.50, 0.38, 0.45 للنوع GCL-Double chirp على التوالي، وهناك تحسن في شفرة GCL بضربها مع Chirp.

678

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الشفرة المقترحة GCL-Chirp و Modulatable Orthogonal-Chirp متعددة الطور، يمكن أن تكون لها أي طول، ما عدا الشفرات Barker-Chirp والشفرة Willard-Chirp فإن الطول محدد نظراً لأن طول الشفرات Barker و Willard محدد.

## INTRODUCTION

In common operation of the JPEG2000 still image compression standard [1], the encoder tiles the image into blocks of  $n \times n$  ( $n$  being a Code division multiple access (CDMA) may be considered as a generalized method for access and switching in communication networks. Code division is applied for both link access and node routing (switching) of the user data. Thus, a switched CDMA network may be formed for the interconnection of end users. One of the basic concepts in communication channel is the idea of allowing several transmitters to send information simultaneously over a communication channel. There are several techniques for providing multiple access which are Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA) and Code Division Multiple Access (CDMA).

In general, modulating a data signal onto a wide-band carrier generates a spread spectrum signal. One of the most widely used techniques to spread spectrum is direct-sequence spread spectrum, in which there is a certain code used to spreading purpose, this sequence code generated has two main types binary and non-binary sequences. In our paper we used Barker, Willard and

Walsh-Hadamard binary sequence codes as mentioned in section (2), and for the non-binary sequences (polyphase sequences) we used modulatable orthogonal polyphase sequence and generalized chirp-like (GCL) polyphase sequence as mentioned in section (3). Then by multiplying the polyphase chirp sequence by these sequences, we obtained some new sets of polyphase sequence with good correlation properties as mentioned in section (4). We can obtain other sets of polyphase sequences by multiplying other binary and non-binary sequences by the polyphase chirp sequence also they have show good correlation properties.

## BINARY SEQUENCES

There are many binary sequences for example : m-sequence, Barker codes, Willard codes, Walsh Hadamard sequence codes. The proposed new polyphase sequences is obtained by multiplying the polyphase chirp sequences by the Walsh Hadamard, Barker & Willard codes.

### a) Walsh Hadamard Sequence Code

Walsh Hadamard bipolar spreading sequences can be used for channel separation in (DS/CDMA) systems. They are easy to generate and orthogonal in the case of perfect synchronization. However, the cross-correlation between two Walsh Hadamard sequences can rise considerably in magnitude if there is a non-zero delay shift between them.

Due to their very regular structure, Walsh Hadamard sequences are characterized with very poor auto-correlation property [1].

The Walsh Hadamard sequences of the length  $N$ ;  $N=2^n$ ,  $n=1,2$ , are often defined using Hadamard matrices  $H_N$ , with:

$$H_2 = \begin{Bmatrix} 1 & 1 \\ 1 & -1 \end{Bmatrix} \dots\dots\dots(1)$$

$$H_N = \begin{Bmatrix} H & H \\ H & -H_N \end{Bmatrix} \dots\dots\dots(2)$$

The resulting matrices  $H_N$  are orthogonal matrices, i.e., for every  $N$  we have:

$$H_N H_N^T = N I_N \dots\dots\dots(3)$$

Where,  $H_N^T$  is the transposed Hadamard matrix of order  $N$ , and  $I_N$  is the  $N \times N$  unity matrix. For example, if  $N=8$ , the Hadamard matrix is:

$$\begin{Bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{Bmatrix}$$

Take, for instance,  $Wal(6,t)$ , number 6 in binary is 110, which in bit reversed form becomes 011, conversion to decimal form gives us that  $WAL(6,t)$  is in the 3<sup>rd</sup> row of the

matrix and is therefore given the pulse sequence  $\{1,1,-1,-1,1,1,-1,-1\}$ . For the series expansion of the function  $f(t)$  a Walsh series can be written:

$$F(t) = a_0 WAL(0,t) + \sum_{n=1}^{N-1} a_n WL(n,t),$$

$$a_n = 1/T \int_0^T f(t)Wal(n,t)dt \dots\dots(4)$$

**b) Barker Code**

Barker codes are short unique codes that exhibit very good correlation properties. These short codes with  $N$  bits, with  $N=3$  to 13, are very well suited for DS spread spectrum [2]. A list of Barker codes is tabulated in table (1).

**c) Willard Codes**

Willard codes under certain conditions offer better performance than Barker codes [2]. A list of Willard codes is tabulated in table (1).

The inverted or bit reversed versions of the codes listed on table (1) can be used since they still maintain the desired autocorrelation properties:

**POLYPHASE SEQUENCES**

Several families of complex spreading sequences have been proposed with some of them allowing for a good compromise between autocorrelation (AC) and crosscorrelation (CC) properties or even achieving orthogonality in the case of perfect synchronization. In this paper we used two types of these polyphase sequences: Generalized Chirp-Like (GCL) sequence and modulated orthogonal sequence.

**Modulatable Orthogonal Polyphase Spreading Sequence**

Suehiro proposed a set of (N-1) orthogonal sequences of period N which can be used for an asynchronous spread spectrum multiple access (SSMA) systems. However, these sequences are not directly modulatable.

We propose a set of (N-1) readily modulatable orthogonal sequences of period  $N^2$ , where N is a prime number. The absolute value of the cross-correlation function between any two of these N-1 sequences is  $1/N$  for every nonzero lag. This realizes the mathematical lower bound for the absolute value peak in the cross-correlation function between two orthogonal sequences. Each of N-1 orthogonal sequences can be modulated by N complex numbers of absolute value 1 such that each of the modulated sequences remains orthogonal. The cross-correlation properties already mentioned also are inherited by the modulated sequence [4].

The resulting N-1 classes of modulated orthogonal sequences have the following properties:

- 1) Each class includes an infinite number of modulated orthogonal sequences of period  $N^2$ .
- 2) The absolute value of the cross-correlation function between any two sequences in different classes is constant and equals the lower bound of the absolute value peak of the cross-correlation function.
- 3) The cross-correlation function between any two sequences in the same class is 0 for every lag that is not a multiple of N.

Let N is a natural number. An N-dimensional normalized discrete

Fourier transformation (DFT) matrix  $F_N$  is defined as:

$$F_N = [f_N(i_0, i_1)]$$

$$f_N(i_0, i_1) = 1/\sqrt{N} \exp(-j\pi(i_0 - i_1)(i_0 + i_1)/N) \quad (5)$$

where  $0 \leq i_0 \leq N-1$ ,  $0 \leq i_1 \leq N-1$ . Then matrix  $F_{Nm}$  is defined as :

$$F_{Nm} = [f_{Nm}(i_0, i_1)]$$

$$f_{Nm}(i_0, i_1) = 1/\sqrt{N} \exp(-j\pi(i_0 - i_1)(i_0 + i_1)/N) \quad (6)$$

where, m is a natural number.

An N-dimensional diagonal matrix B is introduced as :

$$B = [b(i_0, i_1)]$$

Where the absolute value  $b(i_0, i_1)$  of each diagonal element of B is 1, and other elements are 0. The  $(i_0, i_1)$  element of  $F_{Nm} B$  is :

$$f_{Nm}(i_0, i_1) b(i_0, i_1) = b(i_0, i_1) / \sqrt{N} \exp(-j\pi(i_0 - i_1)(i_0 + i_1)/N) \quad (6)$$

A sequence of length  $N^2$  is derived from the  $N \times N$  matrix  $F_{Nm} B$ . Let I be :

$$i = i_0 N + i_1$$

Then  $0 \leq i \leq N^2 - 1$

The  $i^{th}$  element of a sequence of length  $N^2$  composed by linking N rows of  $N \times N$  matrix

$F_{sm} B$  is the  $(i_0, i_1)$  element of  $F_{sm} B$ . Let  $G$  be a sequence composed of linked rows of  $\sqrt{N} F_{sm} B$ ; then :

$$G = (g(i))$$

$$G(i) = \sqrt{N} f_{N/m}(i_0, i_1) b(i_0, i_1)$$

$$G(i) = b(i_0, i_1) \exp(((2\pi\sqrt{-1})/N)mi_0 i_1) \quad (7)$$

$G$  is modulated by  $(i_1, i_1)$ , which are diagonal elements of  $B$ .

A periodic sequence of period  $N^2$  obtained by repeating  $G$  is modulated by the diagonal elements of  $B$ .

For example, when  $N=3$  &  $m=1$ :

$$F_{31} = 1/\sqrt{3} \begin{Bmatrix} 1 & 1 & 1 \\ 1 & W & W \\ 1 & W & W \end{Bmatrix}$$

Where,  $W_N = \exp(2\pi\sqrt{-1})$ . If:

$$B = \begin{vmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{vmatrix}$$

The obtained period sequence is

$$\{b_0, b_1, b_2, b_0, b_1 W_3, b_2 W_3^2, b_0, b_1 W_3^2, b_2 W_3\}$$

If  $B$  is a unit matrix, the obtained sequence corresponds to the sequence without modulation.

And for  $(N^2 - 1)$  sequence :

$$g'(l) = (1/N) \exp(((2\pi\sqrt{-1})/N^2)(c'(l) + Nm'l_0 l_1)) \quad (8)$$

where

$$g(i) = \exp(((2\pi\sqrt{-1})/N^2)(c(l) + Nm'l_0 l_1))$$

and

$$c'(l_1) = \{(c(j-i)_1/m) - (1/m)(\text{mod}N)(j-i)_1\} \quad (9)$$

where,

$$l = j - i = l_0 N + l_1 \quad (10)$$

A matrix  $A$  is a cyclic matrix since  $a(i, j)$  depends on only  $(j-i)(\text{modulo}N^2)$  &  $A$  is a unitary matrix, then :

$$A = F_{N^2}^{-1} \wedge F_{N^2}$$

A cyclic matrix  $A$  represents a polyphase periodic sequence since the absolute values of all its elements  $a(i, j)$  are equal. The periodic sequence represented by  $A$  is also an orthogonal sequence because  $A$  is a unitary matrix.

For example, when  $N=3$ ,  $m=1$ ,  $b(0,0)=W_9^0, b(1,1)=W_9^1$  and

$b(2,2)=W_9^2$ ,  $G=(g(i))$  defined in eq.(7) becomes :

$$G = (g(i)) = (1, W_9^1, W_9^3, 1, W_9^4, 1, 1, W_9^7, W_9^6)$$

Matrix  $A$  is calculated as follows :

$$A = F_9^{-1} \begin{pmatrix} 1 & & & & & & & & \\ & W_9^1 & & & & & & & \\ & & W_9^2 & & & & & & \\ & & & 1 & & & & & 0 \\ & & & & W_9^3 & & & & \\ 0 & & & & & 1 & & & \\ & & & & & & W_9^4 & & \\ & & & & & & & W_9^5 & \\ & & & & & & & & W_9^6 \end{pmatrix} F_9$$

According to the previous equations :

$$c(0) = 0 \pmod{9}, c(1) = 1 \pmod{9}, c(2) = 3 \pmod{9}$$

$$m^* = (-1/m) \pmod{N} = 2$$

$$c'(0) = 0, c'(1) = 0, c'(2) = 8$$

$$g'(l) = (1/3) \exp(((2\pi\sqrt{-1})/9)(c'(l) + 3.2.1_0 1_1))$$

$$G' = (g'(l))$$

$$= \{1, 1, W_9^8, 1, W_9^6, W_9^2, 1, W_9^3, W_9^5\}$$

It is shown that (1/3)G' equals the first row in the obtained cyclic matrix A.

The auto-correlation for the modulatable orthogonal sequence with the length N=9 is shown in fig.(1).

The GCL sequences are derived from the Zadoff-Chu polyphase sequences defined as :

$$a_k = \begin{cases} W_N^{k^2/2+qk} & , \text{ for N even} \\ W_N^{K(K+1)/2+qk} & , \text{ for N odd} \end{cases} \quad (11)$$

$k=0,1,2,\dots,N-1$

$q$  is any integer

In the above definition,  $W_N$  is the complex number of unit magnitude, given by :

$$W_N = \exp(-j2\pi r/N), j = \sqrt{-1} \quad (12)$$

Where  $r$  is any integer relatively prime to  $N$ .  $r$  is relatively prime to  $N$  if the greatest common divisor for  $r$  &  $N$  is 1, i.e.  $(r,N) = 1$ .  $W_N$  represent a primitive  $N^{\text{th}}$  root of unity, because  $(W_N)^N = 1$ .  $W_N$  not equal to 1.

Let  $\{a_k\}, k=0,1,\dots,N-1$  be a Zadoff-Chu sequence of length  $N=sm^2$ , where  $m$  &  $s$  are any positive integers. Let  $\{b_l\}, l=0,1,\dots,m-1$  be any sequence of complex numbers having the absolute values equal 1. The GCL sequence  $\{s_k\}$  is defined as :

$$s_k = a_{k \bmod m} b_{(k/m) \bmod m}, k=0,1,\dots,N-1 \quad (13)$$

Where,  $(k) \bmod m$  means that index  $k$  is reduced modulo  $m$ .

For example, when  $N=8$   $m$  we have that  $s=2$  &  $m=2$ , so from eqs.(11)&(13) the GCL sequence  $\{s_k\}$  will be :

$$\{s_k\} = \{b_0, b_1 W^{0.5}, b_0 W^2, b_1 W^{4.5}, b_0, b_1 W^{4.5}, b_0 W^2, b_1 W^{0.5}\}$$

Where,  $W = \exp(-j2\pi/8), j = \sqrt{-1}$ ,  $r$  is any integer relatively prime to  $N = 8, q=0$ , while  $b_0$  &  $b_1$  are arbitrary complex numbers with magnitudes equal to 1[7].

As shown in the figures, the auto-correlation for the following GCL sequences :

GCL for N = 9

$$\begin{bmatrix} 1 & 4\pi/9 & 6\pi/9 & -12\pi/9 & 4\pi/9 & 0 & -6\pi/9 & 4\pi/9 & 12\pi/9 \end{bmatrix}$$

GCL for N = 16 :

$$\begin{bmatrix} 1 & 7\pi/16 & 12\pi/16 & 15\pi/16 & -\pi & -17\pi/16 & 12\pi/16 & 7\pi/16 & 1 & -9\pi/16 & 12\pi/16 & -\pi/16 & -\pi & -\pi/16 & 12\pi/16 & 23\pi/16 \end{bmatrix}$$

GCL for N = 25 :

$$\begin{bmatrix} 1 & 8\pi/25 & 14\pi/25 & 18\pi/25 & 20\pi/25 & -6\pi/25 & -3\pi/25 & 14\pi/25 & 8\pi/25 & 0 & -10\pi/25 & -22\pi/25 & 14\pi/25 & -2\pi/25 & 30\pi/25 & -40\pi/25 & -12\pi/25 & 14\pi/25 & -12\pi/25 & 10\pi/25 & -20\pi/25 & -2\pi/25 & 14\pi/25 & 28\pi/25 & 8\pi/25 \end{bmatrix}$$

The figure ( 2 ) is the auto-correlation for the GCL sequence when N = 9 , the fig. ( 3 ) for N = 16.

### 3.3 Chirp sequences

Chirp signals are widely used in radar applications for pulse compression and were also proposed for use in digital communications by several authors. They refer to creation of such a waveform where an instantaneous frequency of the signal changes linearly between the lower and upper frequency limits. This is graphically illustrated in fig.(4), which presents the two basic types of chirp pulses and their instantaneous frequency profiles [8]: We can write a formula defining a complex polyphase chirp sequence for the above single chirp pulse :

$$B_n^{(h)} = \exp(j2\pi bn), \quad n = 1, 2, \dots, N \tag{14}$$

$$bn = (n^2 - nN)/2N^2 \tag{15}$$

and h can take any arbitrary nonzero real value, fig.(5-8) is the auto

correlation for the single chirp sequence for N=13,16,32&25 respectively.

The main advantage of chirp sequences compared to other known sets of sequences, lies in that we can easily generate the set for any given length N, on the other hand, most of the known sets of sequences can be generated only for a certain values of N . The values of parameter h for the sequences can be optimized to achieve:

1. Minimum multi-access interference (MAI) by minimizing the mean square aperiodic cross-correlation (RCC).
2. The best system synchronizability by minimizing the mean square aperiodic auto-correlation (RAC).
3. Minimum peak interference by minimizing the maximum value for the aperiodic CCFs, ACCFmax, over the whole set of the sequence [9].

A pulse is referred to as a chirp pulse of the order s, if and only if the first time derivative of its instantaneous frequency is a step function with the number of time intervals is equal to s:

$$\int_0^T f_s(t) dt = 0 \tag{16}$$

Then, such a pulse is called a baseband chirp pulse of the order s. As an example, a baseband chirp pulse of orders 2&4 are shown in fig. (9)&fig.(10), the presented pulses are symmetrical[9].

The formula of the elements (d<sub>n</sub>) & (q<sub>n</sub>) for complex double & quadruple chirp sequences respectively are :

$$d_n = \begin{cases} (2n/N) - (n/N) & , 0 < n \\ (-2n/N) + (3n/N) & , N/2 < n \\ 0 & , \text{otherwise} \end{cases} \quad (17)$$

The complex double chirp sequence elements  $d^n$  are therefore given by :

$$D^n = \exp(j2\pi h_n d_n); n=1,2,..,N \quad (18)$$

And corresponding to the chirp pulse of the order 4, the elements  $q_n$  are expressed as:

$$q_n = \begin{cases} (4n/N) - (n/N) & , 0 < n \\ (-4n/N) + (3n/N) & , N/4 < n \\ (4n/N) - (5n/N) & , 3/2 < n \\ (4n/N) - (7n/N) & , 3N/4 < n \\ 0 & , \text{otherwise} \end{cases} \quad (19)$$

and the elements of the complex quadruple chirp sequences are given by :

$$Q^n = \exp(j2\pi h_n q_n); n=1,2,..,N \quad (20)$$

The auto-correlation for the double and quadruple chirp sequences are shown in fig.( 11 - 14 ) for  $N=16,25,32$ .

Another class of higher order chirp sequences, can be obtained if a superposition of chirp sequences of different order  $s$  is used to create the complex polyphase sequence [10,11].

**PROPOSED SETS OF POLYPHASE SEQUENCES**

As shown in section(3), the single, double and quadruple chirp

sequences ( $B_n, D_n$  &  $Q_n$ ) have good correlation properties, but if we use a superposition to obtain another two sets of chirp sequences which are :

$$W_n = \exp(j2\pi (c_1 b_n + c_2 d_n)) \quad (21)$$

$$Q'_n = \exp(j2\pi (c_1 b_n + c_2 d_n + c_3 q_n)) \quad (22)$$

Where,  $b_n = (n^2 - nN)/2N^2$

$$d_n = \begin{cases} (2n/N) - (n/N) & , 0 < n \\ (-2n/N) + (3n/N) & , N/2 < n \\ 0 & , \text{otherwise} \end{cases}$$

$$q_n = \begin{cases} (4n/N) - (n/N) & , 0 < n \\ (-4n/N) + (3n/N) & , N/4 < n \\ (4n/N) - (5n/N) & , 3/2 < n \\ (4n/N) - (7n/N) & , 3N/4 < n \\ 0 & , \text{otherwise} \end{cases}$$

The new sets of polyphase sequences obtained by multiplying these chirp sequences by other sequences, which have good aperiodic correlation properties as follows:

**a) Chirp – Sequence Multiplied by Other Binary Sequences**

In this paper Barker and Willard sequence codes are used. As a result of the multiplication process, other polyphase sequences obtained with different phases. Multiplying the single chirp polyphase sequence ( $B_n$ ) by Barker code, the correlation properties are better than multiplying ( $B_n$ ) by the Willard code, but the double chirp ( $D_n$ ) and ( $W_n$ ) by Willard has better correlation properties than that of ( $W_n$ ) & ( $D_n$ )



by Barker code, as shown in the fig.(15-18).

**b)Chirp Sequence Multiplying by Other Polyphase Sequences**

In this paper modulatable orthogonal sequence with the sequence lengthy (N=9) and Generalized chirp-like (GCL) sequence with different sequence lengths ( N = 9, 16 & 25) are used :

- On = Bn\*Modulatable for N = 9
- On = Wn\*Modulatable for N = 9
- On = Bn\*Wn for N = 13&16
- On = Bn\*GCL for N = 9,16&25
- On = Wn\*GCL for N = 9,16&25

Sets of polyphase sequences which have good aperiodic correlation properties (auto and cross correlation) obtained as shown in fig.(19-39). The values of  $h, c_1, c_2$  &  $c_3$  are obtained by optimization.

**1. Tests of proposed sets with the auto & cross correlation functions**

In order to compare different sets of spreading sequences, a standard or quantitative measure needed for the judgment. They are based on the correlation functions of the set of sequences, since both the level of multi-access interference and synchronization amiability depend on the cross-correlation between the sequences and the auto-correlation functions of the sequences, respectively [9].

**AUTO-CORRELATION AND CROSS-CORRELATION FUNCTIONS AND THEIR PROPERTIES**

Let us consider two complex sequences  $\{a_n\}$ , and  $\{b_n\}$ , both

having a period N. Their discrete cross-correlation function  $R_{a,b}(\tau)$  is defined as :

$$R_{a,b}(\tau) = \sum_{n=0}^{N-1} a_n b_{n+\tau}^* \quad (23)$$

Where  $b_n^*$  denotes the complex conjugate of  $b_n$ .

The discrete auto-correlation function  $R_a(\tau)$  of the sequence  $\{a_n\}$  is defined as:

$$R_a(\tau) = \sum_{n=0}^{N-1} a_n a_{n+\tau}^* \quad (24)$$

Very often, instead of the above unnormalized correlation functions, their normalized equivalents are used :

$$R_{a,b}(\tau) = (1/N) \sum_{n=0}^{N-1} a_n b_{n+\tau}^* \quad (25)$$

$$R_a(\tau) = (1/N) \sum_{n=0}^{N-1} a_n a_{n+\tau}^* \quad (26)$$

The defined above discrete periodic correlation functions have got several useful properties, below is the short summary of some of the most useful of them :

1.The auto-correlation is an even function of  $\tau$  :

$$R_a(-\tau) = R_a(\tau) \quad (27)$$

2.The peak auto-correlation occurs at zero delay :  $R_a(0) \geq R_a(\tau), \tau \neq 0$  (28)

3.The cross-correlation functions have the following symmetry :

$$R_{a,b}(\tau) = R_{a,b}(-\tau) \quad (29)$$

4. Cross-correlation functions are not, in general, even function, and their peak values can be at different delays for different pairs of the sequences.[9]

#### RESULTS DISCUSSION

Generalized Chirp-Like (GCL)-Chirp polyphase sequence show the best performance and a better correlation properties, for the length  $N=9, 16$  &  $25$  which are under test, the peak side-lobe levels of the normalized auto-correlation function are  $0.15, 0.18$  &  $0.15$  for GCL-Single chirp, and  $0.11, 0.29$  &  $0.21$  for GCL-Double chirp respectively. On the other hand, the peak cross-correlation magnitudes are  $0.49, 0.32$  &  $0.55$  for GCL-Single chirp, and  $0.5, 0.38$  &  $0.45$  for GCL-Double chirp, respectively.

For the Barker-Chirp polyphase sequence, the Barker-Single chirp has the better correlation properties than the Barker-Double chirp according to the cross-correlation for which it is  $0.4$  for Barker-Single chirp and  $0.51$  for Barker-Double chirp sequence. On the other hand, Willard-Double chirp has better auto and cross-correlation properties than the Willard-Single chirp, for which it is  $0.33$  peak side-lobe auto-correlation magnitude and  $0.4$  peak cross-correlation magnitude and for the Willard-Double chirp sequence, the peak side-lobe auto-correlation magnitude is  $0.22$  and peak cross-correlation magnitude is  $0.32$ .

Finally, for the Modulatable-Chirp sequence, the Modulatable-Single chirp polyphase sequence has better cross-correlation properties which is equal to  $0.39$ , but for the

Modulatable-Double chirp sequence is equal to  $0.62$ .

For the proposed new sets of GCL-Chirp and the Modulatable Chirp - polyphase sequences, there is no limitation in sequence length, but for the Barker-Chirp and Willard-chirp, the length is limited according to the limitation of the Barker and Willard sequences length.

#### CONCLUSIONS

In this paper, sets of orthogonal polyphase sequences are proposed. The proposed sets based on utilizing a polyphase chirp sequence. The complex coefficients are obtained from the superposition of base-band chirp sequences and other sequences like Barker, Willard, modulatable orthogonal polyphase sequence and generalized chirp-like (GCL) polyphase sequence.

The resultant polyphase sequences can be optimized to achieve desired correlation properties of the set. It can be considered that a single chirp-Barker polyphase sequence has better correlation properties than a double chirp-Barker polyphase sequence, on the other hand, the double chirp-Willard polyphase sequence has better correlation properties than a single chirp-Willard polyphase sequence.

Modulatable-double chirp polyphase sequence correlation properties are better than the modulatable-single chirp polyphase sequence. GCL-double chirp correlation properties for the length  $N=25$  is better than the GCL-single chirp, but for the sequence length  $N=9$ , GCL-single chirp polyphase sequence has better correlation properties than GCL-double chirp polyphase sequence.

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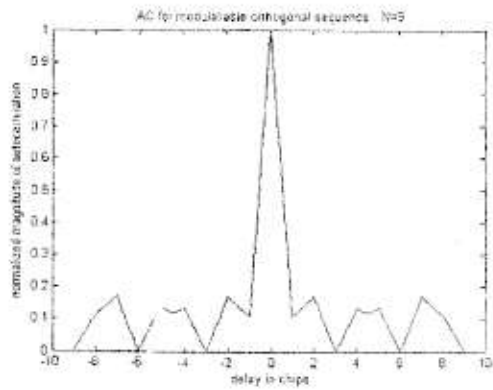


Figure (1) :Auto-correlation for an orthogonal modulated sequence, N=9

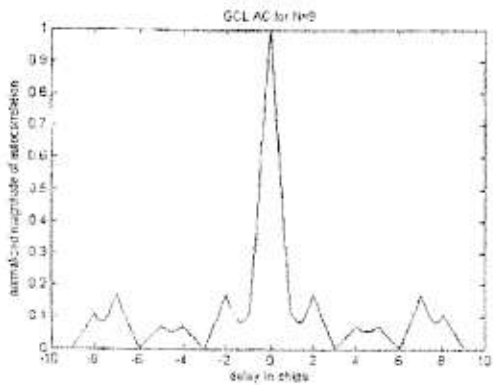


Figure (2): Auto-correlation for GCL, N=9

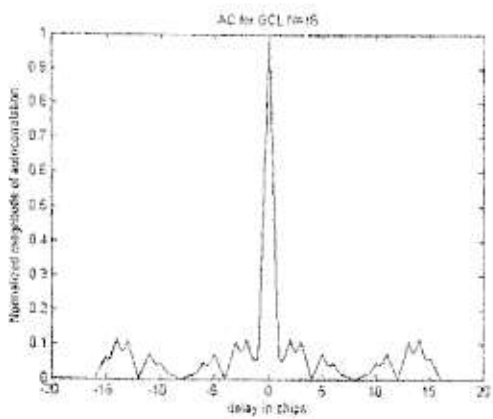


Figure (3) : Auto-correlation for GCL, N=16

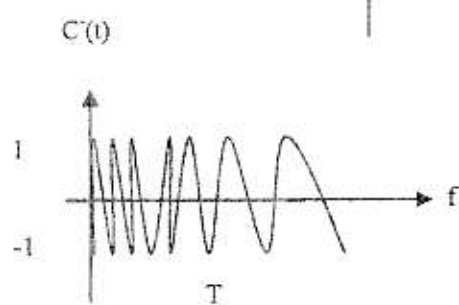
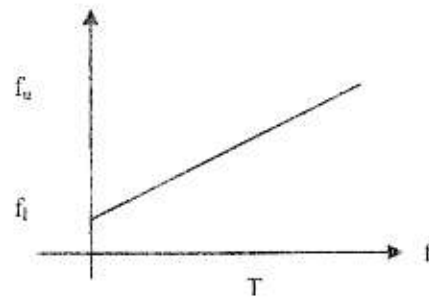
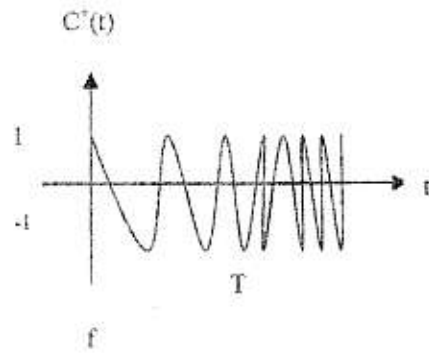


Figure (4):Chirp pulses and their instantaneous frequency profiles.

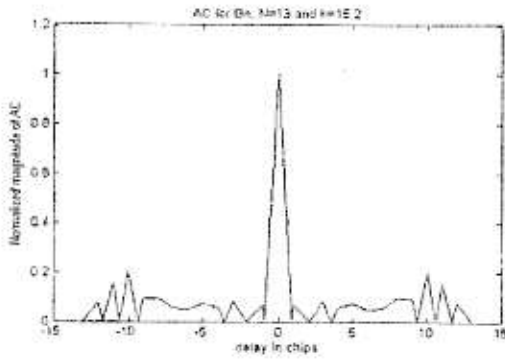


Figure (5) : Auto-correlation for single-chip sequence, N=13

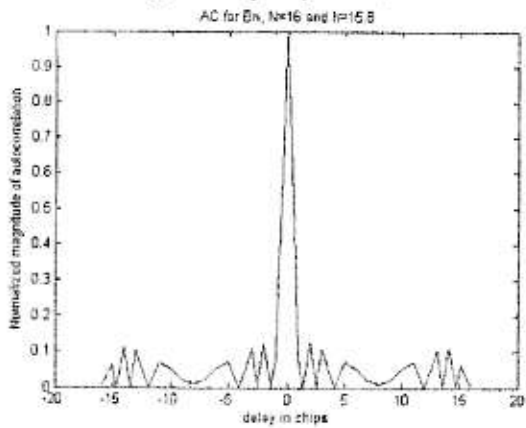


Figure (6) : Auto-correlation for single-chip sequence, N=16

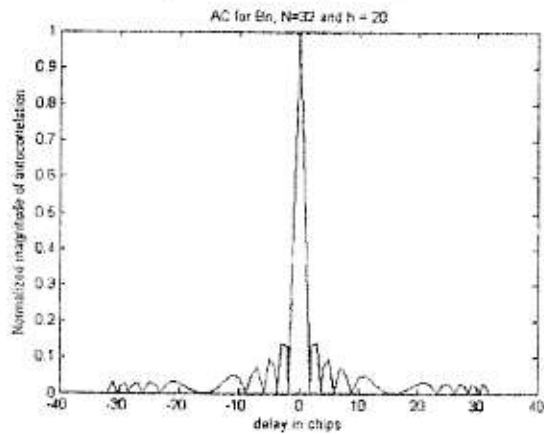


Figure (7) : Auto-correlation for single-chip sequence, N=32

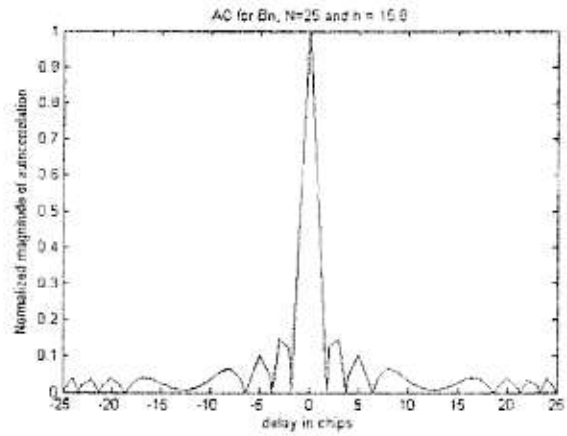


Figure (8) : Auto-correlation for single-chip sequence, N=25

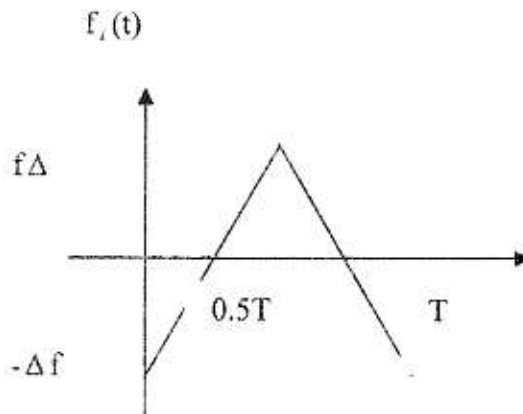


Fig.(9) Example baseband chirp pulses of the order (2)

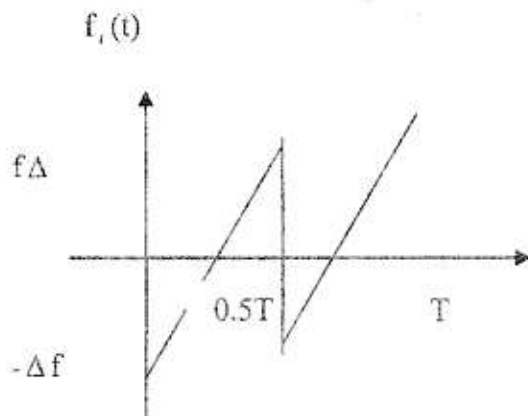


Fig.( 10 ) Example baseband chirp pulses of the order (4)

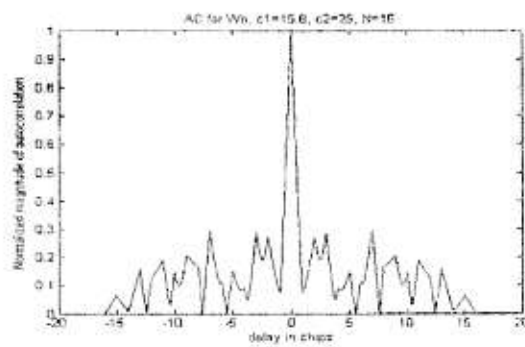


Fig.(11) : Auto-correlation for double-chirp sequence, N=16

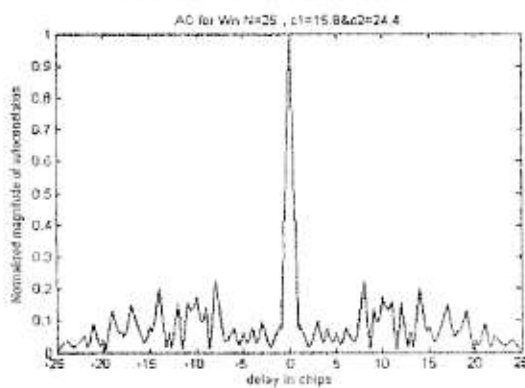


Fig.(12): Auto-correlation for double-chirp sequence, N=25

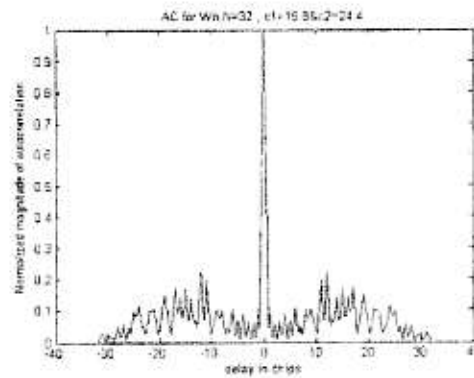


Fig.(13) : Auto-correlation for single-chirp sequence, N=32

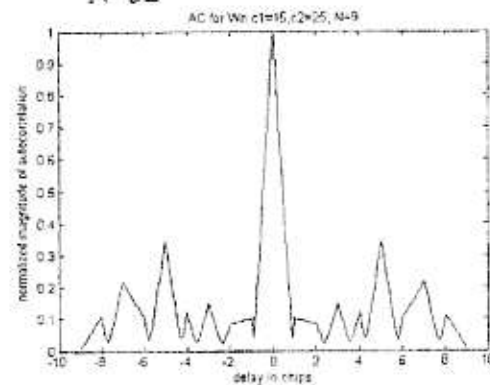


Fig.(14) : Auto-correlation for double-chirp sequence, N=9

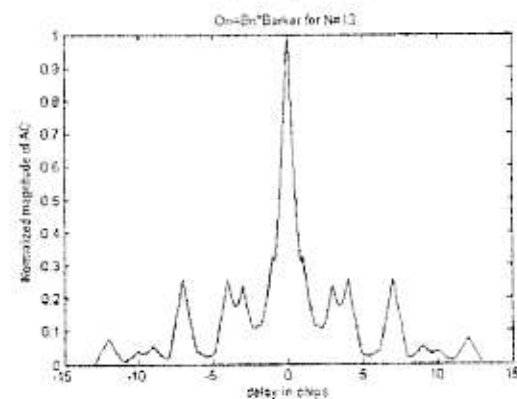


Fig.(15) : Auto-correlation of Bn\*Barker sequence, N=13

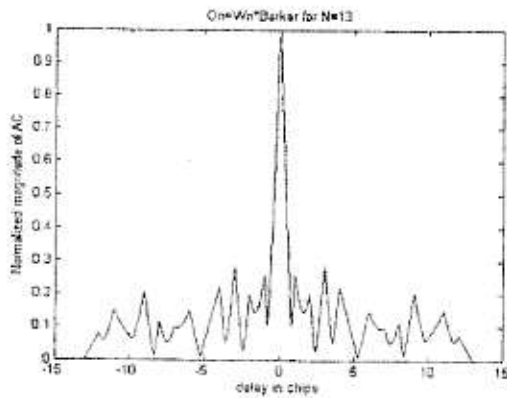


Fig.(16) : Auto-correlation of  $W_n$ \*Barker sequence,  $N=13$

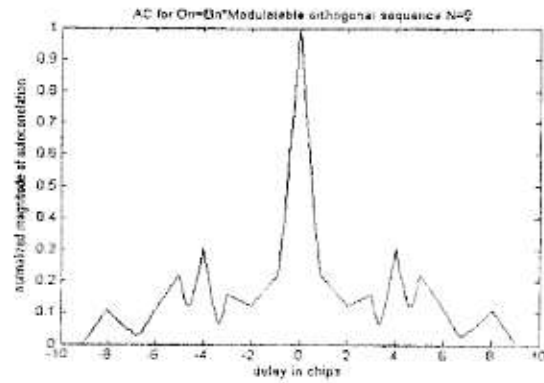


Fig.(19) : Auto-correlation of  $B_n$ \*Modulatable orthogonal sequence,  $N=13$

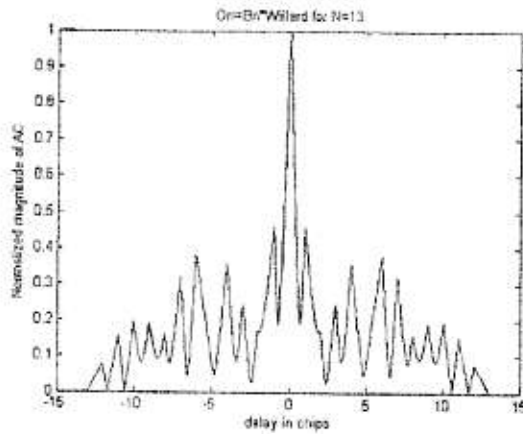


Fig.(17) : Auto-correlation of  $W_n$ \*willard sequence,  $N=13$

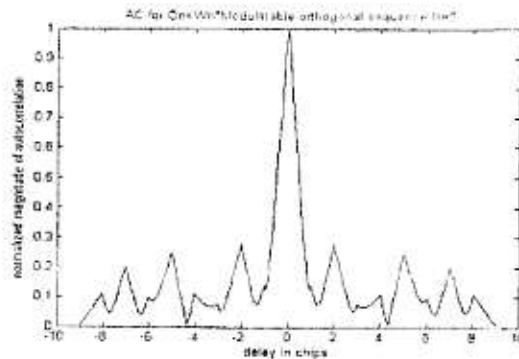


Fig.(20): Auto-correlation of  $W_n$ \*Modulatable orthogonal sequence,  $N=13$

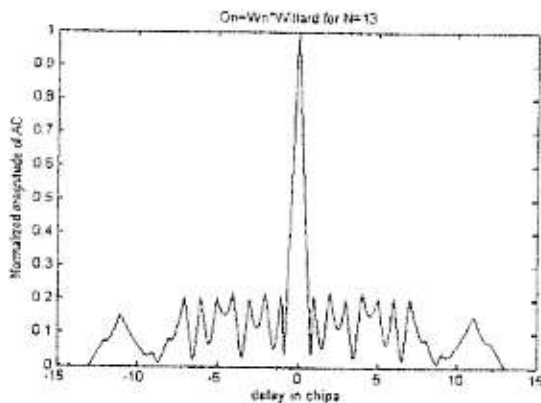


Fig.(18) : Auto-correlation of  $W_n$ \*Barker sequence,  $N=13$

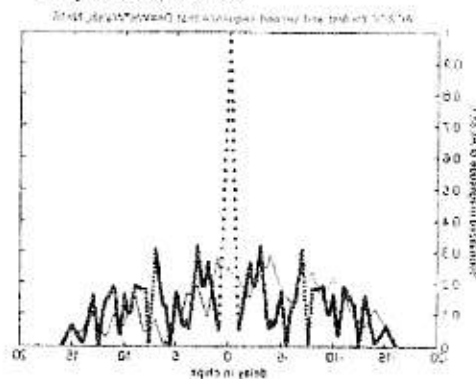


Fig.(21) : Auto&cross correlation for  $W_n$ \*Walsh sequence,  $N=16$

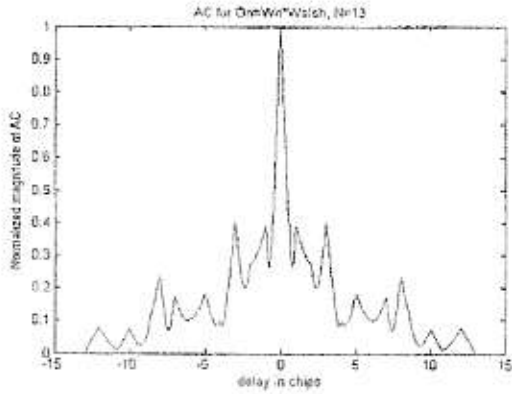


Fig.(22) : Auto-correlation for  $W_n$ \*Walsh sequence,  $N=13$

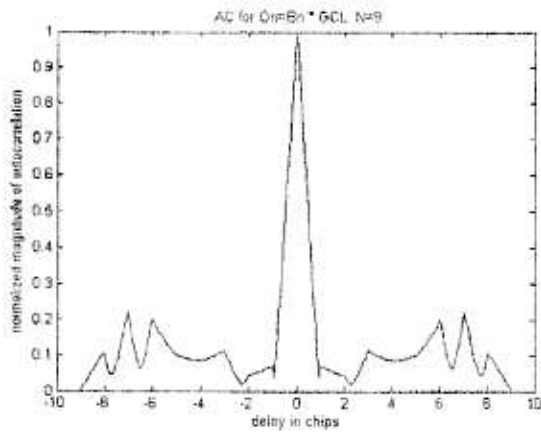


Fig.(23) : Auto-correlation for  $B_n$ \*GCL sequence,  $N=9$

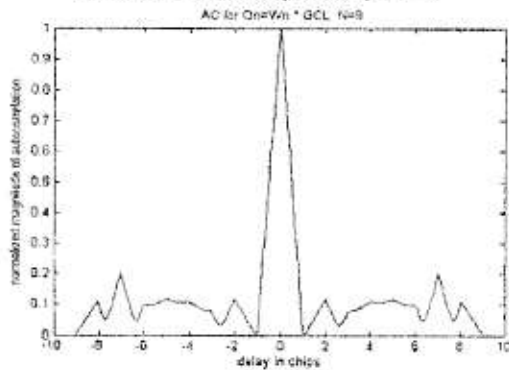


Fig.(24) : Auto-correlation for  $W_n$ \*GCL sequence,  $N=9$

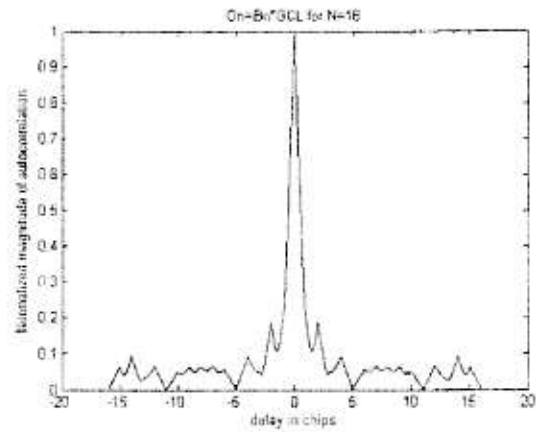


Fig.(25) : Auto-correlation for  $B_n$ \*GCL sequence,  $N=16$

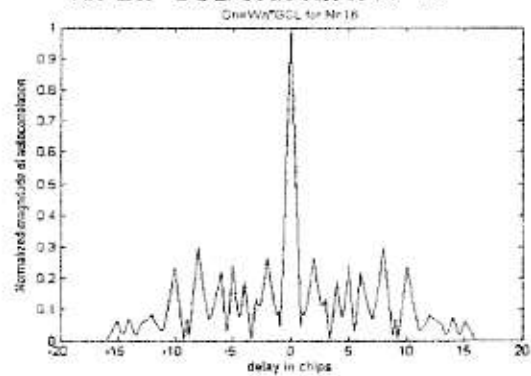


Fig.(26) : Auto-correlation for  $W_n$ \*GCL sequence,  $N=16$

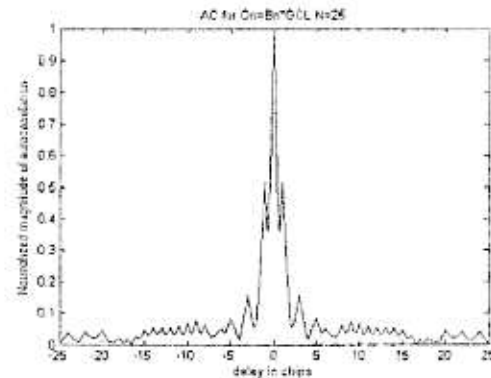


Fig.(27) : Auto-correlation for  $B_n$ \*GCL sequence,  $N=25$



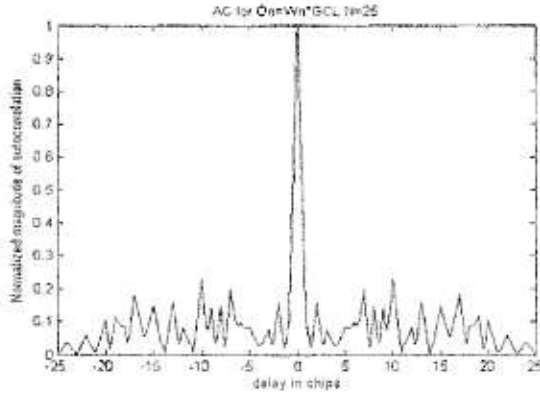


Fig.(28) : Auto-correlation for  $W_n * GCL$  sequence,  $N=25$

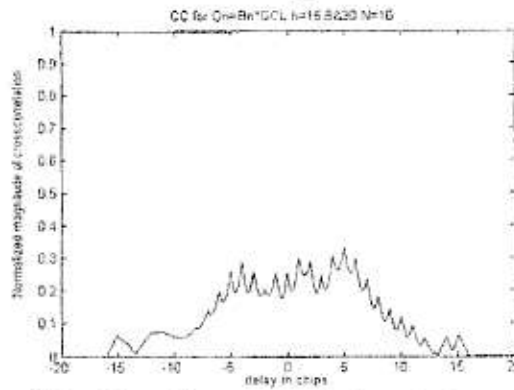


Fig.(31) : Cross correlation between two  $B_n * GCL$  sequences,  $N=16$

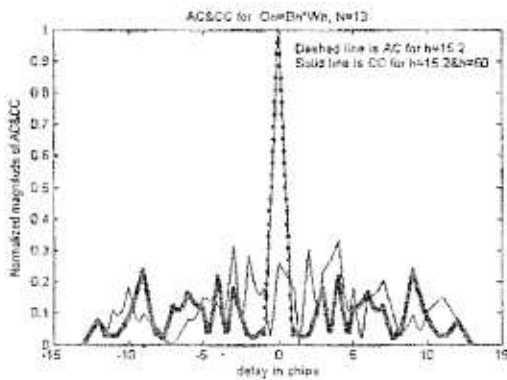


Fig.(29): Auto&cross correlation for  $B_n * W_n$ ,  $N=13$

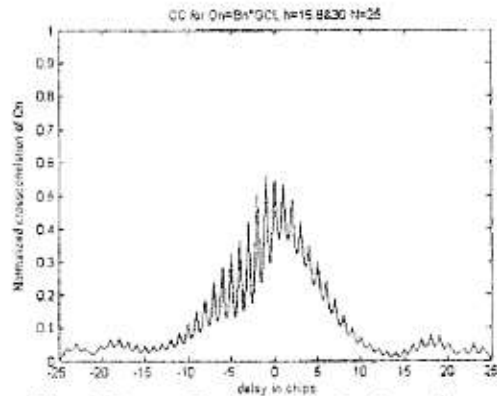


Fig.(32) : Cross correlation between two  $B_n * GCL$  sequences,  $N=25$

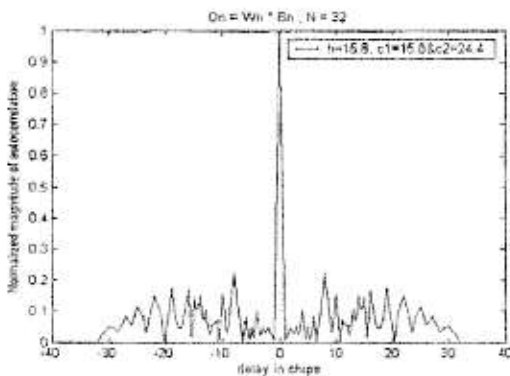


Fig.(30) : Auto-correlation for  $B_n * W_n$ ,  $N=32$

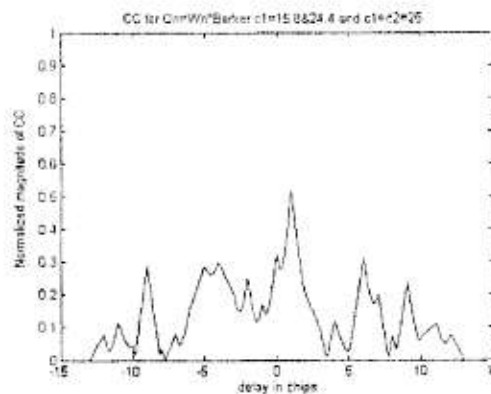


Fig.(33) : Cross-correlation between two  $W_n * Barker$  sequences,  $N=13$

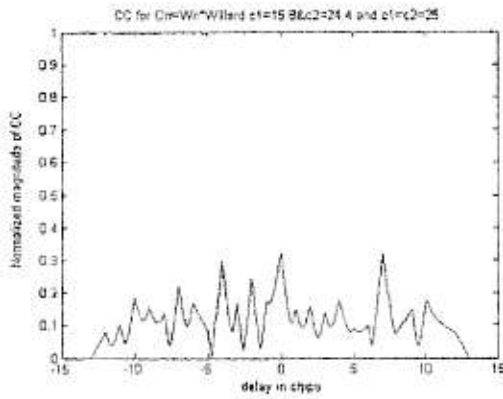


Fig.(34) : Cross-correlation between two  $W_n$ \*Willard sequences,  $N=13$

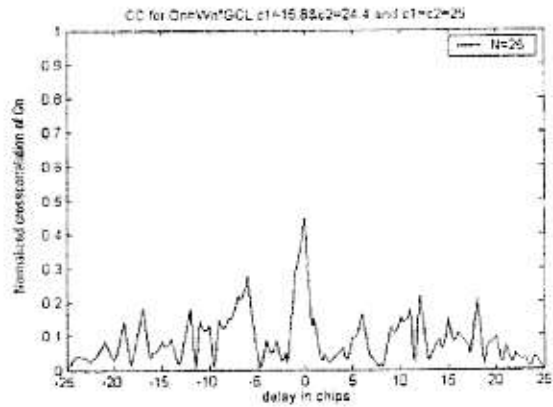


Fig.(37) : Cross-correlation between two  $W_n$ \*GCL sequences,  $N=25$

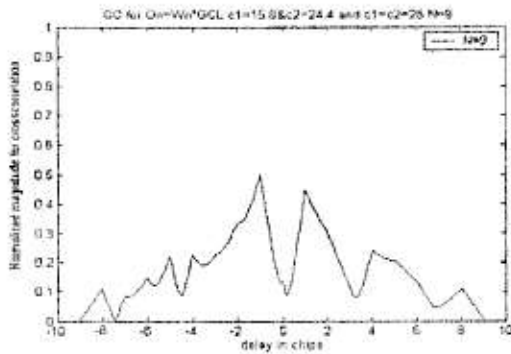


Fig.(35) : Cross-correlation between two  $W_n$ \*GCL sequences,  $N=9$

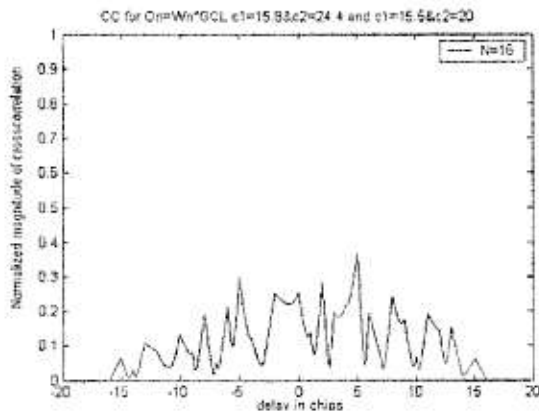


Fig.(36) : Cross-correlation between two  $W_n$ \*GCL sequences,  $N=16$

Table (1) Barker and Willard codes

N	Barker sequence	Willard sequence
3	110	110
4	1110 or 1101	1100
5	11101	11010
7	1110010	1110100
11	11100010010	11101101000
13	1111100110101	1111100101000