

Shear Design Of High And Normal Rc Beams Without Web Reinforcement

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Abstract

This work examines the major parameters that influence the shear strength of reinforced concrete (RC) beams without web reinforcement. These include the shear span/depth (a/d) ratio (between 2.0 and 7.1), concrete compressive strength f'_c (between 20.0 MPa and 101.9 MPa), the longitudinal steel ratio (ρ_w) (between 0.00251 and 0.06620), and beam size ($b_w d$).

271 RC beams failing in shear available in the literature are used to study the effect of the indicated major parameters on the strength of Normal Strength Concrete (NSC) and High Strength Concrete (HSC). Proposed design equations are compared with the existing shear design relationships of the ACI Code 318M-02, BS 8110, Canadian Code, New Zealand Code and Zsutty equation to predict the shear capacity of RC beams.

For all methods considered, the ratio of shear strength of beams V_{rTEST} to the design shear resistance V_{rDES} is calculated. The proposed design equations lead to safe design with a low coefficient of variation (COV). This COV is only 18.6 percent which is significantly less than values obtained for other methods (ranging between 28.5 to 43.3 percent).

الخلاصة

في هذا البحث تم دراسة تأثير المتغيرات الرئيسية على إجهاد القص للعتبات الخرسانية المسلحة غير الحاوية على تسليح القص، والتي تشمل نسبة فضاء القص الى العمق المؤثر (a/d)، مقاومة الإنضغاط (f'_c)، نسبة حديد التسليح (ρ_w) ومساحة مقطع العتبة ($b_w d$). تم استعمال نتائج 271 عتبة متوفرة في (المصادر السابقة)، قد فشلت في إجهاد القص مصنعة من خرسانة اعتيادية المقاومة (NSC) وخرسانة عالية المقاومة (HSC) لغرض اشتقاق معادلات تصميم مقترحة لتحمل إجهاد القص. تم مقارنة نتائج المعادلات المقترحة مع نتائج معادلات التصميم للمواصفات الامريكية، البريطانية، الكندية، النيوزلندية ومعادلة الباحث Zsutty. المعادلات التصميمية المقترحة أعطت أقل

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القيم لمعامل التغيرات (COV) والتي بلغت 18.6% عند مقارنتها مع بقية المعادلات التي أعطت قيم معامل التغيرات تتراوح بين 28.5% و 43.3%.

Keywords : Longitudinal steel ratio, Normal- and High-strength concrete, Shear properties, Size effect, Span-depth ratio, Standards.

Notations

- a = Shear span, distance between concentrated load and face of support, mm.
a/d = Shear span to depth ratio.
A_s = Area of tension reinforcement, mm².
b_w = Web width of beam, mm.
d = Effective depth of the beam, mm.
f_c = Specified compressive strength of (150 x 300 mm) concrete cylinders, MPa.
M_u = Factored moment at section.
V_c = Shear strength provided by concrete of beams without stirrups, N.
V_{r,ACI} = Design shear resistance by Eq.(1).
V_{r,BS} = Design shear resistance by Eq.(2).
V_{r,CAN} = Design shear resistance by Eq.(3).
V_{r,DES} = Design shear resistance.
V_{r,NZ} = Design shear resistance by Eq.(4).
V_{r,PROP} = Design shear resistance by Eq.(8).
V_{r,ZST} = Design shear resistance by Eq.(5).
V_{TEST} = Test shear strength of beam without stirrups.
V_u = Factored shear force at section, N.
ρ_w = Ratio of tension reinforcement = A_s/(b_wd) .
φ = Strength reduction factor.

Introduction

Several methods^[1-4] permit the use of simple cross sectional design for concrete shear resistance when a/d ≤ 2. In two methods^[1,2], a/d < 2 leads to a new category to calculate the concrete resistance by using a

magnifying factor of 2d/a. The Canadian Code^[3], however, requires that the beam with a/d < 2 cannot be designed by the simple method based on the cross-section.

Until recently the ACI Code^[4] permitted the design for beams with low a/d values - the

so called "deep beams". In the latest ACI Code^[5] it is no longer permitted to have simple beam shear design based on the cross-section. This latest code is influenced by reference 6 where there is a "strut-and-tie" design theory in Appendix A of this code essentially taken from this reference .

Factors Affecting Shear Strength of RC Beams

1. Compressive Strength of Concrete (f'_c)

Some codes of practice imply that beam shear strength is proportional to $(f'_c)^{0.5}$ [2,3-5], others assume that it is proportional to $(f'_c)^{1/3}$ [1] and $(f'_c)^{2/3}$ [7]. In these codes of practice it is assumed that the nominal shear strength provided by the concrete section is equal to the shear causing inclined cracking. Taylor^[8] tested beams without shear reinforcement and with different values of concrete compressive strength (ranging from 21 to 42 MPa). It was found that the diagonal cracking load increases with increasing concrete strength. Zsutty^[9,10] used a combination of dimensional and regression analysis to show that both the diagonal cracking and the

ultimate shear capacities are directly proportional to $(f'_c)^{1/3}$.

2. Shear Span to Effective Depth Ratio (a/d)

The possible types of failure of reinforced concrete beams with different a/d ^[11] ratios are illustrated in Fig. 1

- a- True-shear ($a/d < 1$)
- b- Shear-compression or shear-tension ($1 < a/d < 2.5$)
- c- Diagonal tension failure ($2.5 < a/d < 6$)
- d- Flexural failure ($6 < a/d$)

In type (a) and (b) failure modes, the ultimate shear strength significantly exceeds the inclined cracking strength, while in type (c) mode the ultimate strength is approximately equal to the cracking strength.

3. Longitudinal Reinforcement Ratio (ρ_w)

Many researches stated that the tension steel ratio (ρ_w) has a significant influence on beam shear strength. This is based on theoretical considerations as well as on test results. De Cassio and Siess^[12] tested beams with a range of 1.0 % - 3.3 % for ρ_w . It was concluded that the shear capacity is approximately a linear function of ρ_w . Krefeld and Thurston^[13] concluded that the longitudinal reinforcement participates in resisting the

external shear by a modified dowel action which depends on bar size, spacing, depth of cover below the bars and f'_c . Rajagopalan and Ferguson^[14] analyzed results of 38 beam tests with $\rho_w < 1.0\%$ and $a/d > 2.75$. It was found the design equation of ACI Code of that time to be unconservative at low ρ_w ratios.

4. Size Effect

References 15-17 studied the size effect as well as the effect of the maximum aggregate size on the shear strength of longitudinal RC beams by means of a nonlinear fracture mechanics model. Structure size represented by the depth as well as the maximum aggregate size was normalized to intrinsic length parameters of the concrete. This length parameter is proportional to the fracture energy of the concrete. It was found^[18] that the shear strength of RC beams may be equally sensitive to fracture energy as to the tensile strength of the concrete.

Research Significance

HSC and NSC tests of beams of moderate slenderness ($a/d \geq 2.0$) are applied to several simple design methods, including code design originally

developed mainly from NSC research. It is shown that, at least without significant axial loading, some existing design methods lead to conservative design over a wide range of principal variables, while other methods do not. A proposed design method is found that leads to safe design with an improved COV.

Existing Test Results

A wide range of tests on rectangular beams failing in shear without stirrups and with $a/d \geq 2.0$ are supplied from the literature references 15,16,19-28.

Evaluation of Experimental Results

Existing Shear Design Equations

Many design equations were proposed or used in codes^[1-5,7,9-11,16,19,29-35]. Only six simple ones will be considered. Probably the best known is ACI Eq.(11-5)^[5]. The Canadian Code^[3] permits simplified design when $a/d \geq 2$ by simultaneously using two distinct principles. First, concrete contribution is influenced only by $(f'_c)^{0.5}$, as with the widely known ACI Eq.(11-3), for example. Secondly the material reduction factor ϕ has a low value 0.60 for

concrete contribution. Eq.(2) is used to apply test results to these two principles.

In 1979, ACI-ASCE Committee 426^[32] presented a major contribution to shear design. Although ACI Committee 318 has not adopted Committee 426 recommendations, the New Zealand code^[2] has. Canadian code [Eq.(3)] is used to apply test results to this approach. Concrete contribution in the British standard design^[1] is more in line with recent tests, with respect to the influence of longitudinal steel and concrete strength. In addition, there is a size effect that enhances concrete contribution for members with $d < 400$ mm. For these reasons, Eq.(4) is included. Zsutty's equation^[9,10] is also well known and was recommended for further study by Committee 426^[11]. Since the lowest value of a/d or $(M_u/V_u d)$ is limited to 2, Eq.(5) is simplified further by excluding $2.5d/a$ from this equation – $2.5d/a$ being the magnification factor proposed by Zsutty^[10] when $a/d < 2.5$.

To compare between design methods with different material reduction factors, shear resistance force $V_{r\ DES}$ will be used instead of

nominal $V_{n\ DES}$ throughout.

ACI code method^[5]

$$V_{r\ ACI} = \phi V_c = 0.75 [(\sqrt{f'_c} + 120 \rho_w \frac{V_u d}{M_u}) / 7] b_w d \dots (1)$$

British standard method^[1]

$$V_{r\ BS} = 0.79 (100 \rho_w)^{1/3} (f_c/20)^{1/3} (400/d)^{1/4} b_w d / 1.25 \dots (2)$$

In Eq.(2): $f_c = 0.8 f_{cu}$; $(400/d)^{1/4}$ is used when $d < 400$ mm.

Canadian code method^[3]

$$V_{r\ CAN} = 0.6 [0.2 \sqrt{f'_c}] b_w d \dots (3)$$

New Zealand code method^[2]

$$V_{r\ NZ} = 0.85 [(0.07 + 10 \rho_w) \sqrt{f'_c}] b_w d \dots (4)$$

Zsutty's method^[10]

$$V_{r\ ZST} = 0.75 [2.2 (f_c \rho_w d/a)^{1/3}] b_w d \dots (5)$$

$\Phi = 0.75$ is used in line with the latest ACI code^[5].

Proposed Shear Design Equations

Sarsam and Abdulla^[19,33] had made proposals for concrete design essentially on a basis similar to Zsutty's approach^[10]. This is based on regression analysis to produce a relationship whereby three major factors (f_c , ρ_w and d/a) are raised to the same power – 1/3 for reference 2 and 0.38 for

references 19,33.

In this work, however, no limitation is made to any of the empirical constants A, B, C, D and E of Eq.(6)

$$V_r = A (f_c)^B (\rho_w)^C (1/a)^D b_w d^E \dots(6)$$

Nonlinear regression analysis was used in the present work to evaluate the constants A, B, C, D and E.

Thus Eq. (7) was obtained.

$$V_r = 9 (f_c)^{0.4} (\rho_w)^{0.4} (1/a)^{0.8} b_w d^{1.6} \dots(7)$$

In line with the latest ACI code^[5], Eq.(7) is written with a reduction factor $\phi = 0.75$. Also, to obtain a relationship for distributed loading, Eq.(7) is written twice as Eqs.(8a) and (8b). Eq.(8a) applies to principal concentrated loading and (8b) applies equally to uniform loading, as does Eq.(1), for example.

Thus

$$\text{either } V_{r \text{ PROP}} = 0.75 [12 (f_c \rho_w)^{0.4} (d/a)^{0.8}] b_w d^{0.8} \dots(8a)$$

$$\text{or } V_{r \text{ PROP}} = 0.75 [12 (f_c \rho_w)^{0.4} \left(\frac{V_u d}{M_u} \right)^{0.8}] b_w d^{0.8} \dots(8b)$$

Comparison of Design Methods

Table 1 compares six design methods for the 271 beams selected for the present work.

Based on the ratio of

$V_{\text{TEST}}/V_{r \text{ DES}}$, 3 methods are essentially conservative: Eq.(1) has only one low ratio at 0.93; Eq.(5) has only 3 cases with $V_{\text{TEST}}/V_{r \text{ DES}} < 1$ (the lowest being 0.95); Eq.(8) has all values ≥ 1 . The other methods [Eqs.(2-4)] do not qualify as conservative with low values ranging from 0.455 to 0.893. As measures of shear capacity representation, the two lowest COV values are with the proposed and Zsutty's method, at 18.6 and 28.5 percent, respectively. The latter which takes into account f_c , ρ_w and a/d is by far the best of the existing methods, which increasingly have COV values of 34.4, 36.0, 36.5 and 43.4 for the NZ, ACI, BS, and CAN methods respectively. The Highest value of 43.3 for the CAN method reflects the well known research conclusion that using f_c only for shear design does not reflect efficiently the resistance of the cross section^[9,19,29-33].

Influence of Major Parameters

Figs.2-5 show the influence of major parameters (f_c , ρ_w , a/d and size effect $b_w d / (b_w d)_{\min}$) on $V_{\text{TEST}}/V_{r \text{ DES}}$. Figs.2 and 3 show that the two methods with the least scatter are Eqs.(5) and (8). Fig.4 shows a clear tendency for a drop in safety factor with

rising a/d values, for Eqs.(1-4). This is because Eq.(1) underestimates the influence of a/d , while the other 3 [Eqs.(2-4)] do not even recognize the effect of a/d in shear design. These results contrast with the methods by Zsutty and the proposed one, which include a/d in a significant manner.

Fig. 5 shows the influence of $b_w d / (b_w d)_{\min}$ as an indication of the size effect. All existing methods [Eqs.(1-5)] show a significant drop in the factor of safety with increasing beam size. In contrast, with the proposed method [Eq.(8)] there is no such trend. This is because the proposed method takes into account the size effect, including multiplying the resisting stress by $b_w d^{0.8}$ instead of $b_w d$, as in the other five methods. Fig.6 confirms the influence of beam size in lowering the shear stress.

Conclusions

Based on the results of this work, the following conclusions are made.

1. Of the six methods, three are essentially conservative for HSC and NSC beams – ACI, Zsutty and the proposed method. The respective COV values are 36.0, 28.5 and 18.6 percent,

respectively.

2. The New Zealand code, British standard and Canadian code methods are less conservative than those indicated in conclusion 1.
3. Fig.2 shows that f_c values up to 101.8 MPa do not lower the safety factor of the ACI code, Zsutty or the proposed methods.
4. Because they either underestimate the influence of ρ_w (ACI code), or they do not include its influence (Canadian code), both methods show a rise in the safety factor with increasing ρ_w , Fig.3.
5. Fig.4 shows a clear trend for a drop in the safety factor with increasing a/d ratios in four methods – ACI, BS, Canadian and New Zealand codes. This contrast with Zsutty's method and the proposed one.
6. All five existing methods [Eq.(1-5)] show a significant drop in the factor of safety with increasing beam size. In contrast, the proposed design method shows no such trend.

Future Research

Since several design methods, as well as cases in practice, lead to design of beams with stirrups, this type of research is indicated for future

work.

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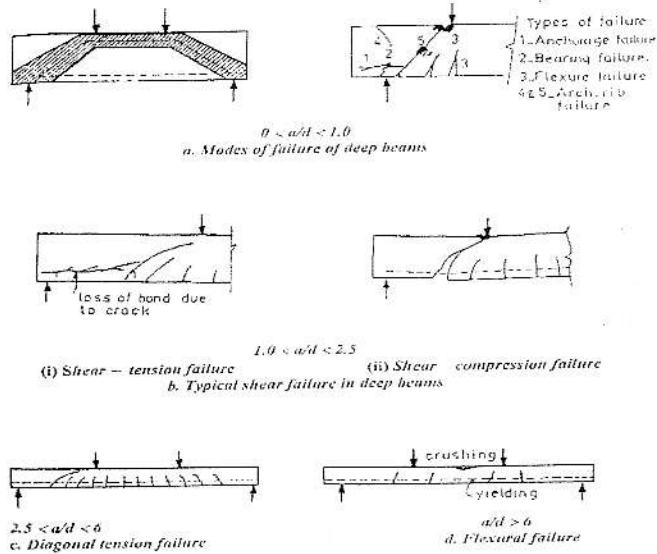


Fig. 1 - Mode of failure of reinforced concrete beams with different a/d ratios⁽¹¹⁾

Table (1): Comparison between V_{TEST} and $V_{r,DESIGN}$ for 271 beams

Ratio	$\frac{V_{TEST}}{V_{r,ACI}}$	$\frac{V_{TEST}}{V_{r,BS}}$	$\frac{V_{TEST}}{V_{r,CAN}}$	$\frac{V_{TEST}}{V_{r,NZ}}$	$\frac{V_{TEST}}{V_{r,ZST}}$	$\frac{V_{TEST}}{V_{r,PROP}}$
Equation used	1	2	3	4	5	8
Mean	2.147	1.506	2.258	1.63	1.524	1.493
Standard Deviation	0.772	0.55	0.977	0.561	0.435	0.278
COV%	35.97	36.51	43.26	34.44	28.51	18.62
Range - Low	0.930	0.893	0.856	0.455	0.954	1.073
High	7.98	6.29	10.52	4.675	4.837	3.14
High	8.583	7.03	12.29	10.27	5.071	2.925
Low			3			
Number < 1*	1	11	4	12	3	0

Notes: Ranges of variables - $f_c=20.1, 101.8$ MPa (ratio of 5.07); $\rho_w=0.0025, 0.0662$ (ratio of 26.37); $a/d=2.0, 7.06$ (ratio of 3.61); $b_w d=15930, 169225$ mm² (ratio of 10.62)

*Number < 1 indicates the number of specimens (out of 271) for which $V_{TEST} < V_{r,DES}$

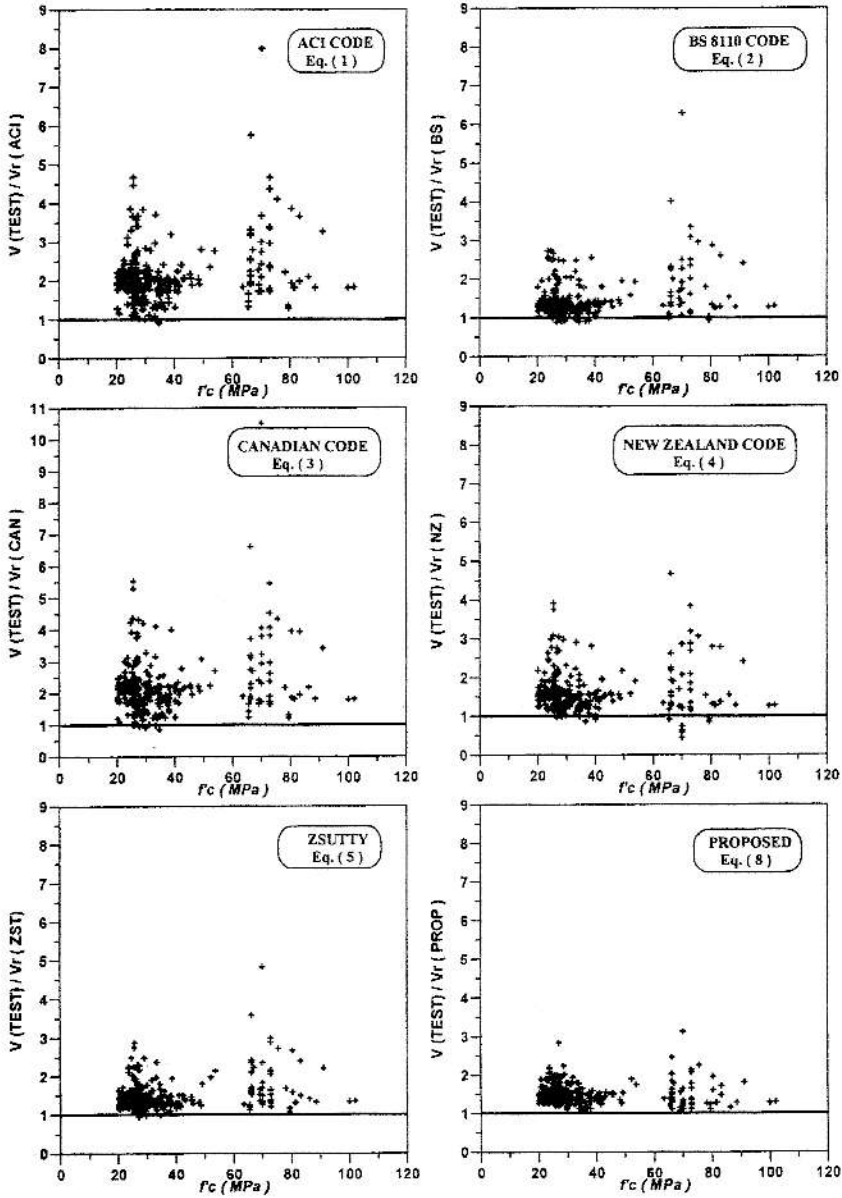


Fig. 2 - Influence of compressive strength f_c on relative shear strength

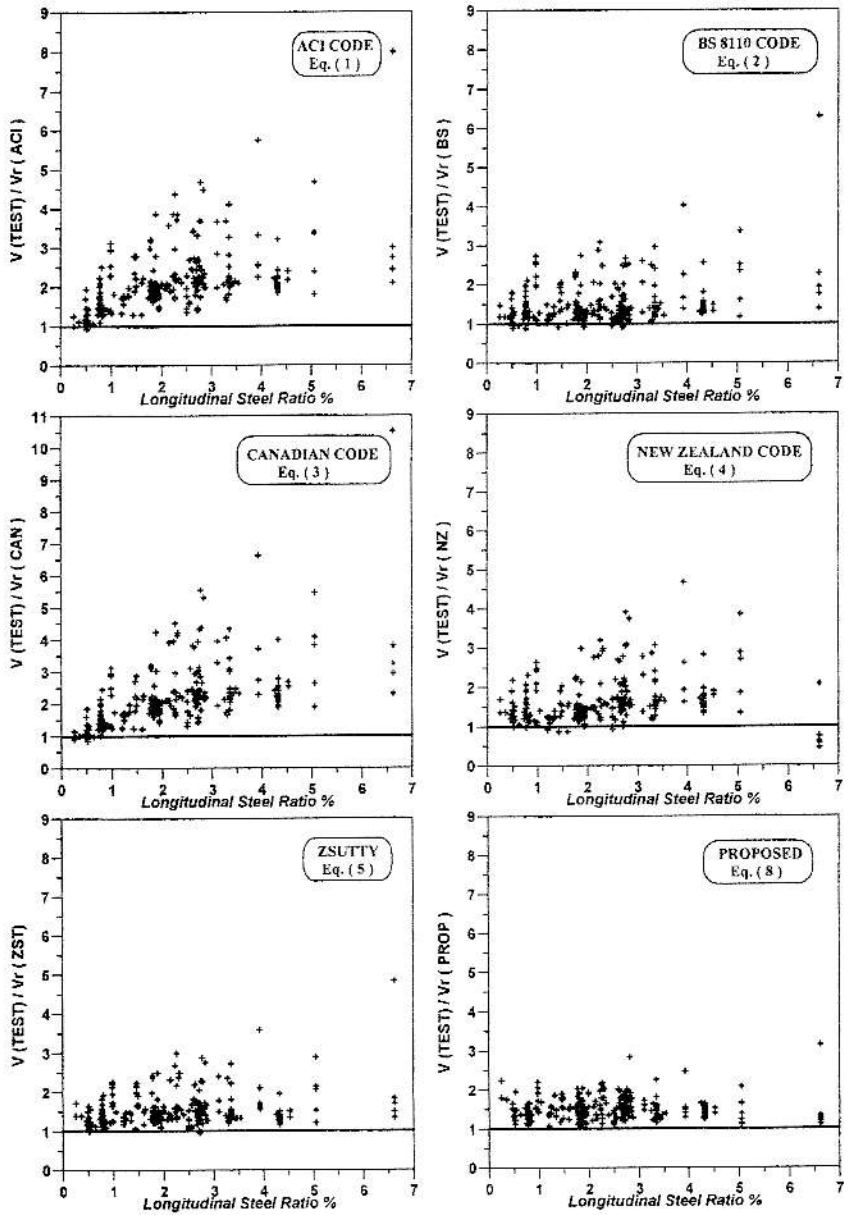


Fig. 3 - Influence of longitudinal steel ratio on relative shear strength

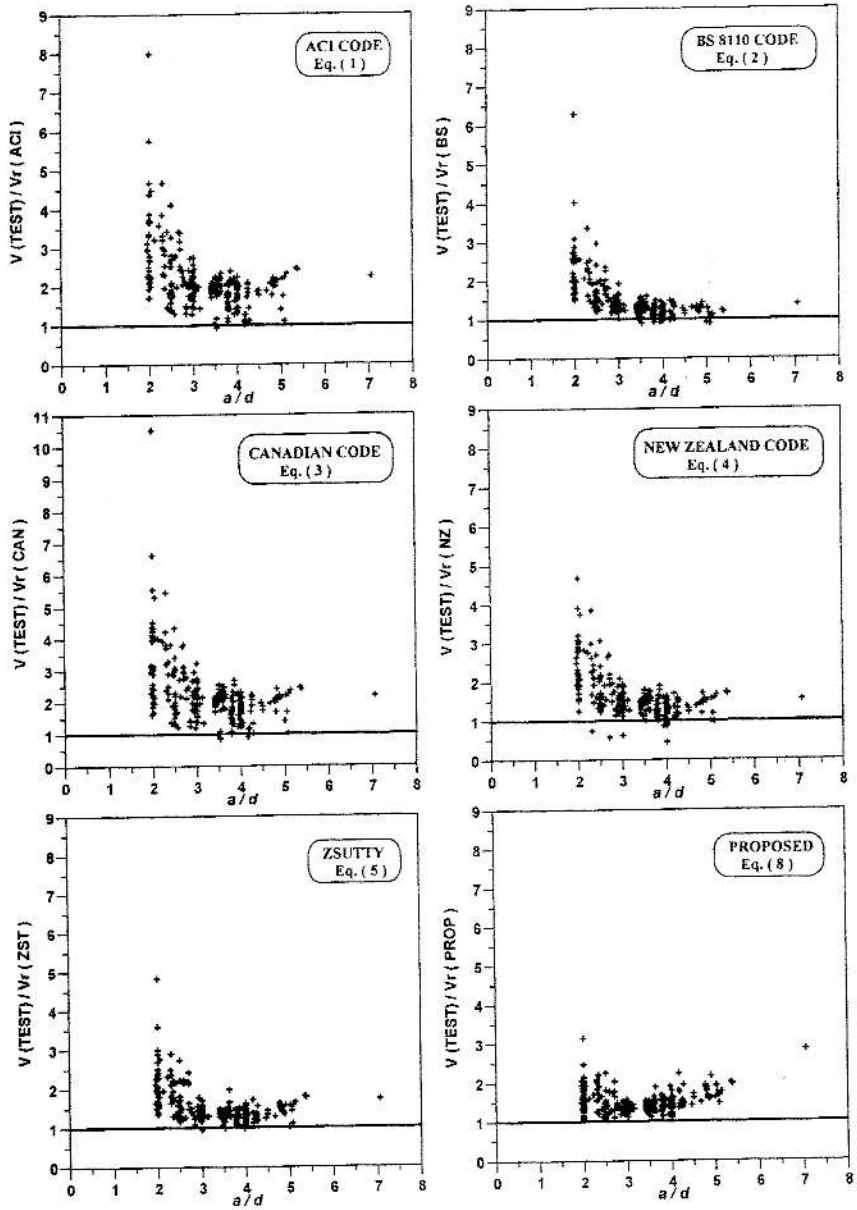


Fig. 4 - Influence of (a / d) ratio on relative shear strength

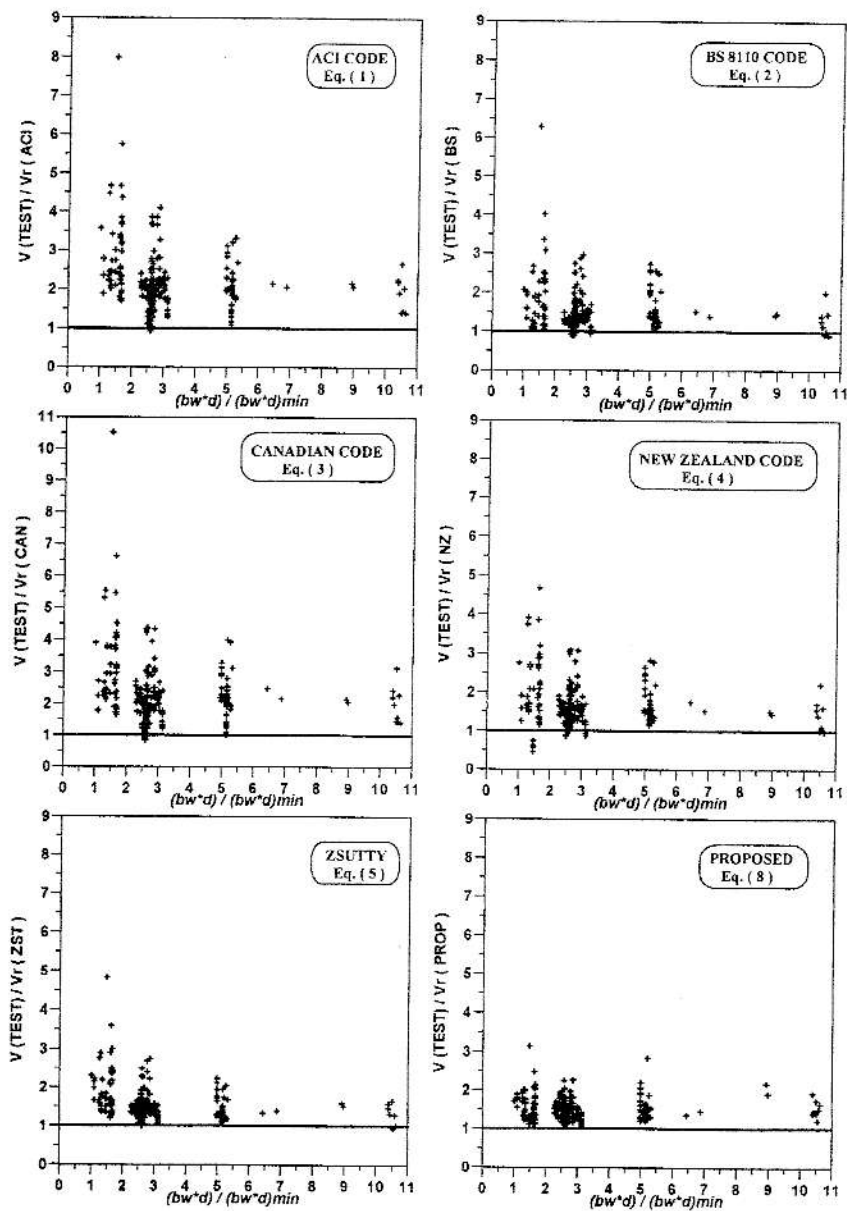


Fig. 5 - Influence of $(bw*d)/(bw*d)_{min}$ on relative shear strength

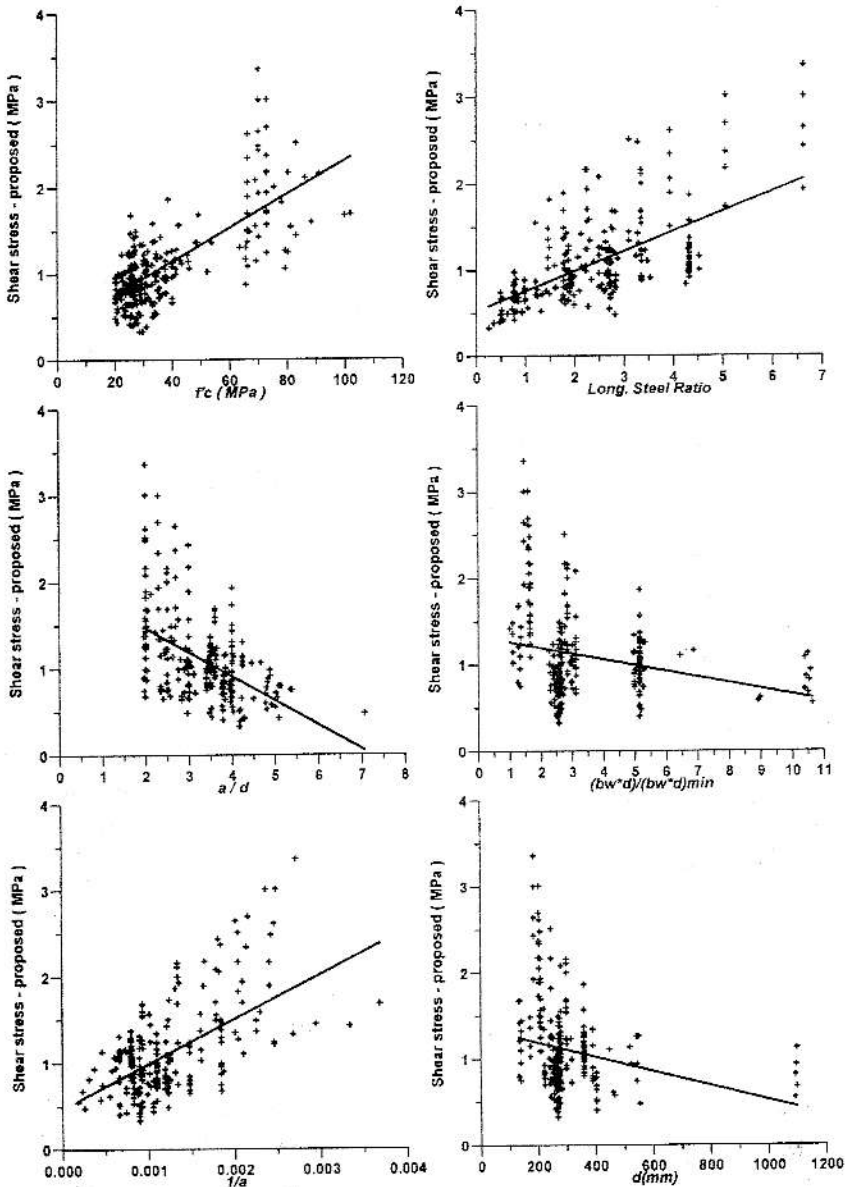


Fig. 6 - Influence of concrete properties on proposed shear strength