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Effects Of WEB Openings On The Analysis Of Cellular Plate Structures Of Varying Depth By Grillage Method

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Abstract

Effect of web openings on the elastic behavior of a tapered box girder is studied. The structures are analyzed by a grillage method including the effect of warping restraint implicitly through the torsional constant. The introduction of web openings results in a significant reduction in the transverse shear and torsional stiffness of the structure.

An appropriate reduction coefficient can be taken from empirical curves derived from a parametric finite element study; these coefficients are used in the grillage analysis.

Results obtained from the grillage method are compared with those gained from finite element solutions (plate/shell) elements by using the software program (MSC/ NASTRAN package), for different proportions of the structure and with different size of opening (12.9% , 25.5% , 36.3%).

Key Words: Cellular structure, Grillage method, plate/shell elements, tapered box beam, Torsion, Warping restraint, Web opening .

الخلاصة

أجريت دراسة تأثير وجود الفتحات في الأغشية العمودية (web opening) على السلوك المرن للمنشآت اللوحية الخلوية متغيرة العمق. تم تحليل المنشأ بطريقة المشبكات (grillage method) مع إدخال تأثير تقييد ألالتواء (warping restraint) من خلال ثابت اللي (torsional constant). أظهرت الدراسة أن وجود ألفتحات يؤدي إلى تقليل ملموس في صلادة القص العرضي(transverse shear stiffness) و صلادة اللي (torsional stiffness). وقد اعتمدت منحنيات تجريبية(empirical curves) لإيجاد معاملات التقليص وهذه المنحنيات اشتقت من خلال دراسة تمت باستخدام العناصر المحددة (finite elements) وهذه المعاملات استخدمت في التحليل بطريقة المشبكات، تمت مقارنة النتائج

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المستحصلة بطريقة المشبكات مع تلك الناتجة من التحليل بطريقة العناصر المحددة باختيار عنصر اللوح/قشرة في التحليل(Plate/shell element) وباستخدام برنامج جاهز NASTRAN. تم التحليل لخواص مختلفة من المنشأ وتحت تأثير تقييد مختلف و لإحجام مختلفة من الفتحات بنسب (%١٢,٩، %٢٥,٥، %٣٦,٣).

Introduction

Cellular plate structures are often used in bridges, double-wall storage tanks, dock gates and as double-layer bottoms in ship construction. To reduce the self-weight of such structures by adding accessories, large openings are cut in the web plates to provide access for inspection and maintenance and to allow any liquid being stored or carried to flow freely between the cells.

Many practical diaphragms installations contain large openings such as those required for roof lights.

In [1973] an approach to diaphragms analysis was developed by Bryan[1] who derived simple expressions for both the strength and stiffness of regular diaphragms of rectangular shape. But Bryan's method was not able to deal with irregular situations such as diaphragms with large openings for roof lights , and the need for a simplified analysis produced By Michael[2][1977].

Bakht and Jaeger[3,4] [1981] showed that the grillage method was the most versatile and efficient on that can realistically model the transverse cell distortion of cellular and voided structures. It was noted that the grillage properties could be obtained by multiplying the appropriate section rigidity of the grillage beam by the relevant beam spacing.

Grillage Idealization Of A Cellular Structure

A grillage is composed of discrete one- dimensional members assembled into two dimensional arrangements, so that the interaction between longitudinal and transverse force system takes place at nodal points. Two problems are involved in establishing such a grillage idealization:

a) The representation of shear stiffness of web plates.

b) The representation of the torsional stiffness of the closed cells.

Evans and shanmugam[5,6] [1979]showed that the introduction of web openings has little influence on the shear lag effect. They applied a simplified grillage approach to the elastic analysis of cellular structure. The introduction of web openings results in a significant reduction in the bending and torsional stiffness of the structure. Such effect can be taken into account by the use of reduction coefficients. Empirical coefficients were derived from a parametric finite element study.

Husain and Meethaq[7] [2004] had considered previously the application of grillage techniques to the elastic analysis of cellular structures of varying depth with inclusion of warping effects. The cellular plate structure is discretized into grillage beams running in two directions (at the same position of the webs). To each grillage beam of I-section of varying depth, flexural and rigidity, transverse shear rigidity and torsional rigidity are specified. These section properties are obtained from the properties of the cellular plate structure and from the spacing of the webs.

For structures without web openings, the effective shear area of I- beam element may be taken as the cross sectional area of the web plate. When a structure has openings in web plates, this leads to a significant reduction in the web shear stiffness and a resulting increase in the deflections of the girder. Secondary shearing effects arising from the Vierndeel action (by torsion) of a girder with large openings tend to increase further the deflections.

These effects are taken into account by introducing an effective shear area coefficient (k_s) defined by

$$A_e = k_s A_w \qquad (1)$$

To allow for the effects of openings on the torsional rigidity, an effective torsional constant coefficient (k_T) is introduced such that

$$J_e = k_T . J_{eff} \qquad (2)$$

Where [5]

$$J_{eff} = \frac{J}{\left(1 - \frac{\mu_\omega}{kL} \tanh kL\right)} \qquad (3\text{-a})$$

$$J_{eff} = \frac{J}{\omega_1} \qquad (3\text{-b})$$

$$\omega_1 = \left[1 - \frac{\mu_\omega . 2(CH - 1)}{kL.SH}\right] \qquad (4)$$

Where

CH= Cosh (kL) , SH= Sinh (kL)

$$\mu_{\omega} = 1 - \frac{J}{I_c} \quad ,$$

$$k = \left(\frac{\mu_{\omega}.G.J}{E.I_{\omega}} \right)^{1/2} \qquad (5)$$

$$I_{\omega} = \oint_A \omega_{(s)}{}^2.dA \qquad (6)$$

Here (J_{eff}) is the effective torsional constant for a single closed cell section under warping restraint. In this analysis, warping restraint effect is included implicitly through the use of the effective torsional constant.

Equation (3-a) is used for a member with one end restraint. Equation (3-b) is used for a member with two end restraints.

The calculation of the effective torsional constant is presented in details in previous paper by Husain and Meethaq[7] [2004]and Al-Dussary[8].

Evans and Shanmugam[6] [1979] presented a parametric study of the effects of openings. They produced design curves for effective torsional constant coefficients (k_T) , Fig.(4) and for shear area coefficient (k_s) as shown in Fig.(5) for girders containing opening .

The proportion of the girders (grillage member) may be defined by

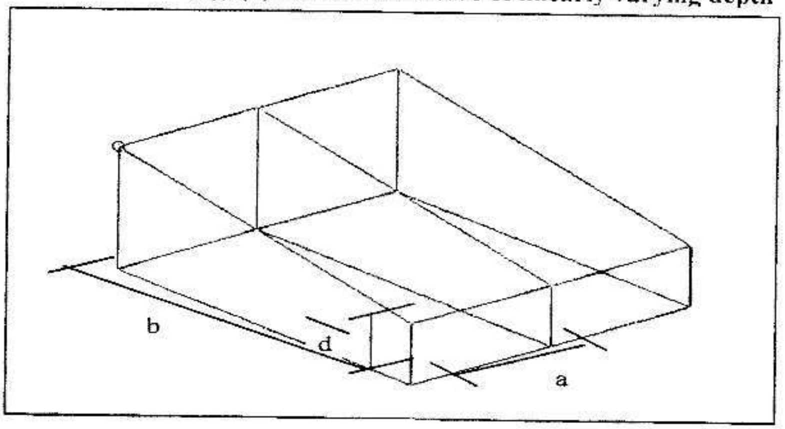
$$\frac{\text{Spacing of transverse webs}}{\text{Web depth}} = \frac{a}{d}$$

$$\frac{\text{Spacing of longitudinal webs}}{\text{Web depth}} = \frac{b}{d}$$

$$\frac{\text{Spacing of longitudinal webs}}{\text{Flange thickness}} = \frac{b}{t}$$

Typical girders (grillage member) in a cellular structure are shown in Fig. (1).

Fig. (1) Cellular structure of linearly varying depth



COMPARISON OF RESULTS FOR CELLULAR STRUCTURES WITH WEB OPENING

Results For A Cellular Structure of Square Plan Form

A cellular structure of linearly varying depth Fig.(2} was analyzed by the grillage method in details in a previous paper of Husain and Meethaq. The results of the analysis were compared with those from the finite elements (plate / shell elements) from MSC/ NASTRAN Package.

The properties of the square structure are

- b/d =2 , a/d =2
- $E= 0.202 \times 10^6$ N/mm^2
- $\nu = 0.3$
- Three percentages of web openings are considered. These are 12.9 %, 25.5% and 36.3%. Rectangular openings with different sizes are cut in each case, between the transverse diaphragms and symmetrically about the web as shown in Fig.(3). To avoid an unrepresentative loss of stiffness of the girder, the depth of openings was not allowed to exceed half the depth of web.
- Two support conditions are studied. In the first, all four edges are simply supported, and in the second only two longitudinal edges are simply supported. Applied load of (300 kN)is used at each internal web intersection was considered.
- The values of k_T and k_S are taken from Fig. (4) and Fig.(5) for the structure with different openings and they are given also in Table (1).

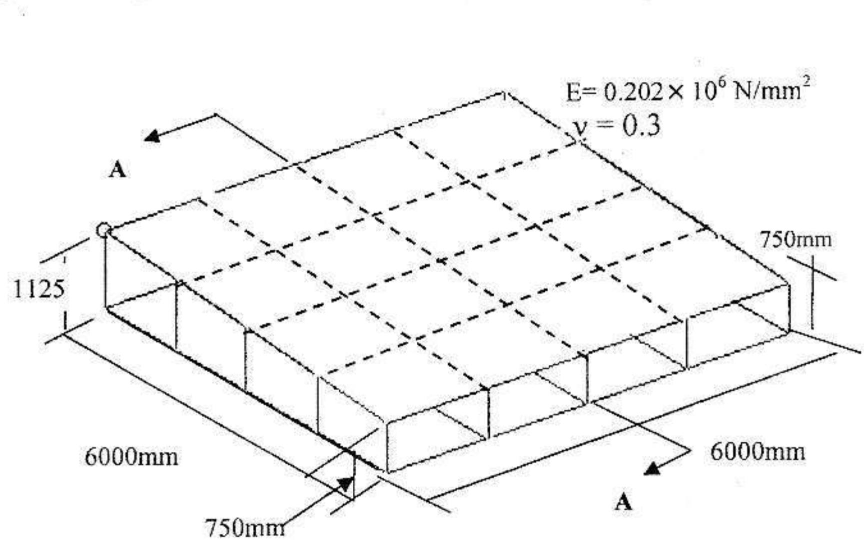


Fig.(2) Details of square cellular structure of varying depth.

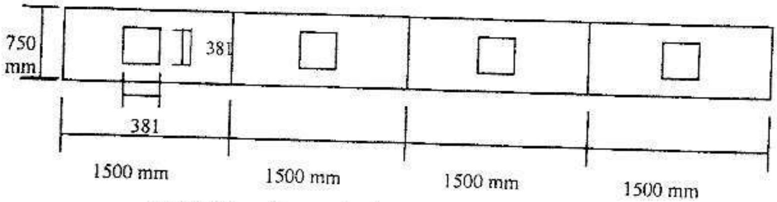
Table (1) Values of k_S and k_T, taken from Fig.(4) and Fig.(5) for grillage idealization of structures.

% web openings	Square structure a/d=2, b/d=2		Rectangular structure a/d=2, b/d=4		Rectangular structure a/d= 4, b/d = 2	
	k_S	k_T	k_S	k_T	k_S	k_T
0.0	1.0	1.0	1.0	1.0	1.0	1.0
12.9	0.52	0.80	0.52	0.90	0.46	0.75
25.5	0.21	0.51	0.21	0.58	0.10	0.32
36.3	0.12	0.38	0.12	0.38	0.05	0.18

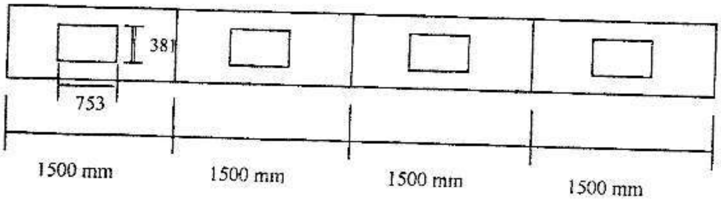
Table (2). Results of deflections and normal stresses for cellular structure of square plan form.

% of web opening	Simply supported at four edges				Simply supported at two edges			
	Finite Element Method		Grillage Method		Finite Element Method		Grillage Method	
	Defl. (mm	Norm. Stress (MPa)	Defl. (mm	Norm. Stress (MPa)	Defl. (mm	Norm. Stress (MPa)	Defl. (mm)	Norm. Stress (MPa)
0	1.20	35.18	1.31	38.06	1.61	28.09	1.84	30.54
12.9	1.38	37.34	1.53	41.12	1.92	50.82	2.19	55.64

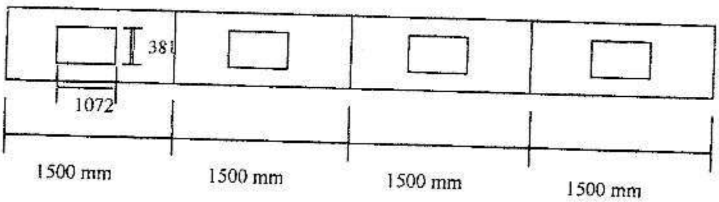
25.5	2.15	89.31	2.34	98.55	2.77	113.02	3.17	125.39
36.3	2.65	107.32	2.90	116.94	3.01	120.70	3.38	134.62



(a) 12.9% web opening in transverse direction.



(b) 25.5% web opening in transverse direction.



(c) 36.3% web opening in transverse direction.

Fig.(3) Dimensions of openings in the transverse diaphragms depending on the least depth of the structure.

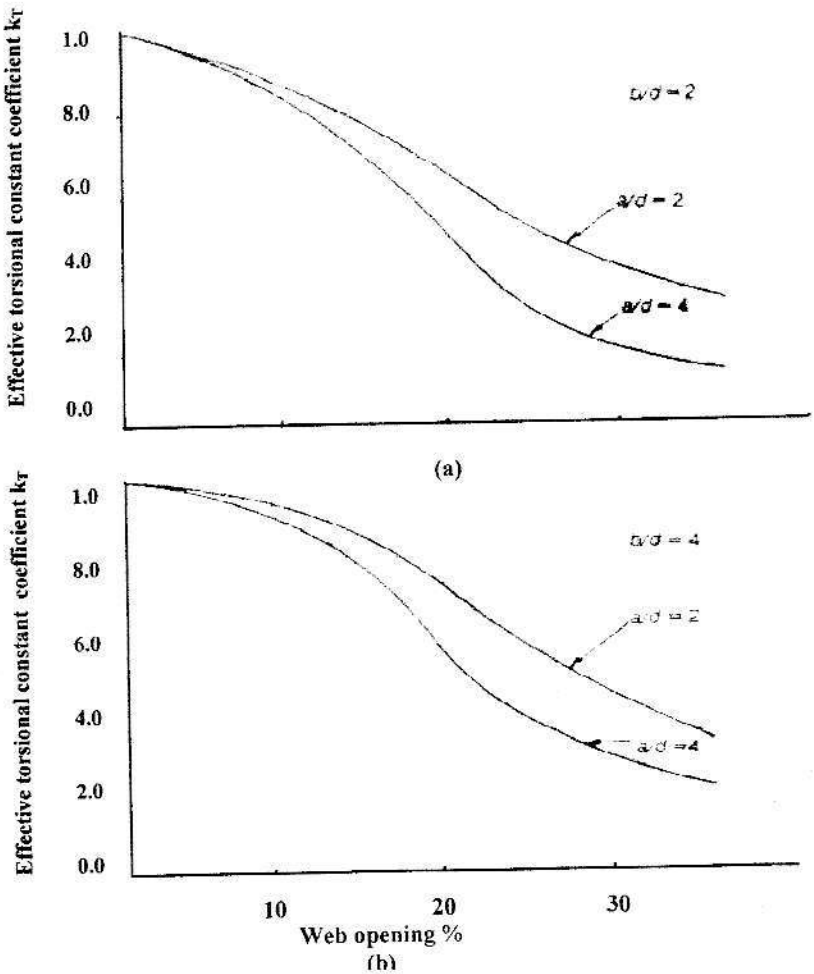


Fig. (4) Variation of effective torsional constant coefficient k_T with percentage of web openings.[6]

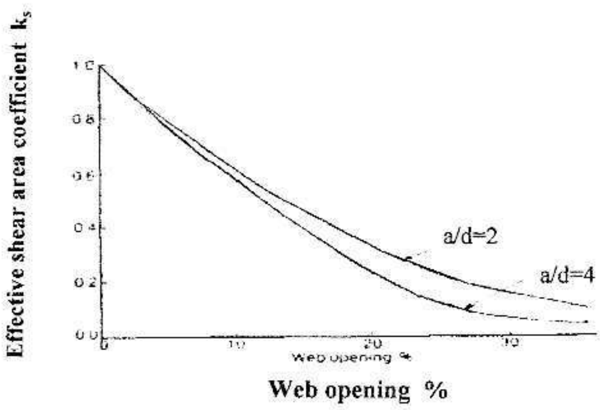


Fig.(5) . Design curves for the shear area coefficient k_s [6]

- The results are shown in Fig.(6) for the vertical deflection of the simply supported structure at four edges with no openings , 12.9% web opening , 25.5% and 36.3% opening.
- The results in Table.(2) show a significant reduction in structure stiffness due to introduction of web openings. The deflection increases by (14.4%) for structures with 12.9% opening and (44% and 54%) increase in deflection are noted for (25.5%) and (36.3%) openings in webs respectively.
- For structure of square plan form and supported at four edges (grillage method), the normal stresses increase by (7.4%) for 12.9% opening. The increase reaches (61.4% and 67.45%) for opening percentages from (25.5%) to (36.3%) respectively.
- The Finite Element mesh involved (144) elements for top plates and the same for bottom plate. (32) Elements for each transverse plate which contained openings. (36) Elements for each longitudinal plate. So the total elements are (888) elements.
- Results were also obtained for the square cellular structure simply supported structure at two opposite edges and subjected to equal vertical loads at the internal web intersection points. Table (2) shows that the deflection increases by (16%) for 12.9% opening, (42%) increase in deflection for 25.5% opening and the last increase is (45.5%) for 36.3% opening in web plates.

- The normal stresses for the cellular structure simply supported at two edges increase by (45%) for (12.9%) opening, (75.6%) increase in normal stresses for (25.5%) opening and (77.3%) for (36.3%) opening

- The results which are obtained from Table (2) show that the change in stresses due to introduction of web opening is much greater than the change in deflection. This is true as the stresses depend on the derivatives of displacements and thus liable to higher errors.

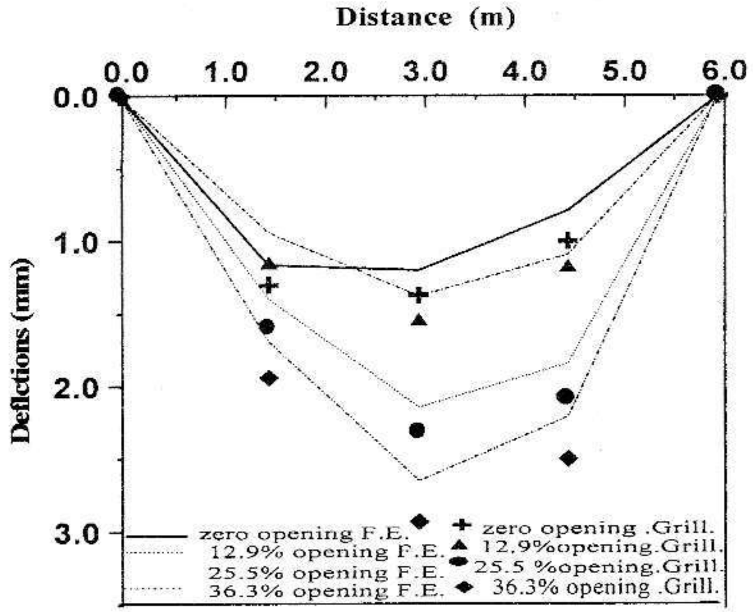


Fig.(6) Variation of vertical deflection at section (A-A) of the cellular structure simply supported at four edges

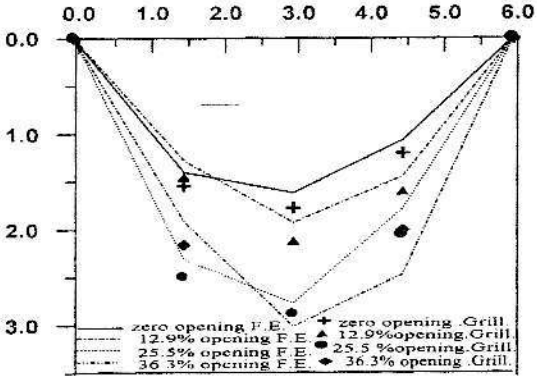


Fig.(7) Variation of vertical deflection at section (A-A) of the cellular structure simply supported at two opposite edges

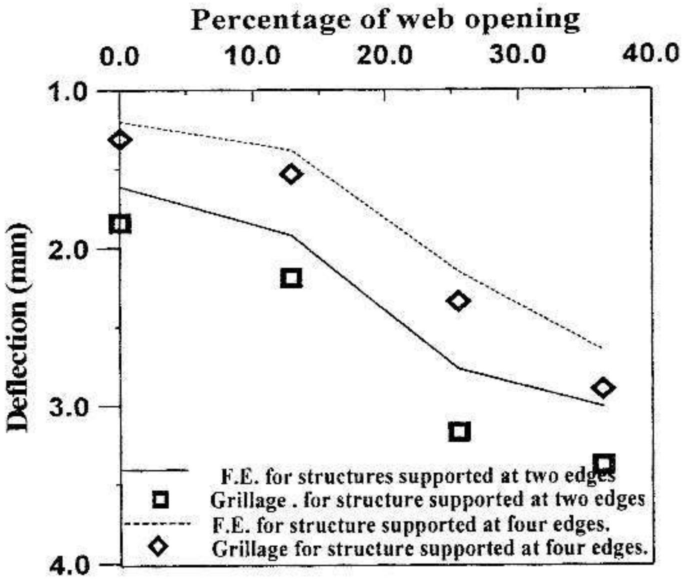


Fig.(8) .Variation of central deflection of the square cellular structure.

- Fig.(8) shows the variation of the central deflection against the percentage web opening for both support conditions. This increase in deflection is due to mainly to the reduction in the transverse shear rigidity of the web.

Results For a Cellular Structure of Rectangular Plan Form.

- The rectangular cellular structure has b/d= 4, a/d= 2, b/t = 300, t = 10 mm, d= 750 mm at one edge and the other is 1500mm. The structure is subjected to equal vertical loads at the internal web intersection points. The dimensions of the structure are shown in Fig. (9)
- The dimensions of the openings for each case are shown in Fig. (3).
- Table (3) shows the results for the structure of rectangular plan form with two support conditions. In the first support condition, the structure is supported at four edges. In the second condition of supporting, the structure supported at two opposite edges.
- The values of shear area coefficient k_s and torsional coefficient k_T are presented in Table (2). These values are taken from Fig.(4) and Fig.(5). And used in the grillage analysis.
- For the structure supported at four edges, the maximum deflection increases by (34%) for 12.9% opening , [54% and 68.2%] increase in deflection for 25.5% and 36.3% opening respectively. The normal stress increase by (54.2%) for 12.9% opening. The increase reaches 68% and 77% for 25.5%and 36.3% of opening.
- In the rectangular cellular structure supported at two opposite edges, the increase in deflections are (41%, 44.8% and 49.7 %) for percentage of opening (12.9%, 25.5% and 36.3%) respectively. While the normal stresses increase from (60.5 %, 63.4 % and 66.5%) for the same stepping of openings.
- The characteristics noted in the results for the square cellular structure are again presented. Although there is little change in the longitudinal and transverse stresses,
- The deflection increases significantly as a result of the reduction in the stiffness arising from the introduction of openings. The grillage stresses tend to exceed those obtained from the finite element method; this is acceptable for a method intended for use as a design tool.

Table (3). Results for rectangular cellular structure of b/d=4 a/d=2 and b/t =300

% of web opening	Simply supported at four edges				Simply supported a t two edges			
	Finite Element Method		Grillage Method		Finite Element Method		Grillage Method	
	Defl. (mm	Norm. Stress (MPa)	Defl. (mm	Norm. Stress (MPa)	Defl. (mm	Norm. Stress (MPa)	Defl. (mm)	Norm. Stress (MPa)
0	1.29	19.39	1.42	21.67	3.63	50.39	4.07	65.60
12.9	1.98	43.86	2.15	47.39	6.08	127.86	6.89	138.24
25.5	2.87	63.22	3.09	68.1	6.44	136.17	7.38	149.22
36.3	4.09	89.34	4.47	95.23	7.04	148.43	8.10	162.67

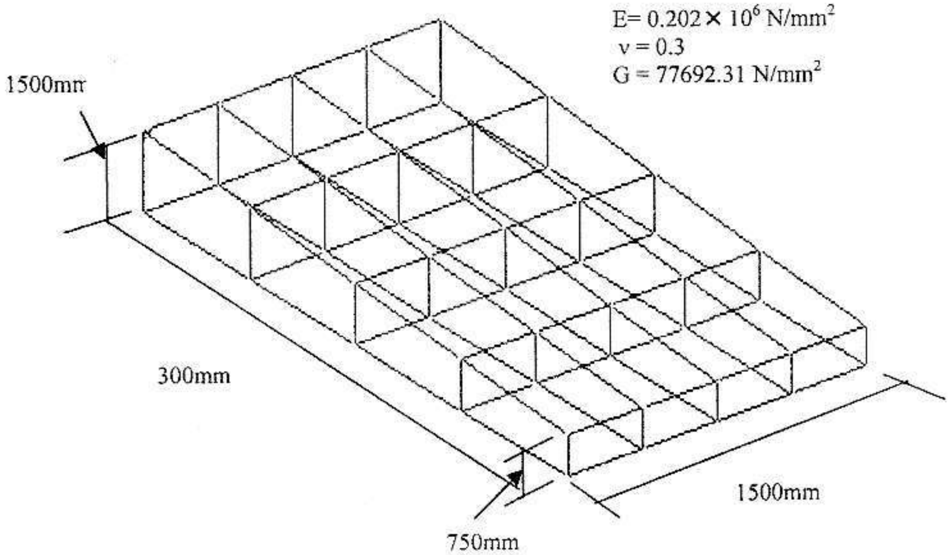


Fig.(9) Dimensions of rectangular cellular structure.

Results For a Larger Cellular Plate Structure of Rectangular Plan Form

Another rectangular cellular plate structure of a/d = 4 , b/d = 2 , and b/t = 150 . The structure is subjected to the same loading as in the previous example. The dimensions of openings for each case are (width= b1=1161mm, height =h1= 250 mm) for 12.9% opening, (b1= 1530 mm, h1=375 mm) for 25.5% opening and (b1= 2178 mm, h1= 375 mm) for 36.3% opening.

Characteristics similar to those noted earlier are again observed. The results in Table.(4) show the maximum vertical deflections and normal stresses .The grillage values of the deflections and the predicated stresses are slightly in excess of the finite element results for the girders with openings.

The reduction in torsional stiffness is again apparent and the reduction becomes most rapid for the models with larger openings. Fig.10)and Fig.(11) present the variation of maximum deflection and normal stresses for the larger rectangular cellular structure.

Table(4) Results For Rectangular Cellular Structure of b/d=2, a/d=4 and b/t =150.

% of web opening	Simply supported at four edges				Simply supported at two edges			
	Finite Element Method		Grillage Method		Finite Element Method		Grillage Method	
	Defl. (mm	Norm. Stress (MPa)	Defl. (mm	Norm. Stress (MPa)	Defl. (mm	Norm. Stress (MPa)	Defl. (mm)	Norm. Stress (MPa)
0	1.62	29.81	1.78	31.97	1.82	34.23	2.05	36.81
12.9	2.52	106.08	2.76	114.16	2.82	117.84	3.25	128.93
25.5	3.03	123.40	3.29	132.58	3.17	128.36	3.58	141.44
36.3	3.21	129.28	3.5	140.46	3.24	130.58	3.68	143.3

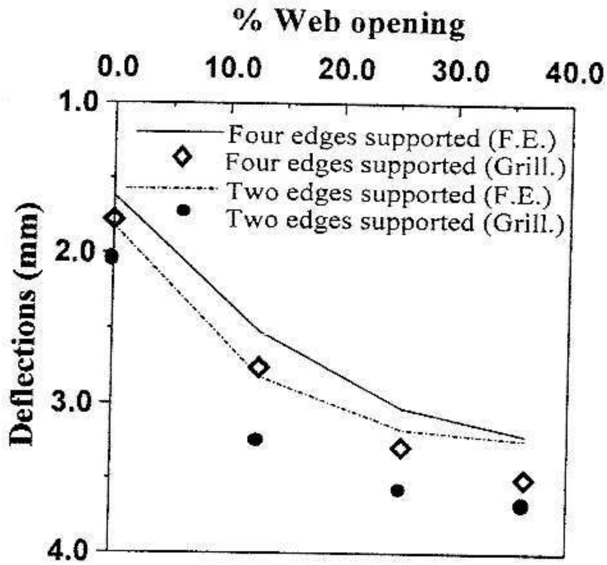


Fig.(10) Variation of vertical deflection for the larger rectangular cellular structure

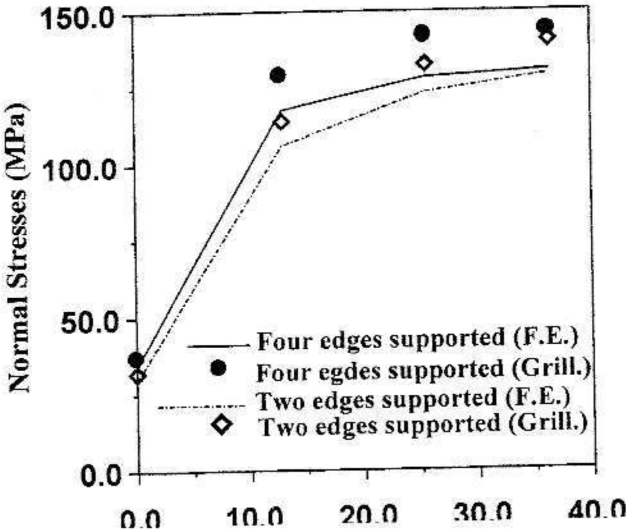


Fig.(11) Variation of normal stresses for different support condition.

CONCLUSIONS

- The introduction of web openings leads to a considerable increase in deflections.

Theses increases in deflections arise mainly from a reduction in the transverse shear and torsional stiffness of the web plates. Such effect can be taken into account by the use of appropriate reduction coefficients, the values of which may be obtained from the empirical curves in Fig. (4)and (5) . Theses values were needed in a grillage analysis of the cellular structure in which the effect of warping restraint is included implicitly through the torsional stiffness of the grillage member.

- The structure of linearly varying depth is analyzed under different proportions, with different support conditions and with different size of openings (12.9%, 25.5% and 36.3%). The results are comparable to those by the finite (plate/ shell) elements.
- This study shows that the change in stresses due to the introduction of web openings is much greater than the change in deflection.

- The two methods (grillage and finite element methods) take the same time for solution of the cellular structure due to efficient computer software.

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