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Shear Capacity Of Rc Beams With Web Reinforcement- A New Approach

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Abstract

In this paper 115 reinforced concrete (RC) beams failing in shear, obtained from the literature, are used to study the effect of the major parameters on the shear strength of normal strength concrete (NSC) and high strength concrete (HSC) beams. These parameters include the shear span/depth (a/d) ratio (between 2.0 and 4.9), concrete compressive strength f'_c (between 22.1 and 125.3 MPa), the longitudinal steel ratio ρ_w (between 0.01233 and 0.06972), stirrup shear strength $\rho_v f_{yv}$ (between 0.204 and 8.053 MPa) and beam size ($b_w d$).

Following the recent ACI 318M-02 Code, all 115 beams are not "deep" ($a/d \geq 2.0$). A proposed design method is introduced in these "non-deep" beams, which shows that increasing $\rho_v f_{yv}$ would lead to a slower increase in shear capacity than a direct proportionality to $\rho_v f_{yv}$. This stirrup effectiveness method K contrasts with the conventional code stirrup design which is based on the usual 45° truss analogy. In these code approaches, doubling (say) $\rho_v f_{yv}$ would lead to twice the increase in beam capacity due to stirrups – in contrast with the proposed design equations.

For all methods considered, the ratio is calculated of shear strength of beams $V_{r\ TEST}$ to the design shear resistance $V_{r\ DES}$. The proposed design equations lead to safe design with a low coefficient of variation (COV). This COV is only 17.5 percent which is significantly less than for other methods ranging between 30.8 to 35.9 percent.

Keywords: Longitudinal steel ratio, Normal and High-strength concrete, Shear properties, Size effect, Span-depth ratio, Standards, Web reinforcement.

الخلاصة

درس البحث تأثير المتغيرات الرئيسية على مقاومة القص لـ 115 عتبة خرسانية مسلحة فشلت بالقص من مصادر سابقة. جميع هذه العتبات فشلت بالقص لخرسانة اعتيادية المقاومة (NSC) وخرسانة عالية المقاومة (HSC). شملت المتغيرات (a/d) نسبة فضاء القص إلى العمق المؤثر (بين 2.0 و 4.9)، (f'_c) مقاومة الانضغاط (بين 22.1 و 125.3 ميكاباسكال) ، (ρ_w) نسبة حديد التسليح الطولي (بين 0.01233 و 0.06972)، ($\rho_v f_{yv}$) إجهاد حديد تسليح القص (بين 0.204 و 8.053 ميكاباسكال) ومساحة مقطع العتبة ($b_w d$). بموجب المدون الجديد ACI 318M-02 فان كافة العتبات (115 عتبة) تعتبر ليست عميقة ($a/d \geq 2.0$). المعادلات المقترحة ركزت على أن مقاومة القص للعتبات تزداد مع إجهاد حديد الأطواق ($\rho_v f_{yv}$) بصورة أبطأ من العلاقة الخطية لقيمة ($\rho_v f_{yv}$). أن طريقة معامل تأثير الأطواق (K) تختلف مع طرق التصميم التقليدية للمدونات المعتمدة على فرضية المسنم ذو °45 ، حيث أن

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الأخيرة (مثلاً) تؤدي إلى مضاعفة مساهمة الأطواق بمقاومة القص في حالة مضاعفة ($\rho_v f_{yv}$) – وهذا يختلف عن المقترحات التصميمية لهذا البحث. تم مقارنة نتائج المعادلات المقترحة مع نتائج معادلات التصميم للمواصفات الأمريكية، البريطانية، الكندية، النيوزلندية ومعادلة الباحث Zsutty. المعادلات التصميمية المقترحة أعطت أقل القيم لمعامل التغاير(COV) والتي بلغت % 17.5 عند مقارنتها مع بقية المعادلات التي أعطت قيم معامل التغاير من % 30.8 إلى 35.9 %.

Notation

a = Shear span, distance between concentrated load and face of support, mm.
a/d = Shear span to depth ratio.
A_s = Area of tension reinforcement, mm^2.
b_w = Web width of beam, mm.
d = Effective depth of the beam, mm.
f'_c = Specified compressive strength of (150 x 300 mm) concrete cylinders, MPa.
f_{yv} = Yield strength of vertical shear reinforcement, MPa.
K = Stirrups effectiveness factor.
M_u = Factored moment at section.
S = Spacing of vertical shear reinforcement, mm.
V_c = Shear strength provided by concrete of beams without stirrups, N.
V_n = Nominal shear strength, N.
$V_{r\,ACI}$ = Design shear resistance as per ACI Code.
$V_{r\,BS}$ = Design shear resistance as per BS Code.
$V_{r\,CAN}$ = Design shear resistance as per Canadian Code.
$V_{r\,DES}$ = Design shear resistance.
$V_{r\,NZ}$ = Design shear resistance as per New Zealand Code.
$V_{r\,PROP}$ = Design shear resistance by proposed equation.
$V_{r\,ZST}$ = Design shear resistance by Zsutty's method.
V_s = Nominal shear strength provided by shear reinforcement, N.
V_{TEST} =Test shear strength of beam with stirrups.
V_u = Factored shear force at section, N.
ρ_w = Ratio of tension reinforcement = $A_s/(b_w d)$.
ρ_v = Ratio of vertical shear reinforcement = $A_v/(b_w S)$.
φ = Strength reduction factor.

Introduction

ACI-ASCE Committee 326[1] was one of the first to suggest that shear resistance of beams with web reinforcement can be calculated as follows:

$$V_n = V_c + K(\rho_v f_{yv}) b_w d \qquad \ldots(1)$$

Where K is the stirrups effectiveness factor, ρ_v is the ratio of vertical shear reinforcement, f_{yv} is the yield strength of the vertical shear reinforcement and V_c is the shear strength provided by concrete.

Bresler and Scordelis[2] found that a small amount of stirrup reinforcement with $\rho_v\ f_{yv}$ values as low as 0.35 MPa, effectively increase the shear strength of RC beams. Haddadian et al.[3] agreed with reference [2] regarding the effect of low to moderate amount of stirrups on the shear strength of beams.

Elzanaty et al.[4] stated that the stirrups not only carry shear themselves but also enhance the strength of the other shear transfer mechanisms. The stirrups provide support for the longitudinal steel and prevent the bars from splitting from the surrounding concrete, hence they greatly increase the strength of the dowel action. At the same time, the stirrups help to contain the shear crack, limiting its propagation and keeping its width small. These effects increase both the shear carried by aggregate interlock and the shear strength of the uncracked compression zone. Stirrups also increase the strength of compression concrete by providing confinement. Although stirrups do not affect the diagonal cracking load, they enhance the concrete contribution by increasing the capacity of the different shear transfer mechanisms. Mphonde and Frantz[5] suggested a stirrup effectiveness factor K=1.6.

In this study 115 RC beams were used to investigate the influence of web reinforcement on the shear strength of RC beams. In these beams nominal stirrup shear stress values ($\rho_v . f_{yv}$) were in the range of (0.2-8.1) MPa . They were tested under one or two-point top loading, ρ_w ranging from 1.23 to 6.97 percent, f'_c ranging from 22.1 to 125.3 MPa and a/d ranging from 2.0 to 4.9. The yield strength of stirrups f_{yv} ranged from 265.9 to 820.0 MPa.

Crack Patterns

For beams with stirrups, the behavior was generally similar to that described for modes of failure for beams without stirrups but the following new points were observed:

a- Reference 4 shows that, the behavior of beams with stirrups was the same as for those without stirrups up to the inclined cracking load. As the load was increased beyond that point, additional flexural and diagonal tension cracks formed, and existing cracks lengthened and widened.

b- Smith and Vantsiotis[6] found that, beams with web reinforcement exhibited considerably less damage at failure than beams without web reinforcement. Beams with web reinforcement exhibited more uniform cracking and smaller crack widths at corresponding load levels and failure.

Research Significance

Existing design methods rely on the assumption that any increase in stirrups leads to a linearly proportional increase in shear resistance. By using a stirrup effectiveness factor K that decreases with increasing amounts of stirrups, proposed equations are presented that lead to an improved shear design. The proposed shear design is safe and leads to significant improvement, where the COV is 42.9 percent lower than the lowest value of 30.8 percent by the method proposed by Zsutty.

Shear Strength Data

Test results from references [4,7,8,9-15] are used to obtain the results of 115 tests on RC beams failing in shear.

Existing Methods for Predicting Shear Strength of RC Beams with Stirrups

Reference [16] presents a proposal for shear design of RC beams without web reinforcement. In this reference five other methods of design are compared[17-21]. These same five methods are also studied in this work.

To compare between design methods with different material reduction factors, shear resistance force $V_{r\ DES}$ will be used instead of nominal $V_{u\ DES}$ throughout.

ACI Code method[17]

$$V_{r\ ACI} = \varphi(V_c + V_s) = 0.75\ [(\sqrt{f'_c} + 120\ \rho_w \frac{V_u d}{M_u})\ /7 + \rho_v\ f_{yv}]\ b_w\ d \qquad \ldots(2)$$

BS Code method[18]

$$V_{r\ BS} = [0.79\ (100\ \rho_w)^{1/3}\ (f_c/20)^{1/3}\ (400/d)^{1/4}\ /1.25 + 0.95\ \rho_v\ f_{yv}\ ]\ b_w d \qquad \ldots(3)$$

In Eq.(3): $f_c = 0.8 f_{cu}$; $(400/d)^{1/4}$ is used when $d < 400$ mm.

Canadian Code method[19]

$$V_{r\ CAN} = [0.6\ (\ 0.2\ \sqrt{f'_c}\ ) + 0.85 \rho_v\ f_{yv}]\ b_w d \qquad \ldots(4)$$

New Zealand Code method[20]

$$V_{r\ NZ} = 0.85\ [(0.07 + 10\rho_w)\ \sqrt{f'_c} + \rho_v\ f_{yv}]\ b_w\ d \qquad \ldots(5)$$

Zsutty's method[21]

$$V_{r\ ZST} = 0.75\ [2.2(f_c\ \rho_w\ d/a)^{1/3} + \rho_v\ f_{yv}]\ b_w\ d \qquad \ldots(6)$$

$\Phi = 0.75$ is used as recommended in the latest ACI code[17].

Proposed Shear Design Equations

The proposed equation is based on the method of truss analogy. This analogy is based on the assumption that a reinforced concrete beam with an inclined cracking can be modeled by a truss. The top and bottom chords of the truss are the concrete compressive zone and longitudinal reinforcement respectively. The diagonal and the vertical struts consist of the beam concrete web and the shear reinforcement respectively. Ritter[1] proposed the following equation to predict the stirrup effectiveness factor K .

$$K=(\sin\alpha \cot\theta + \cos\alpha)\ (\sin\alpha) \ldots(7)$$

where α is the inclination angle of stirrup, θ is the angle of inclined crack.

It is most common for the crack angle θ as assumed to be 45° and α = 90°. These values lead to K= 1. This model is very simple but it ignores the shearing force carried by the shear transfer mechanisms.

Code Eqs. (2-5) and Eq. (6) use the same model with the same modifications. It is assumed that part of the applied shear is carried by the concrete (V_c) and the rest is carried by the shear reinforcement (V_s):

$$V_u \leq \varphi V_n = \varphi(V_c + V_s) \qquad \ldots(8)$$

The shear force (V_s) resisted by the stirrups is calculated assuming that all stirrups crossing the crack will yield and the inclined crack has a horizontal projection of d.

$$V_s = (A_v\ .\ f_{yv}\ .\ d/s).\ (\sin\alpha + \cos\alpha)$$

$$= \rho_v\ f_{yv}\ .\ b_w d\ (\sin\alpha + \cos\alpha) \ldots(9)$$

Mphonde and Frantz[5] found that the stirrup effectiveness factor K equals to 1.6. Schlaich et al.[22] found that, the truss model becomes an appropriate approach with increasing a/d > 2.5.

The proposed equation is based on non-linear multiple regression analysis to find the stirrup effectiveness factor K. First the shear strength carried by concrete (V_c) was calculated by the Sarsam and Al-Bayati[16] as given in the following equation:

$$V_c = 12\ (f'_c\ \rho_w)^{0.4}\ (d/a)^{0.8}\ b_w\ d^{0.8} \quad \ldots(10)$$

Then the shear strength carried by the stirrups was calculated as follows:

$$\varphi V_s = V_u - \varphi V_c \quad \ldots(11)$$

The general equation is formed as:

$$V_u = \varphi\,[V_c + K\,(\rho_v\ f_{yv})\ b_w\ d] \quad \ldots(12)$$

$$V_u = 0.75\ [\ 12\ (f'_c\ \rho_w)^{0.4}\ (1/a)^{0.8}\ d^{0.6} + K\ (\rho_v\ f_{yv})]\ b_w\ d \quad \ldots(13)$$

where K is found by using multiple regression analysis with other variables (f'_c, ρ_w, d/a, ρ_v).

$$K = 69 f'_c{}^{0.14}\ \rho_w{}^{0.4}\ (1/a)^{0.9}\ d^{0.5} (\rho_v\ f_{yv})^{-0.5} \quad \ldots(14)$$

Comparison of Design Methods

Table 1 compares six design methods for the 115 beams.

The following points can be concluded from Table 1:

a- Eq. (2) to Eq.(5) gave close values for shear strength estimations leading to COV of 32.1% to 35.9%. The ranges of the means were slightly more different at 1.43 to 2.0.

b- Among existing design equations both BS Code[18] Eq. (3) and the empirical method proposed by Zsutty[21] Eq. (6) gave the lowest dispersion of their estimation (low COV values of 32.1% and 30.8%). However, Eq.(6) is significantly safer than the BS Code method. The latter has nearly twice the number of unsafe beams, 20 versus 6 for the former.

c- The proposed equation [Eq. (13)] was also compared with five other methods shown in Table 1. Eq. (13) has the lowest COV among all five methods (17.6% versus a range of 30.8% - 35.9% for the others) which is 43% less than the least value of 30.8% corresponding to the Zustty method [Eq.(6)]. Thus Eq. (13) gave the best prediction of shear strength with the least dispersion among all six methods.

d- All existing design equations [Eqs.(2-6)] led to some unsafe predictions, ranging between 3 to 20 cases. By using a strength reduction factor (φ) equal to 0.75, for Eq. (13), the RSSV (Relative Shear Stress Value) will be greater than 1.0. Therefore, it can be used as a safe design equation.

Influence of Major Parameters

Only the proposed method Eq.(13) is conservative for all tests ($V_{TEST}/ V_{r\ DESIGN} \geq 1$) with a relatively low COV value (17.55 percent). For all the 115 beams, Figs.1-5 show the influence of major parameters (f'_c, ρ_w , a/d , $b_w d/(b_w d)_{min}$ and $\rho_v\ f_{yv}$)on $V_{TEST}/V_{r\ DES}$. Figs.1-5 show that applying proposed method Eq.(13) leads to the least scatter. Increasing f'_c, ρ_w (Figs. 1 and 2) up to 125.3 N/mm^2 and 0.0697 simultaneously, causes no drop in the factor of safety (ratio of $V_{TEST}/ V_{t\,DESIGN}$) using the proposed method.

Fig.3 shows a clear tendency for a drop in safety factor with rising a/d values, for Eqs.(2-6). This is because Eqs.(2) and (6) underestimate the influence of a/d, while the other three [Eqs.(3-5)] do not even recognize the effect of a/d in shear design. These results contrast with the proposed method, which includes a/d in a significant manner.

Fig. 4 shows the influence of $b_w d/(b_w d)_{min}$ as an indication of the size effect. All existing methods [Eqs.(2-6)] show a significant drop in the factor of safety with increasing beam size. Fig. 5 shows that the proposed method was conservative up to 8.053 MPa of $\rho_v f_{yv}$.

Conclusions

Based on the results of this work, the following conclusions are made.

1. Eq. (13) and Eq. (14) can be used for a safe, rational and easy method for the design of both HSC and NSC beams with ($2.0 \leq a/d \leq 4.9$) by using a strength reduction factor $\varphi = 0.75$. Eq. (13) gave the lowest COV value of 17.55 % which is 45 % less than the lowest Code value of 32.07 % by the British Standard code Eq. (3) as shown in Table 1, based on test results of 115 beams.
2. Of the six methods, two are essentially conservative for HSC and NSC beams – Zsutty and the proposed method. The COV values are 30.77 and 17.55 percent, respectively.
3. The ACI code, British Standard code, Canadian code and New Zealand code methods are less conservative than indicated in conclusion 2.
4. Fig.1 shows that f'_c up to 125.3 MPa does not lower the safety factor of the proposed method.
5. Because they either underestimate the influence of ρ_w (ACI code), or they do not include its influence (Canadian code), both methods show a rise in the safety factor with increasing ρ_w, Fig.2.
6. Fig.3 shows a clear trend for a drop in the safety factor with increasing a/d ratios in four methods – ACI, BS, Canadian and New Zealand codes. In contrast, Zsutty's method shows a smaller drop in the safety factor with rising a/d ratios, while the proposed Eq.(13) does not show such trend.
7. Fig.4 indicates clearly that all five existing methods [Eq.(2-6)] show a significant drop in the factor of safety with increasing beam size. In contrast, the proposed design method shows no such trend .
8. Fig.5 shows clearly that all existing methods [Eqs.(2-6)], assume a linear strength increase with $\rho_v f_{yv}$. This leads to a drop in the factor of safety with rising $\rho_v f_{yv}$. In contrast, the proposed method shows no drop in the factor of safety with rising $\rho_v f_{yv}$. This is because proposed Eq.(13) leads to a less than linear contribution of $\rho_v f_{yv}$.

Future Research

The use of steel fibers as shear reinforcement in NSC and HSC beams should be studied, since adding steel fibers may enhance the ductility of HSC.

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References

1. ACI-ASCE Committee 326, "Shear and Diagonal Tension," ACI Journal, Vol. 59, No. 1-3, , pp. 1-30, 277-334 and 353-396, January, February and March 1962.
2. Bresler, B., and Scordelis, A.C., "Shear Strength of Reinforced Concrete Beams", ACI Journal, Vol. 60,No. 1, , pp. 51-73. January 1963.
3. Haddadian, M.; Hong, S. T.; and Mattock, A. H., "Stirrup Effectiveness in Reinforced Concrete Beams With Axial Force," Journal of the Structural Division, ASCE, Vol. 97, ST. 9, pp.2277-2297, September 1971.
4. Elzanaty, A. H.; Nilson, A. H.; and Slate, F. O., "Shear Capacity of Reinforced Concrete Beams Using High Strength Concrete," ACI Journal, Vol. 83, No. 2, pp.290-296. March – April 1986.
5. Mphonde, A. G.; and Frantz, G. C., "Shear Tests of High and Low Strength Concrete Beams Without Stirrups," ACI Journal, Vol. 81, No. 4, pp. 350-357. July-August 1984.
6. Smith, K. N.; and Vantsiotis, A. A., "Shear Strength of Deep Beams," ACI Journal, Vol. 79, No. 3, pp. 201-213. May-June 1982.
7. Johnson, M. K.; and Ramirez, J. A., "Minimum Shear Reinforcement in Beams With Higher Strength Concrete," ACI Structural Journal, Vol. 86, No. 4, pp. 376-382. July-August 1989.
8. Roller, J. J.; and Russell, H. G., "Shear Strength of High-Strength Concrete Beams With Web-Reinforcement," ACI Structural Journal, Vol. 87, No. 2, pp.191-198. March-April 1990.
9. Clark, A. R., "Diagonal Tension in Reinforced Concrete Beams," ACI Journal, Vol. 48, No. 10, pp. 145-156. October, 1951.
10. Bazant, Z. P.; and Sun, H-H, "Size Effect in Diagonal Shear failure: Influence of Aggregate size and Stirrups," ACI Materials Journal, Vol. 84, No. 4, pp. 259-272. July-August 1987.
11. Salandra, M. A. and Ahmad, S., H., "Shear Capacity of Reinforced Lightweight High-Strength Concrete Beams," ACI Structural Journal, Vol. 86, No. 6, pp. 697-704. November-December 1989.
12. Mphonde, A. G., "Use of Stirrup Effectiveness in Shear Design of Concrete Beams," ACI Structural Journal, Vol. 86, No. 5, pp. 541-545. September-October 1989.
13. Aziz, O. Q., "Shear Strength Prediction of Crushed Stone Reinforced Concrete Deep Beams," Ph. D. Thesis, University of Technology, Baghdad, Iraq, 189pp. 1997.
14. Sarsam, K. F.; and Al-Musawi, J. M. S., "Shear Capacity of High-Strength Concrete Beams with Web Reinforcement," ACI Structural Journal, Vol. 89, No. 6, pp. 658-664. November-December 1992.
15. Al-Musawi, J. M. S., "Shear Capacity of High-Strength Concrete Beams with Web Reinforcement," MSc thesis, University of Technology, Baghdad, 121 pp. (in Arabic).

1989.

16. Sarsam, K. F.; and Al-Bayati, N. A-M., "Shear Design of High and Normal RC Beams Without Web Reinforcement," Engineering & Technology Journal, Iraq, Vol. 24, No. 15, 2005.
17. ACI Committee 318, "Building Code Requirements for Structural Concrete and Commentary (ACI 318M-02/ACI 318 RM-02)," American Concrete Institute, Farmington Hills, 443 pp. 2002.
18. "Structural Use of Concrete" (BS 8110:1997), British Standards Institution, London 1997.
19. "Design of Concrete Structures for Buildings," (CAN3-A23.3-M84), Canadian Standards Association, Rexdale, Canada, 1984.
20. "Code of Practice & Commentary on: The Design of Concrete Structures (NZS 3101: 1982. Part 1 and 2)," Standards Association of New Zealand, Standards Council, New Zealand, 1982.
21. Zsutty, T. C., "Shear Strength Prediction for Separate Categories of Simple Beam Tests," ACI Journal, Proceeding Vol.68, No.2, pp. 138-143. Feb. 1971.
22. Schlaich, J.; Schäfer, K.; and Jennewein, M. "Toward a Consistent Design of Structural Concrete," PCI Journal, Vol. 32, No. 3, pp. 74-150. May-June, 1987.

Table (1):Comparison between V_{TEST} and $V_{r\ DESIGN}$ for 115 beams

Ratio	$\frac{V_{TEST}}{V_{r\ ACI}}$	$\frac{V_{TEST}}{V_{r\ BS}}$	$\frac{V_{TEST}}{V_{r\ CAN}}$	$\frac{V_{TEST}}{V_{r\ NZ}}$	$\frac{V_{TEST}}{V_{r\ ZST}}$	$\frac{V_{TEST}}{V_{r\ PROF}}$
Equation used	2	3	4	5	6	13
Mean	2.0	1.43	1.93	1.59	1.60	1.55
Standard Deviation	0.68	0.46	0.69	0.57	0.49	0.27
COV%	33.99	32.07	35.86	35.83	30.77	17.55
Range - Low	0.73	0.51	0.66	0.59	0.70	1.00
High	4.61	2.76	4.63	3.68	3.40	2.24
$\frac{High}{Low}$	6.30	5.38	7.04	6.24	4.84	2.23
Number < 1*	3	20	3	9	6	0

Notes: Ranges of variables - f'_c=22.1, 125.3 MPa (ratio of 5.68); ρ_w=0.0123, 0.0697 (ratio of 5.654); $\rho_v\ f_{yv}$=0.204, 8.053 MPa(ratio of 39.475); a/d=2.0, 4.9 (ratio of 2.522); $b_w d$=13919, 348386 mm^2 (ratio of 25.03).

*Number < 1 indicates the number of specimens (out of 115) for which $V_{TEST}<V_{r\ DES}$

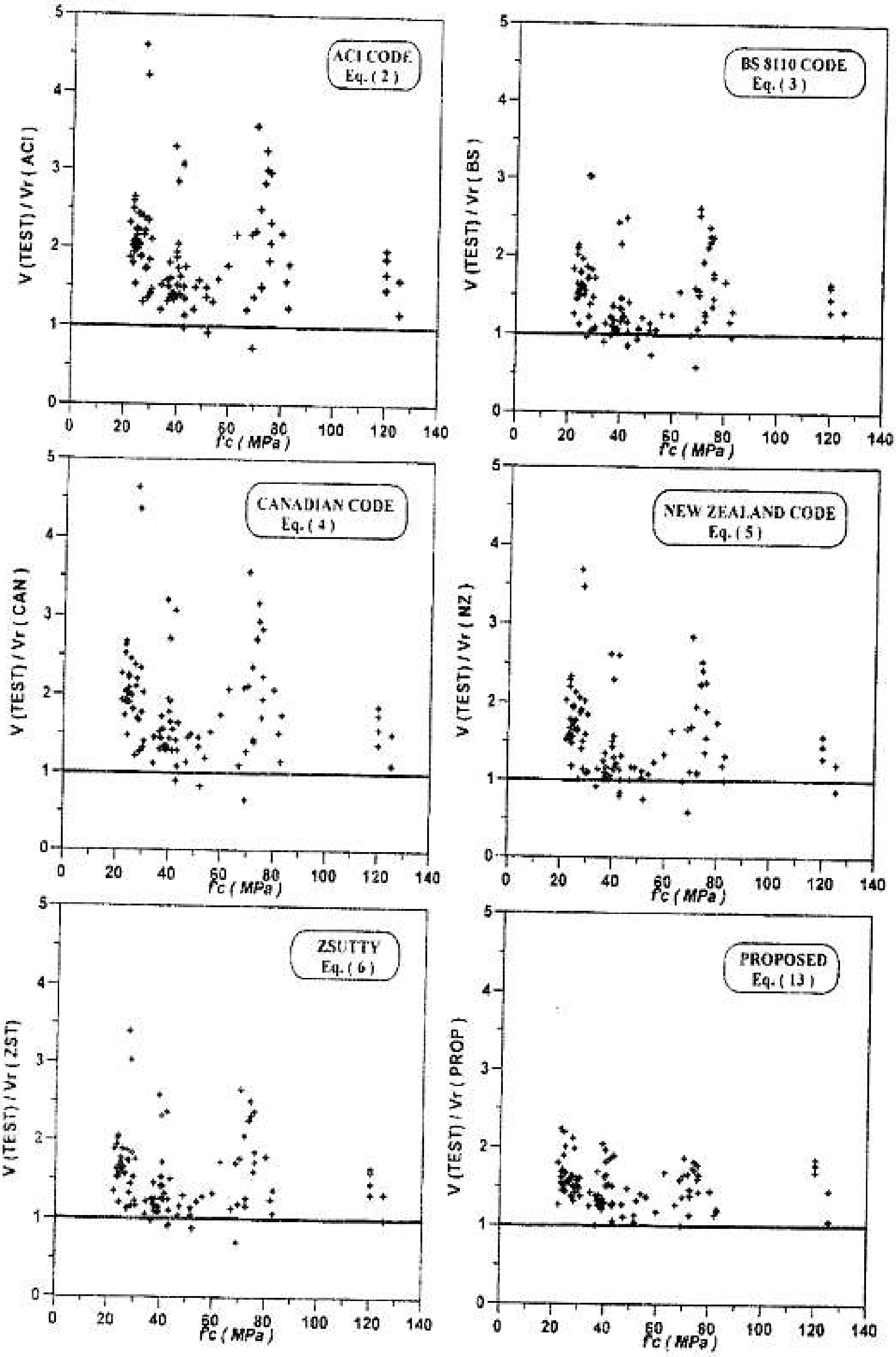

Fig. 1 - Influence of compressive strength f'c on relative shear strength

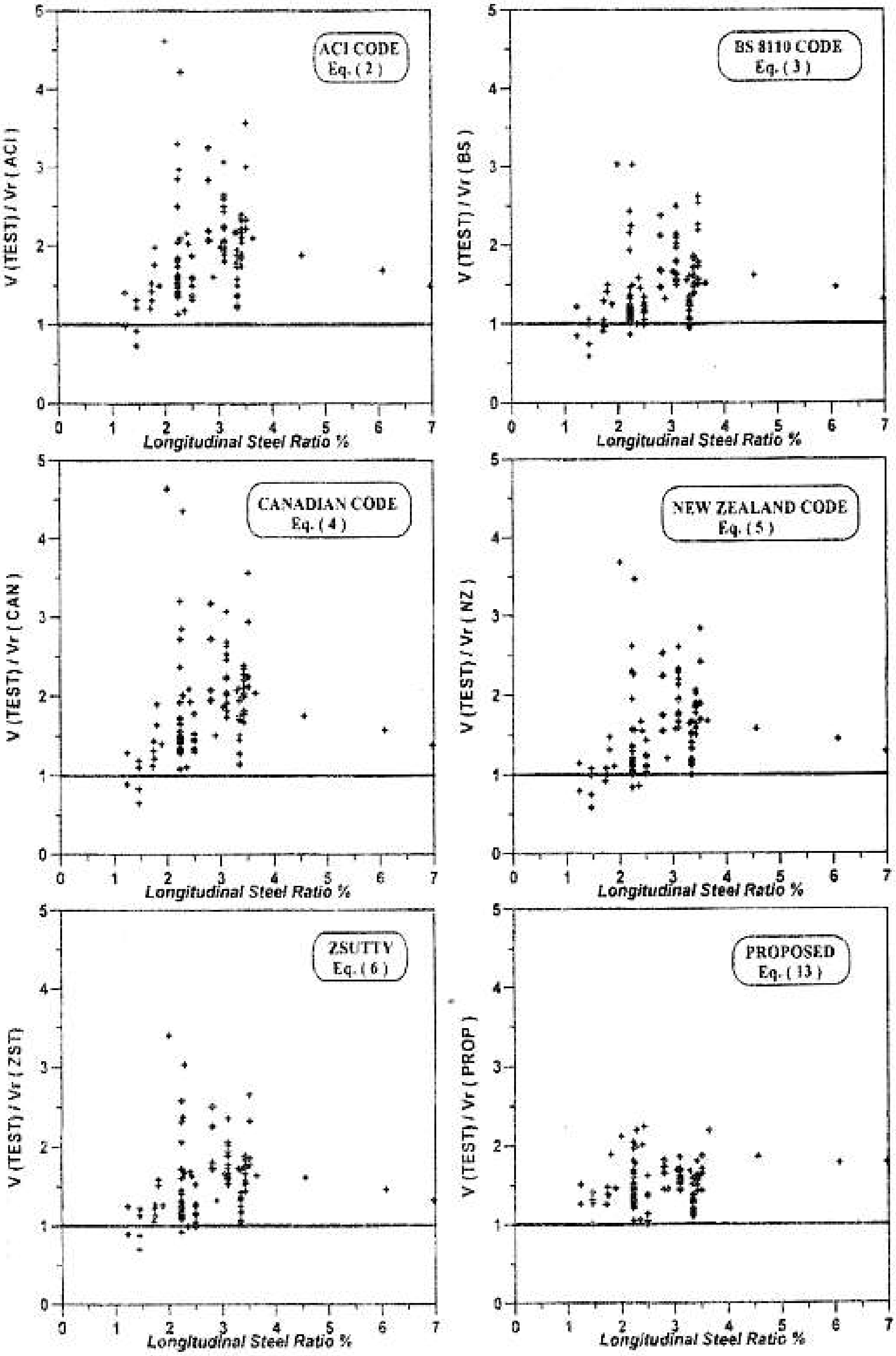

Fig. 2 - Influence of longitudinal steel ratio on relative shear strength

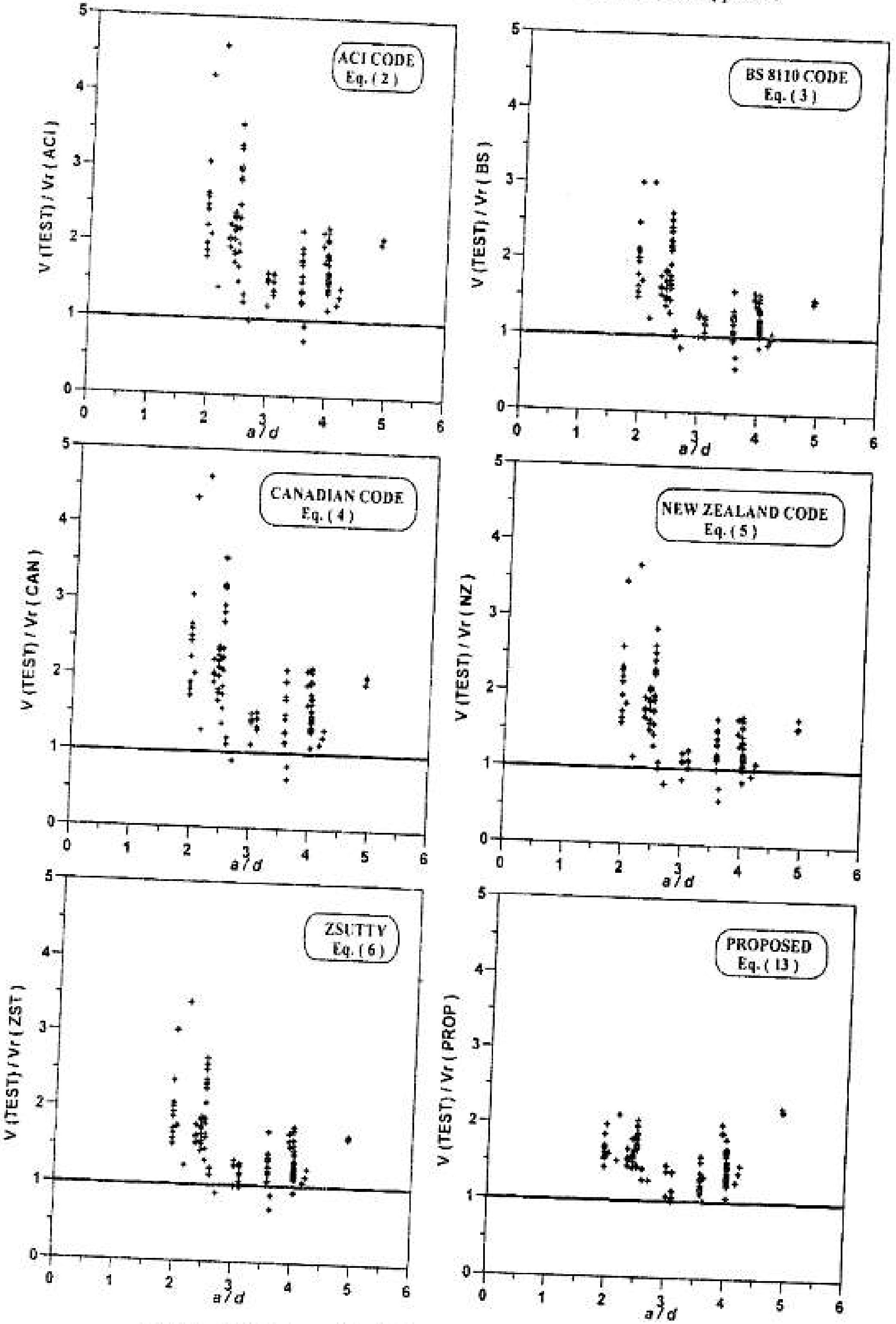

Fig. 3 - Influence of (a / d) on relative shear strength

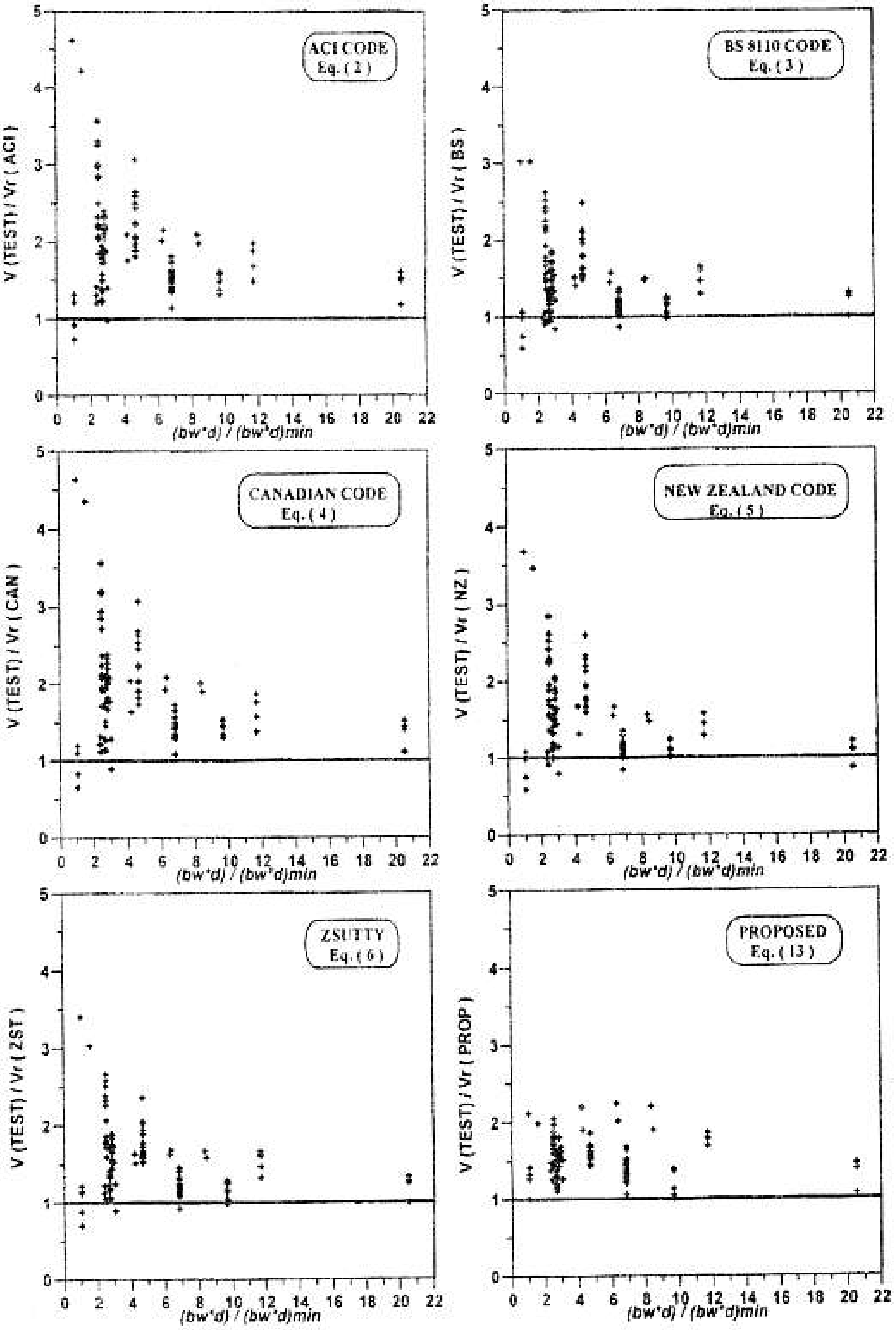

Fig. 4 - Influence of (bw*d) / (bw*d)min on relative shear strength

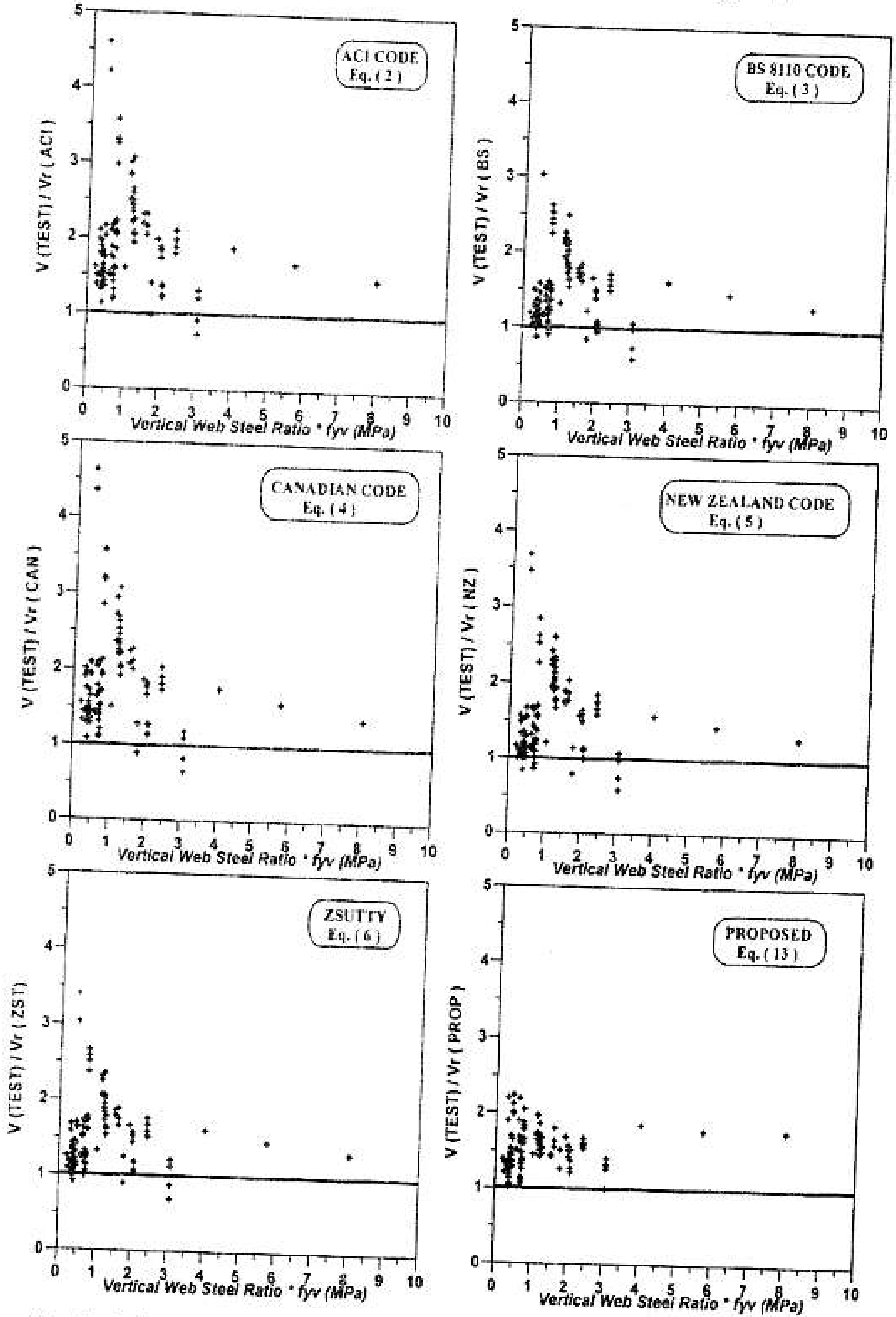

Fig. 5 - Influence of (vertical web steel ratio*fyv) on relative shear strength