Eng. & Technology, Vol.24, No.9, 2005

Trajectory Models of Deep Bed Filtration

Dr. Riyad H. Al-Anbai,* Dr .Mahdi S. Essa,** Afrah J. Al- Ithari**
Received on: 12/6/2004
Accepted on: 19/4/2005

Abstract

In this research an analysis and evaluation is made to five of the trajectory models. These five models were chosen among the others because they represent the new approach in modeling filtration process. The five trajectory models which were analyzed and evaluated are:

1 * O'Melia and Ali model (1978).
2 * Tien *et al.* model (1979).
3 * Chiang and Tien model (1985).
4 * Mackie *et al.* model (1987).
5 * Choo and Tien model (1993).

Comparisons between the predicted values from these trajectory models with the experimental data taken from the work of Chiang (1985), Tanaka (1982), and Mohammed (1989) were made.

Keywords: Deep bed filtration, Transient–state, Deposition, Filter coefficient, Trajectory models.

نماذج تحليل المسار للمرشحات العميقة

الخلاصة

في هذا البحث تم اجراء تحليل و تقييم ل خمسة من النماذج الخاصة بالمرشحات العميقة من نوع تحليل المسار . تم اختيار هذه النماذج من بين النماذج الاخرى لانها تمثل التوجه او الطريقة الحديثة بالنمذجة لعمليات الترشيح . والنماذج التي اخذت بنظر الاعتبار هي:

النموذج المقترح من قبل omelia and ali في عام 1978
النموذج المقترح من قبل Tien et,al في عام 1979
النموذج المقترح من قبل Chiang and Tien في عام 1985
النموذج المقترح من قبل Mackie et,al في عام 1987
النموذج المقترح من قبل Choo and Tien في عام 1993

كما تم اجراء مقارنة بين القيم المتنبأ بها بأستخدام هذه النماذج مع نتائج تجارب المنفذة من قبل Chiang 1985, Tanaka 1982 , Mohammed 1989,. O Melia and Ali 1987
تبين من النتائج ان نموذج Tien et,al ونموذج O melia and Ali قد اعطت اقرب النتائج للتجارب.

Introduction

Deep bed filtration is a physico-chemical process in which the removal of suspended particulate and colloidal matter in a fluid stream is effected by passing the stream through porous media composed of granular substances. The flow of a suspension through a packed bed of grains (sand, garnet, anthracite, activated carbon, etc.) or fibers results in the deposition of the suspended particles on the grain or fiber surfaces and ultimately the separation of these particles from the fluid is made. The particles penetrate into the porous

* Dept. of Building & Construction Eng., UOT. Baghdad-IRAQ
** College of Engineering, University of Babylon

https://doi.org/10.30684/etj.24.9A.5
2412-0758/University of Technology-Iraq, Baghdad, Iraq

medium and deposit at various depths. This is the reason for which this type of filtration is called deep bed (or depth) filtration [**Tien** and **Payatakes**, 1979].

This process is intrinsically transient (non-steady state), as the deposited material changes both the geometry of the interstitial space of the filter and the nature of the collector (filter grains) surfaces. These changes are reflected in the variations of the filtration efficiency and the pressure drop. Typically, an initial increase of the efficiency is observed then followed by a monotonic decrease [**Payatakes** ***et al.***, 1981; **Choo** and **Tien**, 1993].

The main indicators of the dynamic behavior of the operation are the filtration quality history and the pressure drop history required to maintain a uniform throughput [**Ojha** and **Graham**, 1994].

Background

The fundamental equations describing the particle retention on filter bed are the equations proposed by **Iwasaki** (1937). **Iwasaki** based his model upon a detailed and laborious microscopic examination of the penetration and distribution of microorganisms and fine particulate matter in a mixed bed of nonuniform size sand. Although he worked with a slow sand filter, his model could be applicable to rapid sand filters. His basic model consists of the following equation [**Charles** and **Richard**, 1970; **Adin** and **Rebhun**, 1977].

$$\frac{dc}{dz} = -\lambda c \qquad (1)$$

or $$c = c_o e^{-\lambda z} \qquad (2)$$

where,

- C_o : initial concentration of particles, mg / cm^3
- C : suspension concentration, mg / cm^3
- Z : axial distance, cm
- λ : local filter coefficient, cm^{-1}

and the macroscopic material balance for the suspended matter equation

$$u\frac{dc}{dz} + \frac{d\sigma}{dt} = 0 \qquad (3)$$

or $$\frac{d\sigma}{dt} = u\lambda c \qquad (4)$$

where,

- σ : specific deposit , vol. of deposit / vol. of bed, cm^3 / cm^3
- T : time of filtration, sec.
- U : superficial velocity, cm / sec.

The porosity (or void fraction) of the bed, **e**, changes with time as particle accumulation increases within the bed.

$$e = e_o - \frac{\sigma}{1 - e_d} \quad (5)$$

where,

- **E** : local filter porosity
- $\mathbf{E_o}$: initial porosity (porosity of clean filter bed)
- $\mathbf{E_d}$: porosity of deposit

The time variable is corrected by corrected time variable, **Θ**, which is defined as

$$\theta = t - \int_0^z \frac{e}{u} dz \quad (6)$$

The difference between **t** and **Θ** is usually small. If one ignores it, then combining Equations (1) and (2) yields

$$\left(\frac{d\sigma}{d\theta}\right)_z = u\lambda c \quad (7)$$

where,

$$c = c_{in}, \quad z = o, \quad \theta > 0 \quad (8)$$

$$c = 0, \quad \sigma = 0 \quad when\ z > 0, \quad \theta \le 0 \quad (9)$$

$\mathbf{c_{in}}$: Inlet concentration, mg / cm^3

Since the quality of filtrate obtained in a filter bed in a direct consequence of the extent of particle removal achieved in the bed is characterized by the value of filter coefficient, **λ**, the change in particle removal can be expressed by the change in the filter coefficient with the extent of particle deposition, **σ**. [**Tien** ***et al.***; 1979; **Amirtharajah**; 1988] defined as:

$$\frac{\lambda}{\lambda_o} = F_l = f(\alpha, \sigma) \quad (10)$$

or

$$\frac{\lambda}{\lambda_o} = \frac{\eta}{\eta_o} = F_l \quad (11)$$

where,

- **η** : collection efficiency
- **α** : parameter vector characterizing the effect of particle deposition on the filter coefficient.
- $\boldsymbol{\eta_o}$: initial collection efficiency
- $\boldsymbol{\lambda_o}$: initial filter coefficient, cm^{-1}
- $\mathbf{F_l}$: the filter coefficient ratio or collection efficiency ratio

Specific deposit can be determined from the continuity Eq.(4) which was modified to different forms, one of these forms was given by **Chiang** and **Tien** 1985b.

$$\sigma = \int_0^t \frac{u}{L}(c_i - c_{i-1}) dt \quad (12)$$

or

$$\frac{d\sigma_n}{dt} = \frac{u}{L}\eta_o F_l(\alpha, \sigma) \sum_{i=1}^{n-1} (1 - \eta_o F_l) \quad (13)$$

and the other form was given by **Tien** 1989

$$\sigma L = \left[\int_0^t (\bar{c}_{in} - \bar{c}) u\, dt\right] \frac{\pi}{6} d_g^3 \quad (14)$$

where,

$\bar{c}$: Number of concentration, 1/cm^3.

L : Length of (UBE) unite bed element, cm, and it can be calculated from Payatakes *et al.*, 1973, Eq.15.

c_{i-1}: Inlet concentration of UBE No.i, mg/cm^3.

c_i : Outlet concentration of UBE No.i, mg/cm^3.

d_g : Diameter of grain, cm.

$$L = \left[\frac{\pi}{6(1-e)} d_g^3\right]^{1/3} \quad (15)$$

Many other forms based on pressure drop was cited in Anbari, 1997.

REVIEW OF TRAJECTORY MODELS

The idea of determining the rate of particle deposition from particle trajectory was first advanced more than 60 years ago in connection with air filtration. The possibility of extending this concept to liquid filtration was recognized by (**O'Melia** and **Stumm**, 1967; **Yoa**, 1968; **Yoa** *et al*., 1971; and **Rajagopalan** and **Tien**; 1976) as cited in [**Tien** and **Payatakes**, 1979; **Amirtharajah**, 1988].

Many other researchers such as **O'Melia** and **Ali**, 1978; **Tien** *et al*., 1979; **Chiang** & **Tien**; 1985a, b; **Mackie** *et al*.; 1987; and **Choo** and **Tien**, 1993 proposed trajectory models. All these trajectory models were concerned with filter performance during the ripening period expect the model of **Tien** *et al*. (1979). These models do not address the problem of media clogging and permeability reduction

For an entire cycle of filtration **Vigneswaran** and **Chang** 1986 and **Vigneswaran** and **Tulachan** 1988 modified **O'Melia** and **Ali** model (1978), and these two models were tested with experimental results by **Vigneswaran** and **Chang**, 1989. They found that the first model, based on the detachment assumption, can simulate better than the second model does.

An algorithm was developed by **Choo** and **Tien** (1995b) for simulating particle deposition from liquid suspensions flowing through a granular media. The prediction of the extent of deposition and change of media permeability had been considered (entire cycle of filtration).

Finally, **Altnmann** and **Ripperger**, 1997 introduced a microscopic model of the layer formation and the cake growth at the cross flow micro-filtration. The model considers the hydrodynamic, adhesive and friction forces acting on single particle during the filtration process.

INITIAL FILTRATION COEFFICIENT

The clean bed filter coefficient, λ_o, which is constant for a given filter bed and an unchanging suspension, can be expressed in many terms given by many researchers who have tried to establish the relationship between the initial filtration coefficient and the various parameters as cited in **Tien** 1989.

Rajagopalan and **Tien** (1976), for using sphere-in-cell porous media (Happel model), gave the most important equation shown below.

$$\eta_o = 0.72\, A_s N_L^{1/8} N_R^{15/8} + 2.4 \times 10^{-3} A_s N_G^{1.2} N_R^{-0.4} + 4 A_s^{1/3} N_{Pe}^{-2/3}$$

for

$N_R \le 0.18$ and for interaction is favorable (i.e. when polymers is used) (16)

where ,

$$N_L = \frac{4H}{9\pi \mu d_p^2 u} \qquad (17)$$

$$N_R = \frac{d_p}{d_g} \qquad (18)$$

$$N_G = \frac{g(\rho_p - \rho_f) d_p^2}{18 \mu u} \qquad (19)$$

$$N_{Pe} = \frac{u d_g}{D} = \frac{3\pi \mu u d_p d_g}{KT} \qquad (20)$$

$$A_s = \frac{2(1 - P^5)}{w} \qquad (21)$$

$$P = (1 - e_o)^{1/3} \qquad (22)$$

$$w = 2 - 3P + 3P^5 - 2P^6 \qquad (23)$$

where,

- N_R : Relative size group value of N_R ranges between (2×10^{-4} to 1×10^{-1}), [Ives, 1975a]
- N_L : London group
- N_G : Gravity group
- M_{pe} : Peclet number as given as [Ives, 1975a]
- A_s : Happel's number, as given by Happel (1958), cited in Tien, 1989.
- **H** : Hamaker's constant, $H=1\times10^{-13}$ erg for 20°C and $H=5\times10^{-13}$ erg for 25°C [Payatakes *et al*, 1974b].
- d_p : Diameter of Suspension particle, cm
- **g** : Gravitational acceleration = 9.81 m / sec^2
- ρ_p : Density of particles, gm / cm^3
- ρ_f : Density of fluid (water), gm / cm^3
- **D** : Diffusion coefficient
- **K** : **Boltzmann**'s constant (energy per degree) $K = 1.38048\times10^{-16}$ erg / K [**Rajagopalan** and **Tien**, 1976]
- **T** : Absolute temperature, K (Kelvin)
- μ : Viscosity of fluid, gm/cm.sec

In 1979, **Rajagopalan** and **Tien** gives another form to the previous Equation (16), as cited in [**Tien *et al.***, 1979; **Chiang** and **Tien**, 1985b; **Tien, 1989**];

$$\eta_o = (1-e)^{2/3} A_s N_L^{1/8} N_R^{15/8} + 3.375\times10^{-3}(1-e)^{2/3} A_s N_G^{1.2} N_R^{-0.4} + 4A_s^{1/3} N_{Pe}^{-2/3};$$

for $N_R \le 0.18$ and for interaction is favorable (24)

Chiang and **Tien** (1985b) compared between the experimental and theoretical results from Eq.(24). They found a fairly good agreement between them. So this model is used in this research to calculating η_o.

MATHEMATICAL MODELS:

1- The model of O'Melia and Ali 1, 1978"

The model developed by O'Melia and Ali was based on some experimental observations. They suggested that particles removed during the early stage of filter run can serve as collectors for particles reaching the bed at later times. Analytical solutions were given by (Ali, 1977). In this solution ηr and c are considered as step functions rather than continuous functions of time [O'Melia and Ali, 1978]. The resulted equation is as follows:

- c_j Particle concentration in the depth z at j time step, mg / cm3
- η_r Single collector removal efficiency of filter grain and retained particles.
- η_r^{j-1} Single collector removal efficiency for the (j-1) time step
- η_p Efficiency of clean filter grain
- β Fraction of the particles retained directly on the filter grain which act as additional collectors
- γ Particle – grain collision efficiency factor (fraction of the contacts between a filter grain and suspended particle)
- γ_p Contacts between retained particle and suspended particle
- η_n Contact efficiency of retained particle

$$ln\frac{c_j}{c_o} = \frac{-3}{2}\eta_r\gamma(1-e)\frac{z}{d_g}$$
$$\left\{1+\gamma_p\eta_p\bar{\beta}u\frac{\pi}{4}d_p^2\left[\sum_{i=1}^{j} c_{io}(\Delta t)exp.\left(\frac{-3}{2}(1-e)\frac{z}{d_g}\eta_r^{(i-1)}\right)\right]\right\} \quad (25)$$

where

O'Melia and Ali when compared with experimental data took γ=γp=1.

The single collector efficiency at a given time was proposed by O'Melia and Ali, as cited in [Tien, 1989], is as follows:

$$\eta_r^i = \gamma\eta_{ro}\left[1+\left(\bar{\beta}\gamma_p\eta_p\right)u\frac{\pi}{4}d_p^2\Delta t\sum_{k=1}^{i}c^{k-1}\right] \quad (26)$$

O'Melia and Ali model described above contains adjustable parameters which are present in groups: γηro, γpηpβ‾. O'Melia and Ali (1978) suggested that these parameters should be considered as empirical parameters, to be determined from experimental data. The empirical expressions for estimating these parameters were given by many researchers. A summary of these empirical expressions is given in Table (1), [Vigneswaran and Chang, 1986, 1989; Vigneswaran and Tulachan, 1988; Tien, 1989].

When the empirical expressions are used in the present research, only Equation (27) gives the best results among the empirical expressions presented in Table (1) when the predicted results were compared with the experimental data.

$$\bar{\beta}\gamma_p\eta_p = 42.7u^{-0.8}c_{in}^{-1.32}d_g^{-0.41} \quad (27)$$

with γ=1, and ηo is estimated using Rajagopalan and Tien (1976) mode

Equations (25) and (26), can be used in obtaining the relationship between F1=λ/λo vs. σ in the following manner: The corresponding value of the specific deposit can be found from overall mass balance as given by Tien (1989),(Eq. (14)).Thus, the relationship between F1=ηr/γηro and σ is established.

2- The model of Tien et al. ,1979 [Tien *et al.*, 1979; Tien and Payatakes, 1979, Tien, 1989].

This model was developed by postulating an overall picture of the filtration process which views it as consisting of two consecutive stages, as has been observed in the visual work of **Pendse *et al.*** (1978). These two stages are:

1-The first stage is one in which deposition occurs primarily through the direct adhesion of individual particles to filter grains. The consequence of this mode of deposition is the formation of a relatively smooth layer of deposits outside filter grains. This first stage will continue until the local specific deposit, □, reaches a transition value, σ_{tran}(σ_{tran} is the value of specific deposit when $\lambda=\lambda_{max.}$).

2- The second stage is dominated by the formation of particle aggregates, the reentrainment of part of these aggregates, and their redeposition into constriction pores.

The expression, obtained by **Rajagopalan** and **Tien** (1976), correlates collection efficiencies form trajectory calculations was developed by **Tien *et al*,** (1979) to predict the dynamic behavior of deep bed filter over the entire filter cycle as illustrated below(For first stage only). A clogged bed differs from that in the clean state by the increased value of d_g and the decrease value of porosity.

The changes in these parameters are expressed below:

$$e = e_o - \frac{\sigma}{1-e_d} \qquad (28)$$

$$\frac{d_g}{d_{go}} = \left(\frac{1-e}{1-e_o}\right)^{1/3} \qquad (29)$$

where,

$_o$: Initial condition

e_d : Porosity of deposit

$$F_1 = B_1 \frac{A_s}{A_{so}}\left(\frac{1-e}{1-e_o}\right)^{17/24} + B_2 \frac{A_s}{A_{so}}\left(\frac{1-e}{1-e_o}\right)^{4/3} + B_3\left(\frac{A_s}{A_{so}}\right)^{1/3}\left(\frac{1-e}{1-e_o}\right)^{4/9} \qquad (30)$$

or

$$F_1 = B_1 \frac{A_s}{A_{so}}\left[1+\frac{\sigma}{(1-e_o)(1-e_d)}\right]^{17/24} + B_2\left(\frac{A_s}{A_{so}}\right)\left[1+\frac{\sigma}{(1-e_o)(1-e_d)}\right]^{4/3} + B_3\left(\frac{A_s}{A_{so}}\right)^{1/3}\left[1+\frac{\sigma}{(1-e_o)(1-e_d)}\right]^{4/9};$$

$$for\ \sigma < \sigma_{tran} \qquad (31)$$

where,

$$B_1 = \frac{(1-e_o)^{2/3}}{\eta_o} A_{so} N_L^{1/8} N_{Ro}^{15/8} \qquad (32)$$

$$B_2 = \frac{3.375\times10^{-3}}{\eta_o}(1-e)^{2/3} A_{so} N_G^{1.2} N_{Ro}^{-0.4} \qquad (33)$$

$$B_3 = \frac{4 A_{so}^{1/3} N_{Peo}^{-2/3}}{\eta_o} \qquad (34)$$

A_{so} : The value of Happel parameter (A_s) initially [namely $e = e_o$].

3- The model of Chiang and Tien, 1985

[Chiang and Tien, 1985a, 1985b]

In this model, the effect of particle deposition on the dynamic behavior of deep bed filters was examined by considering two limits situations:

A- In the first case, particle collection on filter grains results in the formation of a reasonably smooth deposit layer outside the filter grains

B- For the second limiting situation, it is assumed that the build up of particle deposits can be described by the hypothesis advanced by Tien et al. (1977) and Wang and Tien (1977). The structure of the deposits, once formed, was assumed to be rigid.

Two basic characteristics, singular and random behavior, and shadow effect were used to view the particle deposition that is illustrated in Ithari and Anbari ,2001.

Neither case completely describes the deposition phenomenon taking place in a real filter since physical processes can often be described approximately by a combination of their limiting situations. In addition, two limits do not agree with the experimental data when comprised with Chiang and Tien experimental work; the experimental data fall between those of the two limiting cases, as shown in Fig.(1).

Consequently, Chiang and Tien developed an empirical correlation capable of predicting the effect of particle deposition on collection efficiency (i.e., F1) based on theoretical as well as on experimental results.

$$F_{I,exp.} = \bar{f} F_{I(b)} + (1 - \bar{f}) F_{I(A)} \quad (35)$$

$F_{1,exp.}$: Experimentally determined value by optimization method (discussed later)

$F_{1(A)}$: Theoretical value based on limiting case **A**

$F_{1(B)}$: Theoretical value based on limiting case **B**

$\bar{f}$: Weighting factor

Based on the average value of f, the five runs of Chiang and Tien,1985b work was conducted for $\sigma \geq 2 \times 10\text{-}4$, according to (Chiang and Tien, 1985b) f may be taken to be 0.25 or

$$F_I = 0.25 F_{I(B)} + 0.75 F_{I(A)} \quad (36)$$

F1(A) and F1(B) are combined to obtain F1's as an approximation, the following two expression were given:

$$F_I = \frac{\eta}{\eta_o} = 1 + \sigma^{0.713}(213 - 3310 N_R) \quad (37)$$

or

$$F_I = \frac{\eta}{\eta_o} = 1 + \sigma^{0.755}\left(492 - 1.6 \times 10^4 N_R + 1.46 \times 10^5 N_R^{\ 2}\right) \quad (38)$$

Equation (38) gives a better accuracy than Equation (37) [Chiang and Tien, 1985b]. 4- The model of Mackie et al. 1987 [Mackie et al., 1987; Mackie, 1989].

The transient state model of Mackie et al. was developed from a consideration of the physical processes that occur in deep bed filtration. As a result, the need to rely on empirical parameters that have no physical meaning is eliminated. The model is based on three main elements:

1. Deposited particles themselves act as collectors. This is not a new idea, but previous workers such as Wang & Tien. (1977) calculated the effect of deposited particles by considering each particle individually, where in this model statistical theory was used.
2. It considers both the macroscopic and microscopic effect of deposition on grain. If the grain could be viewed from a distance it would look like Fig.(2a)(dome shape), but if a close view of a section of the grain was taken it would look like that in Fig.(2b).
3. Reductions in removal efficiency results primarily from increases in interstitial velocities as deposits grow.

The entire collector used in this model is the Happels sphere in-cell model. In recasting the result of Mackie et al. for the simple case of monodispersed suspension, the relationship of F1 vs. σ on the basis that interception and sedimentation are the dominant mechanisms of deposition and the effect due to the presence of deposit layers is insignificant, can be determined as follows:

1. When the shape of the deposit is described by a dome shape ($0<\sigma<\Delta\sigma$) then the trajectory parameter for capture particles passing through (1+dp+NR, ϕ) or through (1+3 NR, ϕ) is as follows [Mackie et al., 1987]

$$A(\phi) = \bar{\bar{\psi}}(1 + 3 N_R . \phi) + \frac{1}{2} N_G (1 + 3N_R)^2 \sin^2 \phi \quad (39)$$

where **A(ϕ)** is trajectory parameter.

$$\overline{\overline{\psi}} = \frac{\overline{\psi}}{v/2} =$$

$$sin^2\phi\left[\frac{k_1}{r*} + k_2 r* + K_3 r*^2 + k_4 r*^4\right] \qquad (40)$$

$$r* = \frac{r}{a_s} \qquad (41)$$

$$k_1 = \frac{1}{w} \qquad (42)$$

$$k_2 = \frac{(3 + 2P^5)}{w} \qquad (43)$$

$$k_3 = P\frac{(3 + 2P^5)}{w} \qquad (44)$$

$$k_4 = -\frac{P^5}{w} \qquad (45)$$

$$w = 2 - 3P + 3P^5 - 2P^6 \qquad (46)$$

$$P = (1-e)^{1/3} \qquad (47)$$

r* : Dimensionless radial coordinate
r : Radial coordinate
$\overline{\psi}$: Stream function of **Happel** model
ϕ : Angle of deposit
a_s=d_g/2: Radius of a typical grain (spherical collector), cm

The collection efficiency can be calculate as follows:

$$\eta_\phi = \frac{2A_\phi p^2}{(1+N_G)} \qquad (48)$$

where,

$$A_\phi = max[A(\phi), A_o] \qquad (49)$$

$$A_o = A_{lim} \qquad (50)$$

$$A_{lim} = \overline{\overline{\psi}}(1 + N_R, \pi/2) + \frac{1}{2}N_G(1+N_R)^2 \qquad (51)$$

$$\overline{\eta}_{\sigma=o} = \int_0^{\pi/2}\eta_\phi f(\phi)d\phi \quad (52)$$

$\overline{\eta}_{\sigma=0}$ is mean value of η_ϕ(collection efficiency in presence of deposited particle situated at ϕ with0<ϕ <Л/2).

$$f(\phi) = 2\sin\phi\cos\phi \qquad (53)$$

$$\eta = p_o\eta_o + (1-p_o)\overline{\eta}_{\sigma=o} \qquad (54)$$

where,

p_o : Probability of no particles are deposited
(1-p_o) : Probability of at least one particle is deposited

$$p_o = (1-\overline{p})^{n'} \qquad (55)$$

$$\overline{p} = \left(\frac{d_p}{\pi}\right)\left\{1 - \left(\frac{d_p}{4\pi}\right)\right\} \qquad (56)$$

n': number of particles deposited, which act as additional collectors.

$$n' = \frac{\sigma}{N_R^{\ 3}(1-e_o)} \qquad (57)$$

2. When the shape of the deposit is describe as dendritic and smooth coating deposition {(**k-1**) **Δσ**<σ< **k Δσ**, **k**=2, ...} then the trajectory parameter capture particles passes through

$(1 + e_\sigma\cos\phi - e_\sigma^2\cos^2\phi + x, \phi)$ is:

$$e_\sigma = \frac{4\sigma}{3(1-e_o)(1-e_d)} \qquad (58)$$

$$A(\phi)_\sigma = \overline{\overline{\psi}}(1 + e_\sigma\cos\phi - e_\sigma^2\cos^2\phi + x, \phi) + \frac{1}{2}N_G(1 + e_\sigma\cos\phi - e_\sigma^2\cos^2\phi + x)\sin^2\phi \qquad (59)$$

where,

$$x = d_p + N_R \qquad (60)$$

As before,

$$\eta_{\phi_\sigma} = \frac{2A_{\phi_\sigma}p^2}{(1+N_G)} \qquad (61)$$

$$A_{\phi_\sigma} = max[A(\phi)_\sigma, A_o] \qquad (62)$$

$$\overline{\eta}_{\sigma=(k-1)\Delta\sigma} = \int_0^{\pi/2}\eta_{\phi_\sigma} f(\phi)d\phi \quad (63)$$

$$f(\phi) = 2\sin\phi\cos\phi \qquad (53)$$

$$\eta = p_o\eta_{\sigma=(k-1)\Delta\sigma} + (1-p_o)\overline{\eta}_{\sigma=(k-1)\Delta\sigma} \qquad (64)$$

The number of particles deposited, which act as additional collectors is given by:

$$n' = \frac{\sigma - (k-1)\Delta\sigma}{N_R^{\,3}(1-e)} \qquad (65)$$

Mackie *et al.* stated that the value of $\Delta\sigma$ used in carrying out the calculations described above is not critical and they suggested a value between 10^{-4} to 5×10^{-3}, but **Choo** and **Tien** 1993 gave Eq.(66) for determining $\Delta\sigma$ based on certain physical arguments.

$$\Delta\sigma = (1-e_o)(1-e_d)\,\sigma N_R; \quad for \;\; 10^{-4} < \Delta\sigma < 5\times10^{-3} \qquad (66)$$

In the present research Eq.(66) is applied and if the result is out of range then $\Delta\sigma$is taken to be equal to 0.004.The model described above can then be used to work out the change in λ, i.e.λ/λo only.

5- The model of Choo and Tien, 1993 [Choo and Tien, 1993, 1995a]

Choo and **Tien** (1993) gave the recent model for describing filter media with significant deposition. A porous media which considers the presence of porous deposit layer was formulated by modifying the classical sphere in-cell model. Unlike the previous works the thickness of deposit layer is not assumed to be uniform and more importantly, the deposit layer itself is considered porous and permits the flow of fluid.

Based on this model, an increase in filter coefficient as a function of the extent of deposition can be determined. To facilitate the use of the result of this analysis, an empirical expression relating $\bar{\lambda}/\lambda_o$ with the external deposition and other relevant parameters was established. The numerical results used to obtain the empirical expression cover the following ranges of variables: [**Choo** and **Tien**, 1995a].

$$0 \le \sigma \le 2\times10^{-1}$$
$$10^{-6} \le k_d \le 1$$
$$0.4 \le e_o \le 0.6$$
$$0.6 \le e_d \le 0.8$$
$$0.005 \le N_R \le 0.05$$

where **k**$_d$ is permeability of deposit layer.

The empirical equation was assumed to have the following format:

$$F_1 = \frac{\lambda}{\lambda_o} = 1 + Y\left\{\left(\frac{\lambda}{\lambda_o}\right)_{k_d=0} - 1\right\} + (1-Y)\left\{\left(\frac{\lambda}{\lambda_o}\right)_{k_d\to\infty} - 1\right\} = Y\left(\frac{\lambda}{\lambda_o}\right)_{k_d=0} + (1-Y)\left(\frac{\lambda}{\lambda_o}\right)_{k_d\to\infty} \qquad (67)$$

where $(\lambda/\lambda_o)_{k_d=0}$ and $(\lambda/\lambda_o)_{k_d\to\infty}$ are values of **F**$_1$ corresponding to the two limiting cases; namely the case in which the deposit permeability is zero (i.e. non-permeable deposit layer) and the case with infinitely large deposit permeability. **Y** is a weighting factor.

$$\left(\frac{\lambda}{\lambda_o}\right)_{k_d=0} = 1 + 9.61\sigma\frac{(1-e_o)^{2/3}}{(1-e_d)} \quad (68)$$

$$\left(\frac{\lambda}{\lambda_o}\right)_{k_d\to\infty} = 1 + 0.6794\left(\frac{1}{N_R} - 0.921\right)\frac{\sigma}{(1-e_d)(1-e_o)} + 0.173\left(\frac{1}{N_R^2} + \frac{3}{N_R} - 1.17\times10^{?}\right)\left(\frac{\sigma}{(1-e_d)(1-e_o)}\right)^2 \quad (69)$$

$$Y = \frac{x}{1+x} \quad (70)$$

$$x = 0.598\,k_d^{-0.8}\left(1 + \frac{0.0128}{N_R}\right)\left\{\frac{\sigma}{5(1-e_d)}\right\}^{1.63+\frac{3.5\times10^{-?}}{N_R}} \times \left(\frac{1}{1-e_o}\right)^2 \quad (71)$$

k_d was estimated according to the **Craman-Kozeny** Equation, Eq.(72) below, or twice that value.

$$k_d = 3.5\frac{e_o^3}{(1-e_o)^{0.5}}\left[1 + 57(1-e_o)^3\right]\times10^{-6} \quad (72)$$

Eq.(72) after **Frank** [1975].

LIMITATION OF THE PRESENT STUDY

All the trajectory models that are limited to the considerations of single-medium filter for removal of monodispersed particles and for ripening period only.

When applying the models of **Tien** ***et al.*** (1979), **Mackie** ***et al.*** (1987), and **Choo** and **Tien** (1993), the value of the deposit porosity, **e_d**, is required. In the present study this value (**e_d**) is taken to be equal to 0.7 and 0.8. This particular choice is made after considering the results of a number of previous studies. **Deb** (1969) found experimentally that the porosity of flocs formed from particulate matter equals 0.75. A theoretical study by **Hutchinson** and **Sutherland** (1965) concerning the formation of floc from coagulation indicated that porosity of deposit equals 0.8., as cited in [**Vigneswaran** and **Tualchan**, 1988; **Tien**, 1989]., but when **Vigneswaran** and **Chang** (1989) measured this value experimentally they found that it is in the range of 0.79 to 0.82. **Jung** (1991), and **Jung** & **Tien** (1993) gave a simulation to estimate porosity of deposit, **e_d**, they found it to be equal to 0.8, as cited in [**Choo** and **Tien**, 1995a]. The choice of 0.7 and 0.8 therefore, represents a reasonable compromise.

COMPARISONS:

1-Data

The experimental data (6 runs) used in the present study include those of **Chiang** (1985); **Vigneswaran** (1980), **Tanaka** (1982), and Mohammed (1989) for certain specific conditions which are listed in Tables (2) and (3).

2-Analysis approach

To quantify the fitness of trajectory models to the experimental data a statistical approach which includes the mean square error was used [Draper and Smith, 1981].

3-Comparison of results

a- Comparisons of theoretical results with experimental data obtained by Chiang 1985

Chiang (1985) conducted filtration experiments using monosized filter grains and monodispersed suspensions. Glass bed media, lycopodium and ragweed particle were used with filter depth equal to 0.5 cm.

Comparisons of F_1 vs. σ are shown in Figs. (4 and 5) for different conditions runs Nos. 2, and 4 respectively.

Table (4) shows the fitness of the five trajectory models with the data obtained from Chiang 1985 using the mean square error as a statistical approach.

From Table (4), it can be seen that there is a good agreement between Chiang and Tien model with the experimental data. This can be attributed to the fact that they used the same experimental data to fit their empirical equations of the model. The combination factor, they used, was $\bar{f}$=0.25. There is some agreement between Tien et al model with the experimental data. All the other models predict values which are close to each other but these values differ considerably from the experimental values.

b. Comparison of theoretical results with experimental data obtained by Tanaka 1982

Tanaka (1982) performed an extensive study on effect of polyelectrolyte addition on filter performance. Among the results he obtained were several runs in each of which the time-dependent behavior of effluent quality at three depths of filter [12.7, 30.48, and 50.8 cm] was presented. The major difference in experimental conditions among these runs is the using of flocculent (polyelectrolyte). The type of the polyelectrolyte is given in Table (3).

The comparisons of the two Tanaka experimental runs with the trajectory models prediction of F_1 vs. σ are shown in Figs. (6,7, and 8).

The results of fitting the models to experimental data using the mean square error measure are listed in Table (5).

It can be noticed from Figs.(6) through (8) and Table (5) that O'Melia and Ali model, Mackie *et al.* model, and Tien *et al* model (using e_d=0.7) give good agreement with experimental data, while Chiang and Tien model, and Choo and Tien model do not agree with experimental data.

c. Comparison of theoretical results with experimental data obtained by Mohammed 1989.

Mohammed 1989 conducted a comparison of performance of the conventional and dual-media filters using sand media filter as conventional type.

The results of comparisons of conventional type with the predictions of the five trajectory models are shown in Figs. (9, and 10) for the operative conditions of runs Nos. (4, and 6). These figures give F_1 vs. σ relationships

Table (6) gives values of mean square error for Mohammed experimental data.

It can be seen that O'Melia and Ali model, Tien *et al* model (with e_d=0.7), and the model of Choo and Tien agree well with experimental data. Chiang and Tien model, and Mackie *et al.* model do not agree with experimental data.

CONCLUSION AND VALIDITY OF THE TRAJECTORY MODELS:

The validity of the trajectory models is tested by comparing the predicted histories of filtrate quality with experimental results reported earlier and listed in Tables (2, and 3). The validity analysis and results are summarized as follows:

1. Chiang and Tien model was based on experimental results obtained from Chiang and Tien work, in cooperation with theoretical analysis. Thus, one may argue that the validity of the model extends beyond the conditions used in the experimental work. This claim can be justified by the fact that, unlike earlier empirical correlations, Eq.(38) demonstrates the explicit effect of operating variables (through the dependence of FI on NR=dg/dp) as well as the substantial agreement between predictions and experiments reported from these independent sources. This correlation ignores the effect of surface interaction, especially that resulting from the addition of polyelectrolyte between filter grain and suspended particles themselves. Consequently, one may argue that the combination factor of of Eq.(35), should not be constant but is, instead, dependent upon the surface interaction between particles and filter and between particles themselves.
2. O'Melia and Ali model, and Tien et al. model show very good agreement with experimental data because they both take most of the parameters that affect filter efficiency into consideration as follows.

   A. O'Melia and Ali model takes the following parameters into consideration [co, u, z, dg, dp, e, and the effect of chemical additives].

   B. Tien et al model takes the major removal transport mechanisms index [NR, NG, NL, and NPe] into account. Namely most of the parameters that mentioned earlier were taken when building up this model such as [d_p, d_g, t_c, u, ρ_f, ρ_p, e, e_d, μ, ... etc.].
3. The accuracy of predictions of Mackie *et al.* model depends upon the value of $\Delta\sigma$ which is used in making the predictions.
4. Application of Choo and Tien model requires the knowledge concerning the structure of the deposit layer formed, its porosity, and permeability. The porosity of deposit layer was estimated from previous studies to be 0.7 or 0.8. The permeability may be estimated by using the Garman-Kozeny equation.

One can conclude that this model is easy to be applied but it gives poor agreement with experimental data. This can be attributed to that it was derived according to theoretical analysis but simplified by giving empirical equations. In addition, it ignores many effective parameters on filter performance.

References

1. Adin A., Rebhun M., "A model to predict concentration and head – loss profiles in filtration" AWWA Vol.69, No.8, PP.444 . [1977]
2. Altmann J., Ripperger, S., "Practical deposition and layer formation at cross flow microfiltration" J. of

Membrane Science, Vol.124, No.1, pp.119-128. [1997]
3. Amirtharajah A., "Some theoretical and conceptual views of filtration", AWWA, Vol.80, No.2, pp. 36-46. [1988]
4. Anbari R.H., "Selected alternatives for up-grading existing water treatment plants. A quantitative and qualitative improvement" Thesis for Ph.D. in Environmental Engineering in University of Technology, Iraq. [1997]
5. Charles R.O., Richard H.B., "Theoretical evaluation of filter modeling experiments" Journal of the Sanitary Engineering Division, Vol.96, No.SA2, pp. 7233. [1970]
6. Chiang H.W., Tien C., "Dynamics of deep bed filtration part I: Analysis of two limiting situations", AICHE J Vol.31, No.8, pp.1349-1359. [1985a]
7. Chiang H.W., Tien C., "Dynamics of deep bed filtration part II: Experiment", AICHE J Vol. 31, No.8, pp.1360-1370. [1985b]
8. Choo C.U., Tien C., [1993] "Transient – state modeling of deep bed filtration", Water Science and Technology Vol. 27, No. 10, pp.101-116.
9. Choo C.U., Tien C., "Analysis of the transient behavior of deep bed filtration", Journal of Colloid and Interface Science, Vol.169, pp.13-33. [1995a]
10. Choo C.U., Tien C., "Simulation of hydrosol deposition in granular media"; AICHE J., Vol.4, No.6, pp.1426-1442. [1995b]
11. Draper W.R., Smith H., "Applied regression analysis", Second Edition, John Wiley and Sons. New York, p.294. [1981]
12. Ithari A.J., Anbari R.H., "Conceptual views on deep bed filtration models", J. of Babylon University, Vol.6, No.5. [2000]
13. Mackie R.I., "Rapid gravity filtration-towards a deeper understanding", J. Filtration and Separation, January / February, pp.32-35. [1989]
14. Mackie R.I., Horner R.M.W., Jarvis R.J., "Dynamic modeling of deep bed filtration", AICHE J Vol.33, No.11, pp.1761-1775. [1987]
15. Mohammed O.I., "A comparison between the performance of the conventional and dual-media filters", MSc. Thesis University of Basrah. [1989]
16. O'Melia C.R., Ali W., "The role of retained particles in deep bed filtration", Water Tech. Vol. 10 , pp.167-182. [1978]
17. Ojha C.S.P., Graham N.J.D., "Prediction of deep bed filter performance using recursive algorithms", J. of Envir. Engrg. ASCE, Vol.120, No.4, pp.961-971. [1994]
18. Payatakes A.C., Park H.Y., Petrie J., "A visual study of particle deposition and entrainment during depth filtration of hydrosols with polyelectrolyte", Chem. Eng. Sci., Vol.36, pp.1319-1335. [1981]
19. Payatakes A.C, Tien C., Turain R.M., "A new model for granular porous media I-

model formulation", AICHE J Vol.19, No.1, pp.58-76. [1973]

20. Pendse H., Tien C., Turain, R.M., "Dispersion measurement in clogged filter bed-a diagnostic study on the morphology of particle deposits", AICHE J., Vol.24, P.473. [1978]
21. Rajagopalan R., Tien C., "Trajectory analysis of deep bed filtration with the sphere-in-cell porous media model", AICHE J Vol.22, No.3, pp.523-533. [1976]
22. Tien C., "Granular filtration of aerosols and hydrosolos" Butterworth, Boston, MA. [1989]
23. Tien C., Payatakes A.C., "Advances in deep bed filtration", AICHE J. Vol.25, No.5, pp.737-759. [1979]
24. Tien C., Turian R.M., Pendse H., " Simulation of the dynamic behavior of deep bed filters", AICHE J Vol.25, No.3, pp.385 -395. [1979]
25. Tien C., Wang C.S., Barot D.T., "Chainlike formation of particle deposits in fluid-particles separation", Science Vol.196 , pp.983. [1977]
26. Tobiason J.E., O'Melia, C.R., "Physico-chemical aspects of particle removal in depth filtration", J. AWWA, Vol.80, No.12, pp.54-64. [1988]
27. Vigneswaran S., Chang J.S., "Mathematical modeling of the entire cycle of deep filtration", Wat. Air. Soil Pollut., Vol.29, pp.155-164. [1986]
28. Vigneswaran S., Chang, J.S., "Experimental testing of mathematical models describing the entire cycle of filtration", Wat. Res., Vol.23, No.11, pp.1413-1421. [1989]
29. Vigneswaran S., Tulachan R.K., "Mathematical modeling of transient behavior of deep bed filtration", Wat. Res, Vol.22, No.9, pp.1093-1100. [1988]
30. Wang C.S., Tien C., "Deposition of solid particles on a collector Formulation of New theory", AICHE J Vol. 23, No.6, pp.879-889. [1977]

Table (1): A summary of empirical expressions of O'Melia and Ali parameters

Researchers	Empirical formulations
Perera* (1982)	$\gamma_p \eta_p \overline{\beta} = 42.7u^{-0.8} c_{in}^{-1.32} d_g^{-0.41}$
Chang* (1985)	$\gamma_p \eta_p \overline{\beta} = 2.06 c_{in}^{-0.44} u^{-0.726}$
Vigneswaran and Chang (1986)	$\eta\gamma = 0.0236u^{-0.88}$ $\gamma_p \overline{\beta} = 2.535u^{-1.134}$
Vigneswaran and Tulachan (1988)	$\eta\gamma = 0.023\, u^{-0.88}$ $\gamma_p \overline{\beta} = 1.306\, u^{-1.125}$

u is in $m^3 / m^2\, h$
c is in mg/l
d_g is in cm
•cited in **Tien** 1989

Table (2): Experimental conditions of data used in the present study

Experiment by	d_p (cm)	d_c (cm)	u (cm\sec)	z (cm)	c_o (ppm)	e	N_R	N_G ×10^{-2}	N_L ×10^{-5}	N_{Pe} ×10^{7}
*Chiang No. 2	0.0026	0.0505	0.2	0.5	5	0.41	0.0515	0.9172	0.5852	5.3810
*Chiang No. 4	0.0026	0.0505	0.4	0.5	6	0.41	0.0515	0.4586	0.2926	10.762
*Tanaka No. 41a, 30a	0.00085	0.17	0.3	12.7	5	0.41	0.005	0.7016	3.6501	8.8830
*Tanaka No. 41b, 30b	0.00085	0.17	0.3	30.48	5	0.41	0.005	0.7016	3.6501	8.8830
*Tanaka No. 41c, 30c	0.00085	0.17	0.3	50.8	5	0.41	0.005	0.7016	3.6501	8.8830
Mohammed No. 4	0.0006	0.0478	0.138	40	30	0.42	0.0125	1.9311	15.582	0.8162
Mohammed No. 6	0.0006	0.0478	0.275	40	30	0.42	0.0125	0.9722	7.9669	1.6212

*cited in [**Chiang** and **Tien**, 1985b].

Table (3): Experimental conditions of data used in the present study

Experiment by	Particles in suspension	Flocculent used
*Chiang No. 2 (1985)	Lycopodium	None

***Chiang No. 4 (1985)**	Lycopodium	None
***Tanaka No. 41 a,b,c (1982)**	Kaolin	5 mg/L of PDADMAC587C
***Tanaka No. 30 a,b,c (1982)**	Kaolin	1 mg/L of PEI P-600
Mohammed No. 4 (1989)	Earth fuller	40 mg/L of Al2(SO4).18H2O
Mohammed No. 6 (1989)	Earth fuller	40 mg/L of Al2(SO4).18H2O

Table (4): Mean square error using as a statistical measure for Chiang 1985 experimental results.

Fig. No.	Exp. No.	m.s.e					Best Models
		O'Melia	Tien	Chiang	Mackie	Choo	
4a	2	7.129	5.368	0.953	7.666	7.044	Chiang,Tien
4b	2	7.129	1.441	0.527	7.404	6.011	Chiang,Tien
5a	4	1.322	0.797	0.801	1.280	1.122	Tien, Chiang
5b	4	1.322	4.541	0.801	1.187	0.801	Chiang, Choo

Table (5): Mean square error using as a statistical measure for Tanaka ,1982 experimental results:

Fig. No.	Exp. No.	m.s.e					Best Models
		O'Melia	Tien	Chiang	Mackie	Choo	
6a	41a	0.114	0.891	262.071	0.011	14.379	Mackie, O'Melia, Tien
6b	41a	0.114	12.343	262.071	0.010	19.630	Mackie, O'Melia
6a	30a	0.102	0.213	245.268	0.343	11.225	O'Melia, Tien, Mackie
6b	30a	0.102	9.097	245.268	0.300	15.888	O'Melia, Mackie
7a	41b	0.008	1.080	265.727	0.011	15.219	O'Melia, Mackie,Tien
7b	41b	0.008	12.998	265.727	0.018	20.616	O'Melia, Mackie
7a	30b	0.043	1.390	271.991	0.070	16.952	O'Melia, Mackie,Tien
7b	30b	0.043	13.909	271.991	0.092	22.640	O'Melia, Mackie
8a	41c	0.012	1.015	264.558	0.006	14.963	Mackie, O'Melia,Tien
8b	41c	0.012	12.797	264.558	0.010	20.320	Mackie, O'Melia
8a	30c	0.033	1.337	271.733	0.080	17.129	O'Melia, Mackie,Tien
8b	30c	0.033	13.695	271.733	0.100	22.851	O'Melia, Mackie

Table (6): Mean square error using as a statistical measure for Mohammed 1989 experimental results:

Fig. No.	Exp. No.	m.s.e					Best Models
		O'Melia	Tien	Chiang	Mackie	Choo	
9a	4	0.065	0.633	149.115	177.005	0.947	O'Melia, Tien, Choo
9b	4	0.065	7.242	149.115	350.174	1.611	O'Melia, Choo, Tien
10a	6	0.121	0.547	147.217	252.684	0.791	O'Melia, Tien, Choo
10b	6	0.121	6.576	147.217	499.685	1.405	O'Melia, Choo, Tien

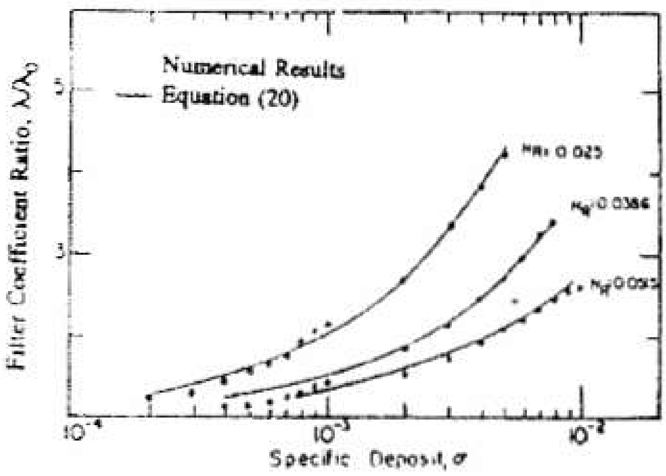
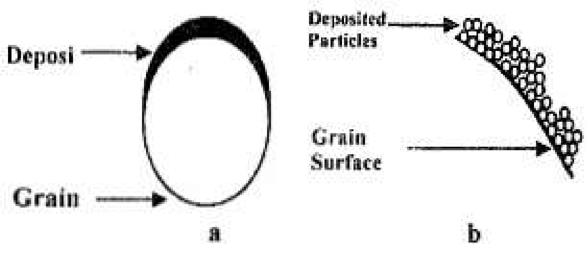


Fig (1): Comparison of predicted F_1 with experimental data (run No. 5,Chiang and Tien 1985b), according to two limiting situations, (after Chiang and Tien, 1985b)

Fig.(2) Deposit on a grain: (a) macroscopic view; (b) microscopic view, [after Mackie *et al.*, 1987]



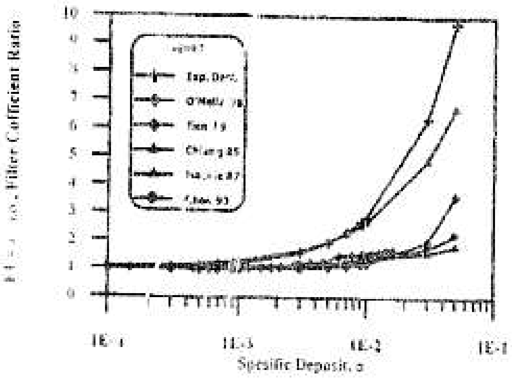


Fig.(4a) "Comparisons between the prediction of five models with experimental data under conditions of run No.2 of Chiang 1985"

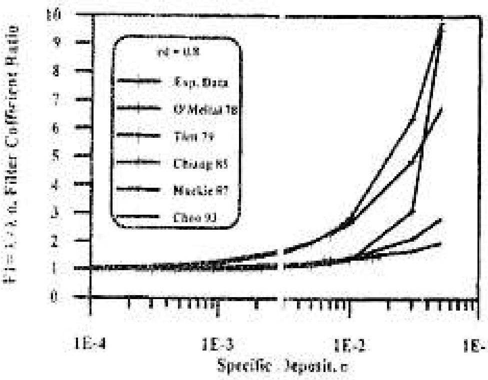


Fig.(4b) " Comparisons between the prediction of five models with experiment al data under conditions of run No. 2 o f Chiang 1985"

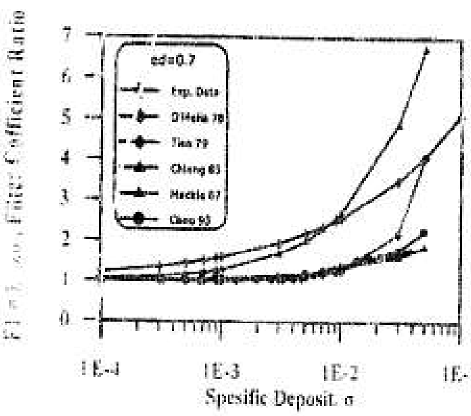


Fig (5a) "Comparisons between the prediction of five models with experimental data under conditions of run No.4 of Chiang 1985"

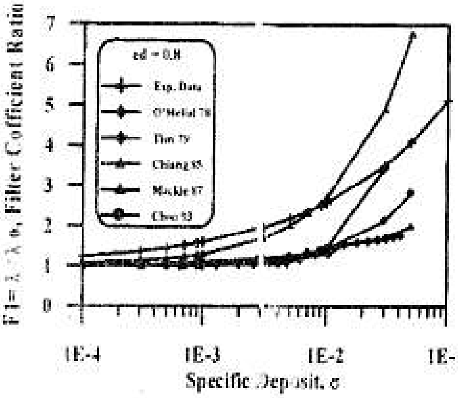


Fig.(5b) " Comparisons between the prediction of five models with experimental data under conditions of run No. 4 of Chiang 1985"

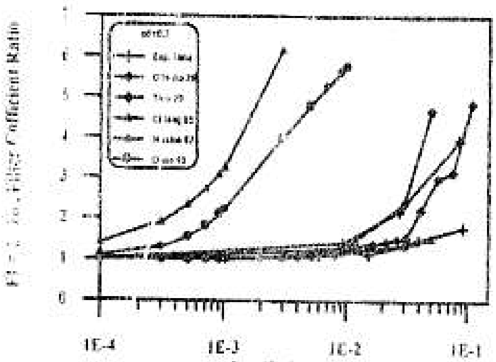


Fig.(6a) " Comparisons between the prediction of five models with experimental data under conditions of run Nos.(41a&30a) of Tanaka 1982

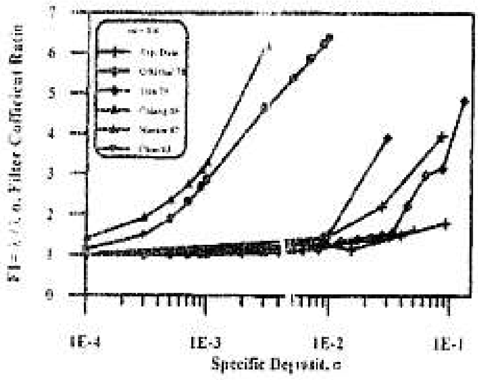


Fig.(6b) " Comparisons between the prediction of five models with experimental data under conditions of run Nos.(41a&30a) of Tanaka 1982"

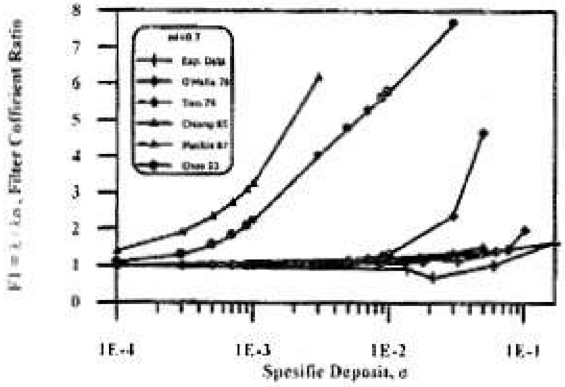


Fig.(7a) "Comparisons between the prediction of five models with experimental data under conditions of run Nos.(41b&30b) of Tanaka 1982

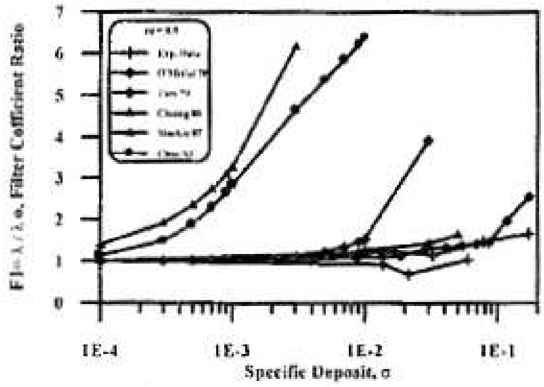


Fig.(7b) " Comparisons between the prediction of five models with experimental data under conditions of run Nos.(41b&30b) of Tanaka 1982"

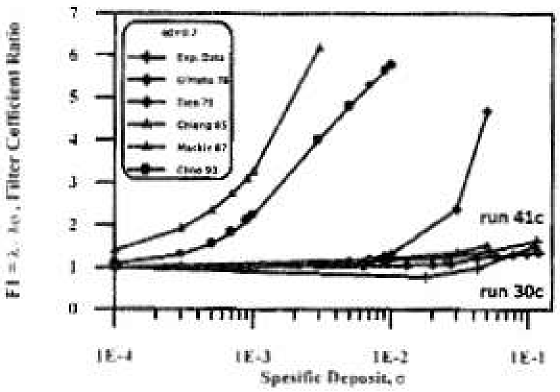


Fig.(8a) "Comparisons between the prediction of five models with experimental data under conditions of run Nos.(41c&30c) of Tanaka 1982"

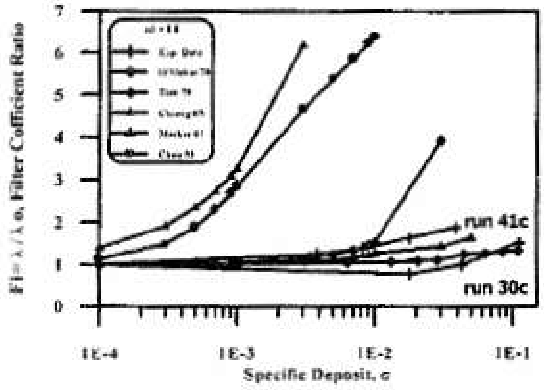


Fig.(8b) "Comparisons between the prediction of five models with experimental data under conditions of run Nos.(41c&30c) of Tanaka 1982"

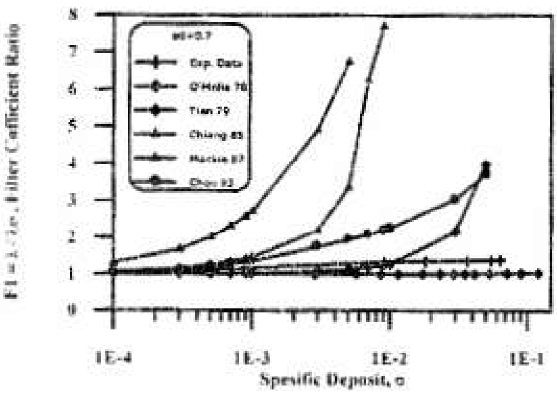


Fig.(9a) "Comparisons between the prediction of five models with experimental data under conditions of run No.4 of Mohammed 1989"

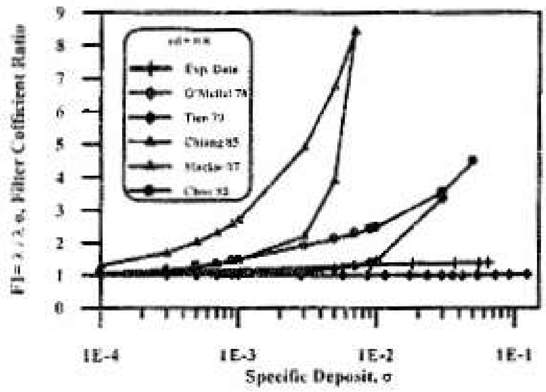
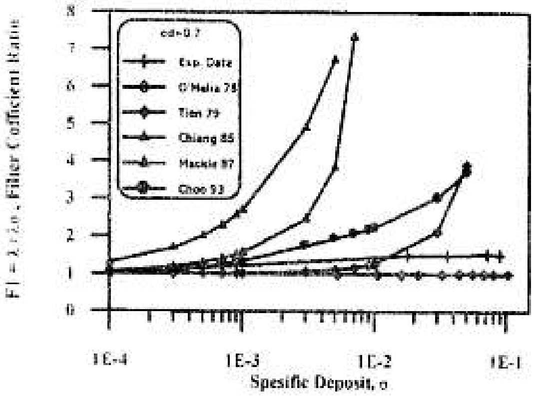
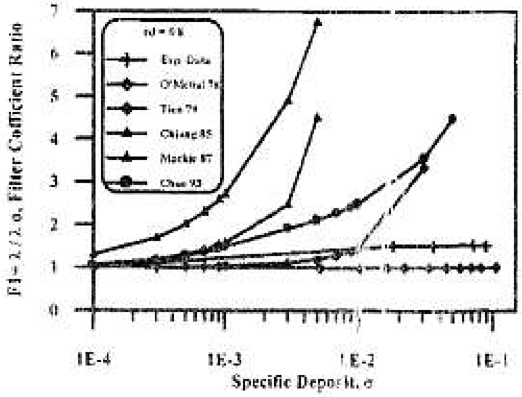


Fig.(9b) " Comparisons between the prediction of five models with experimental data under conditions of run No. 4 of Mohammed 1989"



Fig(10a) "Comparisons between the prediction of five models with experimental data under conditions of run No.6 of Mohammed 1989"



Fig(10b) "Comparisons between the prediction of five models with experimental data under conditions of run No. 6 of Mohammed 1989"