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NC Machining Of Free-Form Blanking Dies Cavities Using Linear Interpolation

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Abstract

This paper presents an analytical study on the path command generation for linear interpolation of blanking dies cavities defined by free form closed curves. These curves are highly used in mechanical applications like dies, cams, molds, etc. An integrated algorithm is presented in this paper to machine these cavities. When the coordinates of the free form curve's nodes are given(based on the shape of the die cavity), the proposed algorithm automatically generates the required tool path in three phases: phase (1) Generation the mathematical representation of the curve that passes through all the given points using the *Four Point Interpolation technique*. Phase (2) Optimum approximation of the generated curve into a sequence of linear segments passed on the intended accuracy tolerance. Phase (3) Tool path generation of the approximated segments using linear interpolation. This research also focuses on the algorithm can control the approximating error efficiently resulting in the fewest number of linear segments.

The closed curve of the die cavity is constructed from more than one curve segments; therefore the conditions of C^1 continuity are adopted in this paper for joining the curve segments. The proposed system software is most characterized by its elimination of any manual intervention involved in closing the boundary curve of the die cavity and in cutter offset determination.

Keywords: CAD/CAM, Interpolation, Tool Path, NC Machining, Free Form Curves and Blanking Dies.

الخلاصة

يتناول البحث دراسة تحليلية لعملية توليد مسار العدة باستخدام الاستكمال الخطي لتجاويف قوالب انتاج الاغفال المعرفة بالمنحنيات المغلقة ذات الاشكال الحرة. تستخدم هذه المنحنيات بشكل واسع في العديد من التطبيقات الميكانيكية مثل قوالب القطع ، الحدبات ، قوالب البلاستك .... الخ . تم في هذا البحث عرض خوارزمية متكاملة لتشغيل تجاويف قوالب انتاج الاغفال ، حيث عند ادخال احداثيات العقد (نقاط السيطرة) للمنحني الحر وبالاعتماد على شكل التجويف "تقوم الخوارزمية " المقترحة لتوليد مسار العدة بشكل مؤتمت وعلى ثلاث أطوار: الطور الاول ويتم خلاله توليد الصيغة الرياضية للمنحني والذي يمر بكل نقاط السيطرة باستخدام تقنية الاستكمال رباعي النقاط، الطور الثاني ويتم خلاله التقريب الامثل للمنحني المولد بمجموعة من القطع المستقيمة بالاعتماد على الدقة المطلوبة من عملية التقريب والطور الثالث يتم فيه توليد مسار العدة للقطع المستقيمة المقربة باستخدام الاستكمال الخطي . تركز البحث على امكانية الخوارزمية المقترحة السيطرة على الخطأ الناتج من عملية التقريب بكفاءة عالية وهذا بدوره ينعكس على نقصان عدد الخطوط المستقيمة . المنحنيات المغلقة المكونة لفجوة القالب تم بناءها من خلال ربط مجموعة من المنحنيات بعضها بالبعض الاخر وبهذا تم اعتماد شروط (c^1) للاستمرارية لربط هذه المنحنيات. من اهم مميزات النظام المقترح خلوه من اية حسابات

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يدوية قد يحتاجها المصمم لغلق منحني الفجوة أو قد يحتاجها المصنع لحساب مقدار ترحيل عدة القطع .

1.Introduction

A major area of interest in the CAD/CAM systems is the integration between them. The most important stage in the integration of computer aided design and computer aided manufacturing is the conversion of the CAD model into tool path of a CNC machine[1].

In truly integrated CAD/CAM systems, apart design is developed and the manufacturing process controlled from start to finish within a single system [2]. Many researches and Algorithms in the die design field were found. Cheok and Nee [4] described the configuration of progressive dies and some problems concerned with the design of these dies. Got Shepherd [5] examined some of the features of solid modeling in designing a progressive dies from construction of the piece parts to designing the die around the strip with the aid of (Pro E) solid modeling software. An integrated CAM system for the manufacturing the upper and lower part of the die was proposed by Choi [6]. From the open literature, most of the researches concerned with the design and manufacturing of blanking dies were limited at the basic primitive shapes for the modeling of die cavities such as cubic shapes, Rectangular shapes, circular shapes,...etc. or a combination between these shapes. This study discussed a strategy for designing and manufacturing of free form blanking die cavities.The proposed integrated software provides users with powerful interactive modification; shape changing capabilities and some manufacturing parameters.

In this paper the CAD model of the die cavity is go to build using four-point interpolation technique [3]. In this technique, each four points or nodes are interpolated into a curve segment and so on. All the generated curve segments are joined (sewing) together taking C^1 continuity conditions as criteria to generate a smooth closed curve which represent the boundary curve of the die cavity. Figure (1) shows the integrated system of closed free form die cavity design and manufacturing. The boundary curve is machined by flat-end tool; therefore the curve must be offset along the normal vector of the curve.

2-Boundary Curve Description of the Die Cavity

The preferred way to represent shapes in computer aided geometric design (CAGD) is with parametric equations since fully closed space curves can be expressed by the vector function of single parameter[3,7].

The parametric free form curve can be defined by [8]:

$$P(u)=[x(u)\ y(u)\ \ z(u)] \qquad (1)$$

Where u is an independent parameter takes the values from $-\infty$ to $+\infty$.But of course it is not possible to plot the curve for all values of $-\infty$ to $+\infty$.The designer should select a proper interval that has some significance to the modeling situation and that has computational convenience.

Passing a curve through a given discrete data points is denoted by *interpolation* [9], the four-point interpolation formula is the technique used for designing any curve segment which is adopted in this paper. The four-point interpolation is a special case of cubic Hermit curve that passes through four given points. We begin by specifying four distinct points in x-y plane [**P1 P2 P3 P4**], assigning to each successive u values, so that u1=0 which gives **P1**, u2=1/3 which gives **P2**, u3=2/3 which gives **P3** and finally u=1 which gives P4. The parametric equation in vector form for four-point interpolation is given by [3]:

$$\mathbf{P}(u)=G1(u)\mathbf{P1}+G2(u)\mathbf{P2}+G3(u)\mathbf{P3}+G4(u)\mathbf{P4} \quad \ldots\ldots\ldots\ldots(2)$$

Where G1, G2, G3 and G4 are the basis functions given by:

$$G1(u)=-4.5u^3+9u^2-5.5u+1;$$
$$G2(u)=13.5u^3-22.5u^2+9u;$$
$$G3(u)=-13.5u^3+18u^2-4.5u;$$
$$G4(u)=4.5u^3-4.5u^2+u; \quad \ldots\ldots\ldots(3)$$

The tangent vector at any point on the curve is given by the first derivative of the parametric equation defining the curve [3,10]. Mathematically the tangent vector is denoted by $\mathbf{P}^{u}(u)$,and from equations (2 and 3) [3] :

$$\mathbf{P}^{u}(u)=G1^{u}(u)\,\mathbf{P1}+G2^{u}(u)\,\mathbf{P2}+G3^{u}(u)\,\mathbf{P3}+G4^{u}(u)\,\mathbf{P4} \quad \ldots\ldots\ldots\ldots(4)$$

Where
$$G1^{u}(u)=-13.5u^2+18u-5.5;$$
$$G2^{u}(u)=41.5u^2-45u+9;$$
$$G3^{u}(u)=-41.5u^2+36u-4.5;$$
$$G4^{u}(u)=13.5u^2-9u+1; \quad \ldots(5)$$

Substituting any value of u from 0 to 1 in the above equation with specified increment of Δu, we can find the tangent vector at any point on the curve as shown in Figure (2).

3.Blending Curves Technique

Inserting a missing curve segment between two existing or disjoint curve segments to form a more complex composite closed curve or to close any open curve is called blending curve [11]. The blending technique is one of the most important tasks in closing the open or disjoint curves. The most important conditions for blending a curve segment between two disjoint curves are the conditions of continuity at a junction between the two disjoint curves, this condition is termed as *geometric continuity*, denoted as C^n . The first-order geometric continuity C^1 which is adopted in this paper requires the following conditions [3,8] see Figure (2):

1. $\mathbf{P}_1(1)=\mathbf{P}_2(0) \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$
   $\mathbf{P}_3(0)=\mathbf{P}_2(1) \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$

2. $\mathbf{P}^{u}_2(0)=\mathbf{P}^{u}_1(1) \ldots\ldots\ldots\ldots\ldots\ldots(8)$
   $\mathbf{P}^{u}_2(1)=\mathbf{P}^{u}_3(0) \ldots\ldots\ldots\ldots\ldots\ldots(9)$

Solving equations (2-9) gives the coordinates of the two missing data points $P_2^{\,3}$ and $P_2^{\,2}$ which are required for blending any curve segment as follows:

$$P_2^{\,3} = A\acute{C} + BP_2^{\,1} - \hat{C}\check{C} - DP_2^{\,4} \quad \ldots..(10)$$

$$P_2^{\,2} = E\check{C} + EP_2^{\,1} - FP_2^{\,3} + G\,P_2^{\,4} \ldots(11)$$

Where A = 0.0842 ; B = 0.4386 ; $\hat{C}$ = 0.0249
D = 0.1449 ; E = 0.1818 ; F = 1.8100
G = 1.0000

$$\acute{C} = -P_1^{\,1} + 5.5P_1^{\,2} - 10P_1^{\,3} + 5.5\,P_1^{\,4}$$
$$\check{C} = -5.5P_3^{\,1} + 9P_3^{\,2} - 4.5P_3^{\,3} + P_3^{\,4}$$

4.New Tool Path Generation Method Using Linear Interpolation

For traditional CNC milling machines, only liner and circular tool movements are available [8]. Thus, desired tool paths that are not defined as linear or circular movement are generally approximated with piecewise line segments or circular arcs [12].

This approximation produces error, which is termed *chordal deviation* (δ) between the original path and the commanded path as shown in Figure (3). Many efficient tool path generation have also been introduced. Kim's algorithm [2] calculates the tool paths of a three dimensional parametric space curves on sculptured surfaces. The tool path in this algorithm was approximated by circular arcs and then approximation of these arcs by linear segments, the tool path calculated for ball-end cutter.

Danielsson algorithm [13], Jordan algorithm [14] and Moss algorithm [15] determine the incremental forward step of parametric curves for three or two axes computer numerical milling machines. These incremental path generation algorithms are based on the approximation of curved segments by circular arcs.

Seung Ryol [16] calculates the tool path of parametric curves and surfaces using the Z-map representation, throughout representing an object as a set of Z-axis aligned vectors, which passing through the grid points on the xy-plane. In this algorithm the tool paths are calculated linearly using three different types of tools.

The key to the new path generation method is based on:

1. The segmentation (approximation of the curve by a sequence of linear segments) is not based a constant increments of parameter Δu.
2. The curvature variance of the free form parametric curve, which is dependent on the degree of the complexity of the die cavity. When the curvature of a curve segment is too low, a little number of linear segments are required, but when a such curve with high curvature a large number of linear segments are required as shown in Figure (4). Thus, in this method of curve approximation, the increments of parameter Δu for each curve segment is different from the other one.

4.1An Algorithm for boundary Curve Approximation and Tool Path Generation

In this work the tool path of closed free form curve is generated for flat end mill of diameter d. Referring to Figure(5),the tool movement is started with the point $\mathbf{P}_{i-1}$ and going forward toward point $\mathbf{P}_{i+1}$ along a straight line joining them so that the chordal deviation (δ) stay within the required tolerance (T), which is dependent on the application of the work piece and the type of milling machine being used.

4.1.1 Problem Statements

The problem statements of boundary curve approximation are as follows:

1) Evaluation of the coordinates of the point $\mathbf{P}_{i+1}$ so that $\delta \leq T$.

2) Evaluation of specific value of (u) on the curve in the range u $\in$ [0,1] and computing tangent vector $\mathbf{P}_i^u$ at this specific value of (u) i.e., at $\mathbf{P}_i$ in Figure (5).

The condition of this specific point is that *the tangent vector at this specific point should be parallel to the line segment joining P_{i-1} and P_{i+1}.*

5.1.2 Solution Approach

1) The programming iteration approach is the strategy adopted in this paper to solve problem (1) presented above. This program is based on a successive scanning scheme for the curve segment, in this scheme and referring to Figure(5) :

a) The initial iteration state is based on the expectation "that the curve segment is approximated by a single linear segment from P_{i-1} to P_{i+1} i.e., $\Delta u=1$".

b) Calculation the coordinates of the point P_i corresponding to its specific value of (u) as will be seen later.

c) Calculation the produced chordal deviation (δ).

The normal distance from the point (P_i) to the line $P_{i-1}P_{i+1}$ along the normal vector (**N**) represents the error (δ) between the curve and the linear segment. From the triangle P_{i-1} P_i a in Figure(5).

$$\text{Cos}(\alpha) = \mathbf{L} \cdot \mathbf{P}_{i-1}\mathbf{P}_{i+1}$$

$$\text{or } \alpha = \text{Cos}^{-1}(\mathbf{L} \cdot \mathbf{P}_{i-1}\mathbf{P}_{i+1}) \text{ .... (12)}$$

Where **L** is the unit vector in the direction of $P_{i-1}P_i$

$$\text{Sin}(\alpha) = \delta / \| \mathbf{L} \| \text{ .... (13)}$$

Where $\| \mathbf{L} \|$ is the magnitude of the vector **L**

Substituting (13) in (12) yields,

$$\delta = \| \mathbf{L} \| \text{Sin}\, [\text{Cos}^{-1}(\mathbf{L} \cdot \mathbf{P}_{i-1}\mathbf{P}_{i+1})] \text{ ...(14)}$$

If the error (δ) calculated by the equation (14) is equal or less than the required tolerance (T), the algorithm is stopped and the segmentation of the curve will be done depending on the value of Δu at P_{i+1} i.e., ($\Delta u= u_{max}=1$). But when (δ) is greater than (T) the point P_{i+1} is transferred to the point $P(u_i)$ towards P_{i-1} by reducing the initial value of ($u= u_{max}$) to the new value (u_i) as shown in Figure(6) where :

$$u_i = u_{max} - v du \text{ ....(15)}$$

Where (v) is an independent variable and v = 1,2,3,..., 1/du .

(du) is the iteration increment which is very small and dependent on the degree of segmentation which is as the designer need.

Here it is selected so that $P_{i+1}(u_{max})$ is very close to $P(u_i)$.

At this stage, and assuming that the chordal deviation between P_{i-1} and $P(u_i)$ at v=2, i.e., ($u_i = u_{max}$ –2du) is equal or less than (T) ,then the optimum value for this segment is (u_i = 1 –2du), but the next step of scanning will be ranging from[u_i = 1 – 2du to 1] and so on until the scanning reaches the critical limit which is [1-du to 1] .

2) Evaluation of specific value of (u) on the curve segment. This point can be evaluated by using the ***mean value theorem (MVT)*** in which [17]: *if y=f (x) is continues at every point of the closed interval [u_i,u_{i+1}] and differentiable at every point of interior (u_i,u_{i+1}) ,then there is at least one number(u) between a and b at which the tangent vector f " (u) is parallel to the chord AB.* Figure(7) shows geometrically the mean value theorem.

Mathematically :

$$P_y (u_{i+1}) - P_y (u_i) / P_x (u_{i+1}) - P_x (u_i) = f''(u) \text{ ....(16)}$$

where $P_y (u_{i+1})$ = y-coordinates of the curve at u_{i+1}

$P_y (u_i)$ = y-coordinates of the curve at u_i

$P_x (u_{i+1})$ = x-coordinates of the curve at u_{i+1}

$P_x (u_i)$ = x-coordinates of the curve at u_i

From Equations 3 and 4

$f''(u)$ = tangent vector of the curve at u which is given by [22]

$f''(u) = [(dy/du) / (dx/du)]_u$ ....(17)

Solving equations (2-5, 16 and 17) yields :

$Qu^2+Ru+S=0$ ....(18)

Where Q = -13.5P1+41.5P2-41.5P3+13.5P4

R = 18P1-45P2+36P3-36P4

S = S1-S2

S1 = -5.5P1+9P2-4.5P3+P4

S2 = Left hand side of equation (16)

Solving equation (18) we get :

u_1 = (-R/2Q) - ½ power of [(R^2 – 4QS) / 2Q]

u_2 = (-R/2Q) + ½ power of [(R^2 – 4QS) / 2Q] ....(19)

since u2 > u1, we select u2 in order to reduce as many as possible the number of linear segments.

Substituting u2 in equations (2) and (3) will get the coordinate of the specific point P_i and substituting this point in Equatin 14 to find the value of chordal deviation (δ)

The new tool path generation algorithm is summarized as follows :

1. Introduce (T)
2. Set $u_{max}=1$; $u_{min}=0$.
3. Set v = 1.
4. Introduce the number of data points (n).
5. Introduce the iteration increment (du).
6. Calculate the number of curve segments (N) by equation (5) .
7. Introduce the required tolerance (T).
8. Do the following steps (N) times for each curve segment:
9. Set $u_i=u_{max}$.
10. Determine the position vector starting with the point P_{i-1} at u_{min} by Equations (3) and (4).
11. Determine the position vector of the point P_{i+1} at u_i by Equations (3) and (4).
12. Apply the mean value theorem to find the point (u) between [u_{min} u_i] by Equations (2,3,19).
13. Evaluate the tangent vector, normal vector and position vector at (u).
14. Evaluate the unit vector **L** between $\mathbf{P}_{i-1}$ and $\mathbf{P}_{i+1}$.
15. Evaluate the angle α by equation (13).
16. Evaluate chordal deviation (δ) by equation (14).
17. If $\delta < T$ or $\delta = T$, then set $\Delta U=u_i$ for the current curve segment and go to step (18). Otherwise, if u_i reaches the critical range, go to step (19). Otherwise, set v = v + 1, go to step (18).
18. If N finished go to step (19). Otherwise, go to step (9).
19. End of algorithm.

4.2 An Algorithm for Cutter Offset Determination

To machine a part contour, the path of the tool center must be defined. Since the tool has finite dimensions, the path must be offset. Once the tangent vector and normal vector are determined of free form curve, an accurate offset can be determined. The tool paths of flat end milling cutter with radius (r) are derived as follows (see Figure 8):

$\check{S}.\check{T} = \cos(\theta)$ ....(20)

$r = | O_{i+1} - P_{i+1}(u) | \cos(\theta)$ ....(21)

$P_{i+1}(u) = O_{i+1} + | O_{i+1} - P_{i+1}(u) | N.$ .... (22)

From equations (20-21), the cutter offset point OC_i, corresponding to the point $P_i(u)$ of a linearly approximated free form curve, are given by

$O_{i+1} = P_{i+1}(u) - (r / \check{S}.\check{T}) . N$ ....(28)

6.Experimental results and Discussion

We have implemented our algorithm using MATLAB programming language [18]. Fig (9.a, b and c) shows an example of free form closed curve design. We first set the 2D data points. After that, we use the algorithm described in section (4) to find the missing data points, which are necessary to close the curve. After the calculation of the missing points, and with the aid of equations (3) and (4) described in section (2) the curve can be designed as shown in Figure (9.c). Table (1) shows the information used in design the curve and the data extracted from the adopted algorithm. Based on these results, we use the algorithm described in section (5.1) for converting each curve segment into a sequence of linear segments, taking the value the value of the required tolerance as (T=0.00004 mm).

Table (2) and (3) shows the optimum values of forward steps of the parameter Δu for each curve segment, their corresponding x and y coordinates of liner segments and corresponding absolute chordal deviation (δ).

Based on these results, Figures (10.a,b and c) gives charts showing the relation between the number of points used and the chordal deviation for each curve segments respectively.

The required tolerance is also plotted on these charts, which exhibit a good approximation scheme adopted in this paper ,since that the maximum chordal deviation for all curve segments stay within the required tolerance.

The CNC program of the designed closed curve consists of (1900) steps for each layer of totally (8) layers of the tool paths. The die block has the dimensions of (400,450,50) mm ,using the algorithm for cutter offset points pointed out in section 5.2 and a flat end milling cutter of (10) mm diameter and(2.5)mm depth of cut for each layer.

Figure(11) shows the application of four point interpolation and point-to-point approximation scheme. The simulated tool paths for roughing and finishing stages are shown in Figures (12,a and 12,b) respectively.

6. Conclusion

Elimination as many as possible the sources of manufacturing inefficiency and increasing the productivity, are the most important tasks in the competition of today's companies. One such sources comes from the long machining programs.

In this study ,we present a way to convert a given scattered points into a CAD model based on interpolation technique to design a free from 2D die cavities ,and an efficient algorithm to select an optimum increments of the parameter Δu which used for converting the boundary curve

segment into linear segments. The study is based on the "the maximum chordal deviation" introduced from approximating scheme.
For automating the procedure given in the previous sections, programs are implemented on an IBM compatible PC using MATLAB programming language.
The values of Δu obtained by our optimization strategy have been tested for an example of blanking die cavity and the results show the effectiveness of the proposed optimization scheme.
In particular, this paper shows the following results:

1-We show how to enclose the open curves using the blending curve technique as the solution of this problem.

2-We describe a programming optimization scheme for extracting the optimum increments Δu for each curve segment.

3-The optimum values of Δu are the value that gives the minimum number of linear segment and maximum chordal deviation, consequently the minimum machining time.

4- The maximum chordal deviation for each linear segment for a given boundary curve segment always still within the required tolerance.

5- We describe a blanking die cavity implementation of our approach. The example illustrates how the optimum increments decompose the curve segments into linear tool paths.

6- Implementation of our cutter offset algorithm on the selected die cavity, shows the capability and usefulness of this algorithm.

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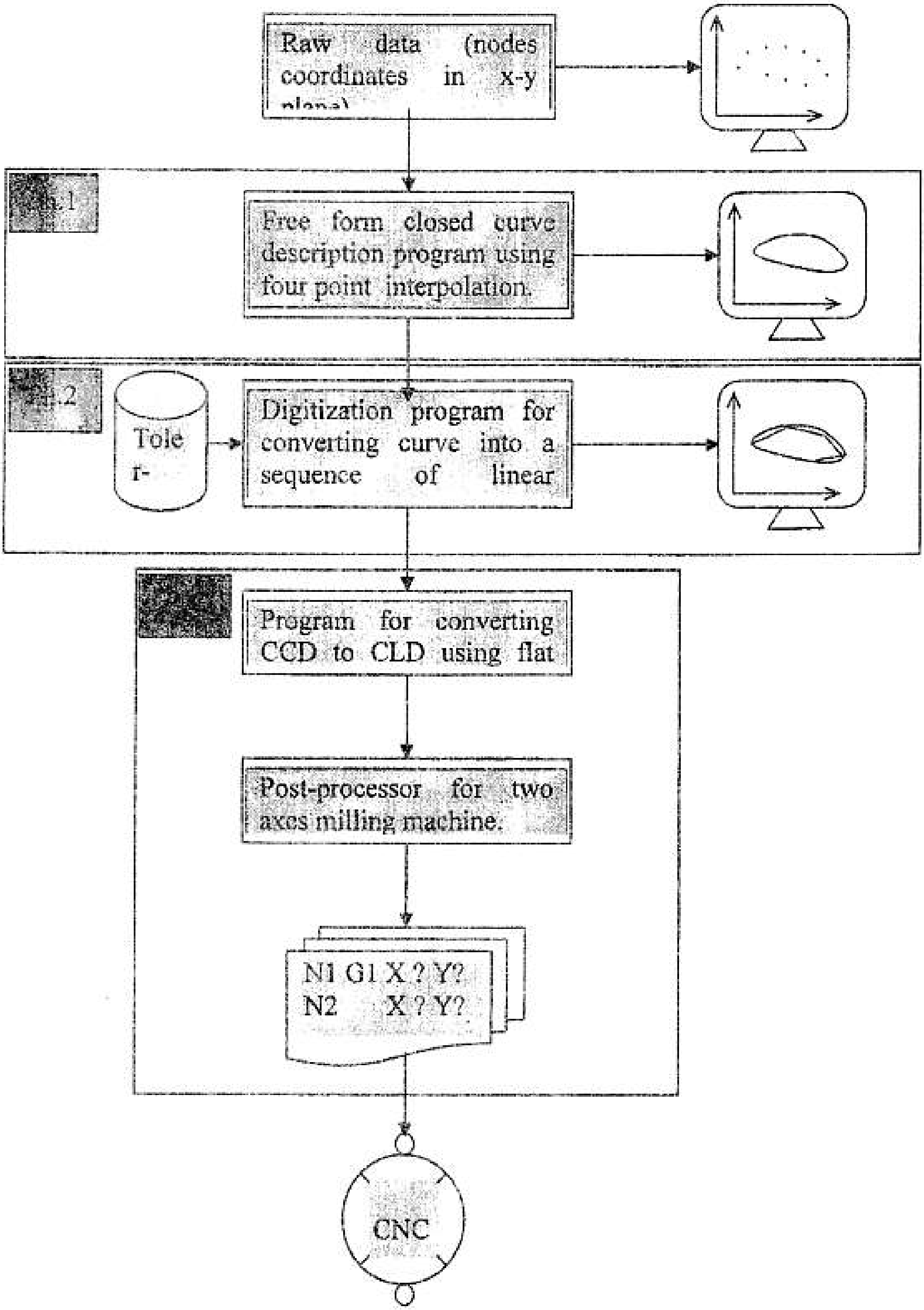
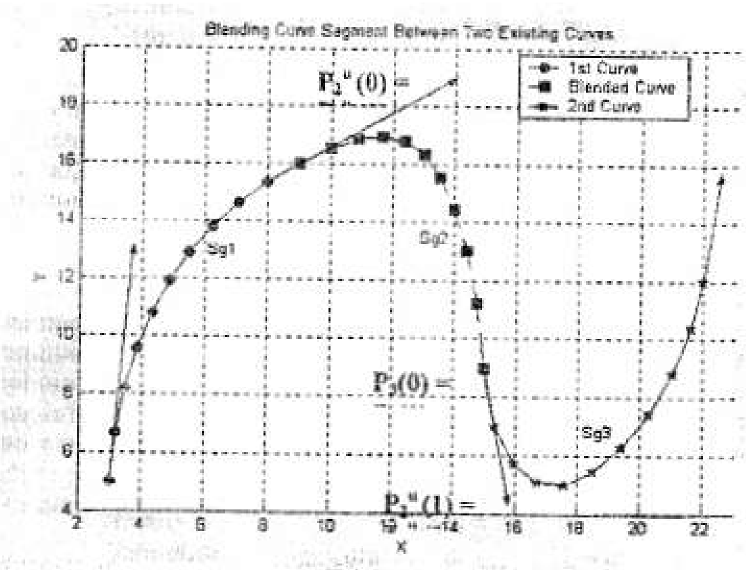
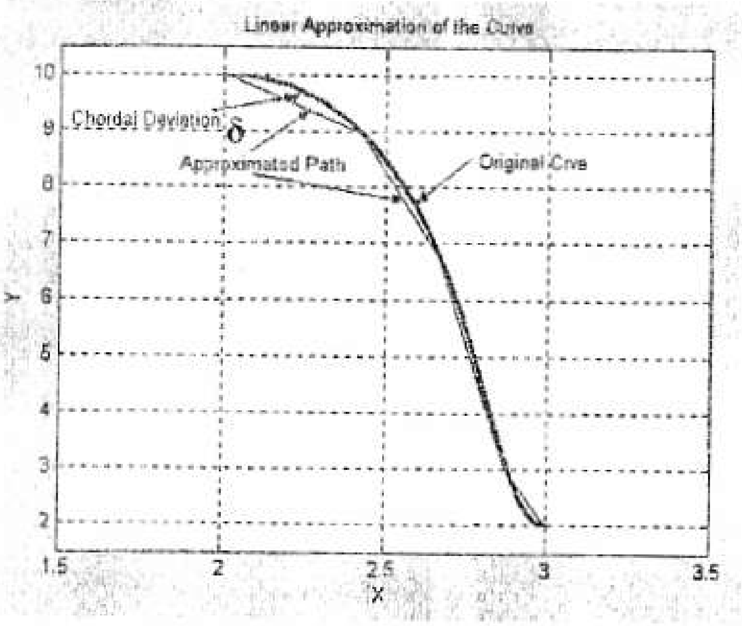


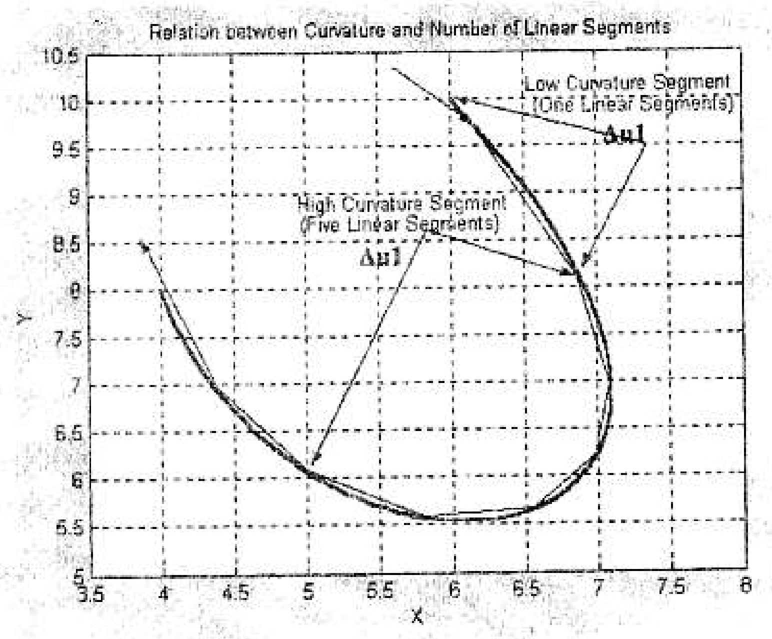
Figure (1) An Overview of the Proposed System.



Figure(2)Schematic Representation of Blending Curve Segment (Sg.2) Between Two Disjoint Segments (Sg.1) and (Sg.3).



Figure(3): Chordal Deviation Produced by Approximation of Curved *Segments With Linear Segments.*



Figure(4): Relation Between Curvature and Number of Linear Segments. As the Curvature Increase, the Number of Linear Segments Increased and Vise Versa i.e., Δu1 > Δu2 .

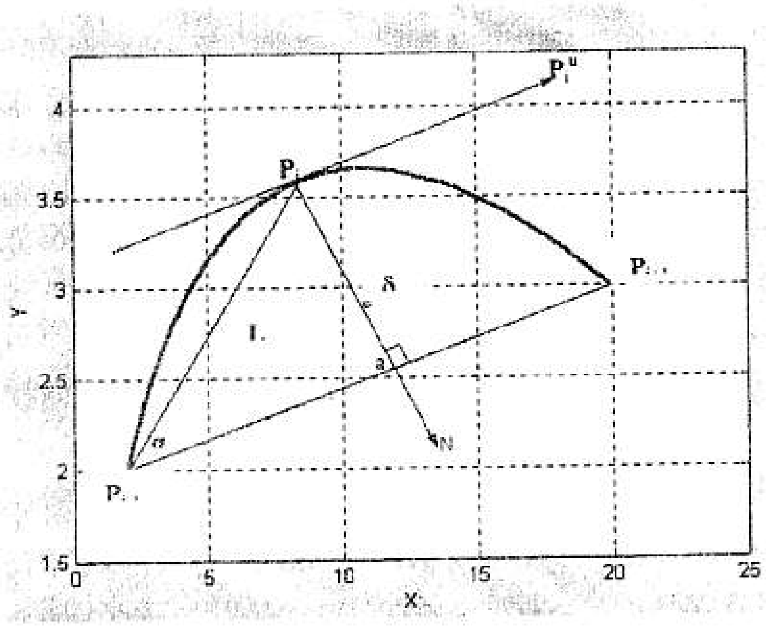
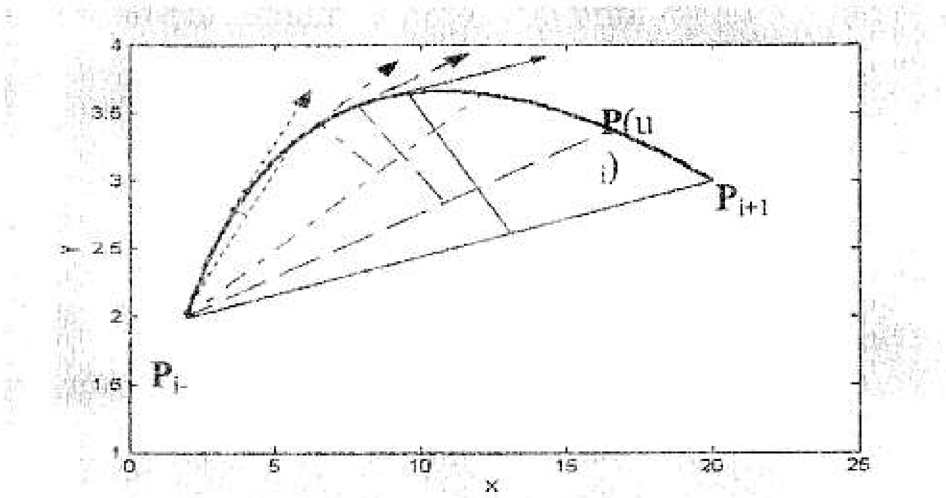


Figure (5): Free Form Curve Approximation and Chordal Deviation Evaluation.



Figure(6) Successive Scanning Scheme to find the Value of Forward Step.

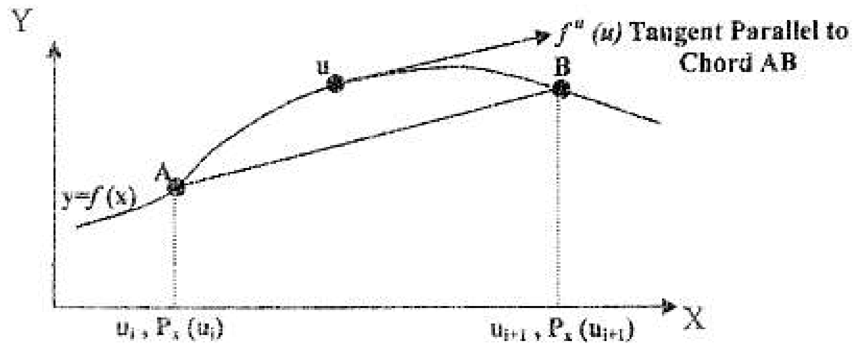


Figure (7) Schematic Representation of Mean Value Theorem.

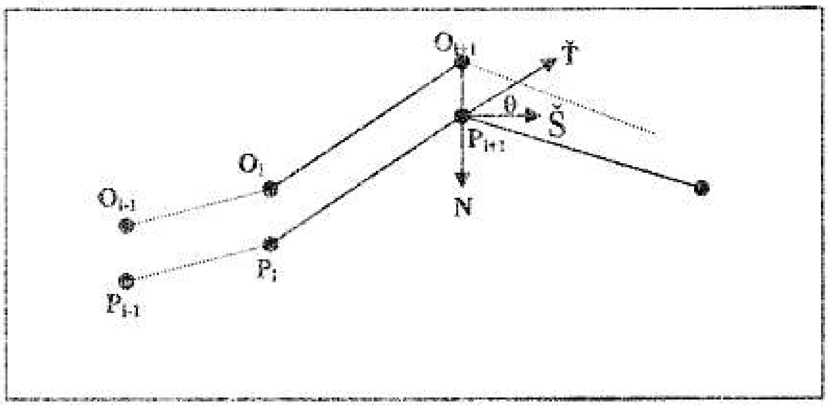


Figure (8) Cutter Offset Determination to Avoid Cutter gouging .

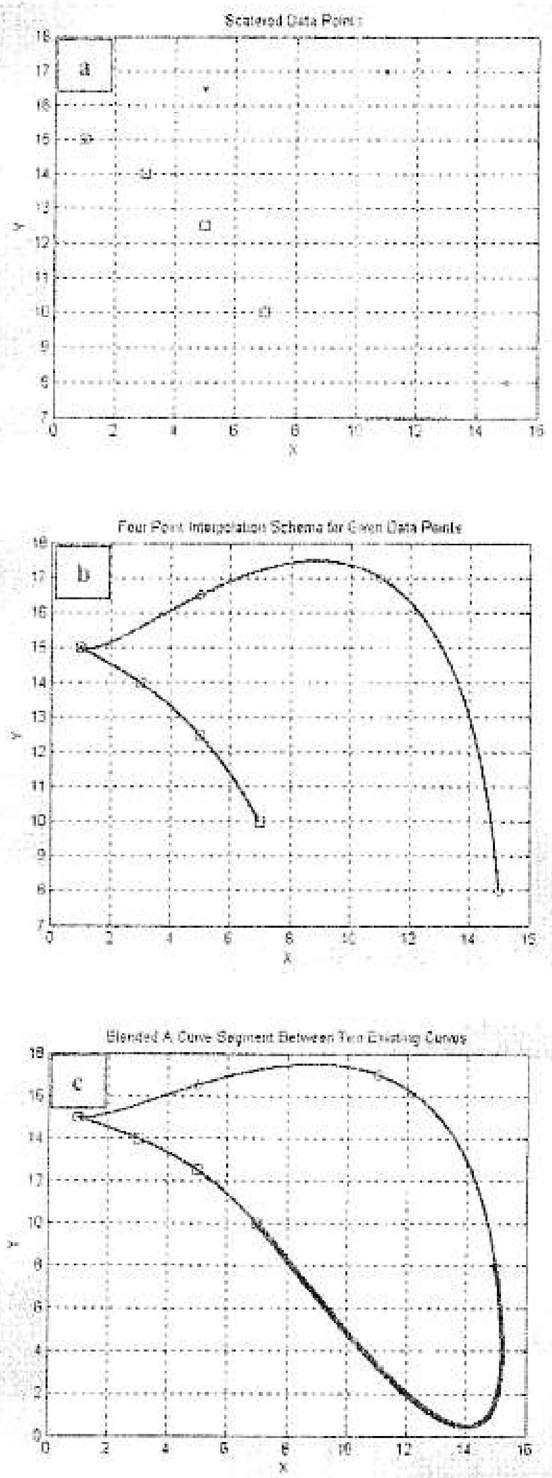


Figure (9) (a) Initial Scattered Data Points. (b) Four Point Interpolation on the Scattered Data Points, (c) Blending Curve Segment Between Tow Existing Curves to Close the Curve.

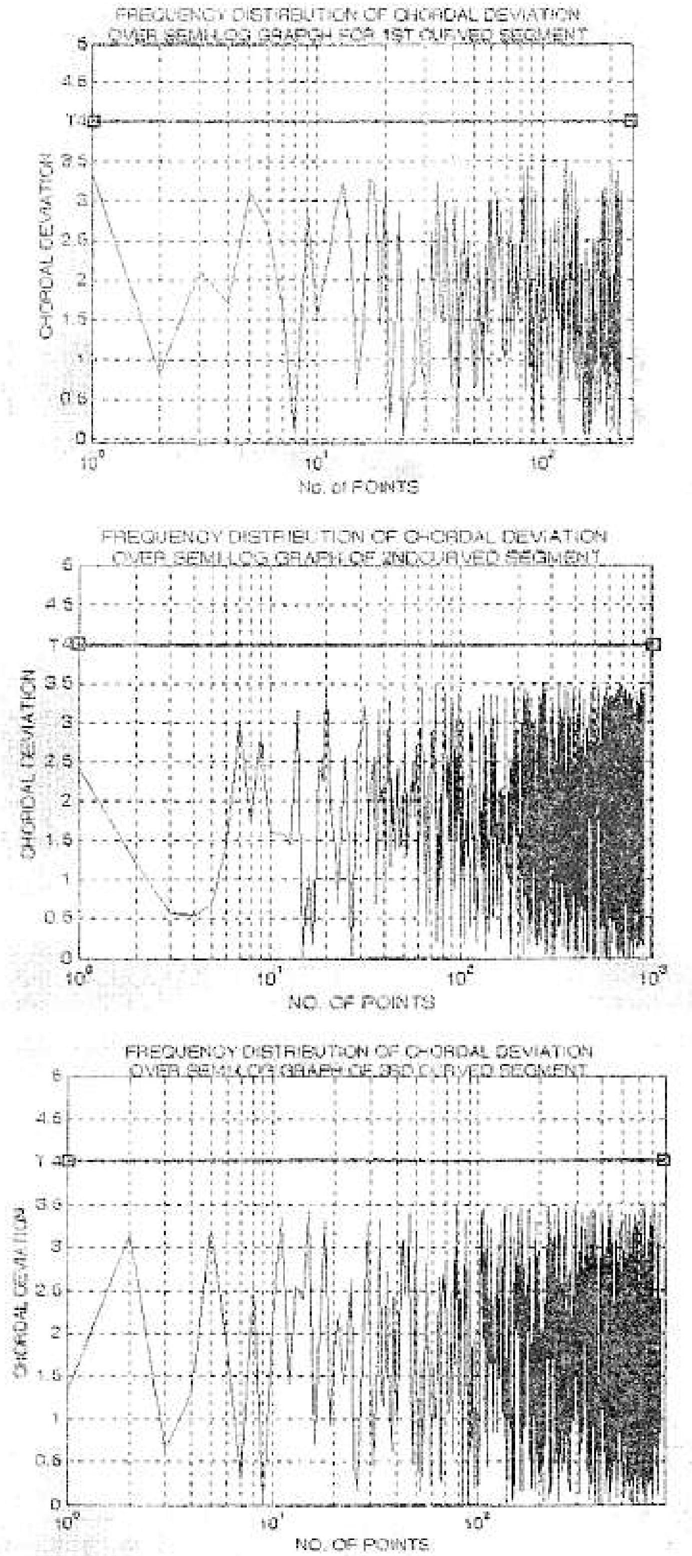


Figure (10) Relation Between the Produced Error and the Required Tolerance (T).

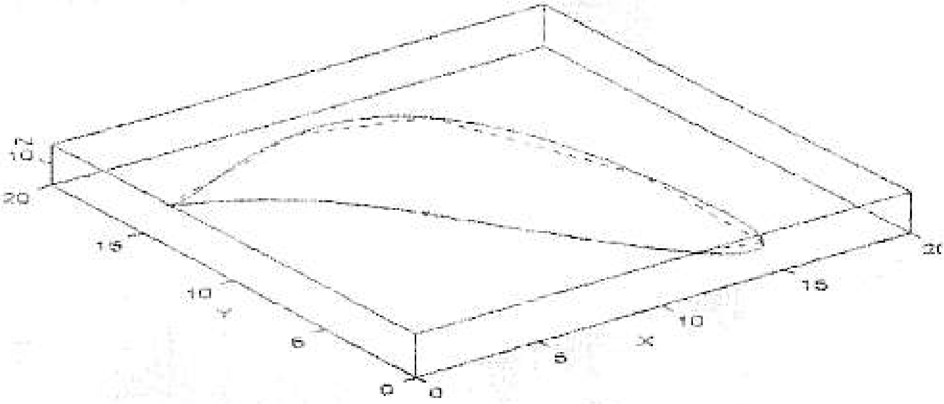


Figure (11) Exaggerated Linear Approximation of Curved Segments.

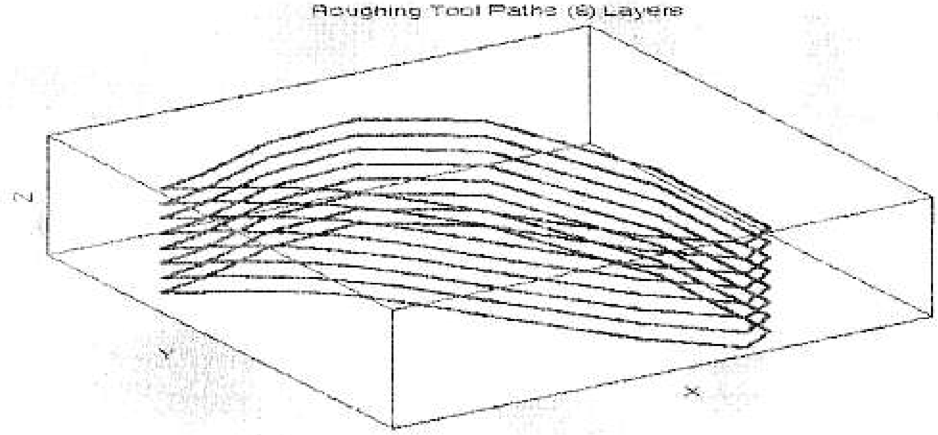


Figure (12) (a), Roughing Tool Paths For Eight Layers of Depth.

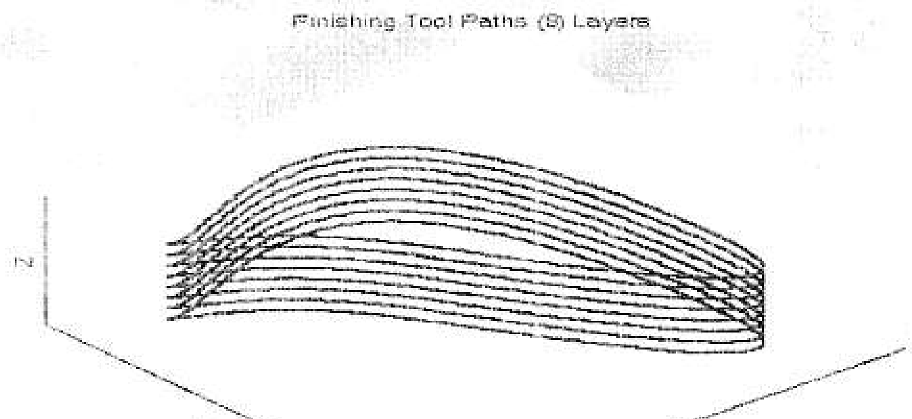


Figure (12) (b), Finishing Tool Paths For Eight Layers of Depth.

Table (1) Initial and Blended Data Points

Blended Points

X	1.00	3.00	5.00	7.00	7.00	10.33	14.11	15.00	15.00	11.00	5.00	1.00
Y	15.00	14.00	12.50	10.00	10.00	4.38	0.44	8.00	8.00	17.00	16.50	15.00
Seg. no.	1	1	1	1	2	2	2	2	3	3	3	3

Blended Segment

Table (2) Results Sample of Proposed System

#	ΔU_{MAX}	Curve Segment	Linear Segment	ΔU	X	Y	$\delta*10^{-5}$
1	0.0045	1st	1st	0.0000	1.0000	15.0000	2.1478
2				0.0280	1.1680	14.9230	
3			2nd	0.0280	1.1680	14.9230	1.3047
4				0.0500	1.4680	14.7844	
5			3rd	0.0500	1.4680	14.7844	2.2700
6				0.0660	1.6960	14.6775	
.....							
1	0.0011	2nd	1st	0.0000	7.0000	10.0000	1.5783
2				0.00125	7.0075	9.9881	
3			2nd	0.00125	7.0075	9.9881	1.9738
4				0.00267	7.0238	9.9622	
5			3rd	0.00267	7.0238	9.9622	2.0162
6				0.00409	7.0431	9.9341	
....							
1	0.0013	3rd	1st	0.0000	15.0000	8.0000	2.1330
2				0.00100	14.9950	8.0497	
3			2nd	0.00100	14.9950	8.0497	2.5765
4				0.00210	14.9842	8.1534	
5			3rd	0.00210	14.9842	8.1534	3.3198
6				0.00320	14.9727	8.2614	
...							