Eng. & Technology, Vol.24, No.7, 2005

An Approach To 3D Surface Curvature Analysis

Dr. Laith A. Mohammed* & Dr. Ghasan A. Al-Kindi**

Received on: 23/11/2003
Accepted on: 12/7/2005

Abstract

This paper reviews the graphical tools available for checking the quality of CAD/CAM (Computer Aided Design / Manufacture) surface models (B-Spline form). It is possible to use these methods by design and manufacture engineers to visually assess the quality of sculptured surfaces and perform the tool path of CAM system. This paper highlights the role of 3D surface curvature analysis in the field of CAD/CAM of sculptured surfaces.

Keywords: CAD/CAM, Surface curvature, Sculptured surfaces

التحليل ثلاثي الأبعاد لتقوس السطح

الخلاصة

يستعرض البحث الطرق المستندة على الرسوم في فحص نوعية السطوح المولدة بأستخدام طريقة (B-Spline) والمستخدمة في تطبيقات التصميم والتصنيع المعانين بالحاسبة. ممكن استخدام هذه الطرق من قبل مهندسي التصميم والتصنيع لتسهيل عملية التحديد المرئي المباشر لنوعية السطوح المعقدة ومن خلالها تحديد مسارات أداة القطع. ركز البحث على دور التحليل ثلاثي الأبعاد لتقوس السطوح المعقدة المستخدمة في مجال تطبيقات التصميم والتصنيع المعانين بالحاسبة في عملية الفحص لهذا النوع من السطوح الهندسية.

Introduction:

Sculptured surfaces in CAD/CAM are typically defined by a vector-valued parametric equation of the form

$r(u,w)=[x(u,w),y(u,w),z(u,w)]$ for $0 \leq u, w \leq 1$.

This definition can be used to generate sequence of points and normals that are used to define numerically controlled (NC) tool cutter paths to drive an NC milling machine. Thus with CAD/CAM the capability exists of matching the computer-based definition and the production tool to machining tolerances. This has resulted in the need to create high-quality surface definitions. Small surface imperfections can have large consequences in downstream the surface manufacturing.

Curvature analysis has applications in modeling as well as in manufacturing. In general, the surface should exhibit continuity of position, tangent vector and curvature. It should also be free of extraneous bumps or wiggles; a quality directly

---

* Dept. of Production & Metallurgy Eng., UOT.
** Dept. of Mechanical Eng., UOT.

https://doi.org/10.30684/etj.24.7A.2
2412-0758/University of Technology-Iraq, Baghdad, Iraq

related to unwanted inflection points.(1)

A surface inflection occurs at a point where the surface crosses its tangent plane. There are several different measures of a surface curvature and while each can be useful, no single type of curvature detects all possible surface anomalies. Thus, it is often necessary to calculate several types of surface curvature to obtain an accurate impression of the surface's behavior.

Standard Procedures:

At any given point on a parametric surface, i.e. a surface of the form $r=r(u,w)$, it is possible to calculate a unit normal vector, fig.(1). (2)

$$n(u,w) = (r^u x r^w)/\left|r^u x r^w\right| \qquad (1)$$

where $\left|r^u x r^w\right| \neq zero$

r^u = partial derivative of (r) with respect to (u).

r^w = partial derivative of (r) with respect to (w).

If a plane containing the normal is rotated about the normal, the intersections of this plane with the surface result in an infinite number of section curves each of which could have a different value for the curvature at the point. There are two mutually perpendicular locations of the normal plane where the curvature is a maximum and a minimum. These curvatures values are defined to be(2):

$$curvature = \frac{\partial'(u,w) * \partial''(u,w)}{(\partial'(u,w))^3} \qquad (2)$$

where: $\partial'(u,w)$ = first derivative of the surface.

$\partial''(u,w)$ = second derivative of the surface.

The maximum principal curvature is

(K'_{max}): $K'_{max} = {}^{K_{max}}/_{1+RK_{max}}$,

and the minimum principal curvature is

(K'_{min}): $K'_{min} = {}^{K_{min}}/_{1+RK_{min}}$.

Similarly, the maximum radius of curvature is

(λ'_{max}): $\lambda'_{max} = {}^{1}/_{K'_{max}}$,

and the minimum radius of curvature is (λ'_{min}):

$\lambda'_{min} = {}^{1}/_{K'_{min}}$, while R represents tool radius.

The Gaussian, average and absolute curvatures are defined as:

G=Gaussian curvature=$K_{max}K_{min}$, and H=Average (Mean) curvature $= \frac{1}{2}(K_{max} + K_{min})$, and Absolute curvature $= |K_{max}| + |K_{min}|$.

Primary curvature is defined as curvature in the direction of constant (u), while secondary curvature is defined as curvature in the direction of constant (w). K_{max} , K_{min} are Principal curvatures at a given point.

Curvature Displaying:

Methods of displaying curvature include (3) normal vectors, contour lines and color. Normal vectors can indicate the surface curvature by a length proportional to the radius of curvature. Contour lines include reference plane and a series of equally spaced planes parallel to it. The intersection of these planes with the surface results in planar curves on the surface. These curves can aid in determining the surface features, e.g. saddle points appear as passes and maxima and minima appear as encircled. Displaying curvature variation as color variation is used as a scale in which the minimum curvature value corresponds to one end of the color spectrum and the maximum curvature value to the other end of the spectrum, with a linear distribution in between. Color change represents a percent change in curvature, i.e. a logarithmic color scale.(4)

Curvature Distribution:

Figure (2) represents a model constructed using B-spline formulation. The curvature values of the surface shown in Fig.(3) which represents 3D curvature distribution of the taken surface.

The normal to a surface at a point is the direction perpendicular to it. The surface normals are based on a bicubic fit of data in *XYZ* space (5). For each vertex, diagonal vectors are computed and crossed to form the normal. Surface normals are local information about surface, so they vary with *u*-*w* position. Surface characterization by using normal vectors to find a suitable segmentation and generate a *CAD* reconstruction, is shown in Fig.(4). Normal plot represents a specified number of quills representing the normals on the surface. The normal quill plot is a valuable tool in detecting a surface's shape profile changes, as shown in Fig.(5).

Figs.(6) to (8) show the curvature distribution using Gaussian Curvature, Mean Curvature and Absolute Curvature respectively. All figures in this paper generated using Matlab programming language based on the formulas described in section 2 in this paper.

Curvature Role in Manufacturing:

Surface curvature plays a key role in selecting a cutting size to avoid gauging in CAM system. Tool size can be selected from the critical curvature. Minimum curvature radius equals largest tool radius, Fig.(9) represents contour map of tool radius distribution mapped on the surface model, (for this surface, the calculation maximum tool radius to machine surface is equal to 10). Curvature analysis provides guidance for an NC programming in choosing cutter dimensions for gauge-free milling of free form surfaces.

A surface trichotomy is a partition of a surface into three types of regions: convex, concave and saddle shapes. The ability of trichotomize sculptured surfaces into these regions is essential to the use of ball nose end cutters in milling free form surfaces. Also, regions with small curvature can be accurately milled faster with larger ball end cutters. Tool changes should be minimized because they are time-consuming operations. Such minimization can be achieved by subdividing the surface into regions with different curvature bounds, each of which may be milled using tools appropriate to that region. Using the trichotomy operator, convex regions within surfaces can be detected and milled in more efficient way and with a better finish.

Results and Discussion:

From the curvature distribution of taken surface, each region may carry the following property, which may help to evaluate surface profile:

1. If the region has a saddle shape, then one of the principal curvatures is positive and the other is negative.
2. If the region is convex both principal curvatures are negative.
3. If the region is concave both principal curvatures are positive.

If the surface is a saddle at (u,v), then principal curvatures have different signs so the magnitude of mean curvature is not a useful measure of such a bound. In the extreme condition when the surface is minimal, mean curvature ≈ 0 regardless of the surface angularity. The magnitude of Gaussian curvature can also be ineffective. Therefore, neither curvature nor average curvature by itself can provide sufficient shape information for subdivision and/or efficient NC applications. Gaussian and mean curvatures are, in general, better than absolute curvature in detecting small regions of uneven curvature. Radius of curvature can be used to perform tool radius map, Fig.(9), of the surface by calculating the maximum radius of the tool that can be used to machine the surface. This is an important tool for determining whether the surface can be offset by a particular radius, or for determining the maximum radius by which the surface can be offset.

Conclusions:

It has been observed that all taken methods of curvature analysis can be used to visually quality confirmation of the sculptured surface. Local surface curvature is used to bound the size of the undetected possible error. Graphical display of curvature values can be useful to indicate error cutting areas which require modification of the machining program, beside the calculation of the value of cutter size required for machining.

References:

1. Seidenberg L.R.; "Surface Curvature Analysis Using Color"; Visualization 92 Conference IEEE Computer

Society & ACM/SIGGRAPH Oct 19-23; 1992, Boston, Ma.

2. Qiulin D. and Davies B.J.; "Surface Engineering Geometry for Computer aided Design and Manufacturing"; John Wiley & sons; 1987; pp.43-46
3. Jerard R.B.; "Methods for Detecting Errors in Numerically Controlled Machining of Sculptured Surfaces"; IEEE Computer Graphics and Applications; January; Vol.9, No.1, 1989, pp. 26-38
4. James M.B.; "Surface Analysis Methods"; IEEE Computer Graphics and Applications; December; Vol.6, No.12, 1986, pp.18-35
5. Cripps R.J. and Ball A.A.; "Visual assessment of free-form surfaces in CAD/CAM"; Proc. Instn. Mech. Engrs.; Vol. 212; Part B; 1998; pp. 207-214

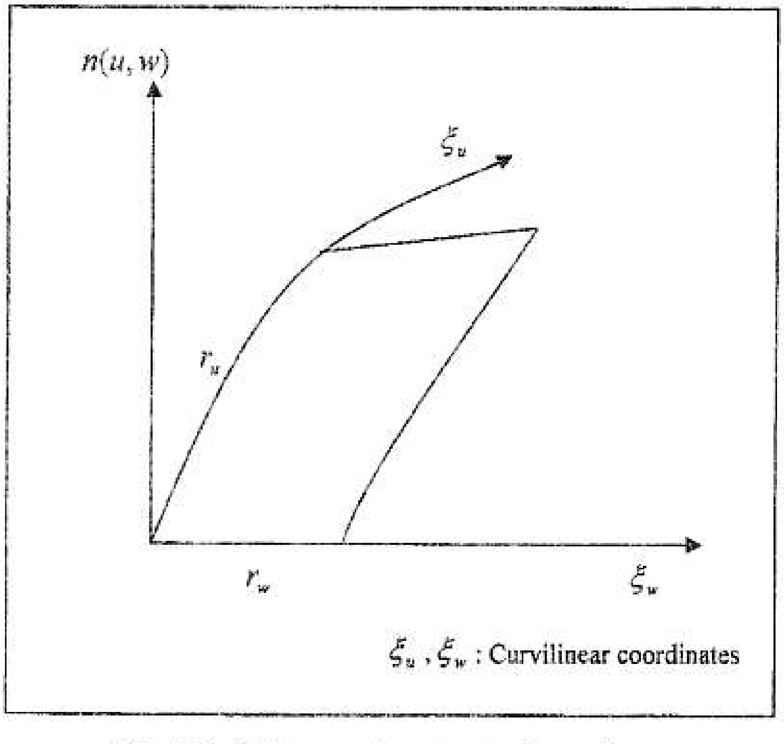

Fig. (1): Unit normal vector to the surface.

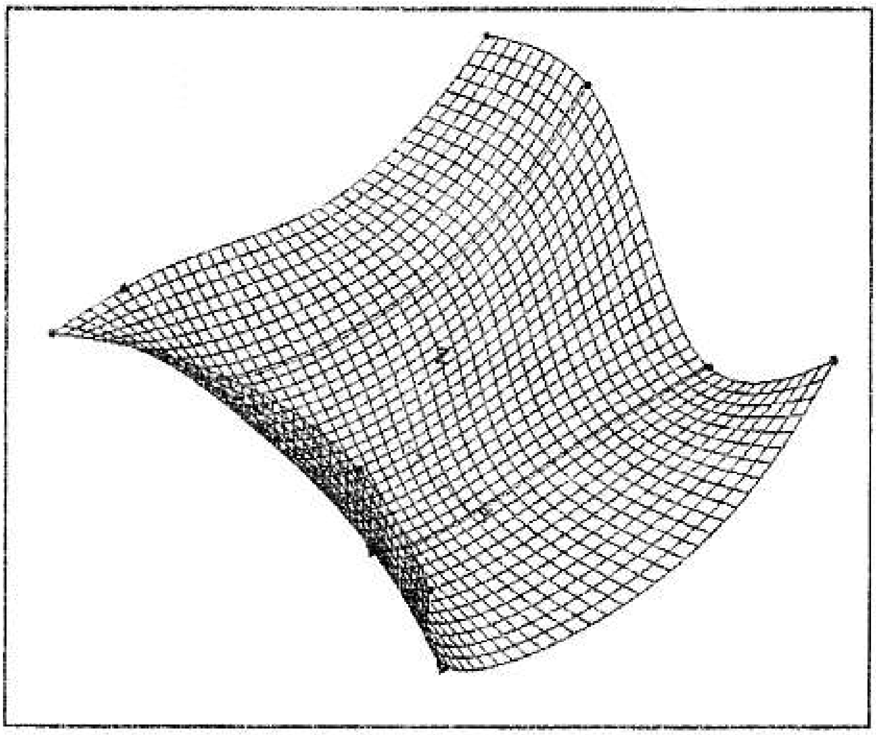

Fig.(2): Sample surface model.

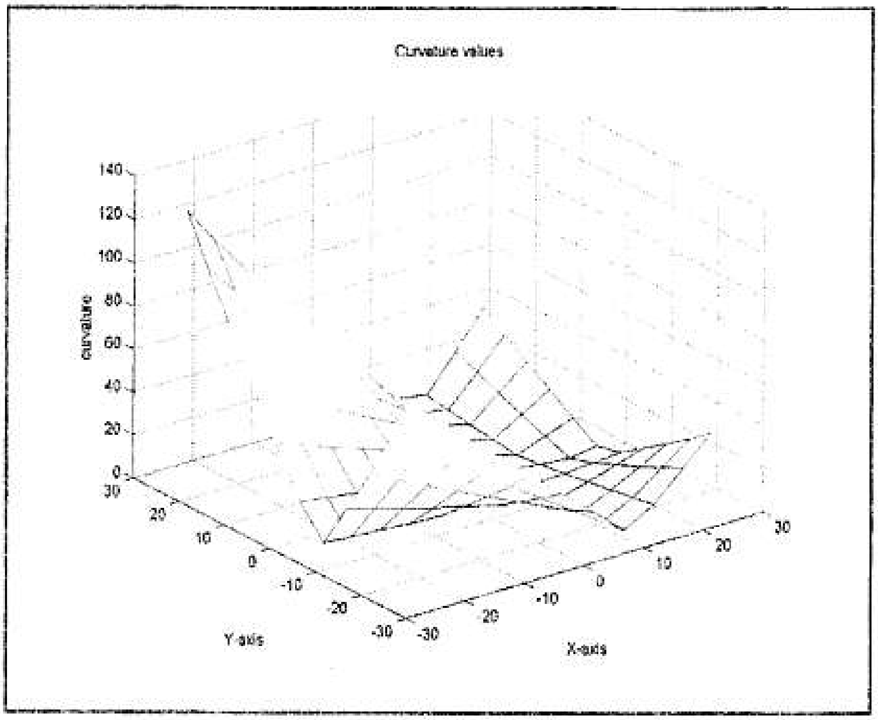

Fig.(3): Surface curvature plot.

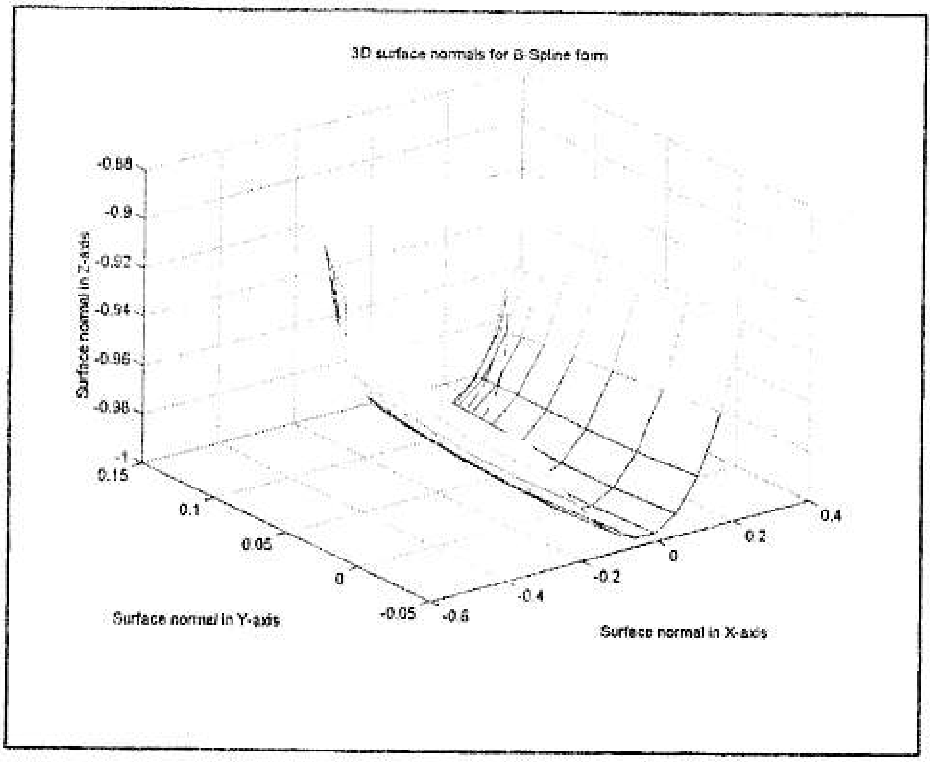

Fig.(4): Surface normal plot.

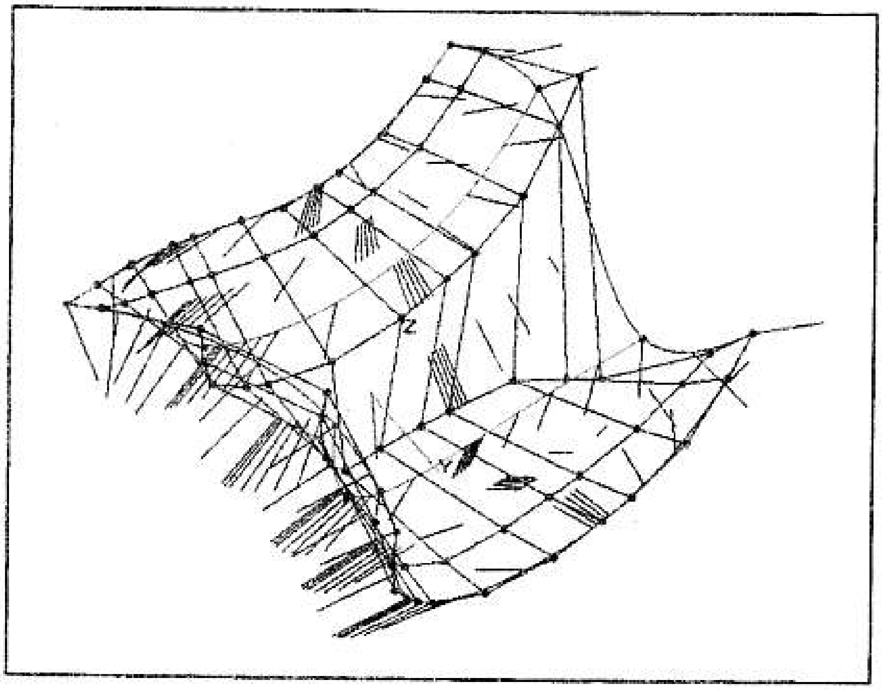

Fig.(5): Normal plot as quills.

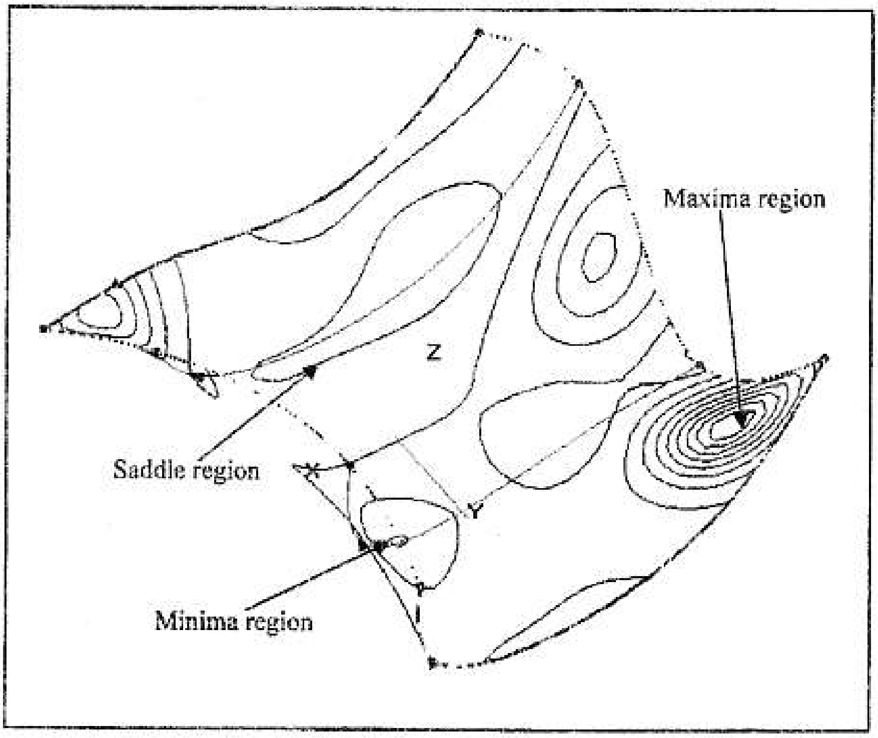

Fig.(6): Gaussian Curvature as contour map.

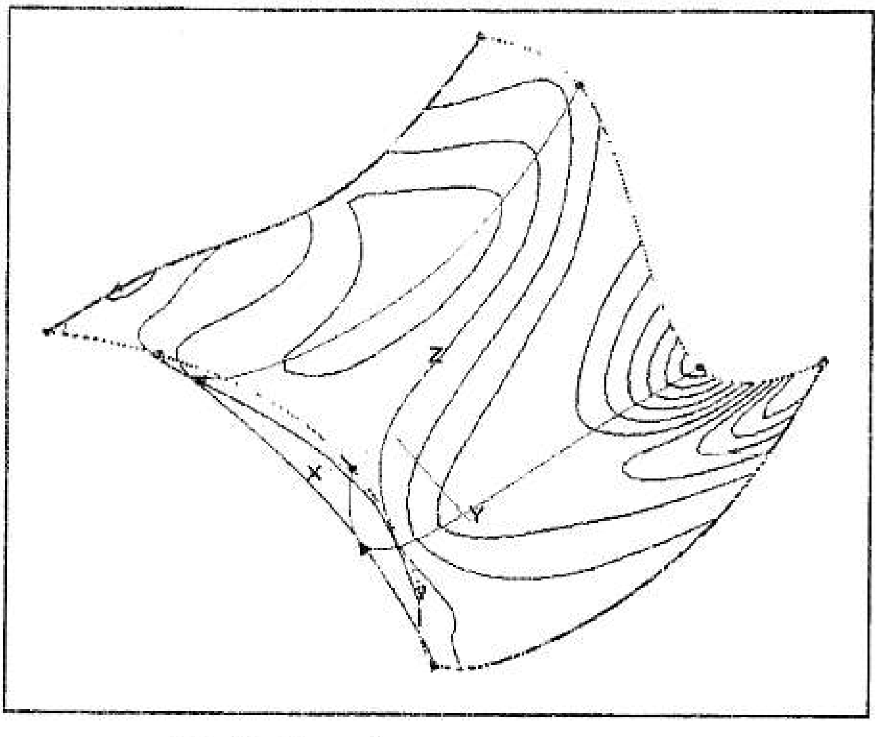

Fig.(7): Mean Curvature as contour map.

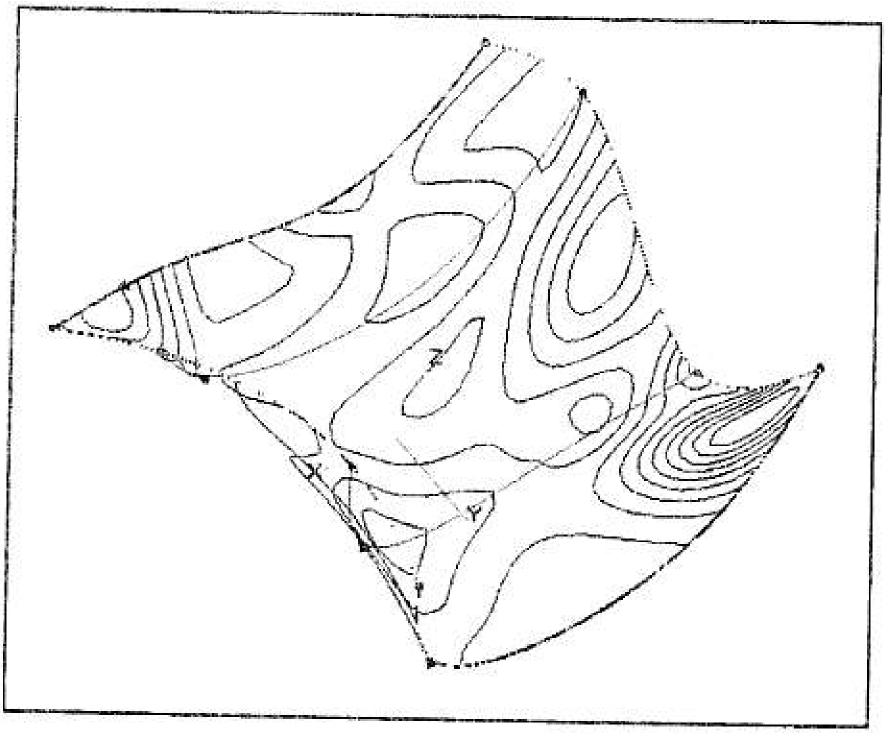

Fig.(8): Absolute Curvature as contour map.

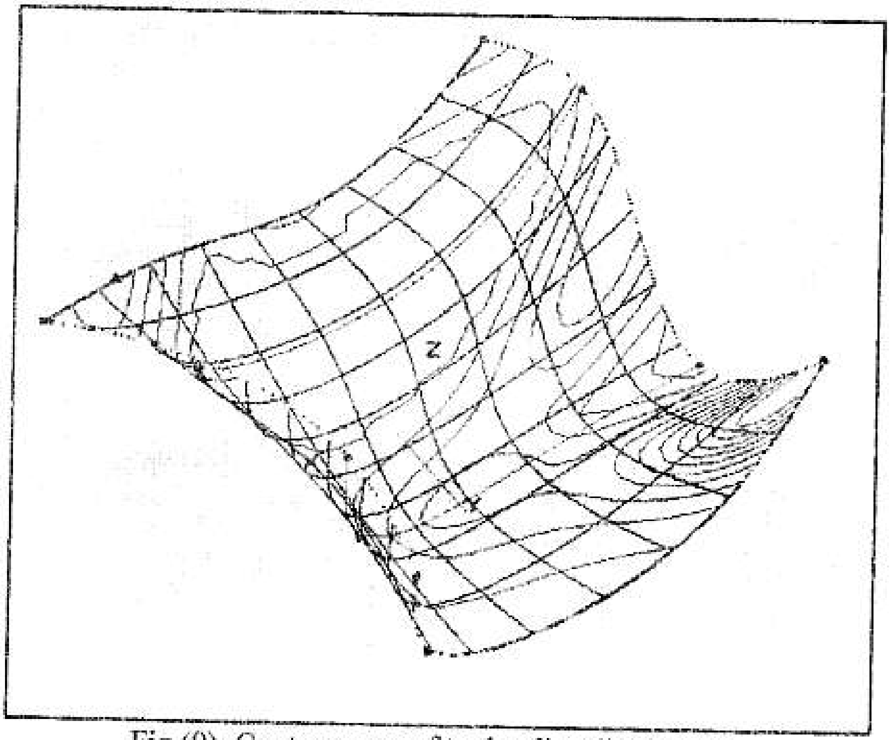

Fig.(9): Contour map of tool radius distribution.